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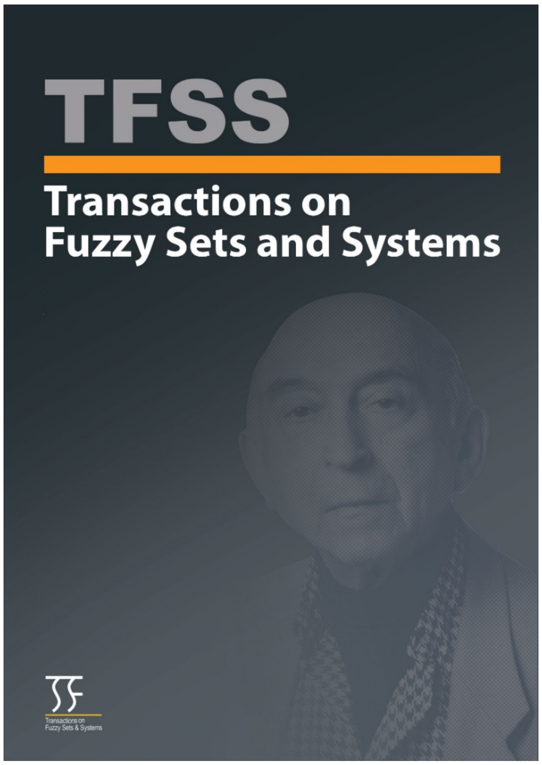

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A Novel Technique for Solving the Uncertainty under the Environment of Neutrosophic Theory of Choice

Tabasam Rashid*, **Aamir Mahboob**, **Ismat Beg**

Abstract. When it comes to solving dynamic programming challenges, it is essential to have a well-structured decision theory. As a result, the decision-makers must operate in a dynamically complicated environment where appropriate and rapid reaction in a cooperative way is the fundamental key to effectively completing the task. We express a theory of decision modeling and axiomatizing a decision-making process. The payoffs and probabilities are represented with simplified neutrosophic sets. We therefore, provide the theory of choice with the implementation of simplified neutrosophic sets. By exploiting the idea of pure strategy, we introduce two steps: in the first step, for each attractive point, some particular event is selected that can bring about a relatively neutrosophic upper payoff with a relatively neutrosophic upper probability or a relatively neutrosophic lower payoff with a relatively neutrosophic upper probability. A decision-maker selects the most favored attractive point in the second stage, based on the focus on all attractive points. Neutrosophic focus theory has been introduced to improve overall performance with more flexibility in complex decision-making. The approach suggested in this work has been implemented in a real-life example to determine its effectiveness. The proposed method is shown to be the most useful for ranking scenarios and addressing dynamic programming problems in decision-making.

AMS Subject Classification 2020: 03E72; 91A30; 91A86; 91B06
Keywords and Phrases: Fuzzy set, Neutrosophic set, Neutrosophic probability, Game theory, Focus theory.

1 Introduction

In various decision-oriented real-life problems, game theory plays a significant role. Nowadays, many such problems are generally described by different uncertainties. Uncertainties occur because of decision-makers the collection of information, perception, belief, opinion, actions, assessment, and finally, due to the problem itself. The definition of fuzzy set [1] with a membership degree initialized the treatment of ambiguity, but it was not sufficient. The concept of the intuitionistic fuzzy set was developed using membership and non-membership grades but struggled to convey truth more accurately. Then, with a new degree of uncertainty, say an indeterminacy degree, in addition to membership and non-membership degrees, neutrosophic logic was developed.

Ambiguities of fact exist everywhere. To explain the uncertainties, fuzzy logic [1] has emerged as one of the essential soft computing methods. From Zadeh to Atanassove [2], the fuzzy notion has been developed from its membership components to an intuitionistic fuzzy notion with non-membership components. For example; in the voting system, when we choose a candidate, one has the option to opt-out or remain independent, in addition to an election or a choice. Intuitionistic characters can not manage such circumstances. In these

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situations, Smarandache [3] introduced and was effectively applied to the neutrosophic set concept. The degree of indeterminacy still occurs in many cases, beyond the stages of acceptance and rejection. So many developments of neutrosophic sets have also been suggested, such as the single-value neutrosophic set [4], the neutrosophic interval set [5], the multi-value neutrosophic set [6], etc.

Game theory is a mathematical analysis in which there are the situation of two contrary ideas exist or strategic decision making. Game theory related to decision-making problems in a mathematical way invented by von Neumann [7]. The research on two-person zero-sum games quickly progressed following Neumann and Morgenstern's pioneering contributions, significantly influencing decision-making and strategic analysis in various disciplines. For example; on nash equilibrium [8, 9] various mathematical models of game theory [10] on decision making and so on. One game consists of multiple players, a set of tactics including a payoff that displays the overall results from every game's play in terms of the rewards won or lost by each based strategy player. According to the probability method, a player who selects a pure strategy randomly selects a row or a column that determines the opportunity for each pure strategy. For players, the probabilities are said to be a mixed strategy. In terms of probability, the measured payoffs represent the probability of each player to obtain and if the game is played a sufficiently large number of time, the player will eventually benefit on average. Due to the ambiguity and vagueness components included as well as what happened throughout the process, the strangeness of the prudence of gamers or decision-makers. We showed the characters of indeterminacy and falsity in matrix form. First of all, for solving fuzzy matrix games, Campos [11] used linear programming models. Later on, Li [12] used Attanasov's intuitionistic fuzzy sets to solve matrix games with different uncertainties. Nowadays, several writers [13, 14, 15, 16, 17] have examined some game models using payoff and probability using maximin, maximax and minmax rules.

After that, the matrix game solution was extended using intuitionistic fuzzy triangular payoff by Bandyopadhyay [18, 19]. He proposes the intuitionistic fuzzy numbers and arithmetic operation of score functions and introduces the matrix game using various strategies. Feng [12] gave the comprehensive idea of a matrix game with the help of intuitionistic payoffs. He also explained trapezoidal intuitionistic fuzzy numbers and interval-valued intuitionistic fuzzy sets and their properties.

Games with neutrosophy set the three contrasting collective grades to be compared: truth-membership, indeterminacy-membership and falsity-membership, whereas intuitionistic games have membership and non-membership degrees. Consequently, it is possible to apply the models and methods of intuitionistic fuzzy games to neutrosophic games. Some authors [20, 21, 22, 23, 24] applied the neutrosophic theory of games in our daily life.

Generally descriptive and normative theories, a decision-maker is believed to maintain a comprehensive understanding while analyzing s lottery game using an aggregated multiplicative model, like that of the SEU. Commonly, cumulative information found from studies using existing techniques makes clear that it is impossible that a risky decision based on weighing and summing procedures is unlikely [25, 26, 27]. Many research show that people assess a lottery by treating every result independently. Wedell [28] showed in his paper that justification for single play decisions is inclined to depend on a single feature of the gamble in which the amount that can be won or lost, the probability of doing so, or other variables are involved in a single attribute. These four characteristics are min payoff, probability of min payoff, max payoff and probability of max payoff suggested in [29]. Furthermore, numerous studies indicate that people judge a lottery based on a specific event associated with this lottery. i.e, they perceive a payoff and its probability [30]. In view of these theories, the Neutrosophic theory of choice claims that is rationally bounded and results with minimal attention, therefore, instead of selecting of all events of a lottery, decision-maker study the event according to the payoff and probability.

In section 2, on the game and neutrosophic set, we give some simple definitions and notations. In section 3, we explore how to pick positive attractive points and how to evaluate the optimal alternative using positive attractive points. An application of neutrosophic set in decision making is discussed and a comprehensive comparison analysis is shown in section 4 to explore the validity and effectiveness. The concluding remarks are given in section 5.

2 Preliminaries

In this section, we deliver concise analysis of neutrosophic set, simplified neutrosophic set, neutrosophic probability, accuracy function and score function. The neutrosophic set allows one to introduce indeterminacy, hesitant or ambiguity irrespective of the knowledge regarding membership and non-membership grades. Therefore, the notion of neutrosophic set is the generalization of fuzzy and intuitionistic fuzzy set. The following definition for a neutrosophic set was given by Smarandache [3].

Definition 2.1. [3] Let X be universe of discourse. A neutrosophic set NS is defined by $T_A(x)$, $I_A(x)$, $F_A(x)$, where $T_A(x)$ is the truth-membership function, $I_A(x)$ is the indeterminancey-membership function and $F_A(x)$ is the falsity-membership function, all of these functions are subset of $]0^-, 1^+[$ with condition $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ for all x belongs to X.

Definition 2.2. [31] A subclass of neutrosophic set is called simplified neutrosophic set (SNS) and it is defined as: $A = \{(x,\ T_A(x),\ I_A(x),\ F_A(x)) \mid x \in X\}$, where $T, I, F \in [0,1]$. For suitability, SNS can be written as: (a, b, c).

In general, if $I_A(x) = 0$, then the above set A can be reduced to intuitionistic fuzzy set, $IFSA = \{(x, T_A(x), F_A(x))|x \in X\}$ and if $I_A(x) = F_A(x) = 0$, then the set A ca be reduced to fuzzy set $FSA = \{(x, T_A(x)|x \in\}$. The relation between fuzzy set, intuitionistic fuzzy set and neutrosophic fuzzy set are shown in Figure 1.

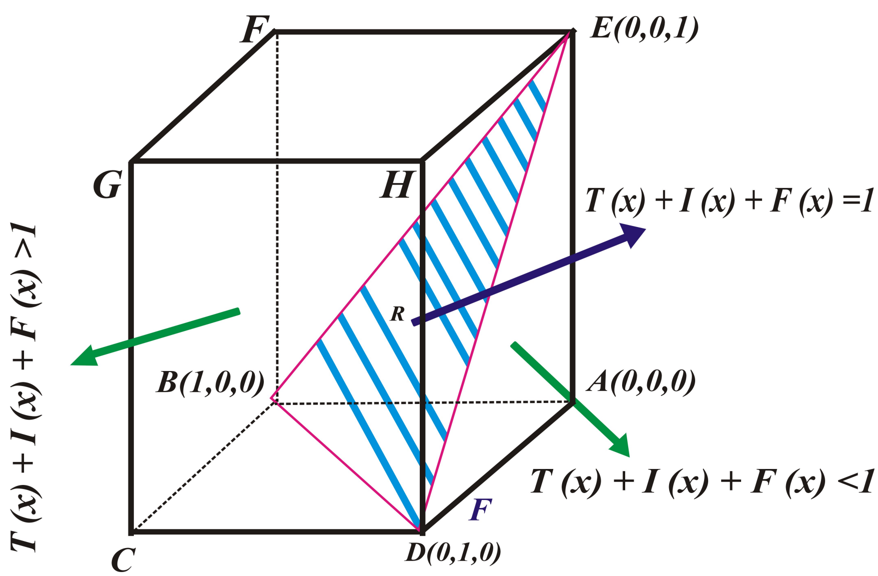

Figure 1: The environment of neutrosophic set

Definition 2.3. [31] Let A be a SNS, then the complement of SNS is denoted by A^c and defined as: $A^c = \{(x,\ F_A(x),\ 1 - I_A(x),\ T_A(x)) \mid x \in X\}$.

Definition 2.4. [31] Let $A = (a_1,\ b_1,\ c_1)$ and $B = (a_2,\ b_2,\ c_2)$ be the SNS, then A contained in B if and only if $a_1 \leq a_2$, $b_1 \geq b_2$ and $c_1 \geq c_2$ for every x in X.

Definition 2.5. [32] Let A be the SNS, then the score function S of a simplified neutrosophic value is defined as:

$$S(A) = \frac{1 + a - 2b - c}{2} \tag{1}$$

where $S(A) \in [-1, 1]$.

Definition 2.6. Let $A = (a_1,\ b_1,\ c_1)$ and $B = (a_2,\ b_2,\ c_2)$ be two simplified neutrosophic sets and $S(A)$ and $S(B)$ be their score functions, then
1) If $S(A) < S(B)$, then A is lesser than B;
2) If $S(A) > S(B)$, then A is greater than B;
3) If $S(A) = S(B)$, then A and B are equal.

Definition 2.7. A matrix game $S = \{Player_1,\ Player_2,\ M_1,\ M_2\}$ is denoted as two-player game;

1). Player$_1$ has countable game plan set M_1 accompanied by p elements.
2). Player$_2$ has countable game plan set M_2 accompanied by q elements.
3). The functions $v_1(m_1,\ m_2)$ and $v_2(m_1,\ m_2)$ are the payoff functions of the player$_1$ and player$_2$ respectively and $(m_1,\ m_2) \in M_1 \times M_2$.

The matrix game will be as: player$_1$ select $m_1 \in M_1$ at the certain time and player$_2$ select $m_2 \in M_2$ at the same time. When each player does this then he/she receives the payoff $v_i(m_1,\ m_2)$. If $M_1 = \{m_1^1, m_2^1, ..., m_p^1\}$, $M_2 = \{m_1^2, m_2^2, ..., m_q^2\}$ are the game plan of player$_1$ and player$_2$ respectively and we replace $a_{ij} = v_1\left(m_i^1, m_j^2\right)$ and $b_{ij} = v_2\left(m_i^1, m_j^2\right)$, then the payoffs can be organize in the form of $p \times q$ matrix.

Definition 2.8. Let S be the set of strategies of two player and M, N are the non-empty subset of set S. A triplet $(M,\ N,\ A)$ describes the strategies of two player for simplified neutrosophic set is defined as: $A = \{\langle (m,n),\ (T_A(m,n),\ I_A(m,n),\ F_A(m,n))\rangle \mid (m,n) \in M \times N\}$, where player$_1$ has M strategies, player$_2$ has N strategies and B be the simplifies neutrosophic set over $M \times N$.

The explanation is as: Player$_1$ choose the $m \in M$ and player$_2$ choose $n \in N$ at the same time and both of them don't know each other preference, at that point the payoff for player$_1$ is represented by $(T_A(m,n),\ I_A(m,n),\ F_A(m,n))$. Results of player$_2$ on the circumstance $(m,\ n)$ is negation of result of player$_1$. Therefore, the neutrosophic payoffs can be organized in matrix from shown in Table 1

Table 1: Gamble matrix

B	n_1	$\dots$	n_q
m_1	$(T_A(m_1,n_1),\ I_A(m_1,n_1),\ F_A(m_1,n_1))$	$\dots$	$(T_A(m_1,n_q),\ I_A(m_1,n_q),\ F_A(m_1,n_q))$
m_2	$(T_A(m_2,n_1),\ I_A(m_2,n_1),\ F_A(m_2,n_1))$		$(T_A(m_2,n_q),\ I_A(m_2,n_q),\ F_A(m_2,n_q))$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
m_p	$(T_A(m_p,n_1),\ I_A(m_p,n_1),\ F_A(m_p,n_1))$	$\dots$	$(T_A(m_p,n_q),\ I_A(m_p,n_q),\ F_A(m_p,n_q))$

For convenance, if we write $a_{ij} = (T_A(m_i,n_j),\ I_A(m_i,n_j),\ F_A(m_i,n_j))$ then the above matrix A can be written as:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1q} \\ a_{21} & a_{22} & \dots & a_{2q} \\ \vdots & \vdots & \vdots & \vdots \\ a_{p1} & a_{p2} & \dots & a_{pq} \end{pmatrix}$$

Definition 2.9. Let $A = \{\langle (m,n)\,,\ (T_A(m,n),\ I_A(m,n),\ F_A(m,n))\rangle \mid (m,n) \in M \times N\}$ be the neutrosophic set of strategies of two person. It satisfied the following properties.

· $\max \Big\{ T_A(m_i,n_j), I_A(m_i,n_j), F_A(m_i,n_j) \Big\} = \Big(T_A(m,n), I_A(m,n), F_A(m,n) \Big)$

· $\min \Big\{ T_A(m_i,n_j), I_A(m_i,n_j), F_A(m_i,n_j) \Big\} = \Big(T_A(m,n), I_A(m,n), F_A(m,n) \Big)$

Example 2.10. Let $M = \{m_1,\ m_2,\ m_3\}$ and $N = \{n_1,\ n_2,\ n_3\}$ be the strategies for player$_1$ and player$_2$ respectively. The neutrosophic payoff is given as:

$$\begin{pmatrix} (0.95,\ 0.2,\ 0.1) & (0.86,\ 0.3,\ 0.2) & (0.76,\ 0.3,\ 0.3) & (1,\ 0,\ 0) & \\ (0.63,\ 0.3,\ 0.3) & (1,\ 0,\ 0) & (0.92,\ 0.2,\ 0.1) & (0.3,\ 0.4,\ 0.6) & \\ (0.43,\ 0.4,\ 0.6) & (0.38,\ 0.5,\ 0.6) & (1,\ 0,\ 0) & (0.98,\ 0.2,\ 0.2) & (0.85,\ 0.3,\ 0.3) \end{pmatrix}$$

let us consider the $a_{11} = (0.95,\ 0.2,\ 0.1)$ and $a_{12} = (0.86,\ 0.3,\ 0.2)$, according to definition (2.4),

$\max(a_{11},\ a_{12}) = \max\Big((0.95,\ 0.2,\ 0.1), (0.86,\ 0.3,\ 0.2) \Big) = (0.95,\ 0.2,\ 0.1) = a_{11}$

$\min(a_{12},\ a_{13}) = \min\Big((0.86,\ 0.3,\ 0.2),\ (0.76,\ 0.3,\ 0.3) \Big) = (0.76,\ 0.3,\ 0.3) = a_{13}$

3 Neutrosophic Evaluation System

3.1 Neutrosophic Attractive Point

Let E be the set of mutually exclusive events and $A = \{A_1,\ A_2, ..., A_p\}$ be the set of action. The neutrosophic probability is given as $\Big(P(T),\ P(I),\ \ P(F)\Big)$. An occurrence can be therefore be defined by $\Big(v(m_i,\ n_j),\ (P(T),\ \ P(I),\ \ P(F))\Big)$. An neutrosophic attractive point with events n_i is described to as a lottery $\{(v(m_1,\ n_1),\ (P(T_1),\ P(I_1),\ P(F_1))), ..., (v(m_{pi},\ n_{pi}),\ (P(T_{pi}),\ P(I_{pi}),\ P(F_{pi})))\}$.

Definition 3.1. Let $E_1, E_2 \in E$ if $p\,(SNS_1) \geq p\,(SNS_2)$ and $v\,(SNS_1) \geq v\,(SNS_2)$ and at least $p\,(SNS_1) > p\,(SNS_2)$ or $v\,(SNS_1) > v\,(SNS_2)$ at that point it is said to be E_1 is neutrosophic dominate E_2 for A_i.

Let us consider the following example to promote the comprehension of the above introduced definition and ideas.

Example 3.2. Let $A = \{A_1,\ A_2,\ A_3\}$ be the set of neutrosophic action, $N^1 = \{n_1^1,\ n_2^1,\ n_3^1\}$ and $N^2 = \{n_1^2,\ n_2^2,\ n_3^2\}$ be the strategies for player$_1$ and player$_2$ respectively. Then the neutrosophic payoff and their against neutrosophic probability is given as in Table 2 and 3:

Table 2: Neutrosophic payoff

$$\begin{pmatrix} (0.95,\ 0.2,\ 0.1) & (0.86,\ 0.3,\ 0.2) & (0.76,\ 0.3,\ 0.3) & (1,\ 0,\ 0) & (0, 0, 0) \\ (0.63,\ 0.3,\ 0.3) & (1,\ 0,\ 0) & (0.92,\ 0.2,\ 0.1) & (0.3,\ 0.4,\ 0.6) & (0, 0, 0) \\ (0.43,\ 0.4,\ 0.6) & (0.38,\ 0.5,\ 0.6) & (1,\ 0,\ 0) & (0.98,\ 0.2,\ 0.2) & (0.85,\ 0.3,\ 0.3) \end{pmatrix}$$

Table 3: Neutrosophic probability

$$\begin{pmatrix} (0.1,\ 0.4,\ 0.8) & (0.4,\ 0.2,\ 0.6) & (0.3,\ 0.5,\ 0.5) & (0.2,\ 0.3,\ 0.5) & (0, 0, 0) \\ (0.15,\ 0.2,\ 0.8) & (0.15,\ 0.2,\ 0.8) & (0.3,\ 0.5,\ 0.5) & (0.4,\ 0.2,\ 0.6) & (0, 0, 0) \\ (0.15,\ 0.2,\ 0.8) & (0.24,\ 0.3,\ 0.7) & (0.35,\ 0.2,\ 0.6) & (0.13,\ 0.3,\ 0.8) & (0.13,\ 0.3,\ 0.8) \end{pmatrix}$$

For A_1, according to definition (2.9), Clearly n_4^1 is neutrosophic dominate n_1^1, because using the definition (2.4) and equation (1), neutrosophic payoff of n_4^1 is $a_{14} = (1,\ 0,\ 0)$ greater than neutrosophic payoff n_1^1 is $a_{11} = (0.95,\ 0.2,\ 0.1)$ and at their corresponding neutrosophic probabilities $b_{14} = (0.2,\ 0.3,\ 0.5)$ is greater than $b_{11} = (0.1,\ 0.4,\ 0.8)$. Also we see that n_2^1 is neutrosophic dominates n_3^1, because neutrosophic payoff $a_{12} > a_{13}$ and neutrosophic probability $b_{12} > b_{13}$. Therefore, $\{n_2,\ n_4\}$ are the neutrosophic dominates for A_1.

For A_2, the analysis shows that n_2^2 is neutrosophic dominate n_1^2, the reason is that $a_{22} > a_{21}$ and $b_{22} = b_{21}$. Similarly, we can see that n_3^2 is neutrosophic dominate n_1^2, because neutrosophic payoff $a_{13} > a_{21}$ and neutrosophic probability $b_{23} > b_{21}$. Now it is clear that neutrosophic probability n_4^2 is higher than the other probabilities for A_2. So, with the help of definition (3.1), n_4^2 is neutrosophic dominated. Therefore, for A_2, the neutrosophic dominated vector is $\left\{n_2^2,\ n_3^2,\ n_4^2\right\}$. For A_3, it is clear that n_3^3 is neutrosophic dominated because neutrosophic payoff and neutrosophic probability are higher than all the other values. Therefore, $\left\{\ n_3^3\right\}$ is the only neutrosophic vector for A_3.

A decision-maker can select the most appealing event from E for each A_i. Obviously, a decision-maker choose the best attractive event from all the events against each activity. In the meantime, it means that then most attractive event is not necessarily extracted from a paired comparison.

Let us suppose that an event using upper value of neutrosophic probability and upper value of neutrosophic payoff would make the decision-maker more attractive. This is the naturally attractive way to characterize the selection process as it employs a relationship superiority that is know as the most generally accepted concept. This principle reflects an attitude of hope when analyzing events. These principles shows that the most desirable case of an alternative A_i is satisfied by overall state of nature E and denoted as $c_+^i(E)$. $c_+^i(E)$ is referred to describe the set of neutrosophic focus points of A_i over E in the event that there are several neutrosophic focus points A_i exist. Let's see how to recognize $c_+^i(E)$.

Definition 3.3. Let X be a space of points (objects) and $B = \Big(P(T),\ P(I),\ P(F)\Big)$ be the neutrosophic probability. A function $\pi : X \to [0,1]$ is called the neutrosophic relatively likelihood function and it is defined as: $\pi(x) = \left(\frac{P^i(T)}{\max\limits_{i\in X} P(T)}, \frac{P^i(I)}{\max\limits_{i\in X} P(I)}, \frac{P^i(F)}{\max\limits_{i\in X} P(F)}\right)$, where $0 \leq \frac{P(T)}{\max\limits_{i\in X} P(T)} + \frac{P(I)}{\max\limits_{i\in X} P(I)} + \frac{P(F)}{\max\limits_{i\in X} P(F)} \leq 3$ for all x belongs to X. Suppose that x_1, x_2 belongs to X, then

$$\pi(x_1) > \pi(x_2) \iff s\left(\frac{P(T(x_1))}{\max\limits_{i\in X} P(T(x_1))}, \frac{P(I(x_1))}{\max\limits_{i\in X} P(I(x_1))}, \frac{P(F(x_1))}{\max\limits_{i\in X} P(F(x_1))}\right) > s\left(\frac{P(T(x_2))}{\max\limits_{i\in X} P(T(x_2))}, \frac{P(I(x_2))}{\max\limits_{i\in X} P(I(x_2))}, \frac{P(F(x_2))}{\max\limits_{i\in X} P(F(x_2))}\right).$$

Definition 3.4. Let a mapping η_i from payoff function to a closed interval zero and one for all A_i is called a satisfaction function and the satisfaction function is dependent to payoff function. i.e., $\eta_i : U_i \to [0,1]$ where, $\max \eta(u_i) = 1$ and if $u_1 > u_2$ then $\eta_i(u_1) > \eta_i(u_2)$.

The above definition is the general form of satisfaction function. The relative position of satisfaction function can be written as:

$$\eta_i(u_i) = \left(\frac{U(T)}{\max\limits_{i\in X} U(T)}, \frac{U(I)}{\max\limits_{i\in X} U(I)}, \frac{U(F)}{\max\limits_{i\in X} U(F)}\right), \text{ where } 0 \leq \frac{U(T)}{\max\limits_{i\in X} U(T)} + \frac{U(I)}{\max\limits_{i\in X} U(I)} + \frac{U(F)}{\max\limits_{i\in X} U(F)} \leq 3 \text{ for all } x \in X.$$

Let us consider the example 3.2, we rewrite the neutrosophic dominates for A_1 are $\left\{n_2^1,\ n_4^1\right\}$, the neutrosophic dominated vector for A_2 is $\left\{n_2^2,\ n_3^2,\ n_4^2\right\}$ and the neutrosophic dominated vector for A_3 is $\left\{\ n_3^3\right\}$. When considering the neutrosophic dominates for A_1 are $\left\{n_2^1,\ n_4^1\right\}$, then their neutrosophic payoff and neutrosophic probabilities are as: $\{[(0.86,\ 0.3,\ 0.2),\ (0.4,\ 0.2,\ 0.6)]\ [(1,\ 0,\ 0),\ (0.2,\ 0.3,\ 0.5)]\}$, using definition 3.3,

$\{[(0.86,\ 0.3,\ 0.2),\ (1.0,\ 0.67,\ 1.0)]\ [(1,\ 0,\ 0),\ (0.5,\ 1.0,\ 0.83)]\}$.

we calculate the attractive point between $\left\{n_2^1,\ n_4^1\right\}$ as: $\min\left(\pi(n_2), \eta(n_2)\right) = \min\left[(0.86,\ 0.3,\ 0.2),\ (1.0,\ 0.67,\ 1.0)\right]$. using equation 1, $\min\left[(0.86,\ 0.3,\ 0.2),\ (1.0,\ 0.67,\ 1.0)\right] = (1.0,\ 0.67,\ 1.0)$, similarly, $\min\left(\pi(n_4), \eta(n4)\right) = \min\left[(1,\ 0,\ 0),\ (0.5,\ 1.0,\ 0.83)\right] = (0.5,\ 1.0,\ 0.83)$. As the attractive point between $\left\{n_2^1,\ n_4^1\right\}$ is the upper value between n_2 and n_4. Therefore, $\max\left(n_2^1,\ n_4^1\right) = \max\Big((1.0,\ 0.67,\ 1.0),\ (0.5,\ 1.0,\ 0.83)\Big) = n_2^1$.

So, the neutrosophic attractive point is n_2^1. It shows that $c_+^1(A_1) = n_2^1$. Similarly, the neutrosophic payoff and neutrosophic probability for A_2 is: $\{[(1,\ 0,\ 0)\,,\ (0.15,\ 0.2,\ 0.8)]\ [(0.92,\ 0.2,\ 0.1)\,,\ (0.3,\ 0.5,\ 0.5)]\ [(0.3,\ 0.4,\ 0.6)\,, (0.4,\ 0.2,\ 0.6)]\}$.
Normalize the above vector using the definition 3.3, $\{[(1,\ 0,\ 0),\ (0.38,\ 0.4,\ 1.0)]\ [(0.92,\ 0.5,\ 0.1),\ (0.75,\ 1.0,\ 0.63)]\ [(0.3,\ 1.0,\ 1.0), (1.0,\ 0.4,\ 0.75)]\}$.

Likewise, we obtain $c_+^2(A_2) = n_2^2$. Because, for A_3, only the singleton set $\{\ n_3^3\}$ is the dominates vector, therefore, $\{\ n_3^3\}$ is the attractive point for A_3.

Therefore, the subsets of A_1, A_2 and A_3 are: $A_1 - \{\ n_2^1\} = \{n_1^1,\ n_3^1,\ n_4^1\}$, $A_2 - \{\ n_2^2\} = \{n_1^2,\ n_3^2,\ n_4^2\}$, $A_3 - \{\ n_3^3\} = \{n_1^3,\ n_2^3,\ n_4^3,\ n_5^3\}$.

Now the same process for the subsets have been done, and we have $c_+^1(A_1 - \{n_2^1\}) = \{n_4^1\}$, $c_+^2(A_2 - \{n_2^2\}) = \{n_1^2\}$, $c_+^3(A_3 - \{n_3^3\}) = \{n_4^3\}$.

3.2 Neutrosophic Ideal Alternatives

In the neutrosophic theory of choice, the first step is to calculate the neutrosophic attractive points and the second step is to calculate the neutrosophic ideal alternatives, these neutrosophic ideal alternatives are based on neutrosophic attractive points. In the problems of the neutrosophic theory of choice, a decision-maker believes that the NS attractive points are the most suitable points. Therefore, the alternatives are chosen that generate the ideal alternative after the selecting of NS attractive points. They are sum up of the following definitions.

Definition 3.5. Let $F \subseteq \bigcup_{j=1}^{n} E^j$, and Q_+ is the set of maximal elements of F, and $NF(F, Q_+) = \{t \in F, (t, e) \notin Q_+ \mid e \in F\}$.

$C_+ = \bigcup_{j=1}^{n} c_+^j\left(E^j\right)$ is the collection of attractive points with relatively high neutrosophic probabilities as well as relatively high neutrosophic payoffs. Suppose that $G = \bigcup_{j=1}^{n} G^j$, where $G^j \subseteq E^j$, then $D_+ = \left\{c_+^i \in NF(G, Q_+) \mid \forall A_i \in A\right\}$ is the neutrosophic set of action whose neutrosophic attractive points are in $NF(F, Q_+)$.

Let us turn Example 3.2. $c_+^1(A_1) = \left\{\ n_2^1\right\}$, $c_+^2(A_2) = n_2^2$, $c_+^3(A_3) = n_3^3$, according to definition 3.4, $C_+ = \left\{n_2^1,\ n_2^2, n_3^3\ \right\} = \{[(0.86,\ 0.3,\ 0.2),\ (0.4,\ 0.2,\ 0.6)]\ [(1.0,\ 0,\ 0)\,,\ (0.15,\ 0.2,\ 0.8)]\ [(1.0,\ 0,\ 0), (0.35,\ 0.2,\ 0.6)]\}$. It is clear that $n_3^3 > n_2^1$, and also $n_2^1 > n_2^2$. Hence $NF(F, Q_+) = \{n_2^1,\ n_3^3\}$. Corresponding to these actions, A_1 and A_3 are their respectively alternatives. Therefore, $D_+ = \{A_1,\ A_3\}$. Now, $\{n_2^1, n_3^3\} = \{[(0.86,\ 0.3,\ 0.2)\,,\ (0.4,\ 0.2,\ 0.6)]\ [(1,\ 0,\ 0)\,,\ (0.35,\ 0.2,\ 0.6)]\}$.

using definition 3.3 and 3.4, $\{n_2^1, n_3^3\} = \{[(0.86,\ 0.3,\ 0.2)\,,\ (1.0,\ 1.0,\ 1.0)]\ [(1,\ 0,\ 0)\,,\ (0.88,\ 1.0,\ 1.0)]\}$.

$\min\left(u\left(n_2^1\right),\ \pi\left(n_2^1\right)\right) = \min\left[(0.86,\ 0.3,\ 0.2)\,,\ (1.0,\ 1.0,\ 1.0)\right] = (1.0,\ 1.0,\ 1.0)$

$\min\left(u\left(n_3^3\right),\ \pi\left(n_3^3\right)\right) = \min\left[(1,\ 0,\ 0)\,,\ (0.88,\ 1.0,\ 1.0)\right] = (1,\ 0,\ 0)$

now the maximum value between the above is the optimal value, so the most attractive point is $\{n_3^3\}$ and hence, A_3 is the optimal alternative.

4 Application of Neutrosophic Set in Decision Making

4.1 Working Rule

In this section, a procedure for neutrosophic theory of choice is shown. The following steps show the algorithm of game problems.
Step 1: Calculate the score values of each neutrosophic payoff and neutrosophic probability.
Step 2: Calculate the neutrosophic dominate points according to definition 2.9 and deleting all other vectors.

Step 3: Apply the definition 3.1 and 3.3 for dominate vectors.
Step 4: Calculate all the neutrosophic attractive points using max(min $(\eta(u_i), \pi(x_i)))$.
Step 5: Collect the neutrosophic set of action for attractive points using definition 3.5.
Step 6: Obtained the optimal strategies from all the neutrosophic attractive points.
The conceptualization of the suggested strategy is shown in figure 2.

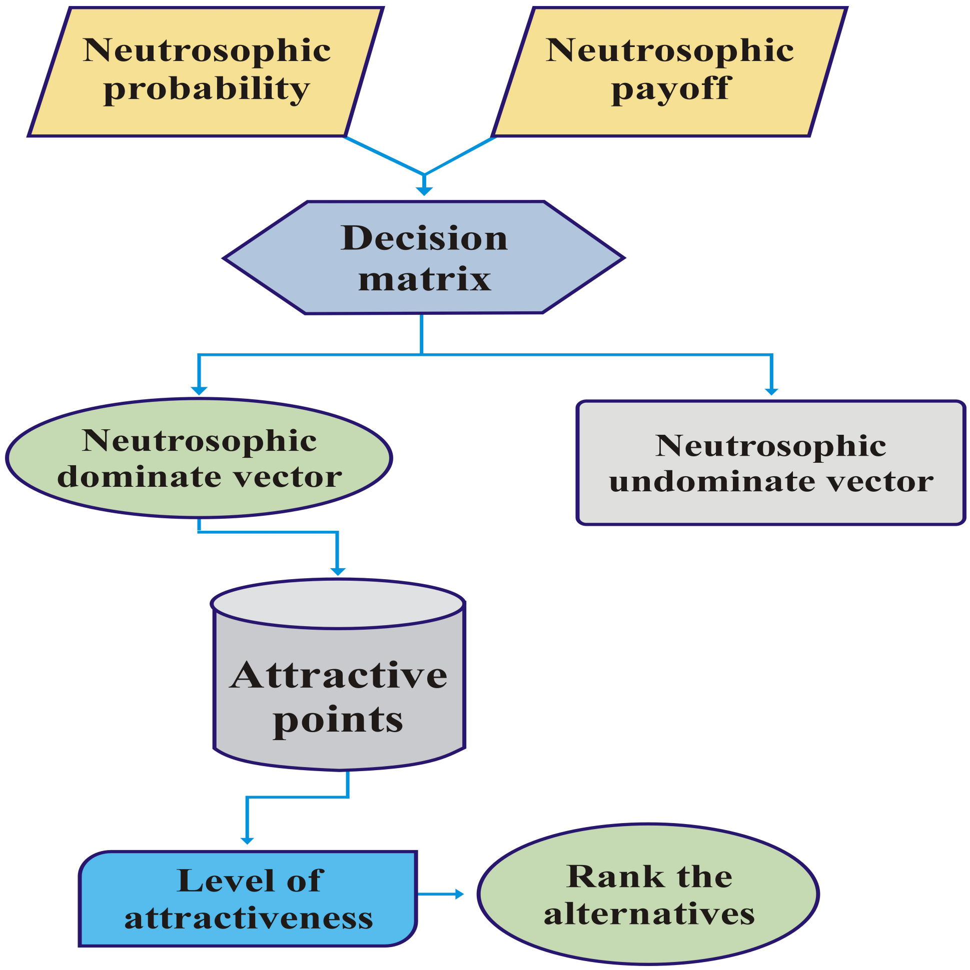

Figure 2: Algorithm of the proposed strategies under the environment of neutrosophic sets

4.2 Case Study

Let's take a real-life example to make the conceptual understanding easy, a person who wants to buy a new mobile phone. He, as decision-maker starts his research. Mainly he evaluates and analysis among the three most popular brands, i.e. Apple, Samsung and LG Mobiles. He compares the five major characteristics of a mobile phone which are the following: 1. camera pixels, 2. Battery power/ timing, 3. processor capacity, 4. mobile RAM & memory capacity, 5. screen resolution & size. The decision-maker collects the information given by the companies and online consumer views on these products. His satisfaction level about each of the characteristic is dependent on the customer's opinions on them. Suppose most of Apple customers are not satisfied with its battery timing, so Apple's probability in this regard is not good, leading to dissatisfaction. However, customers' views about Apple's screen resolution are exceptional, leading to high satisfaction to decision-makers. The neutrosophic satisfaction level for each alternative corresponding to their neutrosophic probabilities are as follows:

Suppose that $\{A_1, A_2, A_3\}$ be the set of neutrosophic action collaborated with the disjoint set of events N^i, where superscript i represents mathematical symbols for the action A_i. Suppose $N^1 = \{n_1^1, n_2^1, n_3^1, n_4^1, n_5^1\}$, $N^2 = \{n_1^2, n_2^2, n_3^2, n_4^2, n_5^2\}$, $N^3 = \{n_1^3, n_2^3, n_3^3, n_4^3, n_5^3\}$. The strategies of neutrosophic payoffs and their corresponding neutrosophic probability associated to each state shown in Table 4 and 5:

Table 4: Strategies of neutrosophic payoff for player.

$$\begin{pmatrix} (0.95,\ 0.2,\ 0.1) & (0.86,\ 0.3,\ 0.2) & (0.76,\ 0.3,\ 0.3) & (1,\ 0,\ 0) & (0,0,0) \\ (0.63,\ 0.3,\ 0.3) & (1,\ 0,\ 0) & (0.92,\ 0.2,\ 0.1) & (0.3,\ 0.4,\ 0.6) & (0,0,0) \\ (0.43,\ 0.4,\ 0.6) & (0.38,\ 0.5,\ 0.6) & (1,\ 0,\ 0) & (0.98,\ 0.2,\ 0.2) & (0.85,\ 0.3,\ 0.3) \end{pmatrix}$$

Table 5: Strategies of neutrosophic probability for player

$$\begin{pmatrix} (0.1,\ 0.4,\ 0.8) & (0.4,\ 0.2,\ 0.6) & (0.3,\ 0.5,\ 0.5) & (0.2,\ 0.3,\ 0.5) & (0,0,0) \\ (0.15,\ 0.2,\ 0.8) & (0.15,\ 0.2,\ 0.8) & (0.3,\ 0.5,\ 0.5) & (0.4,\ 0.2,\ 0.6) & (0,0,0) \\ (0.15,\ 0.2,\ 0.8) & (0.24,\ 0.3,\ 0.7) & (0.35,\ 0.2,\ 0.6) & (0.13,\ 0.3,\ 0.8) & (0.13,\ 0.3,\ 0.8) \end{pmatrix}$$

The neutrosophic dominates vectors for A_1, A_2, and A_3 are $\{n_1^1, n_2^1, n_3^1, n_4^1\}$, $\{n_2^2, n_5^2\}$ and $\{n_1^3, n_4^3\}$ respectively. For $\{n_1^1, n_2^1, n_3^1, n_4^1\}$, the payoff and their corresponding probabilities are:

$$\begin{pmatrix} (1,0,0) & (0.6,0.3,0.3) & (0.5,0.6,0.2) & (0.4,0.6,0.4) \\ (0.1,0.2,0.4) & (0.4,0.3,0.1) & (0.3,0.1,0.1) & (0.1,0.2,0.2) \end{pmatrix}$$

The neutrosophic satisfaction function and neutrosophic relatively likehood functions of above matrix can be written as:

$$\begin{pmatrix} (1,0,0) & (0.6,0.5,0.75) & (0.5,1.0,0.5) & (0.4,1.0,1.0) \\ (0.25,0.67,1.0) & (1,1,0.25) & (0.75,0.33,0.25) & (0.25,0.67,0.5) \end{pmatrix}$$

The optimal action point in neutrosophic theory of choice is; $\max(\min\left(v(n_i^1), \pi(n_i^1)\right)) = \max((0.25, 0.67, 1),$ $(1, 1, 0.25), (0.5, 1, 0.5), (0.4, 1, 1)) = n_2^1$. Therefore, $c_+^1(A_1) = n_2^1$. Similarly, $c_+^2(A_2) = n_2^2$ and $c_+^3(A_3) = n_4^3$. So, $C_+ = \bigcup_{j=1}^{n} c_+^j(A_j) = \{n_2^1, n_2^2, n_4^3\}$. Now the second step is to calculate the ideal alternative, for this, we write the neutrosophic payoff and their corresponding probabilities from Table (4) and (5) for the neutrosophic attractive points.

$$\underbrace{\begin{pmatrix} \overset{n_2^1}{(0.6,0.3,0.3)} & \overset{n_2^2}{(1,0,0)} & \overset{n_4^3}{(1,0,0)} \\ (0.4,0.3,0.1) & (0.2,0.4,0.7) & (0.35,0.4,0.7) \end{pmatrix}}_{\text{Neutrosophic attractive points}}$$

According to definition 3.4, $NF(F, Q_+) = \{n_2^1,\ n_4^3\}$ and $ND_+ = \{A_1,\ A_3\}$. Now we calculate the level of attractive for the alternatives A_1 and A_3 with the help of $\max(\min\left(v(n_2^1, A_1), \pi(n_2^1)\right), \left(v(n_4^3, A_3), \pi(n_4^3)\right))$. Therefore, the optimal action is A_3. Hence, $A_3 > A_1 > A_2$.

The graphical representation of the optimal point is shown in Figure 3. The points A, A', B, B' and C, C' shows the strategies of payoff and their corresponding strategies of probabilities, respectively. These points are the attractive points of the given decision matrix. Moreover, n_2^1,$n_2^1\prime$ and $n_4^3, n_4^3\prime$ represents the relative position of neutrosophic payoff and probabilities, respectively, of the points A, A' B, B' and C, C'.

4.3 Comparison Analysis

A comparative study between the proposed neutrosophic theory of choice and other methods like TOPSIS is discussed and analysis shows that the TOPSIS evaluates each alternative using the weighted of all the

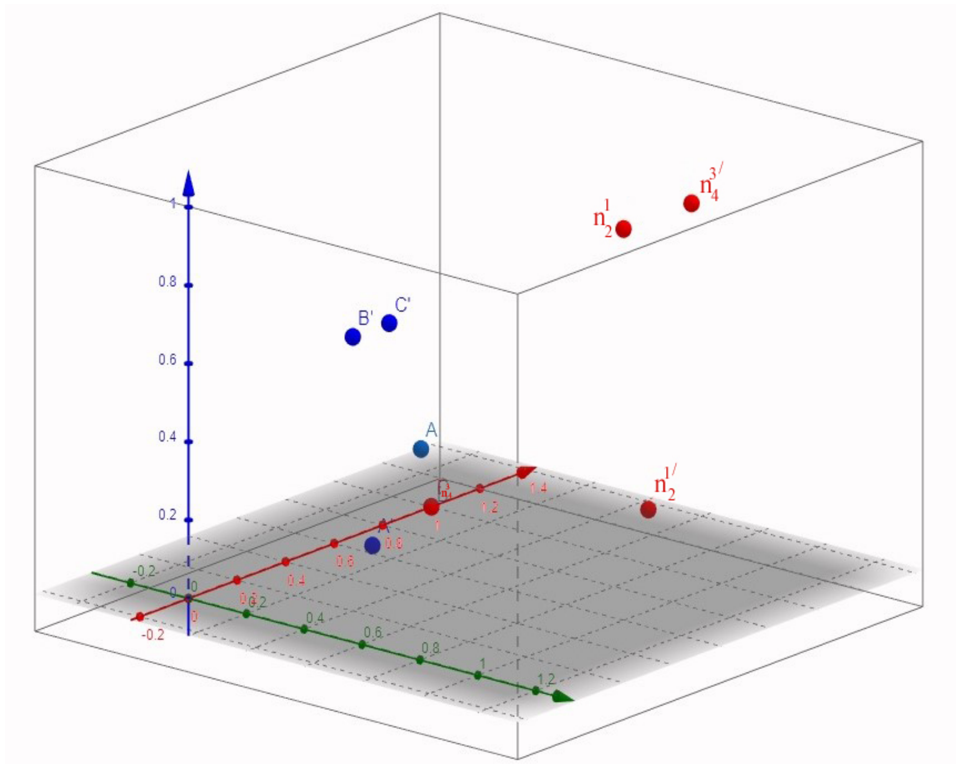

Figure 3: Neutrosophic optimal point

outcomes and then selecting the alternative with maximum relative closeness. Similarly, if we consider the subjective expected utility (SEU), this theory is also based on the weighted average. In, SEU, each alternative is selected by maximum average based on weight. These theories are related to the risk factor. A decision-maker avoids the risk, takes the risk or is neutral, the graph of utility is concave, convex and linear respectively. But the neutrosophic theory of choice, consists of two steps; the first step is to select the event of attractive points and 2nd step is the relationship trade of neutrosophic payoff and neutrosophic relative likelihood probability function. There is no risk factor for the decision-maker. Because in the proposed theory, the satisfaction function is the relative position of payoff, as well as the relative likelihood function, are used to make the decision, which shows the attitude of a decision-maker in uncertain situations. It means when a decision-maker chooses the attractive points. Decision-maker mark the same weight on probability and payoff, or when he tried to obtain an attractive point, the payoff and probability are equal degrees of importance. It shows that the neutrosophic theory of choice is most straight forward and more comfortable than other approaches. Neutrosophic theory of choice deals with totally different ways to select the scenario. SEU and other approaches use the weighted function to deal with the uncertainty, which is not the actual solution of the uncertainty problems. Suppose some alternative is repeated a large number of times. In that case, the obtained result is confidently reaching the maximum value. In contrast, the proposed theory shows a clear solution because of using neutrosophic payoff and relative likelihood function, and psychological evidence clearly supports them. Nowadays, many researchers [25, 26] give evidence gathered from studies using scanpath and different strategies suggesting that it is impossible that a risky decision would be made on a system of weighting and summing. Zhou et al.[27] proved that the proportion task of the information process sequence tends to be more compatible with the summing and weighting method. Therefore, we believe that our outcomes indicate the best results as compared to the weighting and summing process. So, the proposed technique would be an immense addition to decision-making problems.

5 Conclusion

The most generalization of intuitionistic fuzzy sets is the neutrosophic sets, in which ambiguity is introduced through an extra indeterminacy degree. In this research, we have implied neutrosophic frameworks. We chose game theories using neutrosophic logic. We have considered a neutrosophic payoff and probability approach to solving our constructed game phenomena. In this analysis, we have observed an indeterminacy function assuming neutrosophic sets with membership and falsity characteristics. Moreover, in focused recommendation systems, we have declared our proposed game model and have achieved some desirable outcomes. We have found that some have solved the problems in crisp data sets, while we have presented neutrosophic data sets that are more closely linked to the expressions of real-life problems. This can be seen as a limitation of our study's generalization. Some theoretical constructs can, however, be explored in various situations and other real-life issues with different levels of additional measures. In the future, research in multiple fields, such as medical diagnosis, business management optimization, aerospace engineering, space design management, manufacturing industry management, weapons, laboratory research management, wastewater management, optimization of renewable energy sources, supply chain management, can be carried out in game theory under different uncertainties. Game theories neutrosophic attributes can be comprehended utilizing different techniques from neuroscience, mechanical technology, artificial intelligence, humanitarian operations and so on. The neutrosophic focus theory of choice consists of two obligatory portions, from an external source, neutrosophic probability and payoff are given. For the selecting of the neutrosophic attractive points, we can't straightforward allocate the probabilities. Therefore, we use a two-level process to evaluate the probabilities. There are two types of theories for modeling rationality [33], which is substantively rational theory and the second is rational procedural theory. According to many researchers, the second model is more relaxed and latest logical approach. The fundamental principle of all these types of theories is to substitute or ease the portion of the expected utility theory or the expected subjective utility theory axioms. This paper contributes to a basic theory, including some logically satisfying axioms for the rationality procedural and deals with decision-making risk or uncertainty or ignorance. The key point of Neutrosophic theory of choice is that the most relevant occurrence leads to the most favoured attractive point. The Neutrosophic theory of choice dispose of two stages, one is to select an attractive event for each step and then the most occurrence event is chosen from all the attractive points. We have found many cognitive proofs in serval papers [31, 34] that all the evidence consists of the basic principles of the neutrosophic theory of choice. These theories used concave and convex functions to show the gain and loss; these functions are associated with risky situations. Whereas, in the proposed approach, the neutrosophic payoff function has no relation to risk situations. Decision-making models are categorized by Shafir et al.[35] into two groups; one is value-based and the second is reason-based. A value-based model is associated with a numerical value to every option and chooses the maximum value alternative on the other hand reason-based problems describes different goals and reasons that are expected to determine and affect and describes choices in terms of reasons for and against the various alternatives. But, there is no analysis of how these theories related to lottery base decisions. However, we imply the neutrosophic theory of choice that the reason for the alternative is the identify the attractive points of the optimal attractive. It is also possible that sometimes, a decision-maker often does not understand a particular factor when evaluating an ideal alternative [36]. Our proposed neutrosophic theory of choice can be implemented for complicated decisions and real-world problems where some existing approaches may be difficult to solve. In management fields, the proposed theory provides a comprehensive, structured framework for modeling rational thoughts. While it is well-known that behavioral variables are very significant in the study of research, however, it is complicated to integrate the characteristics and qualities of players into mathematical models because of the absence of proper theories. The proposed approach gives a conceptual framework for the development of behavioral models. This study has many limitations, while many recognized irregularities have been announced by this proposed theory, different axioms are proved using the logical procedure.

Conflict of Interest: "The authors declare no conflict of interest."

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