Transactions on Fuzzy Sets and Systems (TFSS)
URL: https://tfss.journals.iau.ir/
Online ISSN: 2821-0131
Vol.2, No.2, (2023), 219-228
DOI: http://doi.org/10.30495/tfss.2023.1980759.1066

Article Type: Original Research Article

Similarity Measure: An Intuitionistic Fuzzy Rough Set Approach

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Abstract. In fuzzy set theory, the concept of a non-membership function and the hesitation margin were not considered while these two concepts have been included along with the membership function for intuitionistic fuzzy sets. It is also to be noted that the intuitionistic fuzzy set is reflected as an extension of the fuzzy set accommodating both membership and non-membership functions together with a hesitation margin. In the intuitionistic fuzzy set theory, the sum of the membership function and the non-membership function is a value between 0 and 1. In recent times, intuitionistic fuzzy rough set theory has emerged as a powerful tool for dealing with imprecision and uncertain information in relational database theory. Measures of similarity between fuzzy rough sets as well as intuitionistic fuzzy rough sets provide wide applications in real-life problems and that is why many researchers paid more attention to this concept. Intuitionistic fuzzy rough set theory behaves like an excellent tool to tackle impreciseness or uncertainties. In this paper, we propose a new approach of similarity measure on an intuitionistic fuzzy rough set based on a set-theoretic approach. The proposed measure is able to give an exact result. In the application part, we consider a real-life problem for selecting a fair play award-winning team in a cricket tournament and describe the algorithm.

AMS Subject Classification 2020: 90C70; 03F55
Keywords and Phrases: Similarity measure, Intuitionistic fuzzy set, Rough set.

1 Introduction

The Rough set theory introduced by Pawlak [13, 14] is an excellent and elegant mathematical tool for the analysis of uncertainty, inconsistency and vague descriptions of objects. The basic idea of a rough set is based upon the approximation of sets by a pair of sets known as lower approximation and upper approximation. Here, the lower and upper approximation operators are based on equivalence relation. However, in many real-life problems, a rough set model cannot be applied due to the restrictions of the requirement of equivalence relation. For this reason, the rough set is generalized to fuzzy sets such as fuzzy rough set and rough fuzzy set [9].

In 1965, L.A.Zadeh [17] first introduced the concept of a Fuzzy set. Atanassov [1] generalized this concept into an intuitionistic fuzzy set in 1983. Since then many authors [3, 4] have been concentrating as well as developing the concepts like algebraic laws of IFSs, basic operations on IFSs, modal operators and normalization of IFSs etc. In fuzzy set theory it is taken into consideration that there exists a membership value for all the elements of the set and we do not consider non-membership values of the elements of the set. But in real-life problems, we feel the existence of hesitation. In fuzzy set theory, if $\mu(x)$ is the degree of membership of an element x, then the degree of non-membership of x is calculated by $1-\mu(x)$. But this concept is not always applicable to all real-life problems and that is why the notion of an intuitionistic fuzzy set is introduced. It may be

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Received: 2 February 2023; **Revised**: 3 October 2023; **Accepted**: 24 October 2023; **Available Online**: 24 October 2023; **Published Online**: 7 November 2023.

How to cite: Bhattacharya J. Similarity measure: An intuitionistic fuzzy rough set approach. *Trans. Fuzzy Sets Syst.* 2023; 2(2): 219-228. DOI: http://doi.org/10.30495/tfss.2023.1980759.1066

mentioned that the intuitionistic fuzzy set theory reduces to fuzzy set theory if the in-deterministic part is zero. Combining the fuzzy sets with the rough sets, Nanda and Majumdar [11] proposed the concept of fuzzy rough sets in 1992. Subsequently, Coker [7] pointed out fuzzy rough sets are the intuitionistic L-fuzzy sets. In this paper, we utilize the concept of the intuitionistic fuzzy rough set [16, 19] model to determine the similarity measure between two given intuitionistic fuzzy rough sets. Furthermore, using this concept we illustrate an example for selecting the procedure of fair play award in a cricket tournament.

2 Preliminaries

Definition 2.1. [2] *Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form A=$\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A, \nu_A : x \to [0,1]$define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, it holds that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*
Furthermore, we have $\pi_A(x)$= 1$-$ $\mu_A(x)$$-$$\nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ that is $\pi_A : X \to [0,1]$ and $0 \leq \pi_A(x) \leq 1$ for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

The definition of rough sets is based upon the approximation of a set by a pair of sets known as a lower and an upper approximation. Let U be the universe of a finite non-empty set of objects. Let $R \subseteq U \times U$ be an equivalence relation on U. The equivalence relation R partitions the set U into disjoint classes and it is denoted as U/R. Let X be a subset of U. Therefore the target set X can be described by a lower and an upper approximation as below, where $\underline{R}X$ and $\overline{R}X$ are $R-lower$ and $R-upper$ approximations of X respectively.
$\underline{R}X = \cup\{X' \in U/R : X' \subseteq X\}$ and $\overline{R}X = \cup\{X' \in U/R : X' \cap X \neq \emptyset\}$
Boundary region of the set X, $BN_R(X)$, is the objects in X that can be distinguished neither as a member nor as a non-member of x employing the relation R. It is denoted as $BN_R(X) = \underline{R}X - \overline{R}X$.
A set X is said to be definable if $\underline{R}X = \overline{R}X$ and the target set is a crisp set i.e., there is no boundary line objects. Similarly, it is said to be rough if $\underline{R}X \neq \overline{R}X$ or equivalently $BN_R(X) \neq \emptyset$.

Definition 2.2. [4] Let U be a non-empty and finite universe of discourse and IFR be an intuitionistic fuzzy relation defined on $U \times U$. The pair (U, IFR) is called an intuitionistic fuzzy rough approximation space. For any $A \in IF(U)$, where IF(U) denotes the intuitionistic fuzzy power set of U, the lower and upper approximations of A with respect to (U, IFR) denoted by $IF\underline{R}(A)$ and $IF\overline{R}(A)$ are defined as follows:
$IF\underline{R}(A) = \{\langle x, \mu_{IF\underline{R}(A)}(x), \nu_{IF\underline{R}(A)}(x)\rangle : x \in U\}$
$IF\overline{R}(A) = \{\langle x, \mu_{IF\overline{R}(A)}(x), \nu_{IF\overline{R}(A)}(x)\rangle : x \in U\}$
Where
$\mu_{IF\underline{R}(A)}(x) = \wedge_{y \in U}[\nu_{IFR}(x,y) \vee \mu_A(y)]$
$\nu_{IF\underline{R}(A)}(x) = \vee_{y \in U}[\mu_{IFR}(x,y) \wedge \nu_A(y)]$
$\mu_{IF\overline{R}(A)}(x) = \vee_{y \in U}[\mu_{IFR}(x,y) \wedge \mu_A(y)]$
$\nu_{IF\overline{R}(A)}(x) = \wedge_{y \in U}[\nu_{IFR}(x,y) \vee \nu_A(y)]$
The pair $(IF\underline{R}(A), IF\overline{R}(A))$ is called the intuitionistic fuzzy rough set associated with A denoted by IFR(A).
Then, an IF rough set $A \in IF(U)$ could be denoted by $A = \{\langle x, \mu_{\underline{A}}(x), \mu_{\overline{A}}(x), \nu_{\underline{A}}(x), \nu_{\overline{A}}(x)\rangle : \forall x \in U\}$.

Definition 2.3. [3] Let U be a non-empty and finite universe of discourse and $A, B \in IF(U)$.
Then (i) The complement of $A = \langle \mu_{\underline{A}}(x), \mu_{\overline{A}}(x), \nu_{\underline{A}}(x), \nu_{\overline{A}}(x)\rangle$ is defined as $A^c = \langle \nu_{\underline{A}}(x), \nu_{\overline{A}}(x), \mu_{\underline{A}}(x), \mu_{\overline{A}}(x)\rangle$, for any $x \in U$.
(ii) $A \subseteq B$ if for any $x \in U$, $\mu_{\underline{A}}(x) \leq \mu_{\underline{B}}(x), \mu_{\overline{A}}(x) \leq \mu_{\overline{B}}(x)$ and $\nu_{\underline{A}}(x) \geq \nu_{\underline{B}}(x), \nu_{\overline{A}}(x) \geq \nu_{\overline{B}}(x)$.

Definition 2.4. [10, 15] Let U be a non-empty and finite universe of discourse and $A \in IFR(U)$. Then $M : A \times A \to [0,1]$ is called the similarity measure on A and $M(x,y)$ is called the similarity degree between the intuitionistic fuzzy rough values $x = (\mu_{\underline{A}}(x), \mu_{\overline{A}}(x), \nu_{\underline{A}}(x), \nu_{\overline{A}}(x), \pi_{\underline{A}}(x), \pi_{\overline{A}}(x))$,
$y = (\mu_{\underline{A}}(y), \mu_{\overline{A}}(y), \nu_{\underline{A}}(y), \nu_{\overline{A}}, \pi_{\underline{A}}(y), \pi_{\overline{A}}(y))$, if M satisfies the following conditions:

1. $0 \leq M(x,y) \leq 1$.

2. $M(x,y) = M(y,x)$.

3. $\forall x \in A, M(x,y) = M(x,z) \Rightarrow M(y,z) = 1$.

4. $M(x,y) = M(x^c, y^c)$, where x^c and y^c are complements of x and y respectively.

5. If $x \leq y \leq z$, then $M(x,z) \leq min\{M(x,y), M(y,z)\}$, $\forall x, y, z \in A$.

3 Similarity Measures

Many researchers [[6], [8], [10], [19], [18], [20]] have paid their concentration to develop the concept of similarity measure between fuzzy sets, intuitionistic fuzzy sets and intuitionistic fuzzy rough sets. On the basis of the set-theoretic approach, Pappis and Karacapilidis [12] defined the similarity measure between fuzzy sets A and B with fuzzy values $a_i \in A$ and $b_i \in B$ as follows.

$$M_p(A,B) = \frac{(|A \cap B|)}{(|A \cup B|)} = \frac{\sum_{i=1}^{n}(a_i \wedge b_i)}{\sum_{i=1}^{n}(a_i \vee b_i)} \tag{1}$$

In [5] Chen defined a similarity measure between two IF sets with IF values x and y (from the set-theoretic point of view) as follows:

$$M_C(x,y) = \frac{(min(\mu(x),\mu(y)) + min(\nu(x),\nu(y)) + min(\pi(x),\pi(y)))}{(max(\mu(x),\mu(y)) + max(\nu(x),\nu(y)) + max(\pi(x),\pi(y)))} \tag{2}$$

Atanassov [3] also gives a similarity measure between fuzzy rough values as follows: Let A be a fuzzy rough set in X, $x = \langle \mu_{\underline{A}}(x), \mu_{\overline{A}}(x) \rangle, y = \langle \mu_{\underline{A}}(y), \mu_{\overline{A}}(y) \rangle$be the fuzzy rough values in A. The degree of similarity between the fuzzy rough values x and y can be evaluated by the function $M_Z(x,y)$.

$$M_Z(x,y) = 1 - \frac{1}{2}(\left|\mu_{\underline{A}}(x) - \mu_{\underline{A}}(y)\right| + \left|\mu_{\overline{A}}(x) - \mu_{\overline{A}}(y)\right|) \tag{3}$$

Gangwal et.al. [10] also mentioned a similarity measure between IF rough values based on a set-theoretic approach as mentioned below.
Let A be an IF rough set in X, $x = \langle \mu_{\underline{A}}(x), \mu_{\overline{A}}(x), \nu_{\underline{A}}(x), \nu_{\overline{A}}(x) \rangle, y = \langle \mu_{\underline{A}}(y), \mu_{\overline{A}}(y), \nu_{\underline{A}}(y), \nu_{\overline{A}}(y) \rangle$ be two IF rough values in A. The degree of similarity between the IF rough values x and y can be defined by the function $M(x,y)$ as follows:

$$M(x,y) = \frac{(\left|\mu_{\underline{A}}(x) \wedge \mu_{\underline{A}}(y) + \mu_{\overline{A}}(x) \wedge \mu_{\overline{A}}(y) + \nu_{\underline{A}}(x) \wedge \nu_{\underline{A}}(y) + \nu_{\overline{A}}(x) \wedge \nu_{\overline{A}}(y)\right|)}{(\left|\mu_{\underline{A}}(x) \vee \mu_{\underline{A}}(y) + \mu_{\overline{A}}(x) \vee \mu_{\overline{A}}(y) + \nu_{\underline{A}}(x) \vee \nu_{\underline{A}}(y) + \nu_{\overline{A}}(x) \vee \nu_{\overline{A}}(y)\right|)} \tag{4}$$

In the above definition, the IF rough values of 4-tuples are used. Instead of IF rough values of 4-tuples, we consider IF rough values of 6-tuples in the rest of the paper.

Definition 3.1. Let A be an IF rough set in X,

$$x = \langle \mu_{\underline{A}}(x), \mu_{\overline{A}}(x), \nu_{\underline{A}}(x), \nu_{\overline{A}}(x), \pi_{\underline{A}}(x), \pi_{\overline{A}}(x) \rangle,$$

$$y = \langle \mu_{\underline{A}}(y), \mu_{\overline{A}}(y), \nu_{\underline{A}}(y), \nu_{\overline{A}}(y), \pi_{\underline{A}}(y), \pi_{\overline{A}}(y) \rangle$$

be two IF rough values in A. The degree of similarity between the IF rough values x and y can be defined by the function $M_J(x,y)$ as follows:

$$M_J(x,y) = \frac{(\mu_{\underline{A}}(x) \wedge \mu_{\underline{A}}(y) + \mu_{\overline{A}}(x) \wedge \mu_{\overline{A}}(y) + \nu_{\underline{A}}(x) \wedge \nu_{\underline{A}}(y) + \nu_{\overline{A}}(x) \wedge \nu_{\overline{A}}(y) + \pi_{\underline{A}}(x) \wedge \pi_{\underline{A}}(y) + \pi_{\overline{A}}(x) \wedge \pi_{\overline{A}}(y))}{(\mu_{\underline{A}}(x) \vee \mu_{\underline{A}}(y) + \mu_{\overline{A}}(x) \vee \mu_{\overline{A}}(y) + \nu_{\underline{A}}(x) \vee \nu_{\underline{A}}(y) + \nu_{\overline{A}}(x) \vee \nu_{\overline{A}}(y) + \pi_{\underline{A}}(x) \vee \pi_{\underline{A}}(y) + \pi_{\overline{A}}(x) \vee \pi_{\overline{A}}(y))} \tag{5}$$

The larger the value of $M_J(x,y)$, the more the similarity between the IF rough values x and y.

Example 3.2. Let x and y be two IF rough values, where $x = \langle 0.6, 0.5, 0.3, 0.4, 0.1, 0.1 \rangle$ and $y = \langle 0.7, 0.65, 0.25, 0.3, 0.05, 0.05 \rangle$. Then the degree of similarity between x and y can be evaluated as

$$M_J(x,y) = \frac{min(0.6,0.7)+min(0.5,0.65)+min(0.3,0.25)+min(0.4,0.3)+min(0.1,0.05)+min(0.1,0.05)}{max(0.6,0.7)+max(0.5,0.65)+max(0.3,0.25)+max(0.4,0.3)+max(0.1,0.05)+max(0.1,0.05)}$$

$$= \frac{0.6+0.5+0.25+0.3+0.05+0.05}{0.7+0.65+0.3+0.4+0.1+0.1} = \frac{1.75}{2.25} \approx 0.7778.$$

Example 3.3. Let x and y be two IF rough values, where $x = \langle 0.6, 0.5, 0.3, 0.4, 0.1, 0.1 \rangle$ and $y = \langle 0.7, 0.65, 0.25, 0.3, 0.05, 0.05 \rangle$.
Then the complementary of x and y can be given by $x^c = \langle 0.3, 0.4, 0.6, 0.5, 0.1, 0.1 \rangle$ and $y^c = \langle 0.25, 0.3, 0.7, 0.65, 0.05, 0.05 \rangle$.
Hence the degree of similarity between x^c and y^c can be evaluated as

$$M_J(x^c,y^c) = \frac{min(0.3,0.25)+min(0.4,0.3)+min(0.6,0.7)+min(0.5,0.65)+min(0.1,0.05)+min(0.1,0.05)}{max(0.3,0.25)+max(0.4,0.3)+max(0.6,0.7)+max(0.5,0.65)+max(0.1,0.05)+max(0.1,0.05)}$$

$$= \frac{0.25+0.3+0.6+0.5+0.05+0.05}{0.3+0.4+0.7+0.65+0.1+0.1} = \frac{1.75}{2.25} \approx 0.7778.$$

From examples 3.2 and 3.3, it is observed that $M_J(x,y) = M_J(x^c,y^c)$.

Example 3.4. Let x and y be two IF rough values, where $x = \langle 0.6, 0.5, 0.3, 0.4, 0.1, 0.1 \rangle$ and $y = \langle 0, 0, 0, 0, 0, 0 \rangle$.

Hence the degree of similarity between x and y can be evaluated as

$$M_J(x,y) = \frac{min(0.6,0)+min(0.5,0)+min(0.3,0)+min(0.4,0)+min(0.1,0)+min(0.1,0)}{max(0.6,0)+max(0.5,0)+max(0.3,0)+max(0.4,0)+max(0.1,0)+max(0.1,0)} = 0.$$

Example 3.5. Let x and y be two IF rough values, where $x = y = \langle 0.6, 0.5, 0.3, 0.4, 0.1, 0.1 \rangle$

Hence the degree of similarity between x and y can be evaluated as

$$M_J(x,y) = \frac{min(0.6,0.6)+min(0.5,0.5)+min(0.3,0.3)+min(0.4,0.4)+min(0.1,0.1)+min(0.1,0.1)}{max(0.6,0.6)+max(0.5,0.5)+max(0.3,0.3)+max(0.4,0.4)+max(0.1,0.1)+max(0.1,0.1)} = 1.$$

Example 3.6. Let x and y be two IF rough values, where $x = \langle 0.6, 0.5, 0.3, 0.4, 0.1, 0.1\rangle$ and $y = \langle 1, 1, 0, 0, 0, 0\rangle$.

Hence the degree of similarity between x and y can be evaluated as

$$M_J(x, y) = \frac{min(0.6,1)+min(0.5,1)+min(0.3,0)+min(0.4,0)+min(0.1,0)+min(0.1,0)}{max(0.6,1)+max(0.5,1)+max(0.3,0)+max(0.4,0)+max(0.1,0)+max(0.1,0)} \approx 0.3793.$$

Theorem 3.7. *Let A be an IF rough set in X where x, y, z be the IF rough values in A. Then the following statements are true:*

1. *$M_J(x, y)$ is bounded i.e.; $0 \leq M_J(x, y) \leq 1$.*

2. *$M_J(x, y) = M_J(y, x)$.*

3. *$\forall x \in X, M_J(x, y) = M_J(x, z) \Rightarrow M_J(y, z) = 1$.*

4. *$M_J(x, y) = M_J(x^c, y^c)$.*

5. *If $x \leq y \leq z$, then $M_J(x, z) \leq min\{M_J(x, y), M_J(y, z)\}$ for $x, y, z \in X$.*

Proof. Let A be an IF rough set in X where $x = \langle \mu_{\underline{A}}(x), \mu_{\overline{A}}(x), \nu_{\underline{A}}(x), \nu_{\overline{A}}(x), \pi_{\underline{A}}(x), \pi_{\overline{A}}(x)\rangle$, $y = \langle \mu_{\underline{A}}(y), \mu_{\overline{A}}(y), \nu_{\underline{A}}(y), \nu_{\overline{A}}(y), \pi_{\underline{A}}(y), \pi_{\overline{A}}(y)\rangle$ and $z = \langle \mu_{\underline{A}}(z), \mu_{\overline{A}}(z), \nu_{\underline{A}}(z), \nu_{\overline{A}}(z), \pi_{\underline{A}}(z), \pi_{\overline{A}}(z)\rangle$ be the IF rough values in A. We may define the order relation of the intuitionistic fuzzy rough values as
$x \leq y \iff (\mu_{\underline{A}}(x) \leq \mu_{\underline{A}}(y),\ \mu_{\overline{A}}(x) \leq \mu_{\overline{A}}(y))$ and $(\nu_{\underline{A}}(x) \geq \nu_{\underline{A}}(y),\ \nu_{\overline{A}}(x) \geq \nu_{\overline{A}}(y))$.

1. The minimum value of (5) is 0 and the maximum is 1. In other cases, the value of the expression (5) must be positive and lesser than one as the value of the numerator is less than the value of denominator. Thus $0 \leq M_J(x, y) \leq 1$.

2. M_J is symmetric as min and max operations are both symmetric.

3. Since $M_J(x, y) = M_J(x, z)$, $\forall x \in X$ then for $x = y$ we get that $1 = M_J(y, y) = M_J(y, z)$.
   Similarly, for $x = z$, we get $1 = M_J(z, z) = M_J(z, y) = M_J(y, z)$.

4. In this case, $x^c = \langle \nu_{\underline{A}}(x), \nu_{\overline{A}}(x), \mu_{\underline{A}}(x), \mu_{\overline{A}}(x), \pi_{\underline{A}}(x), \pi_{\overline{A}}(x)\rangle$
   and $y^c = \langle \nu_{\underline{A}}(y), \nu_{\overline{A}}(y), \mu_{\underline{A}}(y), \mu_{\overline{A}}(y), \pi_{\underline{A}}(y), \pi_{\overline{A}}(y)\rangle$ and hence point 4 holds.

5. Given, $x \leq y \leq z$.
   Substituting $\pi_{\underline{A}}(x), \pi_{\overline{A}}(x), \pi_{\underline{A}}(z), \pi_{\overline{A}}(z)$ we get $M_J(x, z) = \frac{1-\underline{U}_{xz}+1-\overline{U}_{xz}}{1+\underline{U}_{xz}+1+\overline{U}_{xz}}$,
   where $\underline{U}_{xz} = (\mu_{\underline{A}}(z) - \mu_{\underline{A}}(x)) \vee (\nu_{\underline{A}}(x) - \nu_{\underline{A}}(z))$ and $\overline{U}_{xz} = (\mu_{\overline{A}}(z) - \mu_{\overline{A}}(x)) \vee (\nu_{\overline{A}}(x) - \nu_{\overline{A}}(z))$.
   Similarly, $M_J(x, y) = \frac{1-\underline{U}_{xy}+1-\overline{U}_{xy}}{1+\underline{U}_{xy}+1+\overline{U}_{xy}}$, where $\underline{U}_{xy} = (\mu_{\underline{A}}(y) - \mu_{\underline{A}}(x)) \vee (\nu_{\underline{A}}(x) - \nu_{\underline{A}}(y))$ and $\overline{U}_{xy} = (\mu_{\overline{A}}(y) - \mu_{\overline{A}}(x)) \vee (\nu_{\overline{A}}(x) - \nu_{\overline{A}}(y))$.
   Clearly, $(1 - \underline{U}_{xz} + 1 - \overline{U}_{xz}) \leq (1 - \underline{U}_{xy} + 1 - \overline{U}_{xy})$ as $\underline{U}_{xz} \geq \underline{U}_{xy}$ and $\overline{U}_{xz} \geq \overline{U}_{xy}$
   And $(1 + \underline{U}_{xz} + 1 + \overline{U}_{xz}) \geq (1 + \underline{U}_{xy} + 1 + \overline{U}_{xy})$ as $\underline{U}_{xz} \geq \underline{U}_{xy}$ and $\overline{U}_{xz} \geq \overline{U}_{xy}$
   Hence $M_J(x, z) \leq M_J(x, y)$.
   Similarly, it can be shown that $M_J(x, z) \leq M_J(y, z)$.

□

Now, the similarity measure between two given IF rough sets is generalized. Let A and B be two IF rough sets in the universe of discourse $U = \{u_1, u_2, u_3, , u_n\}$, where
$A = \langle \mu_{\underline{A}}(u_1), \mu_{\overline{A}}(u_1), \nu_{\underline{A}}(u_1), \nu_{\overline{A}}(u_1), \pi_{\underline{A}}(u_1), \pi_{\overline{A}}(u_1)\rangle / u_1 +$
$+ \langle \mu_{\underline{A}}(u_n), \mu_{\overline{A}}(u_n), \nu_{\underline{A}}(u_n), \nu_{\overline{A}}(u_n), \pi_{\underline{A}}(u_n), \pi_{\overline{A}}(u_n)\rangle / u_n$
and $B = \langle \mu_{\underline{B}}(u_1), \mu_{\overline{B}}(u_1), \nu_{\underline{B}}(u_1), \nu_{\overline{B}}(u_1), \pi_{\underline{B}}(u_1), \pi_{\overline{B}}(u_1)\rangle / u_1 +$
$+ \langle \mu_{\underline{B}}(u_n), \mu_{\overline{B}}(u_n), \nu_{\underline{B}}(u_n), \nu_{\overline{B}}(u_n), \pi_{\underline{B}}(u_n), \pi_{\overline{B}}(u_n)\rangle / u_n$

Then based on definition 3.1, the degree of similarity between the IF rough sets A and B can be defined as follows:

$$T_J(A,B) = \frac{1}{n}\sum_{i=1}^{n} M_J(\langle \mu_{\underline{A}}(u_i), \mu_{\overline{A}}(u_i), \nu_{\underline{A}}(u_i), \nu_{\overline{A}}(u_i), \pi_{\underline{A}}(u_i), \pi_{\overline{A}}(u_i)\rangle, \langle \mu_{\underline{B}}(u_i), \mu_{\overline{B}}(u_i), \nu_{\underline{B}}(u_i), \nu_{\overline{B}}(u_i), \pi_{\underline{B}}(u_i), \pi_{\overline{B}}(u_i)\rangle)$$

So, $T_J(A,B) =$

$$\frac{1}{n}\sum_{i=1}^{n} \frac{(min(\mu_{\underline{A}}(u_i),\mu_{\underline{B}}(u_i))+min(\nu_{\underline{A}}(u_i),\nu_{\underline{B}}(u_i))+min(\pi_{\underline{A}}(u_i),\pi_{\underline{B}}(u_i))+min(\mu_{\overline{A}}(u_i),\mu_{\overline{B}}(u_i))+min(\nu_{\overline{A}}(u_i),\nu_{\overline{B}}(u_i))+min(\pi_{\overline{A}}(u_i),\pi_{\overline{B}}(u_i))}{(max(\mu_{\underline{A}}(u_i),\mu_{\underline{B}}(u_i))+max(\nu_{\underline{A}}(u_i),\nu_{\underline{B}}(u_i))+max(\pi_{\underline{A}}(u_i),\pi_{\underline{B}}(u_i))+max(\mu_{\overline{A}}(u_i),\mu_{\overline{B}}(u_i))+max(\nu_{\overline{A}}(u_i),\nu_{\overline{B}}(u_i))+max(\pi_{\overline{A}}(u_i),\pi_{\overline{B}}(u_i))}$$

Here $T_J(A,B) \in [0,1]$. The larger the value of $T_J(A,B)$, the more similarity between the IF rough sets A and B.

Theorem 3.8. *Let X be the set of all IF rough sets on the fixed finite universe of discourse U and A, B, C $\in X$. Then the following statements are true:*

1. *T_J is bounded, i.e., $0 \le T_J(A,B) \le 1$.*
2. *$T_J(A,B) = T_J(B,A)$.*
3. *$\forall A \in X$, $T_J(A,B) = T_J(A,C) \Rightarrow T_J(B,C) = 1$.*
4. *$T_J(A,B) = T_J(A^c, B^c)$.*
5. *If $A \subseteq B \subseteq C$, then $T_J(A,C) \le min\{T_J(A,B), T_J(B,C)\}$ for $A, B, C \in X$.*

Proof. Similar to the Theorem 3.7. □

4 Application

In this section, we are considering the selection procedure for a Fair Play award in a cricket tournament. The fair play award is to make sure that the teams show the best behavior and sporting spirit while also being competitive. The award motivates the teams to play the game fairly.
The main factors on which the fair play award for a team depends are described below.

- Teams that uphold the spirit of the game$\rightarrow e_1$
- Teams that respect the opposition team$\rightarrow e_2$
- Teams that show respect towards the laws and rules of cricket$\rightarrow e_3$
- Teams that respect the umpires and officials$\rightarrow e_4$

4.1 Algorithm

The steps of the algorithm of this method are as follows:
First step: Construct an intuitionistic fuzzy rough set for a standard alternative.
Second step: Construct an intuitionistic fuzzy rough set for the available alternatives.
Third step: Calculate the similarity measure.
Fourth step: Arrange alternatives in order to their ranking.
Fifth step: Choose the best alternative.

4.2 Computation

Let U be the universal set where $U = \{TeamA, TeamB, TeamC, TeamD, TeamE, TeamF\}$ and $S = \{e_1, e_2, e_3, e_4\}$ be the parameters.
Also let S be the standard alternative and A, B, C, D, E, and F are the available alternatives.
S $=\langle 1, 1, 0, 0, 0, 0\rangle/e_1+\langle 0.9, 0.9, 0.05, 0.05, 0.05, 0.05\rangle/e_2+\langle 1, 0.9, 0, 0.1, 0, 0\rangle/e_3+\langle 0.95, 0.95, 0.05, 0.05, 0, 0\rangle/e_4$
A $= \langle 0.6, 0.5, 0.2, 0.3, 0.2, 0.2\rangle/e_1 + \langle 0.45, 0.5, 0.25, 0.4, 0.3, 0.1\rangle/e_2 + \langle 0.5, 0.6, 0.4, 0.4, 0.1, 0\rangle/e_3$
$+ \langle 0.75, 0.6, 0.2, 0.2, 0.05, 0.2\rangle/e_4$
B $=\langle 0.8, 0.7, 0.2, 0.2, 0, 0.1\rangle/e_1 + \langle 0.7, 0.5, 0.2, 0.3, 0.1, 0.2\rangle/e_2 + \langle 0.55, 0.65, 0.35, 0.25, 0.1, 0.1\rangle/e_3$
$+ \langle 0.6, 0.4, 0.3, 0.5, 0.1, 0.1\rangle/e_4$
C $= \langle 0.8, 0.9, 0.1, 0.1, 0.1, 0\rangle/e_1 + \langle 0.75, 0.85, 0.15, 0.1, 0.1, 0.05\rangle/e_2 + \langle 0.85, 0.9, 0.1, 0.05, 0.05, 0.05\rangle/e_3$
$+ \langle 0.5, 0.6, 0.3, 0.3, 0.2, 0.1\rangle/e_4$
D $=\langle 0.85, 0.6, 0.1, 0.2, 0.05, 0.2\rangle/e_1 + \langle 0.7, 0.65, 0.2, 0.3, 0.1, 0.05\rangle/e_2 + \langle 0.5, 0.5, 0.4, 0.4, 0.1, 0.1\rangle/e_3$
$+ \langle 0.6, 0.6, 0.3, 0.3, 0.1, 0.1\rangle/e_4$
E $= \langle 0.6, 0.8, 0.2, 0.2, 0.2, 0\rangle/e_1 + \langle 0.65, 0.75, 0.25, 0.2, 0.1, 0.05\rangle/e_2 + \langle 0.7, 0.7, 0.3, 0.3, 0, 0\rangle/e_3$
$+ \langle 0.8, 0.9, 0.1, 0.1, 0.1, 0\rangle/e_4$
F $=\langle 0.9, 0.8, 0.05, 0.15, 0.05, 0.05\rangle/e_1 + \langle 0.7, 0.5, 0.2, 0.3, 0.1, 0.2\rangle/e_2 + \langle 0.8, 0.6, 0.1, 0.3, 0.1, 0.1\rangle/e_3$
$+ \langle 0.7, 0.6, 0.25, 0.35, 0.05, 0.05\rangle/e_4$

Now using the formula of $T_J(A, B)$, we can evaluate

$$T_J(S, A) = \frac{1}{4}[\frac{min(1,0.6)+min(1,0.5)+min(0,0.2)+min(0,0.3)+min(0,0.2)+min(0,0.2)}{max(1,0.6)+max(1,0.5)+max(0,0.2)+max(0,0.3)+max(0,0.2)+max(0,0.2)} +$$

$$\frac{min(0.9,0.45)+min(0.9,0.5)+min(0.05,0.25)+min(0.05,0.4)+min(0.05,0.3)+min(0.05,0.1)}{max(0.9,0.45)+max(0.9,0.5)+max(0.05,0.25)+max(0.05,0.4)+max(0.05,0.3)+max(0.05,0.1)} +$$

$$\frac{min(1,0.5)+min(0.9,0.6)+min(0,0.4)+min(0.1,0.4)+min(0,0.1)+min(0,0)}{max(1,0.5)+max(0.9,0.6)+max(0,0.4)+max(0.1,0.4)+max(0,0.1)+max(0,0)} +$$

$$\frac{min(0.95,0.75)+min(0.95,0.6)+min(0.05,0.2)+min(0.05,0.2)+min(0,0.05)+min(0,0.2)}{max(0.95,0.75)+max(0.95,0.6)+max(0.05,0.2)+max(0.05,0.2)+max(0,0.05)+max(0,0.2)}]$$

$= 0.4450.$

Thus we get,

Similarity Measure	Value
$T_J(S,A)$	0.4450
$T_J(S,B)$	0.4998
$T_J(S,C)$	0.7010
$T_J(S,D)$	0.5155
$T_J(S,E)$	0.6558
$T_J(S,F)$	0.6040

This indicates that team C will receive the fair play trophy.

5 Conclusion

In this paper, we describe an intuitionistic fuzzy rough set model or approach to find the similarity measure between intuitionistic fuzzy rough sets. The main feature of this model is that we have considered and calculated the hesitation margin. We also establish some rules for measuring the degree of similarity between elements and between intuitionistic fuzzy rough sets. Based on this concept we solve a problem related to fair play winner in a cricket tournament. Many such problems also can be solved by applying this method. As the proposed similarity measures have some good properties, they can provide a useful way for measuring the similarity between intuitionistic fuzzy rough sets.

Acknowledgements: The author is most grateful to the referees for carefully reading the article and for their constructive comments and suggestions, which have helped to develop the paper.

Conflict of Interest: The author declares that there is no conflict of interest regarding the publication of this paper.

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