Transactions on Fuzzy Sets and Systems
URL: https://tfss.journals.iau.ir/
Online ISSN: 2821-0131
Vol.1, No.1, (2022), 80-87
DOI: http://doi.org/10.30495/TFSS.2022.690293

Article Type: Original Research Article (Invited Article)

Rough Convergence of Bernstein Fuzzy Triple Sequences

Ayhan Esi*, **Subramanian Nagarajan**

Abstract. The aim of this paper is to introduce and study a new concept of convergence almost surely (a.s.), convergence in probability, convergence in mean, and convergence in distribution are four important convergence concepts of random sequence and also discusses some convergence concepts of the fuzzy sequence: convergence almost surely, convergence in credibility, convergence in mean, and convergence in distribution.

AMS Subject Classification 2020: 40F05; 40J05; 40G05
Keywords and Phrases: Triple sequences, Rough convergence, Convergence almost surely, Convergence in probability, Convergence in mean, Convergence in distribution.

1 Introduction

The idea of rough convergence was first introduced by Phu [13, 14, 15] in finite dimensional normed spaces. He showed that the set LIM_x^r is bounded, closed and convex; and he introduced the notion of rough Cauchy sequence. He also investigated the relations between rough convergence and other convergence types and the dependence of LIM_x^r on the roughness of degree r.

Aytar [1] studied rough statistical convergence and defined the set of rough statistical limit points of a sequence and obtained two statistical convergence criteria associated with this set and prove that this set is closed and convex. Also, Aytar [2] studied that the $r-$imit set of the sequence is equal to the intersection of these sets and that $r-$core of the sequence is equal to the union of these sets. Dundar and Cakan [4] investigated of rough ideal convergence and defined the set the rough ideal limit points of a sequence The notion of $I-$convergence of a triple sequence spaces which is based on the structure of the ideal I of subsets of $\mathbb{N}\times\mathbb{N}\times\mathbb{N}$, where $\mathbb{N}$ is the set of all-natural numbers, is a natural generalization of the notion of convergence and statistical convergence.

Let K be a subset of the set of positive integers $\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ and let us denote the set

$$K_{ij\ell}=\{(m,n,k)\in K : m\leq i, n\leq j, k\leq \ell\}.$$

Then the natural density of K is given by

$$\delta(K)=\lim_{i,j,\ell\to\infty}\frac{|K_{ij\ell}|}{ij\ell},$$

*Corresponding author: Ayhan Esi, Email: aesi23@hotmail.com, ORCID: 0000-0003-3137-3865
Received: 18 February 2022; Revised: 14 March 2022; Accepted: 2 April 2022; Published Online: 7 May 2022.

How to cite: A. Esi and S. Nagarajan, Rough convergence of Bernstein fuzzy triple sequences, *Trans. Fuzzy Sets Syst.*, 1(1) (2022), 80-87.

where $|K_{ij\ell}|$ denotes the number of elements in $K_{ij\ell}$.

The Bernstein operator of order rst is given by

$$B_{rst}(f,x) = \sum_{m=0}^{r}\sum_{n=0}^{s}\sum_{k=0}^{t} f\left(\frac{mnk}{rst}\right)\binom{r}{m}\binom{s}{n}\binom{t}{k} x^{m+n+k}(1-x)^{(m-r)+(n-s)+(k-t)}$$

where f is a continuous (real or complex valued) function defined on $[0,1]$.

Throughout the paper, $\mathbb{R}$ denotes the real of three dimensional space with metric (X,d). Consider a triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ such that $(B_{mnk}(f,x)) \in \mathbb{R}, m,n,k \in \mathbb{N}$.

Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ is said to be statistically convergent to $0 \in \mathbb{R}$, written as $st - \lim\ x = 0$, provided that the set

$$K_\epsilon := \left\{(m,n,k) \in \mathbb{N}^3 : |B_{mnk}(f,x) - f(x)| \geq \epsilon\right\}$$

has natural density zero for any $\epsilon > 0$. In this case, 0 is called the statistical limit of the triple sequence of Bernstein polynomials. i.e., $\delta(K_\epsilon) = 0$. That is,

$$\lim_{r,s,t\to\infty} \frac{1}{rst}\left|\{m \leq r, n \leq s, k \leq t : |B_{mnk}(f,x) - (f,x)| \geq \epsilon\}\right| = 0.$$

In this case, we write $\delta - \lim B_{mnk}(f,x) = f(x)$ or $B_{mnk}(f,x) \to^{S_B} f(x)$.

Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers, $\chi_A-$the characteristic function of $A \subset \mathbb{N}$, $\mathbb{R}$ the set of all real numbers. A subset A of $\mathbb{N}$ is said to have asymptotic density $d(A)$ if

$$d(A) = \lim_{i,j,\ell\to\infty} \frac{1}{ij\ell}\sum_{m=1}^{i}\sum_{n=1}^{j}\sum_{k=1}^{\ell} \chi_A(K).$$

A triple sequence (real or complex) can be defined as a function $x : \mathbb{N}\times\mathbb{N}\times\mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers, and complex numbers respectively. The different types of notions of triple sequence were introduced and investigated at the initial by Sahiner et al. [16, 17], Esi et al. [6, 8, 9, 7, 10, 11, 12], Dutta et al. [5], Subramanian et al. [18], Debnath et al. [3] and many others.

The set of fuzzy real numbers is denoted by $f(x)(\mathbb{R})$, and d denotes the supremum metric on $f(X)\left(\mathbb{R}^3\right)$. Now let r be nonnegative real number. A triple sequence space of Bernstein polynomials of $(B_{mnk}(f,X))$ of fuzzy numbers is $r-$convergent to a fuzzy number $f(X)$ and we write

$$B_{mnk}(f,X) \to^r f(X) \text{ as } m,n,k \to \infty,$$

provided that for every $\epsilon > 0$ there is an integer $m_\epsilon, n_\epsilon, k_\epsilon$ so that

$$d(B_{mnk}(f,X), f(X)) < r + \epsilon \text{ whenever } m \geq m_\epsilon, n \geq n_\epsilon, k \geq k_\epsilon.$$

The set $LIM^r B_{mnk}(f,X) := \left\{f(X) \in f(X)\left(\mathbb{R}^3\right) : B_{mnk}(f,X) \to^r f(X),\text{ as } m,n,k \to \infty\right\}$ is called the $r-$limit set of the triple sequence space of Bernstein polynomials of $(B_{mnk}(f,X))$.

A triple sequence space of Bernstein polynomials of fuzzy numbers which is divergent can be convergent with a certain roughness degree. For instance, let us define

$$B_{mnk}(f,X) = \begin{cases} \eta(X), & \text{if } m,n,k \text{ are odd } integers, \\ \mu(X), & \text{otherwise} \end{cases},$$

where

$$\eta(X) = \begin{cases} X, & \text{if } X \in [0,1], \\ -X+2, & \text{if} X \in [1,2], \\ 0, & \text{otherwise} \end{cases}$$

and

$$\mu(X) = \begin{cases} X-3, & \text{if } X \in [3,4], \\ -X+5, & \text{if} X \in [4,5], \\ 0, & \text{otherwise} \end{cases}.$$

Then we have where

$$LIM^r B_{mnk}(f,X) = \begin{cases} \phi, & \text{if } r < \frac{3}{2}, \\ [\mu - r_1, \eta + r_1], & \text{otherwise} \end{cases},$$

where r_1 is a nonnegative real number with

$$[\mu - r_1, \eta + r_1] := \left\{ B_{mnk}(f,X) \in f(X)\left(\mathbb{R}^3\right) : \mu - r_1 \leq B_{mnk}(f,X) \leq \eta + r_1 \right\}.$$

The ideal of rough convergence of a triple sequence space of Bernstein polynomials can be interpreted as follows:

Let $(B_{mnk}(f,Y))$ be a convergent triple sequence space of Bernstein polynomials of fuzzy numbers. Assume that $(B_{mnk}(f,Y))$ can not be determined exactly for every $(m,n,k) \in \mathbb{N}^3$. That is, $(B_{mnk}(f,Y))$ cannot be calculated so we can use approximate value of $(B_{mnk}(f,Y))$ for simplicity of calculation. We only know that $(B_{mnk}(f,Y)) \in [\mu_{mnk}, \lambda_{mnk}]$, where $d(\mu_{mnk}, \lambda_{mnk}) \leq r$ for every $(m,n,k) \in \mathbb{N}^3$. The triple sequence space of Bernstein polynomials of $(B_{mnk}(f,X))$ satisfying $(B_{mnk}(f,X)) \in [\mu_{mnk}, \lambda_{mnk}]$, for all m,n,k. Then the triple sequence space of Bernstein polynomials of $(B_{mnk}(f,X))$ may not be convergent, but the inequality

$$d(B_{mnk}(f,X), f(X)) \leq d(B_{mnk}(f,X), B_{mnk}(f,Y)) + d(B_{mnk}(f,Y), f(Y)) \leq r + d(B_{mnk}(f,Y), f(Y))$$

implies that the triple sequence space of Bernstein polynomials of $(B_{mnk}(f,X))$ is $r-$convergent.

A fuzzy number X is a fuzzy subset of the real $\mathbb{R}^3$, which is normal fuzzy convex, upper semi-continuous, and the X^0 is bounded where $X^0 ;= cl\left\{x \in \mathbb{R}^3 : X(x) > 0\right\}$ and cl is the closure operator. These properties imply that for each $\alpha \in (0,1]$, the $\alpha-$level set X^α defined by

$$X^\alpha = \left\{x \in \mathbb{R}^3 : X(x) \geq \alpha\right\} = \left[\underline{X}^\alpha, \overline{X}^\alpha\right]$$

is a non-empty compact convex subset of $\mathbb{R}^3$.

The supremum metric d on the set $L\left(\mathbb{R}^3\right)$ is defined by

$$d(X,Y) = \sup_{\alpha \in [0,1]} \max\left(\left|\underline{X}^\alpha - \underline{Y}^\alpha\right|, \left|\overline{X}^\alpha - \overline{Y}^\alpha\right|\right).$$

Now, given $X, Y \in L\left(\mathbb{R}^3\right)$, we define $X \leq Y$ if $\underline{X}^\alpha \leq \underline{Y}^\alpha$ and $\overline{X}^\alpha \leq \overline{Y}^\alpha$ for each $\alpha \in [0,1]$.

We write $X \leq Y$ if $X \leq Y$ and there exists an $\alpha_0 \in [0,1]$ such that $\underline{X}^{\alpha_0} \leq \underline{Y}^{\alpha_0}$ or $\overline{X}^{\alpha_0} \leq \overline{Y}^{\alpha_0}$.

A subset E of $L\left(\mathbb{R}^3\right)$ is said to be bounded above if there exists a fuzzy number μ, called an upper bound of E, such that $X \leq \mu$ for every $X \in E$. μ is called the least upper bound of E if μ is an upper bound and $\mu \leq \mu^{'}$ for all upper bounds $\mu^{'}$.

A lower bound and the greatest lower bound is defined similarly. E is said to be bounded if it is both bounded above and below.

The notions of least upper bound and the greatest lower bound have been defined only for bounded sets of fuzzy numbers. If the set $E \subset L\left(\mathbb{R}^3\right)$ is bounded then its supremum and infimum exist.

The limit infimum and limit supremum of a triple sequence space (X_{mnk}) is defined by

$$\lim_{m,n,k \to \infty} \inf\ X_{mnk} := \inf\ A_X.$$

$$\lim_{m,n,k \to \infty} \sup\ X_{mnk} := \inf\ B_X.$$

where

$$A_X := \left\{ \mu \in L\left(\mathbb{R}^3\right) : \text{The set } \left\{(m,n,k) \in \mathbb{N}^3 : X_{mnk} < \mu\right\} \text{is infinite} \right\}$$
$$B_X := \left\{ \mu \in L\left(\mathbb{R}^3\right) : \text{The set } \left\{(m,n,k) \in \mathbb{N}^3 : X_{mnk} > \mu\right\} \text{is infinite} \right\}.$$

Now, given two fuzzy numbers $X, Y \in L\left(\mathbb{R}^3\right)$, we define their sum as $Z = X + Y$, where $\underline{Z}^\alpha := \underline{X}^\alpha + \underline{Y}^\alpha$ and $\overline{Z}^\alpha := \overline{X}^\alpha + \overline{Y}^\alpha$ for all $\alpha \in [0,1]$.

To any real number $a \in \mathbb{R}^3$, we can assign a fuzzy number $a_1 \in L\left(\mathbb{R}^3\right)$, which is defied by

$$a_1(x) = \begin{cases} 1, & \text{if } x = a, \\ 0, & \text{otherwise} \end{cases}.$$

An order interval in $L\left(\mathbb{R}^3\right)$ is defined by $[X, Y] := \left\{Z \in L\left(\mathbb{R}^3\right) : X \leq Z \leq Y\right\}$, where $X, Y \in L\left(\mathbb{R}^3\right)$.

A set E of fuzzy numbers is called convex if $\lambda\mu_1 + (1-\lambda)\mu_2 \in E$ for all $\lambda \in [0,1]$ and $\mu_1, \mu_2 \in E$.

2 Main Results

Definition 2.1. A rough triple sequence of fuzzy variables of Bernstein polynomials of $(B_{mnk}(f,X))$ of real numbers is said to be rough convergent almost surely to the fuzzy variables of real number $B_{mnk}(f,X)$ if and only if there exists a set A with $Cr(A) = 1$ such that

$$\lim_{m,n,k\to\infty} \left|B_{mnk}(f, X(\theta), f(X))\right| = 0 \tag{1}$$

for every $\theta \in A$. In that case we write $B_{mnk}(f,X) \to f(X)$ almost surely.

Definition 2.2. A rough triple sequence of fuzzy variables of Bernstein polynomials of $(B_{mnk}(f,X))$ of real numbers is said to be rough converges in credibility to the fuzzy variable of Bernstein polynomials if

$$\lim_{m,n,k\to\infty} Cr\left\{\left|B_{mnk}((f,X), f(X))\right| \geq \beta + \epsilon\right\} = 0 \tag{2}$$

for every $\epsilon > 0$.

Definition 2.3. A rough triple sequence of fuzzy variables of Bernstein polynomials of $(B_{mnk}(f,X))$ of real numbers is said to be convergent in mean to the fuzzy variables $f(X)$ if

$$\lim_{m,n,k\to\infty} E\left[\left|B_{mnk}((f,X), f(X))\right|\right] = 0.$$

Example 2.4. Rough convergent almost surely does not imply rough convergence in credibility.

Let us consider $\theta = \{\theta_{111}.\theta_{222}, \ldots\}$, $\text{Pos}\{\theta_{111}\} = 1$ and $\text{Pos}\{\theta_{uvw}\} = \frac{(u-1)(v-1)(w-1)}{uvw}$ for $u, v, w = 2, 3, 4, \ldots$, and the rough triple sequence of Bernstein polynomials of fuzzy variables are defined by

$$B_{mnk}(f, X(\theta)) = \begin{cases} mnk & \text{if } m = u, n = v, k = w \\ 0 & \text{otherwise} \end{cases}$$

for $m, n, k = 1, 2, 3, \ldots$.

Then the triple sequence of Bernstein polynomials of $B_{mnk}(f,X)$ rough converges almost surely to $f(X)$, we have

$$Cr\left\{\left|B_{mnk}((f,X), f(X))\right| \geq \beta + \epsilon\right\} = \frac{(m-1)(n-1)(k-1)}{3(mnk)} \not\to 0.$$

That is, the rough triple sequence of Bernstein polynomials of $B_{mnk}(f,X)$ does not rough converges in credibility to $f(X)$.

Example 2.5. Rough convergence incredibility does not imply rough convergence almost surely.

Let us consider $\theta = \{\theta_{111}.\theta_{222}, \ldots\}$, $\mathrm{Pos}\{\theta_{uvw}\} = \frac{1}{uvw}$ for $u, v, w = 1, 2, 3, \ldots$, and the rough triple sequence of Bernstein polynomials of fuzzy variables are defined by

$$B_{mnk}\left(f, X\left(\theta_{uvw}\right)\right) = \begin{cases} \frac{(u+1)(v+1)(w+1)}{uvw} & \text{if } u = m, m+1, m+2, \cdots;\ v = n, n+1, n+2, \cdots; \\ & w = k, k+1, k+2, \cdots. \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

for $m, n, k = 1, 2, 3, \ldots$, and $m, n, k = 0$.

We have

$$Cr\left\{\left|B_{mnk}\left(\left(f, X\right), f\left(X\right)\right)\right| \geq \beta + \epsilon\right\} = \frac{1}{2\left(mnk\right)} \longrightarrow 0.$$

Thus the triple sequence of Bernstein polynomials of $B_{mnk}\left(f, X\right)$ rough converges in incredibility to $f\left(X\right)$. Hence $B_{mnk}\left(f, X\right) \not\to f\left(X\right)$ almost surely.

Example 2.6. Rough convergence in mean does not imply convergence almost surely.

Let us consider the rough triple sequence of Bernstein polynomials of fuzzy variables defined by the equation (3) which does not rough converge almost surely to $f\left(X\right)$. Hence

$$E\left[\left|B_{mnk}\left(\left(f, X\right), f\left(X\right)\right)\right|\right] = \frac{\left(m+1\right)\left(n+1\right)\left(k+1\right)}{3\left(m^2n^2k^2\right)} \to 0.$$
$$\Longrightarrow B_{mnk}\left(f, X\right) \text{ rough converges in mean to } f\left(X\right).$$

Example 2.7. Rough convergence almost surely does not imply rough convergence in mean.

Let us consider $\theta = \{\theta_{111}.\theta_{222}, \ldots\}$, $\mathrm{Pos}\{\theta_{uvw}\} = \frac{1}{uvw}$ for $u, v, w = 1, 2, 3, \ldots$, and the rough triple sequence of Bernstein polynomials of fuzzy variables are defined by

$$B_{mnk}\left(f, X\left(\theta_{uvw}\right), f\left(X\right)\right) = \begin{cases} mnk & \text{if } u = m,\ v = n,\ w = k \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

for $m, n, k = 1, 2, 3, \ldots$, and $f\left(X\right) = 0$. Then the rough triple sequence of Bernstein polynomials of $B_{mnk}\left(f, X\right)$ converges almost surely. Thus

$$E\left[\left|B_{mnk}\left(\left(f, X\right), f\left(X\right)\right)\right|\right] \cong \frac{1}{3} \not\to 0.$$

Hence the rough triple sequence of Bernstein polynomials of $B_{mnk}\left(f, X\right)$ does not rough converge in mean to $f\left(X\right)$.

Theorem 2.8. *Let $\left(x_{mnk}\right)$ be a triple sequence of rough variables and f be a nonnegative Borel measurable function. If f is even increasing on $[0, \infty)$, then for any number $t > 0$, we have*

$$Tr\left\{|x| \geq t\right\} \leq \frac{E\left[f\left(x\right)\right]}{f\left(t\right)} \tag{5}$$

Proof. It is clear that $Tr\left\{|x| \geq f^{-1}\left(\eta\right)\right\}$ is a monotone decreasing function from η on $[0, \infty)$. It follows from the nonnegativity of $f(x)$ that

$$\begin{aligned} E[f(x)] &= \int_0^\infty Tr\{f(x) \geq \eta\}\, d\eta \\ &= \int_0^\infty Tr\left\{|x| \geq f^{-1}\eta\right\} d\eta \\ &\geq \int_0^{f(t)} Tr\left\{|x| \geq f^{-1}(\eta)\right\} d\eta \\ &\geq \int_0^{f(t)} d\eta \,\cdot\, Tr\left\{|x| \geq f^{-1}(f(t))\right\} \\ &= f(t) \,\cdot\, Tr\{|x| \geq t\}. \end{aligned}$$

□

Theorem 2.9. *Let (x_{mnk}) be a triple sequence of rough variables. Then for any given numbers $t > 0$ and $p > 0$, we have*

$$Tr\{|x| \geq t\} \leq \frac{E[|x^p|]}{t^p} \tag{6}$$

Proof. It is follows from Theorem 2.8 when $f(x) = |x|^p$. □

Theorem 2.10. *Rough triple sequence of Bernstein polynomials of $B_{mnk}(f, X)$ of fuzzy variables of a real number. If it is rough convergence in mean then it is rough convergence in credibility.*

Proof. It follows from Theorem 2.9 that,

$$Cr\{|B_{mnk}((f, X), f(X))| \geq \beta + \epsilon\} \leq \frac{E[|B_{mnk}((f, X), f(X))|]}{\beta + \epsilon} \to 0 \text{ as } m, n, k \to \infty.$$

Thus $B_{mnk}(f, X)$ converges in credibility to $f(X)$. □

3 Conclusion

In this paper, we introduced and studied a new concept of convergence almost surely (a.s.), convergence in probability, convergence in mean, and convergence in distribution are four important convergence concepts of random sequence and also discusses some convergence concepts of the fuzzy sequence, convergence almost surely, convergence in credibility, convergence in mean, and convergence in distribution for triple sequence space of Bernstein polynomials of rough convergence of fuzzy numbers. For the reference sections, consider the following introduction described the main results are motivating the research.

Conflict of Interest: The authors declare no conflict of interest.

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Ayhan Esi
Department of Basic Eng. Sci. (Math. Sect.)
Malatya Turgut Ozal University
Malatya, Turkey
E-mail: aesi23@hotmail.com

Nagarajan Subramanian
Department of Mathematics
SASTRA University
Thanjavur, India
E-mail: nsmaths@gmail.com