Transactions on Fuzzy Sets and Systems (TFSS)
URL: https://tfss.journals.iau.ir/
Online ISSN: 2821-0131
Vol.2, No.1, (2023), 1-21
DOI: http://doi.org/10.30495/tfss.2022.1963756.1042

Article type: Original Research Article

Similarity of Country Rankings on Sustainability Performance

John N Mordeson, **Sunil Mathew***

Abstract. In this paper, we extend the fuzzy similarity measure of two rankings to any finite number of rankings. We provide a method to convert a measure for a finite number of rankings to a number that represents the number for two rankings. We apply our results to the sustainability ranking of countries by the Environmental Performance Index (EPI). The 2020 EPI provides a summary of the state of sustainability around the world. It uses 32 performance indicators across 11 categories. These indicators provide a way to determine problems, set targets, track trends, understand outcomes, and identify best policy practices. The EPI ranks 180 countries on environmental health and ecosystem vitality. The EPI provides a method in support of efforts to meet the targets of the UN Sustainable Development Goals. The EPI determined that the Global West region ranked the highest. The purpose of our project is to find the similarity of the 11 rankings of the countries for eight different regions.

AMS Subject Classification 2020: 03B52; 03E72
Keywords and Phrases: Fuzzy similarity measures, Sustainability, Environmental health, Ecosystem vitality, Performance index.

1 Introduction

We use fuzzy similarity measures to provide a measure of the similarity of rankings. Previously, the similarity of only two rankings at a time could be determined. We extend this situation so that the similarity of any finite number of rankings can be determined at one time. We convert the similarity measures we find to a similarity measure that represents the usual situation of two rankings. Further reading concerning fuzzy similarity measures can be found in [1, 2, 3].

We apply our results to the rankings provided by the Environmental Performance Index (EPI), [4] The 2020 EPI provides a data-driven summary of the state of sustainability around the world. It uses 32 performance indicators across 11 categories. The EPI ranks 180 countries on environmental health and ecosystem vitality. The purpose of our project is to find the similarity of the 11 rankings of the countries for eight different regions. Countries often find it useful to compare their results to their geographic neighbors rather than the entire world. We also determine the similarity of other related rankings. Many pertinent references can be found in [4].

Let n be an integer such that $n \geq 2$. Let X be a finite set and $\mathcal{FP}(X)$ the fizzy power set of X. Let $\mathcal{FP}(X)^n$ denote the Cartesian product of $\mathcal{FP}(X)$ of dimension n. We let $\wedge$ denote minimum and $\vee$ denote maximum.

***Corresponding author:** Sunil Mathew, Email: sm@nitc.ac.in, ORCID: 0000-0001-8478-9245
Received: 21 July 2022; Revised: 4 August 2022; Accepted: 1 September 2022; Published Online: 7 May 2023.

How to cite: J. N. Mordeson and S. Mathew, Similarity of Country Rankings on Sustainability Performance, *Trans. Fuzzy Sets Syst.*, 2(1) (2023), 1-21.

2 Similarity Measures

Definition 2.1. *Let S be a function of $\mathcal{FP}(X)^n$ into $[0,1]$. Then S is called an n-**dimensional fuzzy similarity measure** on $\mathcal{FP}(X)$ if the following properties hold.*

$\forall(\mu_1, ..., \mu_n) \in \mathcal{FP}(X)^n$,

(1)$S(\mu_1, ..., \mu_n) = S(\mu_{\pi(1)}, ..., \mu_{\pi(n)})$ *for any permutation of* π *of* $\{1, 2, ..., n\}$;

(2)$S(\mu_1, ..., \mu_n) = 1$ *if and only if* $\mu_1 = ... = \mu_n$;

(3) *If* $\mu_{i_1} \subseteq \mu_{i_2} \subseteq \mu_{i_3}$, *then* $S(..., \mu_{i_1}, ..., \mu_{i_3}, ...) \leq S(..., \mu_{i_1}, ..., \mu_{i_2}, ...) \wedge S(..., \mu_{i_2}, ..., \mu_{i_3}, ...)$;

(4) *If* $S(\mu_1, ..., \mu_n) = 0$, *then for all* $x \in X$, *there exists* $i \in \{1, ..., n\}$ *such that* $\mu_i(x) = 0$.

Example 2.2. Let $\mu_1, ..., \mu_n$ be fuzzy subsets of X. Then M and S are n-dimensional fuzzy similarity measures, where

$$\begin{aligned} M(\mu_1, ..., \mu_n) &= \frac{\sum_{x\in X} \mu_1(x) \wedge ... \wedge \mu_n(x)}{\sum_{x\in X} \mu_1(x) \vee ... \vee \mu_n(x)}, \\ S(\mu_1, ..., \mu_n) &= 1 - \frac{\sum_{x\in X}(\vee\{\mu_j(x)|j = 1, ..., n\} - \wedge\{\mu_j(x)|j = 1, ..., n\})}{\sum_{x\in X}(\vee\{\mu_j(x)|j = 1, ..., n\} + \wedge\{\mu_j(x)|j = 1, ..., n\})}. \end{aligned}$$

Suppose we consider n countries and that they have been ranked twice 1 through n with no ties. We wish to consider their rankings using the above fuzzy similarity operations. We can accomplish this by mapping the countries to their rank divided by n. For example, let $\mathcal{C}$ denote a set of n countries and if a country C is ranked i, then we define the fuzzy subset μ of $\mathcal{C}$ by $\mu(C) = \frac{i}{n}$. Let μ and ν be two such fuzzy subsets of $\mathcal{C}$. Then

$$M(\mu, \nu) = \frac{\sum_{i=1}^n \mu(C_i) \wedge \nu(C_i)}{\sum_{i=1}^n \mu(C_i) \wedge \nu(C_i)} = \frac{\sum_{i=1}^n n\mu(C_i) \wedge n\nu(C_i)}{\sum_{i=1}^n n\mu(C_i) \wedge n\nu(C_i)}.$$

Consequently, there is no loss in generality in assuming that we are measuring the similarity of two rankings using the integers, $1, ..., n$. This notion can be extended from 2 rankings to m rankings, where $m \geq 2$.

Let m and n be positive integers such that $2 \leq m < n$. Then there exists positive integers q and r such that $n = qm + r$, where $0 \leq r < m$.

Proposition 2.3. $\sum_{i=1}^q (n - i + 1) = \frac{2qn+q-q^2}{2}$.

Proof. We have $\sum_{i=1}^q (n - i + 1) + \sum_{i=1}^q i = \sum_{i-1}^q (n + 1) = q(n + 1)$. Thus

$$\begin{aligned} \sum_{i=1}^q (n - i + 1) &= q(n + 1) - \sum_{i=1}^q i = q(n + 1) - \frac{(q + 1)q}{2} \\ &= \frac{2qn + 2q - q^2 - q}{2} \\ &= \frac{2qn + q - q^2}{2}. \end{aligned}$$

□

In the following, we let $a_{i1}, ..., a_{im}$ be integers between 1 and $n, i = 1, ..., n$.

The values in Propositions 2.4 and 2.5 below are determined as follows: Consider an $m \times m$ matrix with each column and row containing an n and also a 1. Then maximum of each row is n and the minimum of each row is 1. Consider another $m \times m$ matrix with each column and row containing and $n - 1$ and also a 2. We continue q times so that each column and row contains an $n-q+1$ and also a q. At this point the q matrices together (one on top of the other) is an $mq \times m$ matrix. Since we need an $n \times m$ matrix so that each column has all entries from $1, ..., n$, we can adjoin an $r \times m$ matrix to obtain the needed $n \times m$ matrix. (Note that if $r = 0$, then $mq = n$ and we are done.) With respect to the $mq \times m$ matrix, we have $n, n-1, ..., n-q+1$ for maximum values and $1, 2, ..., q$ as minimum values. For the $r \times m$ matrix, we place $n-q$ in each row and also $q+1$ in each row. Now for the $mq \times m$ matrix, the maximum values add to $m\frac{2qn+q-q^2}{2}$ and the minimum values add to $m(\frac{(q+1)q}{2})$. We need r more maximum values $n-q$ and r more minimum values $q+1$. Thus for the $n \times m$ matrix, we have that the maximum values add to $m\frac{2qn+q-q^2}{2} + r(n-q)$ and the minimum values add to $m(\frac{(q+1)q}{2}) + r(q+1)$.

Note also that we have $q(m-2)$ open locations from the $qm \times m$ matrix. For $r > 0$, we then have $q(m-2)+r(m-2)$ open locations in all. Now $q(m-2)+r(m-2) \geq n-2q = qm+r-2q$ since $r(m-2) \geq r$ for $m \geq 3$ or $r = 0$. That is, there is room for the $n-2q$ remaining numbers, $q+1, q+2, ..., q+n-2q = n-q$.

Note that if $n-q = q+1$, then $n = 2q+1$ so that $m = 2$.

Proposition 2.4. $\sum_{i=1}^{n}(a_{i1} \wedge ... \wedge a_{im}) \geq m(\frac{(q+1)q}{2}) + r(q+1)$.

Proof. We have by the immediately preceding discussion that

$$\begin{aligned}\sum_{i=1}^{n}(a_{i1} \wedge ... \wedge a_{im}) &\geq m(\sum_{i=1}^{q} i) + r(q+1)\\ &= m(\frac{(q+1)q}{2}) + r(q+1).\end{aligned}$$

□

Proposition 2.5. $\sum_{i=1}^{n}(a_{i1} \vee ... \vee a_{im}) \leq m\frac{2qn+q-q^2}{2} + r(n-q)$.

Proof. We have by Proposition 2.3 and the discussion above that

$$\begin{aligned}\sum_{i=1}^{n}(a_{i1} \vee ... \vee a_{im}) &\leq m\sum_{i=1}^{q}(n-i+1) + r(n-q)\\ &= m\frac{2qn+q-q^2}{2} + r(n-q).\end{aligned}$$

□

Theorem 2.6. *The smallest value* $\frac{\sum_{i=1}^{n}(a_{i1} \wedge ... \wedge a_{im})}{\sum_{i=1}^{n}(a_{i1} \vee ... \vee a_{im})}$ *can be is* $\frac{m(\frac{(q+1)q}{2}) + r(q+1)}{m\frac{2qn+q-q^2}{2} + r(n-q)}$.

Proof. The smallest value $\sum_{i=1}^{n}(a_{i1} \wedge ... \wedge a_{im})$ can be is $m(\frac{(q+1)q}{2}) + r(q+1)$. The largest value $\sum_{i=1}^{n}(a_{i1} \vee ... \vee a_{im})$ can be is $m\frac{2qn+q-q^2}{2} + r(n-q)$. Hence the smallest value $\frac{\sum_{i=1}^{n}(a_{i1} \wedge ... \wedge a_{im})}{\sum_{i=1}^{n}(a_{i1} \vee ... \vee a_{im})}$ can be is $\frac{m(\frac{(q+1)q}{2}) + r(q+1)}{m\frac{2qn+q-q^2}{2} + r(n-q)}$.
□

We next simplify this term. We have since $q = \frac{n-r}{m}$ that

$$\begin{aligned}
\frac{m(\frac{(q+1)q}{2}) + r(q+1)}{m\frac{2qn+q-q^2}{2} + r(n-q)} &= \frac{m(q+1)q + 2r(q+1)}{m(2nq+q-q^2) + 2r(n-q)} \\
&= \frac{m(q+1) + 2r(1+\frac{1}{q})}{m(2n+1-q) + 2r(\frac{n}{q}-1)} \\
&= \frac{m(\frac{n-r}{m}+1) + 2r(1+\frac{m}{n-r})}{m(2n+1-\frac{n-r}{m}) + 2r(m+\frac{r}{q}-1)} \\
&= \frac{n-r+m+2r(1+\frac{m}{n-r})}{2nm+m-n+r+2r(m+\frac{r}{q}-1)} \\
&= \frac{1-\frac{r}{n}+\frac{m}{n}+\frac{2r}{n}(1+\frac{m}{n-r})}{2m+\frac{m}{n}-1+\frac{r}{n}+\frac{2r}{n}(m+\frac{r}{q}-1)}
\end{aligned}$$

which approaches $\frac{1}{2m-1}$ as n approaches ∞.

Note also

$$\begin{aligned}
\frac{m(\frac{(q+1)q}{2}) + r(q+1)}{m\frac{2qn+q-q^2}{2} + r(n-q)} &= \frac{m(q+1)q + 2r(q+1)}{m(2nq-q^2) + 2r(n-q)} \\
&= \frac{m(1+\frac{1}{q}) + 2r(\frac{1}{q}+\frac{1}{q^2})}{m(\frac{2n}{q}-1) + 2r(\frac{m}{q}+\frac{r}{q^2}-\frac{1}{q})} \\
&= \frac{m(1+\frac{1}{q}) + 2r(\frac{1}{q}+\frac{1}{q^2})}{m(\frac{2(mq+r)}{q}-1) + 2r(\frac{m}{q}+\frac{r}{q^2}-\frac{1}{q})} \\
&\to \frac{m}{m(2m-1)} = \frac{1}{2m-1} \text{ as } q \to \infty
\end{aligned}$$

and that $n \to \infty$ implies $q \to \infty$.

Proposition 2.7. $[\frac{m(q+1)q}{2} + r(q+1)]/[\frac{m(2qn+q-q^2)}{2} + r(n-q)] \geq \frac{q+1}{2n+1-q}$.

Proof. We have

$$\begin{aligned}
n+1 &\geq 0 \\
2n+1-q &\geq n-q \\
\frac{2r}{m}(2n+1-q) &\geq \frac{2r}{m}(n-q) \\
2nq+q-q^2+\frac{2r}{m}(2n+1-q) &\geq 2nq+q-q^2+\frac{2r}{m}(n-q) \\
(q+\frac{2r}{m})(2n+1-q) &\geq 2qn+q-q^2+\frac{2r}{m}(n-q) \\
\frac{q+\frac{2r}{m}}{2qn+q-q^2+\frac{2r}{m}(n-q)} &\geq \frac{1}{2n+1-q} \\
\frac{(q+1)q+\frac{2r}{m}(q+1)}{2qn+q-q^2+\frac{2r}{m}(n-q)} &\geq \frac{q+1}{2n+1-q}.
\end{aligned}$$

□

Proposition 2.8. $\frac{q+1}{2n+1-q} \geq \frac{1}{2m-1}$.

Proof. Since $r < m$, we have

$$\begin{aligned} m &\geq r+1 \\ qm+m &\geq qm+r+1 \\ qm+m &\geq n+1 \\ 2qm+2m-1 &\geq 2n+1 \\ 2qm-q+2m-1 &\geq 2n+1-q \\ (q+1)(2m-1) &\geq 2n+1-q \\ \frac{q+1}{2n+1-q} &\geq \frac{1}{2m-1}. \end{aligned}$$

□

Corollary 2.9. $\frac{\sum_{i=1}^{n}(a_{i1}\wedge\ldots\wedge a_{im})}{\sum_{i=1}^{n}(a_{i1}\vee\ldots\vee a_{im})} \geq \frac{1}{2m-1}$.

Proof. The result follows from Theorem 2.6 and Propositions 2.7 and 2.8. □

Note that if $m = 2$, then $\frac{1}{2m-1} = \frac{1}{3}$ and Corollary 2.9 corresponds to the known result in [2, p. 12].

Note: Let $c > a > 0$. Then $ac > aa$ and so $2ac > ac + aa = a(c+a)$. Hence $\frac{2a}{c+a} > \frac{a}{c}$.

Now $S = \frac{2\sum_{i=1}^{n}\wedge\{a_{ij}|j=1,\ldots,m\}}{\sum_{i=1}^{n}(\vee\{a_{ij}|j=1,\ldots,m\}+\wedge\{a_{ij}|j=1,\ldots,m\})} > \frac{\sum_{i=1}^{n}\wedge\{a_{ij}|j=1,\ldots,m\}}{\sum_{i=1}^{n}(\vee\{a_{ij}|j=1,\ldots,m\}} = M$ if $\sum_{i=1}^{n}(\vee\{a_{ij}|j = 1,\ldots,m\} > \wedge\{a_{ij}|j = 1,\ldots,m\})$.

Theorem 2.10. $S = \frac{2M}{1+M}$.

Proof.

$$\begin{aligned} S &= \frac{2\sum_{i=1}^{n}\wedge\{a_{ij}|j=1,\ldots,m\}}{\sum_{i=1}^{n}(\vee\{a_{ij}|j=1,\ldots,m\}+\wedge\{a_{ij}|j=1,\ldots,m\})} \\ &= \frac{2\sum_{i=1}^{n}\wedge\{a_{ij}|j=1,\ldots,m\}}{\sum_{i=1}^{n}(\vee\{a_{ij}|j=1,\ldots,m\}+\sum_{i=1}^{n}\wedge\{a_{ij}|j=1,\ldots,m\})} \\ &= \frac{\frac{2\sum_{i=1}^{n}\wedge\{a_{ij}|j=1,\ldots,m\}}{\sum_{i=1}^{n}(\vee\{a_{ij}|j=1,\ldots,m\}}}{1+\frac{\sum_{i=1}^{n}\wedge\{a_{ij}|j=1,\ldots,m\})}{\sum_{i=1}^{n}(\vee\{a_{ij}|j=1,\ldots,m\}}} = \frac{2M}{1+M}. \end{aligned}$$

□

Corollary 2.11. *The smallest value S can be is $\frac{2a}{1-a}$, where a is the smallest value M can be.*

Proof. Suppose there exist $a_{ij}, i = 1,\ldots,n; j = 1,\ldots,m$ such that $S < \frac{2a}{1+a}$. Then

$$\frac{2M}{1+M} < \frac{2a}{1+a}$$

and so $2M + 2Ma < 2a + 2Ma$. Thus $M < a$, a contradiction. □

Now let m and $\widehat{m}$ be integers with $n > \widehat{m} > m \geq 2$. Let $\widehat{M}$ and M be determined by $\widehat{m}$ and m, respectively for a given fixed n and a ranking with $\widehat{m}$. We wish to determine a relationship between $\widehat{M}$ and an M using a smaller m. We construct a straight line passing through the points $(\frac{1}{2\widehat{m}-1}, \frac{1}{2m-1})$ and $(1,1)$. Note that $\widehat{M}=1$ and $M=1$ are the largest values that can be obtained. The slope of the straight line is

$$s = \frac{1-\frac{1}{2m-1}}{1-\frac{1}{2\widehat{m}-1}} \tag{1}$$

and $M = s\widehat{M} + 1 - s$. It follows that $s = \frac{(2m-2)(2\widehat{m}-1)}{(2\widehat{m}-2)(2m-1)} = \frac{(m-1)(2\widehat{m}-1)}{(\widehat{m}-1)(2m-1)}$.

Example 2.12. Let $\widehat{m}=3, m=2$, and $n=5$. Suppose X_1, X_2, and X_3 are three rankings.

	X_1	X_2	X_3	$\vee$	$\wedge$
C_1	5	2	1	5	1
C_2	1	5	4	5	1
C_3	3	1	5	5	1
C_4	4	3	2	4	2
C_5	2	4	3	4	2
Col Sum				23	7

Thus $\widehat{M} = \frac{7}{23}$. We next determine the conversion to M. By (1), $s = \frac{1-\frac{1}{3}}{1-\frac{1}{5}} = \frac{2}{3}\frac{5}{4} = \frac{5}{6}$. Thus $M = \frac{5}{6}\widehat{M} + \frac{1}{6}$. For $\widehat{M} = \frac{7}{23}$, we obtain $M = \frac{29}{69}$.

We next consider S. Once again, we assume that $n > \widehat{m} > m \geq 2$. We have that $\frac{2(\frac{1}{2m-1})}{1+\frac{1}{2m-1}} = \frac{\frac{2}{2m-1}}{\frac{2m}{2m-1}} = \frac{1}{m}$. We consider the straight line passing through $(\frac{1}{\widehat{m}}, \frac{1}{m})$ and $(1,1)$. The slope of the line is $\frac{1-\frac{1}{m}}{1-\frac{1}{\widehat{m}}} = \frac{\widehat{m}(m-1)}{m(\widehat{m}-1)}$. Hence the desired straight line is $S = \frac{\widehat{m}(m-1)}{m(\widehat{m}-1)}\widehat{S} + 1 - \frac{\widehat{m}(m-1)}{m(\widehat{m}-1)}$.

Since our applications below use $\widehat{m}$ values of 3, 4, 7 and 11, we will be interested in converting these $\widehat{m}$ values to values for $m=2$.

Let $\widehat{m}=3$ and $m=2$. Then $M = \frac{5}{6}\widehat{M} + \frac{1}{6}$ and $S = \frac{3}{4}\widehat{S} + \frac{1}{4}$. Let $\widehat{m}=4$ and $m=2$. Then $M = \frac{7}{9}\widehat{M} + \frac{2}{9}$ and $S = \frac{2}{3}\widehat{S} + \frac{1}{3}$.

Let $\widehat{m}=7$ and $m=2$. Then $M = \frac{13}{18}\widehat{M} + \frac{5}{18}$ and $S = \frac{7}{12}\widehat{S} + \frac{5}{12}$.

Let $\widehat{m}=11$ and $m=2$. Then $M = \frac{21}{30}\widehat{M} + \frac{9}{30}$ and $S = \frac{11}{20}\widehat{S} + \frac{9}{20}$.

We next consider the case, where $m \geq n$. Then it is possible for $\sum_{i=1}^{n}(a_{i1} \vee ... \vee a_{im}) = n^2$ and $\sum_{i=1}^{n}(a_{i1} \wedge ... \wedge a_{im}) = n$. Consequently, the smallest value M can be is $\frac{1}{n}$. Thus the smallest value $\widehat{S}$ can be is once again $\frac{2\frac{1}{n}}{1+\frac{1}{n}} = \frac{2}{n+1}$. We next consider the conversion of $\widehat{M}$ and $\widehat{S}$ values to $m=2$ values when $\widehat{m} \geq n$.

Let $\widehat{m}=11$ and $m=2$. Assume $\widehat{m} \geq n$. Consider the straight line through $(\frac{1}{n}, \frac{1}{3})$ and $(1,1)$. Then $\frac{1-\frac{1}{3}}{1-\frac{1}{n}} = \frac{2n}{3(n-1)}$. Hence $M = \frac{2n}{3(n-1)}\widehat{M} + 1 - \frac{2n}{3(n-1)}$. Consider the straight line through $(\frac{2}{n+1}, \frac{1}{3})$ and $(1,1)$. Then $\frac{1-\frac{1}{2}}{1-\frac{2}{n+1}} = \frac{n+1}{2(n-1)}$. Thus $S = \frac{n+1}{2(n-1)}\widehat{S} + 1 - \frac{n+1}{2(n-1)}$.

For the converted values for $m=2$, we say that the similarity determined by M is very weak if the value is < 0.4, weak if the value is between 0.4 and 0.55, medium if the value is between 0.55 and 0.7, strong if the value is between 0.7 and 0.85, and very strong if the value is between 0.85 and 1. Theorem 2.10 can be used to determine a similar description for S.

3 Environmental Performance Index

As a composite index, the EPI distills data on many indicators of sustainability into a single number For the 2020 EPI, 32 indicators of environmental performance were assembled, [4]. The data was used to construct indicators on a $0-100$ scale, from worst to best. For each country, the scores were weighed and aggregated for indicators into 11 issue categories:

Air Quality (AQ),

Sanitation and Drinking Water (SDW),

Heavy Metals (HM).

Waste Management (WM),

Biodiversity and Habitat (BH),

Ecosystem Services (ES),

Fisheries (F),

Climate Change (CC),

Pollution Emissions (EM),

Water Resources (WR),

Agriculture (A).

These issue category scores were combined into two policy objectives, Environmental Health (EH) and Ecosystem Vitality (EV) and then consolidated into over all EPI.

The two policy objectives were weighted in [4] with respect to their importance as were the 11 issue categories. The weights are given in the following equations.

$EPI = 0.4EH + 0.6EV$,

$EH = 0.20AQ + 0.16SDW + 0.02HM + 0.02WM$

$EV = 0.15BH + 0.06ES + 0.06F + 0.24CC + 0.03PE + 0.03WR + 0.03A$.

We present the fuzzy similarity measures of the country rankings in terms of all 11 categories, the fuzzy similarity measures of the country rankings for EPI in terms of EH and EV, the fuzzy similarity measures of the country rankings for EH in terms of 4 of the 11 categories and for EV in terms of 7 of the 11 categories. We also present all the country rankings for only the region Global West. Country rankings for the other regions can be found in [3] or can be provided by request of the authors.

4 Rankings and Similarity Measures

In this section, we provide the country rankings for specific regions as given in [4] and their corresponding similarity measures. The number of rankings of the countries is greater than $m = 2$. We convert the similarity measures we find to the case where the smallest value they can be is 0 and we also convert them to the case $m = 2$ by using the equations given at the end of Section 2.

Global West

Country Rankings

In the following situation, From Table 1, we have $n = 22, m = 11, q = 2$, and $r = 0$.

Table 1: Global West Rankings

Country	AQ	SDW	HM	WM	BH	ES	F	CC	PE	WR	A	$\vee$	$\wedge$
Denmark	12	12.5	1.5	2.5	11	12	5	1	7	2.5	1	12.5	1
Luxembourg	11	9	5.5	11	6.5	9		6	7	5.5	16	16	5.5
Switzerland	9	3.5	8	4	21	3		4	7	8	13	21	3
United Kingdom	13	3.5	9	13	3	14	16	2	7	5.5	10	16	2
France	10	15	13	12	2	7	9	3	7	13	5	15	2
Austria	2.5	16	11	9	6.5	8		11	7	9	3	16	3
Finland	1	3.5	1.5	6	13	21	7	8	17	2.5	11	21	1
Sweden	2.5	10	3	2.5	16	20	10	7	7	2.5	6	20	3
Norway	5	3.5	10	7.5	17	11	13	5	14	17	18	18	3.5
Germany	18	8	12	5	1	5	4	9	15	7	7	18	1
Netherlands	16	3.5	7	1	9.5	4	6	17	7	2.5	17	17	1
Australia	2.5	19	16	20	9.5	16	19	13	18	10	12	20	2.5
Spain	20	14	19	15	4	19	2	12	7	11	19	20	2
Belgium	19	17	20	7.5	5	10	15	14	7	15	14.5	20	5
Ireland	8	12.5	14	19	19	17	14	16	7	12	14.5	19	7
Iceland	4	3.5	5.5	16	20	1.5	12	22	22	21	22	22	1.5
New Zealand	6	22	18	21	8	15	18	21	19	14	8	22	6
Canada	7	18	4	17	22	13	11	18	7	16	4	22	4
Italy	22	11	15	18	12	6	3	15	20	19	9	22	3
Malta	21	7	22	10	14	1.5	8	20	21	22	20	22	7
United States	15	20	17	22	18	18	17	10	7	18	2	22	2
Portugal	14	21	21	14	15	22	1	19	16	20	21	22	1
Col. Sum												423.5	67

$M = \frac{67}{423.5} = 0.158$ and $S = \frac{2(0.158)}{1+0.158} = \frac{0.316}{1.158} = 0.273$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{11(3)}{11(50-1)} = \frac{3}{49} = 0.061$. The smallest S can be is $\frac{2(0.061)}{1+0.061} = \frac{0.122}{1.061} = 0.115$. Now $\frac{M-0.061}{1-0.061} = \frac{0.158-0.061}{0.939} = \frac{0.097}{0.939} = 0.103$ and $\frac{S-0.115}{1-0.115} = \frac{0.273-0.115}{0.885} = \frac{0.158}{0.885} = 0.179$.

For $\widehat{m} = 11$ and $m = 2$, the value 0.158 converts to is $\frac{21(0.158)+9}{30} = \frac{12.318}{30} = 0.411$ and the value 0.273 converts to is $\frac{11(0.273)+9}{20} = \frac{12.003}{20} = 0.600$. We see that the similarity is weak.

EPI Rankings

Table 2: Global West-EPI Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Denmark	1	9	1	9	1
Luxembourg	2	7	2	7	2
Switzerland	3	5	5	5	3
United Kingdom	4	9	3	9	3
France	5	12	6	12	5
Austria	6	15.5	4	15.5	4
Finland	7	1	10	10	1
Sweden	8	3	9	9	3
Norway	9	2	13.5	13.5	2
Germany	10	14	7	14	7
Netherlands	11	13	11.5	13	11
Australia	12	11	13.5	13.5	11
Spain	13	17	8	17	8
Belgium	14	19	11.5	19	11.5
Ireland	15	6	19	19	6
Iceland	16	4	22	22	4
New Zealand	17	15.5	18	18	15.5
Canada	18	9	20	20	9
Italy	19	20	15	20	15
Malta	20	18	16	20	16
Untied States	21	22	17	22	17
Portugal	22	21	21	22	21
Col. Sum				329.5	176

From Table 2, $M = \frac{176}{329.5} = 0.543$ and $S = \frac{2(0.543)}{1+0.543} = \frac{1.068}{1.543} = 0.692$. Here $n = 22, m = 3, q = 7$, and $r = 1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{84+8}{399+15} = \frac{92}{414} = 0.222$ and the smallest value S can be is $\frac{2(0.222)}{1+0.222} = \frac{0.444}{1.222} = 0.363$. Now $\frac{M-0.222}{1-0.222} = \frac{0.543-0.222}{0.778} = \frac{0.321}{0.778} = 0.413$ and $\frac{S-0.363}{1-0.363} = \frac{0.692-0.363}{0.637} = \frac{0.329}{0.637} = 0.516$.

For $\widehat{m} = 3$ and $m = 2$, the value 0.543 converts to is 0.619 and the value 0.692 converts to is 0.769. We have that the similarity is medium.

Global west-EH Rankings

Look at Table 3. $M = \frac{136}{358.5} = 0.379$ and $S = \frac{2(0.379)}{1+0.379} = \frac{0.758}{1.379} = 0.550$. Here $n = 22, m = 4, q = 5$, and $r = 2$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{60+12}{2(200)+34} = \frac{72}{434} = 0.166$ and the smallest value S can be is $\frac{2(0.166)}{1+0.166} = \frac{0.332}{1.166} = 0.297$. Now $\frac{M-0.166}{1-0.166} = \frac{0.379-0.166}{0.834} = \frac{0.213}{0.834} = 0.255$ and $\frac{S-0.297}{1-0.297} = \frac{0.550-0.297}{0.703} = \frac{0.253}{0.703} = 0.360$.

For $\widehat{m} = 4$ and $m = 2$, the value 0.379 converts to is 0.517 and the value 0.550 converts to is 0.700. The similarity here is medium.

Table 3: EH Rankings

Country	AQ	SDW	HM	WM	$\vee$	$\wedge$
Denmark	12	12.5	1.5	2.5	12.5	1.5
Luxembourg	11	9	5.5	11	11	5.5
Switzerland	9	3.5	8	4	9	3.5
United Kingdom	13	3.5	9	13	13	3.5
France	10	15	13	12	15	10
Austria	2.5	16	11	9	16	2.5
Finland	1	3.5	1.5	6	6	1
Sweden	2.5	10	3	2.5	10	2.5
Norway	5	3.5	10	7.5	10	7.5
Germany	18	8	12	5	18	5
Netherlands	16	3.5	7	1	16	1
Australia	2.5	19	16	20	20	2.5
Spain	20	14	19	15	20	14
Belgium	19	17	20	7.5	20	7.5
Ireland	8	12.5	14	19	19	8
Iceland	4	3.5	5.5	16	16	3.5
New Zealand	6	22	18	21	22	6
Canada	7	18	4	17	18	4
Italy	22	11	15	18	22	11
Malta	21	7	22	10	22	7
United States	15	20	17	22	22	15
Portugal	14	21	21	14	21	14
Col. Sum					358.5	136

EV Rankings

From Table 4, $M = \frac{74.5}{399} = 0.187$ and $S = \frac{2(0.187)}{1+0.187} = \frac{0.374}{1..187} = 0.315$. Here $n = 22, m = 7, q = 3$, and $r = 1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{42+4}{7(63)+19} = \frac{46}{460} = 0.100$ and the smallest value S can be is $\frac{2(0.100)}{1+0.100} = \frac{0.200}{1.100} = 0.182$. Now $\frac{M-0.100}{1-0.100} = \frac{0.187-0.100}{0.900} = \frac{0.087}{0.900} = 0.097$ and $\frac{S-0.182}{1-0.182} = \frac{0.315-0.182}{0.818} = \frac{0.133}{0.818} = 0.163$.

For $\widehat{m} = 7$ and $m = 2$, the value 0.187 is converts to is 0.413 and 0.315 is converts to is 0.600. The similarity here is weak.

Southern Asia

Country Rankings

Here $n = 8$ and $m = 11$ and so $m > n$.

We have $M = \frac{11}{59} = 0.186$ and $S = \frac{2(11)}{59+11} = \frac{22}{70} = 0.314$. The smallest value M can be is $\frac{1}{8} = 0.125$. The smallest value S can be is $\frac{2(\frac{1}{8})}{1+\frac{1}{8}} = \frac{2}{9} = 0.222$. Now $\frac{M-0.125}{1-0.125} = \frac{0.186-0.125}{0.875} = \frac{0.061}{0.875} = 0.070$ and $\frac{S-0.222}{1-0.222} = \frac{0.314-0.222}{0.778} = \frac{0.092}{0.778} = 0.118$.

For $\widehat{m} = 11$ and $m = 2$, the value 0.186 converts to is $\frac{2(8)}{3(7)}(0.186) + \frac{5}{21} = \frac{7.976}{21} = 0.380$ and the value 0.314 converts to is $\frac{9}{2(7)}(0.314) + \frac{5}{14} = \frac{7.826}{14} = 0.559$.

Table 4: Global West-EV Rankings

Country	BH	ES	F	CC	PE	WR	A	$\vee$	$\wedge$
Denmark	11	12	5	1	7	2.5	1	12	1
Luxembourg	6.5	9		6	7	5.5	16	16	5.5
Switzerland	21	3		4	7	8	13	21	3
United Kingdom	3	14	16	2	7	5.5	10	16	2
France	2	7	9	3	7	13	5	13	2
Austria	6.5	8		11	7	9	3	11	3
Finland	13	21	7	8	17	2.5	11	21	2.5
Sweden	16	20	10	7	7	2.5	6	20	2.5
Norway	17	11	13	5	14	17	18	18	5
Germany	1	5	4	9	15	7	7	15	1
Netherlands	9.5	4	6	17	7	2.5	17	17	2.5
Australia	9.5	16	19	13	18	10	12	19	9.5
Spain	4	19	2	12	7	11	19	19	2
Belgium	5	10	15	14	7	15	14.5	15	5
Ireland	19	17	14	16	7	12	14.5	19	7
Iceland	20	1.5	12	22	22	21	22	22	1.5
New Zealand	8	15	18	21	19	14	8	21	8
Canada	22	13	11	18	7	16	4	22	4
Italy	12	6	3	15	20	19	9	20	3
Malta	14	1.5	8	20	21	22	20	22	1.5
United States	18	18	17	10	7	18	2	18	2
Portugal	15	22	1	19	16	20	21	22	1
Col. Sum								399	74.5

Southern Asia-EPI Rankings

In Table 5, we have $n = 8, m = 3, q = 2$, and $r = 2$.

Table 5: Southern Asia-Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Bhutan	1	3	1	3	1
Sri Lanka	2	2	4	4	2
Maldives	3	1	8	8	1
Pakistan	4	8	2	8	2
Nepal	5	5	3	5	3
Bangladesh	6	4	6	6	4
India	7	7	5	7	5
Afghanistan	8	6	7	8	6
Col. Sum				49	24

We have $M = \frac{24}{49} = 0.490$ and $S = \frac{2(24)}{49+24} = \frac{48}{73} = 0.658$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{9+6}{48+12} = \frac{15}{57} = 0.263$ and the smallest value S can be is $\frac{2(\frac{15}{57})}{1+\frac{15}{57}} = \frac{30}{72} = 0.417$. Now $\frac{M-0.263}{1-0.263} = \frac{0.490-0.263}{0.737} = \frac{0.227}{0.737} = 0.308$ and $\frac{S-0.417}{1-0.417} = \frac{0.658-0.417}{0.583} = \frac{0.241}{0.583} = 0.414$.

For $\widehat{m} = 3$ and $m = 2$, 0.490, the value 0.490 converts to is $\frac{5}{6}(0.490) + \frac{1}{6} = 0.575$ and the value 0.658 converts to is $\frac{3}{4}(0.658) + \frac{1}{4} = 0.7435$.

EH Rankings

Here $n = 8, m = 4, q = 2$, and $r = 0$. Now $M = \frac{20}{50} = 0.4$ and $S = 1 - \frac{30}{70} = \frac{4}{7} = 0.57$ The smallest M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{m\frac{(q+1)q}{2}}{m\frac{2qn+q-q^2}{2}} = \frac{12}{60} = 0.2$ and the smallest S can be is $\frac{2a}{1+a} = \frac{2(0.2)}{1+0.2} = \frac{0.4}{1.2} = 0.333$. Hence $\frac{M-0.2}{1-0.2} = \frac{0.4-0.2}{0.8} = \frac{1}{4} = 0.25$ and $\frac{S-0.333}{1-0.333} = \frac{0.57-0.333}{0.667} = \frac{0.237}{0..667} = 0.335$.

For $\widehat{m} = 4$ and $m = 2$, the value 0.4 converts to is $\frac{2.8+2}{9} = 0.533$ and the value 0.57 converts to is $\frac{1.14+1}{3} = 0.713$.

EV Rankings

Here $n = 8, m = 7, q = 1$, and $r = 1$. Now $M = \frac{12}{56} = \frac{3}{14} = 0.214$ and $S = 1 - \frac{44}{68} = \frac{24}{68} = \frac{6}{17} = 0.353$. The smallest M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{9}{63} = \frac{1}{7} = 0.143$ and the smallest S can be is $\frac{2a}{1+a} = \frac{2}{8} = 0.250$. Hence $\frac{M-0.143}{1-0.143} = \frac{0.214-0.143}{0.857} = \frac{0.071}{0.857} = 0.083$ and $\frac{S-0.250}{1-0.250} = \frac{0.353-0.250}{0.750} = \frac{0.103}{0.750} = 0.137$.

For $\widehat{m} = 7$ and $m = 2$, the value 0.214 converts to is $\frac{13(0.214)+5}{18} = \frac{7.782}{18} = 0.432$ and the values 0.353 converts to is $\frac{7(0.353)+5}{12} = \frac{7.471}{12} = 0.623$.

Former Soviet States

Country Rankings

In the following situation, we have $n = 12, m = 11, q = 1$, and $r = 1$.

$M = \frac{18}{131.5} = 0.137$ and $S = \frac{2M}{1+M} = \frac{2(0.137)}{1+0.137} = \frac{0.274}{1.137} = 0.241$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{11+2}{11(12)+11} = \frac{13}{143} = 0.091$.

The smallest S can be is $\frac{2(0.091)}{1+0.091} = \frac{0.182}{1.091} = 0.167$. Now $\frac{M-0.091}{1-0.091} = \frac{0.137-0.091}{0.909} = \frac{0.046}{0.909} = 0.051$ and $\frac{S-0.167}{1-0.167} = \frac{0.241-0.167}{0.833} = \frac{0.074}{0.833} = 0.089$.

For $\widehat{m} = 11$ and $m = 2$, the value 0.137 converts to is $\frac{21(0.137)+9}{30} = \frac{11.86}{30} = 0.395$ and the value 0.241 converts to is $\frac{11(0.241)+9}{20} = \frac{11.651}{20} = 0.583$.

EPI Rankings

In the Table 6, $n = 12, m = 3, q = 4$, and $r = 0$.

$M = \frac{49.5}{107.5} = 0.460$ and $S = \frac{2(0.46)}{1+0.46} = \frac{0.92}{1.46} = 0.630$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{30}{3(42)} = \frac{5}{21} = 0.238$ and the smallest value S can be is $\frac{2(0.238)}{1+0.238} = \frac{0.476}{1.238} = .384$. Now $\frac{M-0.238}{1-0.238} = \frac{0.460-0.238}{0.762} = \frac{0.222}{0.762} = 0.0.291$ and $\frac{S--.384}{1-0.384} = \frac{0.630-0.384}{0.616} = \frac{0.246}{0.616} = 0.399$.

For $\widehat{m} = 3$ and $m = 2$, the value 0.460 converts to is 0.550 and the value 0.630 converts to is 0.7225.

EH Rankings

In Table 6, $n = 12, m = 4, q = 3$, and $r = 0$.

Table 6: Former Soviet States- Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Belarus	1	1	4	4	1
Armenia	2	6	1	6	1
Russia	3	2	7	7	2
Ukraine	4	3	5.5	5.5	3
Azerbaijan	5	10	2	10	2
Kazakhstan	6	7	8	8	6
Moldova	7	4	10	10	4
Uzbekistan	8	11	3	11	3
Turkmenistan	9	5	11.5	11.5	5
Georgia	10	8	11.5	11.5	8
Kyrgyzstan	11	9	9	11	9
Tajikistan	12	12	5.5	12	5.5
Col. Sum				107.5	49.5

$M = \frac{58.5}{98.5} = 0.594$ and $S = \frac{2(0.594)}{1+0.594} = \frac{1.188}{1.594} = 0.745$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{24}{144-3} = \frac{24}{141} = 0.170$ and the smallest value S can be is $\frac{2(0.170)}{1+0.170} = \frac{0.340}{1.170} = 0.291$. Now $\frac{M-0.170}{1-0.170} = \frac{0.594-0.170}{0.830} = \frac{0.424}{0.830} = 0.511$ and $\frac{S-0.291}{1-0.291} = \frac{0.745-0.291}{0.709} = \frac{0.454}{0.709} = 0.640$.

For $\widehat{m} = 4$ and $m = 2$, the value 0.594 converts to is 0.684 and the value 0.745 converts to is 0.830.

EV Rankings

In the Table 6, $n = 12, m = 7, q = 1$, and $r = 5$.

$M = \frac{24}{129} = 0.186$ and $S = \frac{2(0.186)}{1+0.186} = \frac{0.372}{1.186} = 0.314$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{7+10}{84+55} = \frac{17}{139} = 0.122$ and the smallest value S can be is $\frac{2(0.122)}{1+0.122} = \frac{0.244}{1.122} = 0.217$. Now $\frac{M-0.122}{1-0.122} = \frac{0.186-0.122}{1-0.122} = \frac{0.064}{0.878} = 0.073$ and $\frac{S-0.217}{1-0.217} = \frac{0.314-0.217}{0.783} = \frac{0.097}{0.783} = 0.124$.

For $\widehat{m} = 7$ and $m = 2$, the value 0.186 is converts to is 0.412 and the value 0.314 is converts to is 0.600.

Greater Middle East

Table 7 is used in the following.

Country Rankings

In the following situation, $n = 16, m = 11, q = 1$, and $r = 5$.

$M = \frac{36.5}{236} = 0.155$ and $S = \frac{2(0.155)}{1+0.155} = \frac{0.310}{1.155} = 0.268$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{11+10}{176+75} = \frac{21}{251} = 0.084$. The smallest S can be is $\frac{2(0.084)}{1+0.084} = \frac{0.168}{1.084} = 0.155$. Now $\frac{M-0.084}{1-0.084} = \frac{0.155-0.084}{0.916} = \frac{0.071}{0.916} = 0.078$ and $\frac{S-0.155}{1-0.155} = \frac{0.268-0.155}{1-0.155} = \frac{0.113}{0.845} = 0.134$.

For $\widehat{m} = 11$ and $m = 2$, the value 0.155 converts to is $\frac{21(0.155)+9}{30} = \frac{12.255}{} = 0.408$ and the value 0.268 converts to is $\frac{11(0.268)+9}{20} = \frac{11.948}{20} = 0.597$.

EPI Rankings

Table 7: Greater Middle East-Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Israel	1	1	2	2	1
UAE	2	5	1	5	1
Kuwait	3	3	4	4	3
Jordan	4	2	5	5	2
Bahrain	5	8.5	3	8.5	3
Iran	6	10	8	10	6
Tunisia	7	8.5	9	9	7
Lebanon	8	6	13	13	6
Algeria	9	7	12	12	7
Saudi Arabia	10	11	11	11	10
Egypt	11	14	6	14	6
Morocco	12	15	7	15	7
Iraq	13	13	14	14	13
Oman	14	12	15	15	12
Qatar	15	4	16	16	4
Sudan	16	16	10	16	10
Col. Sum				169.5	98

$M = \frac{98}{169.5} = 0.578$ and $S = \frac{2(0.578)}{1+0.578} = \frac{1.156}{1.578} = 0.733$. Here $n = 16, m = 3, q = 5$, and $r = 1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{45+5}{210+11} = \frac{50}{221} = 0.226$ and the smallest S can be $\frac{2(0.226)}{1+0.226} = \frac{0.452}{1.226} = 0.369$. Now $\frac{M-0.226}{1-0.226} = \frac{0.578-0.226}{0.774} = \frac{0.352}{0.774} = 0.455$ and $\frac{S-0.369}{1-0.369} = \frac{0.733-0.369}{0.631} = \frac{0.364}{0.631} = 0.577$.

For $\widehat{m} = 3$ and $m = 2$, the value 0.578 converts to is 0.648 and the value 0.733 converts to is 0.800.

EH Rankings

$M = \frac{79.5}{184.5} = 0.431$ and $S = \frac{2(0.431)}{1+0.431} = \frac{0.862}{1.431} = 0.602$. Here $n = 16, m = 4, q = 4$, and $r = 0$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{40}{2(128-12)} = \frac{40}{232} = 0.172$. The smallest value S can be is $\frac{2(0.172)}{1+0.172} = \frac{0.342}{1.172} = 0.292$. Now $\frac{M-0.172}{1-0.172} = \frac{0.431-0.172}{0.828} = \frac{0.259}{0.828} = 0.313$ and $\frac{S-0.292}{1-0.292} = \frac{0.602-0.292}{0.708} = \frac{0.310}{0.708} = 0.438$.

For $\widehat{m} = 4$ and $m = 2$, the value 0.431 converts to 0.557and the value 0.602 converts to 0.747.

EV Rankings

$M = \frac{47}{227} = 0.207$ and $S = \frac{2(0.207)}{1+0.207} = \frac{0.414}{1.207} = 0.343$. Here $n = 16, m = 7, q = 2$, and $r = 2$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{21+6}{7(31)+28} = \frac{27}{245} = 0.110$. The smallest value S can be is $\frac{2(0.110)}{1+0.110} = \frac{0.220}{1.110} = 0.198$. Now $\frac{M-0.110}{1-0.110} = \frac{0.207-0.110}{0.890} = \frac{0.097}{0.890} = 0.109$ and $\frac{S-0.198}{1-0.198} = \frac{0.343-0.198}{0.802} = \frac{0.145}{0.802} = 0.181$.

For $\widehat{m} = 7$ and $m = 2$, the value 0.207 is converts to is 0.427 and the value 0.343 is converts to is 0.617.

Eastern Europe

Country Rankings

In the following situation (Table 8), we have $n = 19, m = 11, q = 1$, and $r = 8$.

$M = \frac{45.5}{320} = 0.142$ and $S = \frac{2(0.142)}{1+0.142} = \frac{0.284}{1.142} = 0.249$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} =$

$\frac{11+16}{209+144} = \frac{27}{353} = 0.076$. The smallest S can be $\frac{2(0.076)}{1+0.076} = \frac{0.152}{1.076} = 0.141$. Now $\frac{M-0.076}{1-0.076} = \frac{0.142-0.076}{0.924} = \frac{0.066}{0.924} = 0.071$ and $\frac{S-0.141}{1-0.141} = \frac{0.249-0.141}{0.859} = \frac{0.108}{0.859} = 0.126$. Now $\frac{M-0.076}{1-0.076} = \frac{0.142-0.076}{0.924} = \frac{0.066}{0.924} = 0.071$ and $\frac{S-0.141}{1-0.141} = \frac{0.249-0.141}{0.859} = \frac{0.108}{0.859} = 0.126$.

For $\widehat{m} = 11$ and $m = 2$, the value 0.142 converts to is $\frac{21(0.142)+9}{30} = \frac{11.982}{30} = 0.399$ and the value 0.249 converts to is $\frac{11(0.249)+9}{20} = \frac{11.739}{20} = 0.587$.

EPI Rankings

Table 8: Eastern Europe-Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Slovenia	1	4	2	4	1
Czech Republic	2	5	3	5	2
Greece	3	2	12	12	2
Slovakia	4	6	4	6	4
Estonia	5	3	14	14	3
Cyprus	6	1	15	15	1
Romania	7	14	1	14	1
Hungary	8	11	5	11	5
Croatia	9	8	6	9	6
Lithuania	10	7	9	10	7
Latvia	11	10	7	11	7
Poland	12	9	10	12	9
Bulgaria	13	13	11	13	11
N. Macedonia	14	19	8	19	8
Serbia	15	15	13	15	13
Albania	16	17	16	17	16
Montenegro	17	16	18	18	16
Bosnia and Herzegovina	18	18	17	18	17
Turkey	19	12	19	19	12
Col. Sum				242	141

$M = \frac{141}{242} = 0.583$ and $S = \frac{2(0.583)}{1+0.583} = \frac{1.166}{1.583} = 0.737$. Here $n = 19, m = 3, q = 6$, and $r = 1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{63+7}{3(99)+13} = \frac{70}{310} = 0.226$ and the smallest S can be is $\frac{2(0.226)}{1+0.226} = \frac{0.452}{1.226} = 0.369$. Now $\frac{M-0.226}{1-0.226} = \frac{0.583-0.226}{0.774} = \frac{0.357}{0.774} = 0.461$ and $\frac{S-0.369}{1-0.369} = \frac{0.737-0.369}{0.631} = \frac{0.368}{0.631} = 0.583$.

For $\widehat{m} = 3$ and $m = 2$, the value 0.583 converts to is 0.6525and the value 0.737 converts to is 0.803.

EH Rankings

$M = \frac{115}{258.5} = 0.445$ and $S = \frac{2(0.445)}{1+0.445} = \frac{0.890}{1.445} = 0.616$. Here $n = 19, m = 4, q = 4$, and $r = 3$. The smallest M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{40+15}{280+45} = \frac{55}{325} = 0.169$. The smallest value S can be is $\frac{2(0.169)}{1+0.169} = \frac{0.338}{1.169} = 0.289$. Now $\frac{M-0.169}{1-0.169} = \frac{0.445-0.169}{0.831} = \frac{0.276}{0.831} = 0.332$ and $\frac{S-0.289}{1-0.289} = \frac{0.616-0.289}{0.711} = \frac{0.327}{0.711} = 0.460$.

For $\widehat{m} = 4$ and $m = 2$, the value 0.445 converts to is 0.568 and the value 0.616 converts to is 0.744.

EV Rankings

$M = \frac{50.5}{309.5} = 0.163$ and $S = \frac{2(0.163)}{1+0.163} = \frac{0.326}{1.163} = 0.280$. Here $n = 19, m = 7, q = 2$, and $r = 5$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{21+15}{7(37)+85} = \frac{36}{344} = 0.105$. The smallest value S can be is $\frac{2(0.105)}{1+0.105} = \frac{0.210}{1.105} = 0.190$. Now $\frac{M-0.105}{1-0.105} = \frac{0.163-0.105}{0.895} = \frac{0.058}{0.895} = 0.065$ and $\frac{S-0.190}{1-0.190} = \frac{0.280-0.190}{0.810} = \frac{0.090}{0.810} = 0.111$.

For $\widehat{m} = 7$ and $m = 2$, the value 0.163 is converts to is 0.395 and the value 0.280 is converts to is 0.580.

Asia Pacific

Country Rankings

In the following situation (Table 9), we have $n = 25, m = 11, q = 2$, and $r = 3$.

$M = \frac{94}{524.5} = 0.179$ and $S = \frac{2(0.179)}{1+0.179} = \frac{0.358}{1.179} = 0.304$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{33+9}{11(49)+69} = \frac{42}{608} = 0.069$. The smallest S can be is $\frac{2(0.069)}{1+0.069} = \frac{0.138}{1.069} = 0.129$. Now $\frac{M-0.069}{1-0.069} = \frac{0.179-0.069}{0.931} = \frac{0.110}{0.931} = 0.118$ and $\frac{S-0.129}{1-0.129} = \frac{0.304-0.129}{0.871} = \frac{0.175}{0.871} = 0.201$.

For $\widehat{m} = 11$ and $m = 2$, the value 0.179 converts to is $\frac{21(0.179)+9}{30} = \frac{12.759}{30} = 0.425$ and the value 0.304 converts to is $\frac{11(0.304)+9}{20} = \frac{12.344}{20} = 0.617$.

EPI Rankings

$M = \frac{256}{398} = 0.643$ and $S = \frac{2(0.643)}{1+0.643} = \frac{1.286}{1.643} = 0.783$. Here $n = 25, m = 3, q = 8$, and $r = 1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{108+9}{516+17} = \frac{117}{533} = 0.220$ and the smallest S can be is $\frac{2(0.220)}{1+0.220} = \frac{0.440}{1.220} = 0.361$. Now $\frac{M-0.220}{1-0.220} = \frac{0.643-0.220}{0.780} = \frac{0.423}{0.780} = 0.542$ and $\frac{S-0.361}{1-0.361} = \frac{0.783-0.361}{0.639} = \frac{0.422}{0.639} = 0.660$.

For $\widehat{m} = 3$ and $m = 2$, the value 0.643 converts to is 0..7025 and the value 0.783 converts to is 0.837.

EH Rankings

$M = \frac{227}{403} = 0.582$ and $S = \frac{2(0.582)}{1+0.582} = \frac{1.164}{1.582} = 0.736$. Here $n = 25, m = 4, q = 6$, and $r = 1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{84+7}{2(270)+19} = \frac{91}{559} = 0.163$ and the smallest value S can be is $\frac{2(0.163)}{1+0.163} = \frac{0.326}{1.163} = 0.280$. Now $\frac{M-0.163}{1-0.163} = \frac{0.582-0.163}{0.837} = \frac{0.419}{0.837} = 0.501$ and $\frac{S-0.280}{1-0.280} = \frac{0.736-0.280}{0.720} = \frac{0.450}{0.720} = 0.625$.

For $\widehat{m} = 4$ and $m = 2$, the value 0.582 converts to is 0.675 and the value 0.736 converts to is 0.824.

EV Rankings

$M = \frac{97.5}{519} = 0.188$ and $S = \frac{2(0.188)}{1+0.188} = \frac{0.376}{1.188} = 0.316$. Here $n = 25, m = 7, q = 3,$. and $r = 4$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{42+16}{7(72)+88} = \frac{58}{592} = 0.098$ and the smallest value S can be is $\frac{2(0.098)}{1+0.098} = \frac{0.196}{1.098} = 0.179$. Now $\frac{M-0.098}{1-0.098} = \frac{0.188-0.098}{0.902} = \frac{0.090}{0.902} = 0.100$ and $\frac{S-0.179}{1-0.179} = \frac{0.316-0.179}{0.821} = \frac{0.137}{0.821} = 0.167$.

For $\widehat{m} = 7$ and $m = 2$, the value 0.188 converts to is 0.414 and the value 0.316 converts to is 0.601.

Latin America and the Caribbean

Country Rankings

From Table 10, we have $n = 32, m = 11, q = 2$, and $r = 10$.

$M = \frac{96}{880} = 0.109$ and $S = \frac{2(0.109)}{1+0.109} = \frac{0.218}{1.109} = 0.197$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{33+30}{704-11+300} = \frac{63}{983} = 0.064$. The smallest S can be is $\frac{2(0.064)}{1+0.064} = \frac{0.128}{1.064} = 0.120$. Now $\frac{M-0.064}{1-0.064} = \frac{0.109-0.064}{0.936} = \frac{0.045}{0.936} = 0.048$ and $\frac{S-0.120}{1-0.120} = \frac{0.197-0.120}{0.880} = \frac{0.077}{0.880} = 0.087$.

Table 9: Asia Pacific- Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Japan	1	1	1	1	1
South Korea	2	3	2	3	2
Singapore	3	2	11	11	2
Taiwan	4	5	3	5	3
Brunei Darussalam	5	4	9	9	4
Malaysia	6	6	8	8	6
Thailand	7	7	7	7	7
Tonga	8	8	5	8	5
Philippines	9	13	10	13	9
Indonesia	10	17	6	17	6
Kiribati	11	24	4	24	4
China	12.5	10	18	18	10
Samoa	12.5	9	20	20	9
Timor-Leste	14	18	13	18	13
Laos	15	21	12	21	12
Fiji	16	12	19	19	12
Cambodia	17	16	14	17	14
Viet Nam	18	11	24	24	11
Micronesia	19	15	17	19	15
Papua New Guinea	20	19	16	20	16
Mongolia	21	20	15	21	15
Marshall Islands	22	14	23	23	14
Vanuatu	23	22	22	23	22
Solomon Islands	24	25	21	24	21
Myanmar	25	23	25	25	23
Col. Sum				398	256

For $\widehat{m} = 11$ and $m = 2$, the value 0.109 converts to is $\frac{21(0.109)+9}{30} = \frac{11.289}{30} = 0.375$ and the value 0.197 converts to is $\frac{11(0.197)+9}{20} = \frac{11.167}{20} = 0.558$.

EPI Rankings

$M = \frac{372}{690} = 0.538$ and $S = \frac{2(0.538)}{1+0.538} = \frac{1.076}{1.538} = 0.700$. Here $n = 32, m = 3, q = 10$, and $r = 2$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{165+22}{825+22} = \frac{187}{847} = 0.221$ and the smallest value S can be is $\frac{2(0.221)}{1+0.221} = \frac{0.442}{1.221} = 0.362$. Now $\frac{M-0.221}{1-0.221} = \frac{0.538-0.221}{0.779} = \frac{0.317}{0.779} = 0.407$ and $\frac{S-0.362}{1-0.362} = \frac{0.700-0.362}{0.638} = \frac{0.338}{0.638} = 0.530$.

For $\widehat{m} = 3$ and $m = 2$, the value 0.538 converts to is 0.615 and the value 0.700 converts to is 0.775.

EH Rankings

$M = \frac{331}{713.5} = 0.464$ and $S = \frac{2(0.464)}{1+0.464} = \frac{0.928}{1.464} = 0.634$. Here $n = 32, m = 4, q = 8$, and $r = 0$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{144}{2(512-56)} = \frac{144}{912} = 0.158$ and the smallest S can be is $\frac{2(0.158)}{1+0.158} = \frac{0.316}{1.158} = 0.273$. Now $\frac{M-0.158}{1-0.158} = \frac{0.464-0.158}{0.842} = \frac{0.306}{0.842} = 0.363$ and $\frac{S-0.273}{1-0.273} = \frac{0.634-0.273}{0.727} = \frac{0.361}{0.727} = 0.497$.

For $\widehat{m} = 4$ and $m = 2$, the value 0.464 converts to is 0.583 and the value 0.634 converts to is 0.756.

Table 10: Latin America and the Caribbean- Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Chile	1	2	10.5	10.5	1
Columbia	2	7	5	7	2
Mexico	3	15	1	15	1
Costa Rica	4	4	12	12	4
Argentina	5	5	14	14	5
Brazil	6	13	4	13	4
Ecuador	7	12	6	12	6
Venezuela	8	18	3	18	3
Uruguay	9	1	29	29	1
Antigua and Barbuda	10	6	17	17	6
Cuba	11.5	10	13	13	10
St. Vincent and Grenadines	11.5	21	7	21	7
Jamaica	13	20	9	20	9
Trinidad and Tobago	14	8	22	22	8
Panama	15	11	16	16	11
Paraguay	16	16.5	15	16.5	15
Dominican Republic	17	27	2	27	2
Barbados	18	3	30	30	3
Suriname	19	26	8	26	8
Dominica	20	16.5	20	20	16.5
Bolivia	21	28	10.5	28	10.5
Peru	22	21	19	22	19
Bahamas	23	9	28	28	9
El Salvador	25	23	18	25	18
Grenada	25	19	23	25	19
Saint Lucia	25	14	25	25	14
Belize	27	24	21	27	21
Nicaragua	28	25	26	28	25
Honduras	29	30	24	30	24
Guyana	30	29	27	30	27
Guatamala	31	31	31	31	31
Haiti	32	32	32	32	32
Col. Sum				690	372

EV Rankings

$M = \frac{121}{887} = 0.136$ and $S = \frac{2(0.136)}{1+0.136} = \frac{0.272}{1.136} = 0.239$. Here $n = 32, m = 7, q = 4$, and $r = 4$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{70+20}{854+112} = \frac{90}{966} = 0.093$ and the smallest value S can be is $\frac{2(0.093)}{1+0.093} = \frac{0.186}{1.093} = 0.170$. Now $\frac{M-0.093}{1-0.093} = \frac{0.136-0.093}{0.907} = \frac{0.043}{0.907} = 0.047$ and $\frac{S-0.170}{1-0.170} = \frac{0.239-0.170}{0.830} = \frac{0.069}{0.830} = 0.083$.

For $\widehat{m} = 7$ and $m = 2$, the value 0.136 converts to is 0.376 and the value 0.239 converts to is 0.556.

Sub-Saharan Africa

Country Rankings

We do not use WM, F, or WR in the following calculations. This is due to a lack of data.

From Table 11 and Table 12, we have $n = 46, m = 8, q = 5$, and $r = 6$.

$M = \frac{297}{1890.5} = 0.157$ and $S = \frac{2(0.157)}{1+0.157} = \frac{0.314}{1.157} = 0.271$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)} = \frac{8(6)(5)/2+6(6)}{1620+246} = \frac{156}{1866} = 0.084$. The smallest S can be is $\frac{2(0.084)}{1+0.084} = \frac{0.168}{1.084} = 0.155$. Now $\frac{M-0.084}{1-0.084} = \frac{0.157-0.084}{0.916} = \frac{0.073}{0.916} = 0.080$ and $\frac{S-0.155}{1-0.155} = \frac{0.271-0.155}{0.845} = \frac{0.116}{0.845} = 0.137$.

For $\widehat{m} = 8$ and $m = 2$, the value 0.157 converts to is $\frac{15}{21}(0.157) + \frac{6}{21} = \frac{8.355}{21} = 0.398$ and the value 0.271 converts to is $\frac{8}{14}(0.271) + \frac{6}{14} = \frac{8.168}{14} = 0.583$.

EPI Rankings

Table 11: Sub-Saharan Africa-Rankings

Country	EPI	EH	EV	$\vee$	$\wedge$
Seychelles	1	2	1	2	1
Gabon	2	7	2	7	2
Mauritius	3	1	32.5	32.5	1
South Africa	4	3	6	6	3
Botswana	5	27	3	27	3
Namimbia	6	16.5	5	16.5	5
Burkino Faso	7.5	31	7	31	7
Malawi	7.5	11	10	11	7.5
Equatorial Guinea	9	8	12	12	8
Sao Tome Principe	10	5	15	15	5
Zimbabwe	11	16.5	9	16.5	9
Central African Republic	12	45	4	45	4
Dem. Rep. Congo	13	18	11	18	11
Uganda	14	12.5	17	17	12.5
Kenya	15.5	12.5	18.5	18.5	12.5
Zambia	15.5	23	14	23	14
Ethiopia	17	14	18.5	18.5	14
Mozambique	18	6	27	27	6
Eswatini	19.5	37	13	37	13
Rwanda	19.5	15	21	21	15
Cameroon	21	44	8	44	8
Cabo Verde	22	4	35	35	4
Comoros	23	10	32.5	32.5	10
Tanzania	24	9	36	36	9
Nigeria	25	43	16	43	16
Niger	26.5	38	22	38	22
Republic of Congo	26.5	36	23	36	23
Senegal	28	24.5	28	28	24.5
Eritea	29	40	20	40	20

Table 12: Sub-Saharan Africa-Rankings-continued

Country	EPI	EHS	EVE	$\vee$	$\wedge$
Benin	30	26	29	30	26
Angola	31	24.5	31	31	24.5
Togo	32	39	26	39	26
Mali	33	32	30	33	30
Guinea-Bissau	34	41	25	41	25
Dijibouti	35	28.5	37	37	28.5
Lesotho	36	46	24	46	24
Gambia	37	21	40	40	21
Maritania	38	30	38	38	30
Ghana	39	28.5	39	39	28.5
Burundi	40	20	42	42	20
Chad	41	42	34	42	34
Madagascar	42	19	45	45	19
Guinea	43	35	41	43	35
Cote d'lvoire	44	33	44	44	33
Sierra :Leone	45	34	43	45	34
Liberia	46	22	46	46	22
Col. Sum				1415	750.5

$M=\frac{750.5}{1415}=0.530$ and $S=\frac{2(0.530)}{1+0.530}=\frac{1.060}{1.530}=0.693$. Here $n=46, m=3, q=15$, and $r=1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)}=\frac{360+16}{1755++31}=\frac{376}{1786}=0.211$ and the smallest value S can be is $\frac{2(0.211)}{1+0.211}=\frac{0.422}{1.211}=0.348$. Now $\frac{M-0.211}{1-0.211}=\frac{0.530-0.211}{0.789}=\frac{0.319}{0.789}=0.404$ and $\frac{S-0.348}{1-0.348}=\frac{0.693-0.348}{0.652}=\frac{0.345}{0.652}=0.529$.

For $\widehat{m}=3$ and $m=2$, the value 0.530 converts to is 0.608 and the value 0.693 converts to is 0.770.

EH Rankings

$M=\frac{640.5}{1502}=0.426$ and $S=\frac{2(0.426)}{1+0.426}=\frac{0.852}{1.426}=0.597$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)}=\frac{360+16}{1755++31}=\frac{376}{1786}=0.211$ and the smallest value S can be is $\frac{2(0.211)}{1+0.211}=\frac{0.422}{1.211}=0.348$. Now $\frac{M-0.211}{1-0.211}=\frac{0.426-0.211}{1-0.211}=\frac{0.215}{0.789}=0.272$ and $\frac{S-0.348}{1-0.348}=\frac{0.597-0.348}{0.652}=\frac{0.249}{0.652}=0.382$.

For $\widehat{m}=3$ and $m=2$, the number 0.426 converts to is $\frac{5}{6}(0.426)+\frac{1}{6}=\frac{3.130}{6}=0.522$ and the number 0.597 converts to is $\frac{3}{4}(0.596)+\frac{1}{4}=\frac{2.788}{4}=0.622$.

EV Rankings

$M=\frac{399.5}{1808}=0.221$ and $S=\frac{2(0.221)}{1+0.221}=\frac{0.442}{1.221}=0.362$. Here $n=46, m=5, q=9$, and $r=1$. The smallest value M can be is $\frac{m\frac{(q+1)q}{2}+r(q+1)}{m\frac{2qn+q-q^2}{2}+r(n-q)}=\frac{225+10}{1890+37}=\frac{235}{1927}=0.122$ and the smallest value S can be is $\frac{2(0.122)}{1+0.122}=\frac{0.244}{1.122}=0.217$. Now $\frac{M-0.122}{1-0.122}=\frac{0.221-0.122}{0.888}=\frac{0.099}{0.888}=0.111$ and $\frac{S-0.217}{1-0.217}=\frac{0.362-0.217}{0.783}=\frac{0.145}{0.783}=0.185$.

For $\widehat{m}=5$ and $m=2$, the number 0.221 converts to is $\frac{9}{12}(0.221)+\frac{3}{12}=\frac{4.989}{12}=0.416$ and the number 0.362 converts to is $\frac{5}{8}(0.362)+\frac{3}{8}=\frac{4.81}{8}=0.601$.

5 Conclusion

We extended the fuzzy similarity measure of two rankings to any finite number of rankings. We provided a method to convert a measure for a finite number of rankings to a number that represents the number for two rankings. We apply our results to the sustainability ranking of countries by the Environmental Performance Index.

Conflict of Interest: The authors declare no conflict of interest.

References

[1] J. N. Mordeson and S. Mathew, Sustainable Development Goals: Analysis by Mathematics of Uncertainty, *Springer, Berlin*, (2021).

[2] J. N. Mordeson and S. Mathew, Mathematics of Uncertainty for Coping with World Challenges, Climate Change, World Hunger, Modern Slavery, Coronavirus, Human Trafficking, *Springer, Berlin*, (2021).

[3] X. Wang, B. De Baets, and E. Kerre, A comparative study of similarity measures, *Fuzzy Sets and Systems*, 73, (1995) 259-268.

[4] Z. A. Wending, J. W. Emerson, A. de Sherbinin, and D. C. Etsy, (lead authors), *Performance Index 2020*, Global Metrics for the environment: Ranking country performance on sustainability issues, Yale Center for Environmental Law & Policy, Yale University, Center for International Earth Science Information Network, Columbia University, (2020).

John N. Mordeson
Department of Mathematics
Emeritus Professor of Mathematics
Creighton University
Omaha, USA
E-mail: mordes@creighton.edu

Sunil Mathew
Department of Mathematics
Associate Professor of Mathematics
National Institute of Technology calicut
Calicut, India
E-mail: sm@nitc.ac.in