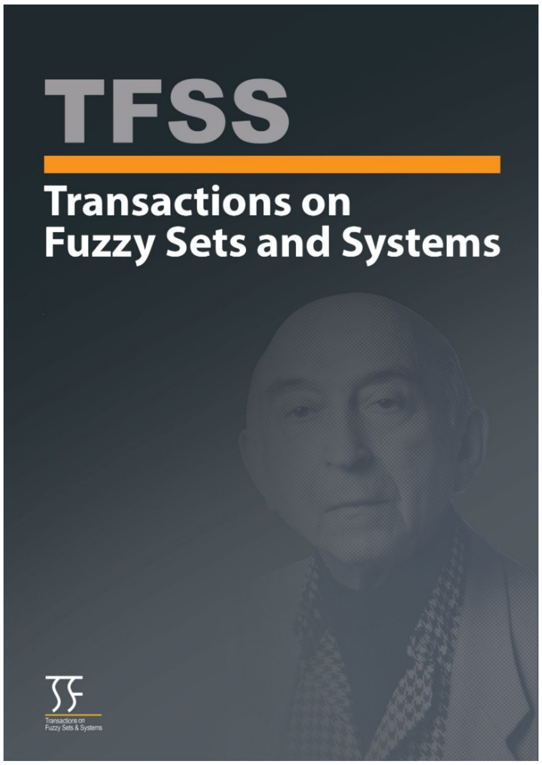
Transactions on Fuzzy Sets and Systems



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ISSN: **2821-0131**

https://sanad.iau.ir/journal/tfss/

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Vol.4, No.1, (2025), 129-158. DOI: https://doi.org/10.71602/tfss.2025.1189142

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Transactions on Fuzzy Sets and Systems (TFSS)
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Article Type: Original Research Article

Fuzzy MABAC Deep Learning for Diagnosis of Alzheimers Disease: Analysis of Complex Propositional Linear Diophantine Fuzzy Power Aggregation Insights

Zeeshan Ali

(This paper is dedicated to Professor "John N. Mordeson" on the occasion of his 91st birthday.)

Abstract. Alzheimers disease is an unpredictable and progressive neurodegenerative disorder that initially affects memory thinking and behavior. Some key features of Alzheimers disease are memory loss, cognitive decline, behavioral changes, disorientation, and physical symptoms. In this article, we design the procedure of a multi-attributive border approximation area comparison deep learning algorithm for the diagnosis of Alzheimers Disease. For this, first, we goal to design the model of complex propositional linear Diophantine fuzzy information with their basic operational laws. In addition, we analyze the model of complex propositional linear Diophantine fuzzy power average operator, complex propositional linear Diophantine fuzzy weighted power average operator, complex propositional linear Diophantine fuzzy power geometric operator, complex propositional linear Diophantine fuzzy weighted power geometric operator, and also initiate their major properties. Additionally, the key role of this paper is to arrange relevant from different sources for diagnosing Alzheimers disease under the consideration of the designed technique. Finally, we compare both (proposed and existing) ranking information to address the supremacy and strength of the designed models.

AMS Subject Classification 2020: 03B52; 68T27; 68T37; 94D05; 03E72
Keywords and Phrases: Alzheimers Disease, Complex Propositional linear Diophantine fuzzy sets, MABAC deep learning methods, Power aggregation operators.

1 Introduction

Diagnosing Alzheimers disease is very ambiguous and uncertain, connected with memory loss and changing behavior because of progressive neurodegenerative disorder [1]. The analysis of Alzheimers disease has been done by different scholars according to consider the information of crisp data [2], but to analyze the best one among the collection of data, we needed a soft and valuable technique that can help us in the evaluation of the procedure of decision-making models [3]. A lot of data has been lost in numerous decision-making procedures because of limited information and due to this, various problems are unsolved [4]. For this, Zadeh [5] prepared the fuzzy sets (FSs). FSs theory developed with just a function, called truth degree, defined from fixed sets to unit intervals. In addition, it is quite complex to deal with genuine life problems in the presence of just FS theory, because truth and falsity, yes and no, supporting and supporting against information are

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Received: 1 November 2024; **Revised**: 5 December 2024; **Accepted**: 10 December 2024; **Available Online**: 26 December 2024; **Published Online**: 7 May 2025.

How to cite: Zeeshan Ali. Fuzzy MABAC Deep Learning for Diagnosis of Alzheimers Disease: Analysis of Complex Propositional Linear Diophantine Fuzzy Power Aggregation Insights. *Transactions on Fuzzy Sets and Systems.* 2025; 4(1): 129-158. DOI: https://doi.org/10.71602/tfss.2025.1189142

the key parts of various real-life scenarios. For this, the model of FSs is not suitable, therefore, Atanassov [6, 7] designed the intuitionistic FSs (IFSs). IFSs designed with two different functions but with the same range, called truth degree and falsity degree with a characteristic that is the sum of both functions belonging to a unit interval.

In genuine life situations, all experts are independent and they are not restricted to following the condition of IFSs, because the provided information of experts will exceed form unit interval. For this, the model of Pythagorean FSs (PFSs) was designed by Yager [8]. PFSs are constructed with truth and falsity degrees with a characteristic that is the sum of the squares of both functions belonging to a unit interval. In addition, Yager [9] designed the q-rung orthopair FSs (q-ROFSs) in 2016. The model of q-ROFSs has also developed with truth and falsity information with a model that is the sum of the q-power of both functions belonging to the unit interval. These techniques are very useful and dominant because of their characteristics and due to this reason, many scholars have utilized them in various fields. Riaz and Hashmi [10] organized the linear Diophantine FSs (LDFSs) with a truth and falsity function $\left(\mathcal{F}^{\omega}_{rp}(\tau), \Finv^{\omega}_{rp}(\tau)\right)$ with parameters $\left(\zeta^{\omega}_{rp}(\tau), \Gamma^{\omega}_{rp}(\tau)\right)$.The prominent characteristics of LDFSs, such as $\zeta^{\omega}_{rp}(\tau) * \mathcal{F}^{\omega}_{rp}(\tau) + \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \in [0,1]$, where $\zeta^{\omega}_{rp}(\tau) + \Gamma^{\omega}_{rp}(\tau) \in [0,1]$. The model of LDFSs is more powerful and more dominant because of their features, the condition of LDFSs is developed based on linear Diophantine equation $ax + by = c$.

Ramot et al. [11] designed the complex FSs (CFSs), the function in CFSs is developed in the form of complex-valued information, where the real and unreal parts of the truth function are limited to unit interval. In various situations, we will cope with complex problems with the help of two-dimensional information, called the complex-valued truth function. Further, Alkouri and Salleh [12] designed the complex IFSs (CIFSs) with complex-valued functions, the condition of CIFSs is that the sum of both functions (for both functions, real and unreal) belongs to the unit interval. Ullah et al. [13] derived the complex PFSs (CPFSs), the projecting condition of CPFSs is the sum of the square of both functions (for both functions, real and unreal) belonging to the unit interval. In 2019, Liu et al. [14] invented the complex q-ROFSs (Cq-ROFSs), the projecting condition of Cq-ROFSs is the sum of the q-power of both functions (for both functions, real and unreal) belonging to the unit interval. In 2020, Ali and Mahmood [15] evaluated the Maclaurin Symmetric mean operators for Cq-ROFSs. In 2022, Kamaci [16] designed the invented the complex LDFSs (CLDFSs), such as $\tilde{H} = \left\{\left(\tau, \left(\mathcal{F}^{\omega}_{rp}(\tau), \mathcal{F}^{\omega}_{ip}(\tau)\right), \left(\Finv^{\omega}_{rp}(\tau), \Finv^{\omega}_{ip}(\tau)\right), \left(\zeta^{\omega}_{rp}(\tau), \zeta^{\omega}_{ip}(\tau)\right), \left(\Gamma^{\omega}_{rp}(\tau), \Gamma^{\omega}_{ip}(\tau)\right)\right) : \tau \in \mathbb{X}\right\}$, where the model of complex-valued membership (non-membership) function is defined by: $\left(\mathcal{F}^{\omega}_{rp}, \mathcal{F}^{\omega}_{ip}\right) : \mathbb{X} \to [0,1], \left(\left(\Finv^{\omega}_{rp}, \Finv^{\omega}_{ip}\right) : \mathbb{X} \to [0,1]\right)$ with $\zeta^{\omega}_{rp}(t) * \mathcal{F}^{\omega}_{rp}(t) + \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \in [0,1], \left(\zeta^{\omega}_{ip}(\tau) * \mathcal{F}^{\omega}_{ip}(\tau) + \Gamma^{\omega}_{ip}(\tau) * \Finv^{\omega}_{ip}(\tau) \in [0,1]\right)$ and $\zeta^{\omega}_{rp}(\tau) + \Gamma^{\omega}_{rp}(\tau) \in [0,1], \left(\zeta^{\omega}_{ip}(\tau) + \Gamma^{\omega}_{ip}(\tau) \in [0,1]\right)$, where, the model of complex-valued parameters is defined by: $\zeta^{\omega}_{rp}, \zeta^{\omega}_{ip}, \Gamma^{\omega}_{rp}, \Gamma^{\omega}_{ip} : \mathbb{X} \to [0,1]$ where $\varepsilon^{\omega}_{rp}(\tau) = 1 - \left(\zeta^{\omega}_{rp}(\tau) * \mathcal{F}^{\omega}_{rp}(\tau) + \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau)\right), \varepsilon^{\omega}_{ip}(\tau) = 1 - \left(\zeta^{\omega}_{ip}(\tau) * \mathcal{F}^{\omega}_{ip}(\tau) + \Gamma^{\omega}_{ip}(\tau) * \Finv^{\omega}_{ip}(\tau)\right)$, called the refusal function.

In 1980, Gottwald [17] designed the fuzzy propositional logic, a modified version of the FSs theory. In 1988, Atanassov [18] derived the intuitionistic fuzzy propositional calculus with two variants. In 2020, Wang et al. [19] presented the intuitionistic fuzzy propositional logic with novel plausible reasoning-based decision-making models. In 2024, Kahraman [20] introduced propositional PFSs with analytical hierarchal process extensions. In addition, Pamucar and Cirovic [21] invented the (multi-attributive border approximation area comparison) MABAC technique for classical set theory. Further, Yager [22] evaluated the power averaging (PoA) technique. In 2009, Xu and Yager [23] introduced the power geometric (PoG) technique for classical set theory. Jiang et al. [24] derived the power operators for IFSs. Wei and Lu [25] examined the power operators for PFSs. Garg et al. [26] initiated the power operators for Cq-ROFSs. Liu et al. [27] derived the power Dombi operators for CPFSs. Rani and Garg [28] evaluated the power operators for CIFSs. Ali [29] presented the power interaction operator for CIFSs. Ali et al. [30] described the power operators for complex intuitionistic fuzzy soft sets. Moslem [31] designed the parsimonious spherical fuzzy AHP models. Moslem et al. [32] evaluated the fuzzy analytical hierarchy model. Moslem and Pilla [33] invented the spherical fuzzy group decision-making techniques. Acharya et al. [34] designed the stability analysis for neutrosophic fuzzy information. Singh et al. [35] evaluated the malaria disease model in crisp and fuzzy information. Momena et al. [36] initiated the generalized dual hesitant hexagonal fuzzy decision-making techniques. Acharya et al. [37] constructed the neutrosophic differential equation with decision-making techniques. During the assessment of the existing models, we noticed or missed that the technique of complex propositional linear Diophantine fuzzy sets (CPLDFS) needed to be introduced because the above techniques are special cases of proposed models. In addition, we also noticed that to propose the technique of power operators and MABAC for CPLDFSs. The key and major contributions of the designed techniques are listed below:

1. To design the procedure of a MABAC deep learning algorithm for the diagnosis of Alzheimers Disease.

2. To design the model of complex propositional linear Diophantine fuzzy (CPLDF) information with their basic operational laws.

3. To analyze the model of CPLDF power average (CPLDFPoA) operator, CPLDF weighted power average (CPLDFWPoA) operator, CPLDF power geometric (CPLDFPoG) operator, CPLDF weighted power geometric (CPLDFWPoG) operator, and also initiate their major properties.

4. To arrange relevant from different sources for diagnosing Alzheimers disease under the consideration of the designed technique.

5. To compare both (proposed and existing) ranking information to address the supremacy and strength of the designed models. The graphical interpretation of the designed technique is derived in the form of Figure 1.

abstract of the proposed theory..png abstract of the proposed theory.bb

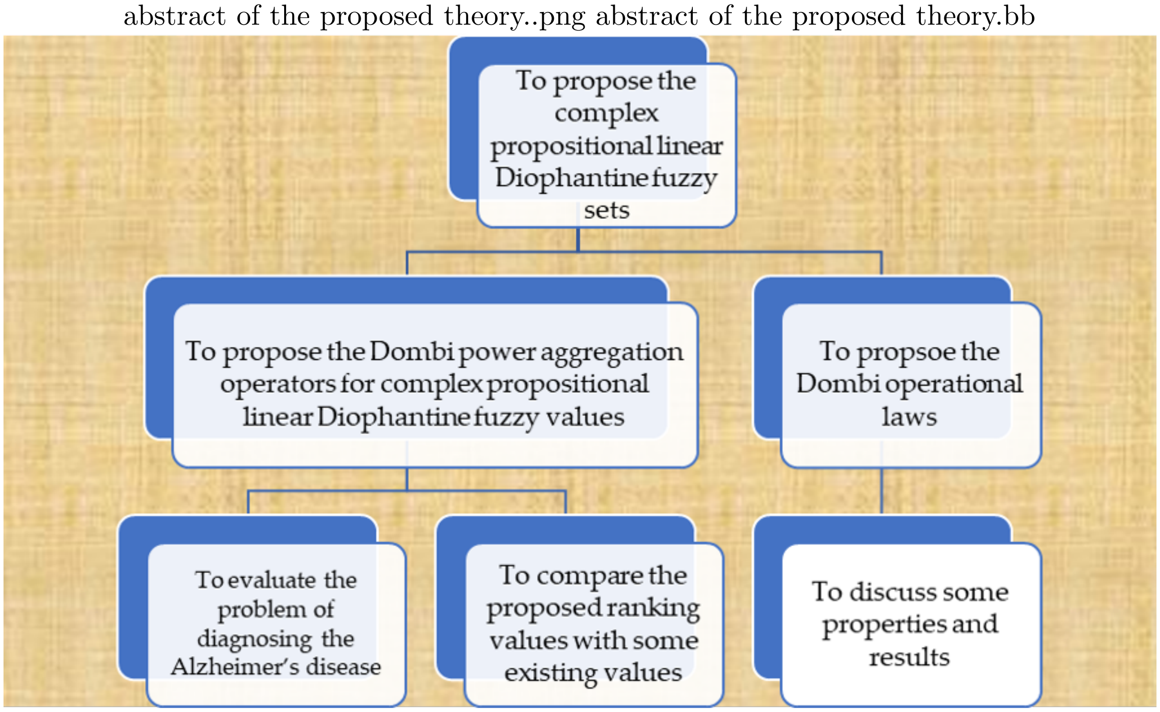

Figure 1: Graphical abstract of the proposed theory

This article is organized in the following ways: In Section 2, we explained the revised techniques of CLDFSs with basic definitions. In addition, we also reviewed the PA operator, and PG operator for the group of any positive integers. In Section 3, we designed the model of CPLDF information with their basic operational laws. In Section 4, we analyzed the model of CPLDFPoA, CPLDFWPoA, CPLDFPoG, and CPLDFWPoG operators, and also initiated their major properties. In Section 5, we designed the procedure of a MABAC deep learning algorithm for the diagnosis of Alzheimers Disease. In Section 6, we arranged relevant from different sources for diagnosing Alzheimers disease under the consideration of the designed technique. In Section 7, we compared both (proposed and existing) ranking information to address the supremacy and strength of the designed models. Some concluding remarks are described in Section 8.

2 Preliminaries

The model of complex linear Diophantine fuzzy information is the reformed version of numerous techniques and very reliable ideas for controlling imprecise and inexact data. This section goals to explain the revised techniques of CLDFSs with basic definitions. In addition, we also reviewed the PA operator, and PG operator for the group of any positive integers.

Definition 2.1. [16] A methodology of CLDFSs for $\mathbb{X}$ (universal set), is designed and deliberated by:

$$\tilde{H}=\left\{\left(\tau,\left(\mathcal{F}_{rp}^{\omega}(\tau),\mathcal{F}_{ip}^{\omega}(\tau)\right),\left(\Finv_{rp}^{\omega}(\tau),\Finv_{ip}^{\omega}(\tau)\right),\left(\zeta_{rp}^{\omega}(\tau),\zeta_{ip}^{\omega}(\tau)\right),\left(\Gamma_{rp}^{\omega}(\tau),\Gamma_{ip}^{\omega}(\tau)\right)\right):\tau\in\mathbb{X}\right\}$$

Where the model of complex-valued membership (non-membership) function is defined by:

$$\left(\mathcal{F}_{rp}^{\omega},\mathcal{F}_{ip}^{\omega}\right):\mathbb{X}\to[0,1],\left(\left(\Finv_{rp}^{\omega},\Finv_{ip}^{\omega}\right):\mathbb{X}\to[0,1]\right)$$

with $\zeta_{rp}^{\omega}(\tau)*\mathcal{F}_{rp}^{\omega}(\tau)+\Gamma_{rp}^{\omega}(\tau)*\Finv_{rp}^{\omega}(\tau)\in[0,1],\left(\zeta_{ip}^{\omega}(\tau)*\mathcal{F}_{ip}^{\omega}(\tau)+\Gamma_{ip}^{\omega}(\tau)*\Finv_{ip}^{\omega}(\tau)\in[0,1]\right)$ and $\zeta_{rp}^{\omega}(\tau)+\Gamma_{rp}^{\omega}(\tau)\in[0,1],\left(\zeta_{ip}^{\omega}(\tau)+\Gamma_{ip}^{\omega}(\tau)\in[0,1]\right)$, where, the model of complex-valued parameters is defined by: $\zeta_{rp}^{\omega},\zeta_{ip}^{\omega},\Gamma_{rp}^{\omega},\Gamma_{ip}^{\omega}:\mathbb{X}\to[0,1]$ where $\varepsilon_{rp}^{\omega}(\tau)=1-\left(\zeta_{rp}^{\omega}(\tau)*\mathcal{F}_{rp}^{\omega}(\tau)+\Gamma_{rp}^{\omega}(\tau)*\Finv_{rp}^{\omega}(\tau)\right),\varepsilon_{ip}^{\omega}(\tau)=1-\left(\zeta_{ip}^{\omega}(\tau)*\mathcal{F}_{ip}^{\omega}(\tau)+\Gamma_{ip}^{\omega}(\tau)*\Finv_{ip}^{\omega}(\tau)\right)$, called the refusal function. The simple version of CLDFN is mentioned in the following form, such as:

$$\tilde{H}_{\&}=\left(\left(\mathcal{F}_{rp}^{\omega_{\&}},\mathcal{F}_{ip}^{\omega_{\&}}\right),\left(\Finv_{rp}^{\omega_{\&}},\Finv_{ip}^{\omega_{\&}}\right),\left(\zeta_{rp}^{\omega_{\&}},\zeta_{ip}^{\omega_{\&}}\right),\left(\Gamma_{rp}^{\omega_{\&}},\Gamma_{ip}^{\omega_{\&}}\right)\right),\&=1,2,\cdots,\backepsilon$$

In addition, we goal to describe numerous operational laws for the above existing models, such as algebraic operational laws, briefly discussed below.

Definition 2.2. [16] Let $\tilde{H}_{\&}=\left(\left(\mathcal{F}_{rp}^{\omega_{\&}},\mathcal{F}_{ip}^{\omega_{\&}}\right),\left(\Finv_{rp}^{\omega_{\&}},\Finv_{ip}^{\omega_{\&}}\right),\left(\zeta_{rp}^{\omega_{\&}},\zeta_{ip}^{\omega_{\&}}\right),\left(\Gamma_{rp}^{\omega_{\&}},\Gamma_{ip}^{\omega_{\&}}\right)\right),\&=1,2$ be two CLDFN. Thus

$$\tilde{H}_1\oplus\tilde{H}_2=\begin{pmatrix}\left(\mathcal{F}_{rp}^{\omega_1}+\mathcal{F}_{rp}^{\omega_2}-\mathcal{F}_{rp}^{\omega_1}\mathcal{F}_{rp}^{\omega_2},\mathcal{F}_{ip}^{\omega_1}+\mathcal{F}_{ip}^{\omega_2}-\mathcal{F}_{ip}^{\omega_1}\mathcal{F}_{ip}^{\omega_2}\right),\left(\Finv_{rp}^{\omega_1}\Finv_{rp}^{\omega_2},\Finv_{ip}^{\omega_1}\Finv_{ip}^{\omega_2}\right),\\\left(\zeta_{rp}^{\omega_1}+\zeta_{rp}^{\omega_2}-\zeta_{rp}^{\omega_1}\zeta_{rp}^{\omega_2},\zeta_{ip}^{\omega_1}+\zeta_{ip}^{\omega_2}-\zeta_{ip}^{\omega_1}\zeta_{ip}^{\omega_2}\right),\left(\Gamma_{rp}^{\omega_1}\Gamma_{rp}^{\omega_2},\Gamma_{ip}^{\omega_1}\Gamma_{ip}^{\omega_2}\right)\end{pmatrix}$$

$$\tilde{H}_1\otimes\tilde{H}_2=\begin{pmatrix}\left(\mathcal{F}_{rp}^{\omega_1}\mathcal{F}_{rp}^{\omega_2},\mathcal{F}_{ip}^{\omega_1}\mathcal{F}_{ip}^{\omega_2}\right),\left(\Finv_{rp}^{\omega_1}+\Finv_{rp}^{\omega_2}-\Finv_{rp}^{\omega_1}\Finv_{rp}^{\omega_2},\Finv_{ip}^{\omega_1}+\Finv_{ip}^{\omega_2}-\Finv_{ip}^{\omega_1}\Finv_{ip}^{\omega_2}\right),\\\left(\zeta_{rp}^{\omega_1}\zeta_{rp}^{\omega_2},\zeta_{ip}^{\omega_1}\zeta_{ip}^{\omega_2}\right),\left(\Gamma_{rp}^{\omega_1}+\Gamma_{rp}^{\omega_2}-\Gamma_{rp}^{\omega_1}\Gamma_{rp}^{\omega_2},\Gamma_{ip}^{\omega_1}+\Gamma_{ip}^{\omega_2}-\Gamma_{ip}^{\omega_1}\Gamma_{ip}^{\omega_2}\right)\end{pmatrix}$$

$$\tilde{\eta}_{\Theta}\tilde{H}_{\&}=\begin{pmatrix}\left(1-\left(1-\mathcal{F}_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\mathcal{F}_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right),\left(\left(\text{Ⅎ}_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},\left(\text{Ⅎ}_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right),\\ \left(1-\left(1-\zeta_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\zeta_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right),\left(\left(\Gamma_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},\left(\Gamma_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right)\end{pmatrix}$$

$$\left(\tilde{H}_{\&}\right)^{\tilde{\eta}_{\Theta}}=\begin{pmatrix}\left(\left(\mathcal{F}_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},\left(\mathcal{F}_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right),\left(1-\left(1-\text{Ⅎ}_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\text{Ⅎ}_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right),\\ \left(\left(\zeta_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},\left(\zeta_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right),\left(1-\left(1-\Gamma_{rp}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\Gamma_{ip}^{\omega_{\&}}\right)^{\tilde{\eta}_{\Theta}}\right)\end{pmatrix}$$

Moreover, we target to revise the information of score value and accuracy value, for evaluating the relationship among any two complex linear Diophantine fuzzy numbers.

Definition 2.3. [16] Let $\tilde{H}_{\&}=\left(\left(\mathcal{F}_{rp}^{\omega_{\&}},\mathcal{F}_{ip}^{\omega_{\&}}\right),\left(\text{Ⅎ}_{rp}^{\omega_{\&}},\text{Ⅎ}_{ip}^{\omega_{\&}}\right),\left(\zeta_{rp}^{\omega_{\&}},\zeta_{ip}^{\omega_{\&}}\right),\left(\Gamma_{rp}^{\omega_{\&}},\Gamma_{ip}^{\omega_{\&}}\right)\right),\&=1$ be a CLDFN. Thus

$$SC\left(\tilde{H}_{\&}\right)=\frac{1}{4}\left(\left(\mathcal{F}_{rp}^{\omega_{\&}}+\mathcal{F}_{ip}^{\omega_{\&}}\right)-\left(\text{Ⅎ}_{rp}^{\omega_{\&}}+\text{Ⅎ}_{ip}^{\omega_{\&}}\right)+\left(\zeta_{rp}^{\omega_{\&}}+\zeta_{ip}^{\omega_{\&}}\right)-\left(\Gamma_{rp}^{\omega_{\&}}+\Gamma_{ip}^{\omega_{\&}}\right)\right)\in[-1,1]$$

$$AC\left(\tilde{H}_{\&}\right)=\frac{1}{4}\left(\left(\mathcal{F}_{rp}^{\omega_{\&}}+\mathcal{F}_{ip}^{\omega_{\&}}\right)+\left(\text{Ⅎ}_{rp}^{\omega_{\&}}+\text{Ⅎ}_{ip}^{\omega_{\&}}\right)+\left(\zeta_{rp}^{\omega_{\&}}+\zeta_{ip}^{\omega_{\&}}\right)+\left(\Gamma_{rp}^{\omega_{\&}}+\Gamma_{ip}^{\omega_{\&}}\right)\right)\in[0,1]$$

Thus, if $SC\left(\tilde{H}_1\right)>SC\left(\tilde{H}_2\right)\Rightarrow\tilde{H}_1>\tilde{H}_2$, then if $AC\left(\tilde{H}_1\right)>AC\left(\tilde{H}_2\right)\Rightarrow\tilde{H}_1>\tilde{H}_2$. Further, we goal to discuss the technique of PoA and PoG techniques.

Definition 2.4. [22, 23] Let $\tilde{H}_{\&},\&=1,2,\cdots,\ni$, be a group of non-negative information. Then

$$PoA\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right)=\frac{\left(1+\tilde{\eta}\left(\tilde{H}_1\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_1\oplus\frac{\left(1+\tilde{\eta}\left(\tilde{H}_2\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_2\oplus\cdots\oplus\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\ni}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_{\ni}$$
$$=\sum_{\&=1}^{\ni}\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_{\&}$$

signified the PoA operators, and the technique

$$PoG\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right)=\left(\tilde{H}_1\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_1\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}\otimes\left(\tilde{H}_2\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_2\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}\otimes\cdots\otimes\left(\tilde{H}_{\ni}\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\ni}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}$$
$$=\prod_{\&=1}^{\ni}\left(\tilde{H}_{\&}\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}$$

called PoG operator with $\tilde{\eta}\left(\tilde{H}_{\&}\right)=\sum_{i\neq\&=1}^{\ni}S\left(\tilde{H}_i,\tilde{H}_{\&}\right)$, and $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)=1-D\left(\tilde{H}_i,\tilde{H}_{\&}\right)$, thus

1. $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)\in[0,1]$.
2. $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)=S\left(\tilde{H}_{\&},\tilde{H}_i\right)$.
3. When $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)\geq S\left(\tilde{H}_k,\tilde{H}_l\right)$, then $D\left(\tilde{H}_i,\tilde{H}_{\&}\right)\leq D\left(\tilde{H}_k,\tilde{H}_l\right)$.

3 CPLDFSs: Complex Propositional Linear Diophantine Fuzzy Sets

This section goals to explain the new techniques of CPLDFSs with basic definitions. Further, we designed some algebraic operational laws for CPLDFSs.

Definition 3.1. A methodology of CPLDFSs for $\mathbb{X}$ (universal set), is designed and deliberated by:

$$\tilde{H} = \left\{ \left(\tau, \left(\mathcal{F}^{\omega}_{rp}(\tau), \mathcal{F}^{\omega}_{ip}(\tau) \right), \left(\Finv^{\omega}_{rp}(\tau), \Finv^{\omega}_{ip}(\tau) \right), \left(\zeta^{\omega}_{rp}(\tau), \zeta^{\omega}_{ip}(\tau) \right), \left(\Gamma^{\omega}_{rp}(\tau), \Gamma^{\omega}_{ip}(\tau) \right) \right) : \tau \in \mathbb{X} \right\}$$

In addition, we define the truth and parameter function according to their real and imaginary parts, such as

$$\mathcal{F}^{\omega}_{rp}(\tau) = \mathbb{L}^{1}_{rp} \left(\Finv^{\omega}_{rp}(\tau) \right), \left(\mathcal{F}^{\omega}_{ip}(\tau) = \mathbb{L}^{1}_{ip} \left(\Finv^{\omega}_{ip}(\tau) \right) \right)$$

and

$$\zeta^{\omega}_{rp}(\tau) = \mathbb{L}^{2}_{rp} \left(\Gamma^{\omega}_{rp}(\tau) \right), \left(\zeta^{\omega}_{ip}(\tau) = \mathbb{L}^{2}_{ip} \left(\Gamma^{\omega}_{ip}(\tau) \right) \right)$$

Then

$$\mathbb{L}^{2}_{rp} \left(\Gamma^{\omega}_{rp}(\tau) \right) * \mathbb{L}^{1}_{rp} \left(\Finv^{\omega}_{rp}(\tau) \right) + \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \leq 1$$
$$\Rightarrow \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \left(1 + \mathbb{L}^{1}_{rp}\mathbb{L}^{2}_{rp} \right) \leq 1 \Rightarrow \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \leq \frac{1}{1 + \mathbb{L}^{1}_{rp}\mathbb{L}^{2}_{rp}}$$

Similarly, we have imaginary parts, such as

$$\Gamma^{\omega}_{ip}(\tau) * \Finv^{\omega}_{ip}(\tau) \leq \frac{1}{1 + \mathbb{L}^{1}_{ip}\mathbb{L}^{2}_{ip}}$$

thus

$$\varepsilon^{\omega}_{rp}(\tau) = 1 - \left(\zeta^{\omega}_{rp}(\tau) * \mathcal{F}^{\omega}_{rp}(\tau) + \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \right) = 1 - \left(\mathbb{L}^{2}_{rp} \left(\Gamma^{\omega}_{rp}(\tau) \right) * \mathbb{L}^{1}_{rp} \left(\Finv^{\omega}_{rp}(\tau) \right) + \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \right)$$
$$\Rightarrow 1 - \left(\Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \left(1 + \mathbb{L}^{1}_{rp}\mathbb{L}^{2}_{rp} \right) \right)$$

then

$$1 - \varepsilon^{\omega}_{rp}(\tau) = \Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) \left(1 + \mathbb{L}^{1}_{rp}\mathbb{L}^{2}_{rp} \right)$$

and

$$\Gamma^{\omega}_{rp}(\tau) * \Finv^{\omega}_{rp}(\tau) = \frac{1 - \varepsilon^{\omega}_{rp}(\tau)}{\left(1 + \mathbb{L}^{1}_{rp}\mathbb{L}^{2}_{rp} \right)}$$

Similarly, we have

$$\Gamma^{\omega}_{ip}(\tau) * \Finv^{\omega}_{ip}(\tau) = \frac{1 - \varepsilon^{\omega}_{ip}(\tau)}{\left(1 + \mathbb{L}^{1}_{ip}\mathbb{L}^{2}_{ip} \right)}$$

But if we use the condition of IFSs, thus we have

$$\mathbb{L}^{1}_{rp} \left(\Finv^{\omega}_{rp}(\tau) \right) + \Finv^{\omega}_{rp}(\tau) \leq 1$$
$$\Rightarrow \Finv^{\omega}_{rp}(\tau) \left(1 + \mathbb{L}^{1}_{rp} \right) \leq 1 \Rightarrow \Finv^{\omega}_{rp}(\tau) \leq \frac{1}{1 + \mathbb{L}^{1}_{rp}}$$

Similarly, we have imaginary parts, such as

$$\Finv_{ip}^{\omega}(\tau) \leq \frac{1}{1+\mathbb{L}_{ip}^{1}}$$

Thus, we have the condition of refusal function in IFSs, such as

$$\varepsilon_{rp}^{\omega}(\tau) = 1-\left(\mathcal{F}_{rp}^{\omega}(\tau)+\Finv_{rp}^{\omega}(\tau)\right) = 1-\left(\mathbb{L}_{rp}^{1}\left(\Finv_{rp}^{\omega}(\tau)\right)+\Finv_{rp}^{\omega}(\tau)\right) = 1-\left(\Finv_{rp}^{\omega}(\tau)\left(1+\mathbb{L}_{rp}^{1}\right)\right)$$

then

$$1-\varepsilon_{rp}^{\omega}(\tau) = \Finv_{rp}^{\omega}(\tau)\left(1+\mathbb{L}_{rp}^{1}\right)$$

and

$$\Finv_{rp}^{\omega}(\tau) = \frac{1-\varepsilon_{rp}^{\omega}(\tau)}{\left(1+\mathbb{L}_{rp}^{1}\right)}, \left(\zeta_{rp}^{\omega}(\tau) = \frac{1-\varepsilon_{rp}^{\omega}(\tau)}{\left(1+\mathbb{L}_{rp}^{2}\right)}\right)$$

Similarly, we have

$$\Finv_{ip}^{\omega}(\tau) = \frac{1-\varepsilon_{ip}^{\omega}(\tau)}{\left(1+\mathbb{L}_{rp}^{1}\right)}, \left(\zeta_{ip}^{\omega}(\tau) = \frac{1-\varepsilon_{ip}^{\omega}(\tau)}{\left(1+\mathbb{L}_{ip}^{2}\right)}\right)$$

if $\varepsilon_{rp}^{\omega}(\tau) = \varepsilon_{ip}^{\omega}(\tau) = 0$, thus $\Finv_{rp}^{\omega}(\tau) = \frac{1}{\left(1+\mathbb{L}_{rp}^{1}\right)}$ and $\Finv_{ip}^{\omega}(\tau) = \frac{1}{\left(1+\mathbb{L}_{ip}^{1}\right)}$. Then

$$\mathcal{F}_{rp}^{\omega}(\tau) = \mathbb{L}_{rp}^{1}\left(\frac{1}{\left(1+\mathbb{L}_{rp}^{1}\right)}\right), \left(\mathcal{F}_{ip}^{\omega}(\tau) = \mathbb{L}_{ip}^{1}\left(\frac{1}{\left(1+\mathbb{L}_{ip}^{1}\right)}\right)\right)$$

and

$$\zeta_{rp}^{\omega}(\tau) = \mathbb{L}_{rp}^{2}\left(\frac{1}{\left(1+\mathbb{L}_{rp}^{2}\right)}\right), \left(\zeta_{ip}^{\omega}(\tau) = \mathbb{L}_{ip}^{2}\left(\frac{1}{\left(1+\mathbb{L}_{ip}^{2}\right)}\right)\right)$$

Then

$$\tilde{H} = \left\{\left(\begin{array}{c}\tau, \left(\left(\mathbb{L}_{rp}^{1}\left(\frac{1}{1+\mathbb{L}_{rp}^{1}}\right), \mathbb{L}_{ip}^{1}\left(\frac{1}{1+\mathbb{L}_{ip}^{1}}\right)\right), \left(\frac{1}{1+\mathbb{L}_{rp}^{1}}, \frac{1}{1+\mathbb{L}_{rp}^{1}}\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2}\left(\frac{1}{1+\mathbb{L}_{rp}^{2}}\right), \mathbb{L}_{ip}^{2}\left(\frac{1}{1+\mathbb{L}_{ip}^{2}}\right)\right), \left(\frac{1}{1+\mathbb{L}_{rp}^{2}}, \frac{1}{1+\mathbb{L}_{rp}^{2}}\right)\right)\end{array}\right) : \tau \in \mathbb{X}\right\}$$

Thus, we have the following final shape, such as

$$\tilde{H}_{\&} = \left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right)\end{array}\right), \& = 1, 2, \cdots, \backepsilon.$$

Definition 3.2. For any $\tilde{H}_{\&} = \begin{pmatrix} \left(\left(\left(\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \mathbb{L}^{1_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{ip}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{ip}}\right)\right)\right)\right), \\ \left(\left(\left(\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \mathbb{L}^{2_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{ip}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{ip}}\right)\right)\right)\right) \end{pmatrix}, \& = 1, 2$, we have

$$\tilde{H}_1 \oplus \tilde{H}_2 = \begin{pmatrix} \begin{pmatrix} \mathbb{L}^{1_1}_{rp}\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{1_1}_{rp}}\right) + \mathbb{L}^{1_2}_{rp}\left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{1_2}_{rp}}\right) - \mathbb{L}^{1_1}_{rp}\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{1_1}_{rp}}\right)\mathbb{L}^{1_2}_{rp}\left(\frac{1-\varepsilon_{rp}}{1+\mathbb{L}^{1_2}_{rp}}\right), \\ \mathbb{L}^{1_1}_{ip}\left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{1_1}_{ip}}\right) + \mathbb{L}^{1_2}_{ip}\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{1_2}_{ip}}\right) - \mathbb{L}^{1_1}_{ip}\left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{1_1}_{ip}}\right)\mathbb{L}^{1_2}_{ip}\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{1_2}_{ip}}\right) \end{pmatrix}, \\ \left(\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{1_1}_{rp}}\right)\left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{1_2}_{rp}}\right), \left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{1_1}_{ip}}\right)\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{1_2}_{ip}}\right)\right), \\ \begin{pmatrix} \mathbb{L}^{2_1}_{rp}\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{2}_{rp}}\right) + \mathbb{L}^{2_2}_{rp}\left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{2_2}_{rp}}\right) - \mathbb{L}^{2_1}_{rp}\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{2_1}_{rp}}\right)\mathbb{L}^{2_2}_{rp}\left(\frac{1-\varepsilon_{rp}}{1+\mathbb{L}^{2_2}_{rp}}\right), \\ \mathbb{L}^{2_1}_{ip}\left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{2_1}_{ip}}\right) + \mathbb{L}^{2_2}_{ip}\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{2_2}_{ip}}\right) - \mathbb{L}^{2_1}_{ip}\left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{2_1}_{ip}}\right)\mathbb{L}^{2_2}_{ip}\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{2_2}_{ip}}\right) \end{pmatrix}, \\ \left(\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{2_1}_{rp}}\right)\left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{2_2}_{rp}}\right), \left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{2_1}_{ip}}\right)\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{2_2}_{ip}}\right)\right) \end{pmatrix}$$

$$\tilde{H}_1 \otimes \tilde{H}_2 = \begin{pmatrix} \left(\mathbb{L}^{1_1}_{rp}\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{1_1}_{rp}}\right)\mathbb{L}^{1_2}_{rp}\left(\frac{1-\varepsilon_{rp}}{1+\mathbb{L}^{1_2}_{rp}}\right), \mathbb{L}^{1_1}_{ip}\left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{1_1}_{ip}}\right)\mathbb{L}^{1_2}_{ip}\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{1_2}_{ip}}\right)\right), \\ \begin{pmatrix} \left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{1_1}_{rp}}\right) + \left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{1_2}_{rp}}\right) - \left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{1_1}_{rp}}\right)\left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{1_2}_{rp}}\right), \\ \left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{1_1}_{ip}}\right) + \left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{1_2}_{ip}}\right) - \left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{1_1}_{ip}}\right)\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{1_2}_{ip}}\right) \end{pmatrix}, \\ \left(\mathbb{L}^{2_1}_{rp}\left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{2_1}_{rp}}\right)\mathbb{L}^{2_2}_{rp}\left(\frac{1-\varepsilon_{rp}}{1+\mathbb{L}^{2_2}_{rp}}\right), \mathbb{L}^{2_1}_{ip}\left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{2_1}_{ip}}\right)\mathbb{L}^{2_2}_{ip}\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{2_2}_{ip}}\right)\right), \\ \begin{pmatrix} \left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{2_1}_{rp}}\right) + \left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{2_2}_{rp}}\right) - \left(\frac{1-\varepsilon_{rp_1}^{\omega}}{1+\mathbb{L}^{2_1}_{rp}}\right)\left(\frac{1-\varepsilon_{rp_2}^{\omega}}{1+\mathbb{L}^{2_2}_{rp}}\right), \\ \left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{2_1}_{ip}}\right) + \left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{2_2}_{ip}}\right) - \left(\frac{1-\varepsilon_{ip_1}^{\omega}}{1+\mathbb{L}^{2_1}_{ip}}\right)\left(\frac{1-\varepsilon_{ip_2}^{\omega}}{1+\mathbb{L}^{2_2}_{ip}}\right) \end{pmatrix} \end{pmatrix}$$

$$\tilde{\eta}_{\Theta}\tilde{H}_{\&} = \begin{pmatrix} \left(\left(1-\left(1-\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, 1-\left(1-\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right), \left(\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, \left(\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right), \\ \left(\left(1-\left(1-\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, 1-\left(1-\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right), \left(\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, \left(\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right) \end{pmatrix}$$

$$\left(\tilde{H}_{\&}\right)^{\tilde{\eta}_{\Theta}} = \begin{pmatrix} \left(\left(\left(\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, \left(\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right), \left(1-\left(1-\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, 1-\left(1-\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right), \\ \left(\left(\left(\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, \left(\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right), \left(1-\left(1-\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}, 1-\left(1-\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right) \end{pmatrix}$$

Definition 3.3. For any $\tilde{H}_{\&} = \left(\begin{array}{c}\left(\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{array}\right), \& = 1,$ we have

$$S\left(\tilde{H}_{\&}\right) = \frac{1}{4}\left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right) + \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right) - \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right) + \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right) + \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right) + \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right) - \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right) + \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{array}\right) \in [-1, 1]$$

$$A\left(\tilde{H}_{\&}\right) = \frac{1}{4}\left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right) + \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right) + \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right) + \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right) + \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right) + \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right) + \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right) + \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{array}\right) \in [-1, 1]$$

Thus, if $SC\left(\tilde{H}_1\right) > SC\left(\tilde{H}_2\right) \Rightarrow \tilde{H}_1 > \tilde{H}_2$, if $SC\left(\tilde{H}_1\right) = SC\left(\tilde{H}_2\right)$, then if $AC\left(\tilde{H}_1\right) > AC\left(\tilde{H}_2\right) \Rightarrow \tilde{H}_1 > \tilde{H}_2$.

4 CPLDF Power Aggregation Insights

This section is famous for the analysis of the power operators for CPLDFSs, called the CPLDFPoA operator, CPLDFWPoA operator, CPLDFPoG operator, CPLDFWPoG operator, and their genuine properties.

Definition 4.1. Let $\tilde{H}_{\&} = \left(\begin{array}{c}\left(\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{array}\right), \& = 1, 2, \cdots, \ni,$ be a group of CPLDF information. Then

$$CPLDFPoA\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) = \frac{\left(1+\tilde{\eta}\left(\tilde{H}_1\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_1 \oplus \frac{\left(1+\tilde{\eta}\left(\tilde{H}_2\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_2 \oplus \cdots$$
$$\oplus \frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\ni}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_{\ni} = \oplus_{\&=1}^{\ni}\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_{\&}$$

Signified the CPLDFPoA operators with $\tilde{\eta}\left(\tilde{H}_{\&}\right) = \sum_{i\neq\&=1}^{\ni} S\left(\tilde{H}_i, \tilde{H}_{\&}\right)$, and $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) = 1 - D\left(\tilde{H}_i, \tilde{H}_{\&}\right)$, thus

1. $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) \in [0, 1]$.
2. $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) = S\left(\tilde{H}_{\&}, \tilde{H}_i\right)$.
3. When $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) \geq S\left(\tilde{H}_k, \tilde{H}_l\right)$, then $D\left(\tilde{H}_i, \tilde{H}_{\&}\right) \leq D\left(\tilde{H}_k, \tilde{H}_l\right)$.

Theorem 4.2. *Let* $\tilde{H}_{\&} = \left(\begin{array}{c} \left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right) \end{array}\right), \& = 1, 2, \cdots, \ni.,$ *be a group of CPLDF information. Then, using the information in Def. (6), we evaluate the information in Def. (8), such as*

$$CPLDFPoA\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right)$$

$$= \left(\begin{array}{c} \left(\begin{array}{c} \left(1 - \prod_{\&=1}^{\ni}\left(1 - \mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, 1 - \prod_{\&=1}^{\ni}\left(1 - \mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right), \\ \left(\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, \prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right) \end{array}\right), \\ \left(\begin{array}{c} \left(1 - \prod_{\&=1}^{\ni}\left(1 - \mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, 1 - \prod_{\&=1}^{\ni}\left(1 - \mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right), \\ \left(\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, \prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right) \end{array}\right) \end{array}\right)$$

Property 4.3. Let $\tilde{H}_{\&} = \left(\begin{array}{c} \left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right) \end{array}\right), \& = 1, 2, \cdots, \ni.,$ be a group of CPLDF information.

1. If $\tilde{H}_{\&} = \tilde{H}, \& = 1, 2, \cdots, \ni$, thus $CPLDFPoA\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) = \tilde{H}$, called the idempotency.
2. If $\tilde{H}_{\&} \leq \tilde{H}'_{\&}, \& = 1, 2, \cdots, \ni$, thus $CPLDFPoA\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) \leq CPLDFPoA\left(\tilde{H}'_1, \tilde{H}'_2, \cdots, \tilde{H}'_{\ni}\right)$, called the monotonicity.
3. If $\tilde{H}_{-} = min\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right)$, and $\tilde{H}_{+} = max\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right), \& = 1, 2, \cdots, \ni$, thus $\tilde{H}_{-} \leq CPLDFPoA\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) \leq \tilde{H}_{+}$, called the boundedness.

Definition 4.4. Let $\tilde{H}_{\&} = \left(\begin{array}{c} \left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right) \end{array}\right), \& = 1, 2, \cdots, \ni.,$ be a group of CPLDF information. Then

$$CPLDFWPoA\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) = \frac{\aleph_1\left(1 + \tilde{\eta}\left(\tilde{H}_1\right)\right)}{\sum_{\&=1}^{\ni}\aleph_{\&}\left(1 + \tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_1 \oplus \frac{\aleph_2\left(1 + \tilde{\eta}\left(\tilde{H}_2\right)\right)}{\sum_{\&=1}^{\ni}\aleph_{\&}\left(1 + \tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_2 \oplus \cdots$$

$$\oplus \frac{\aleph_{\ni}\left(1 + \tilde{\eta}\left(\tilde{H}_{\ni}\right)\right)}{\sum_{\&=1}^{\ni}\aleph_{\&}\left(1 + \tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_{\ni} = \oplus_{\&=1}^{\ni}\frac{\aleph_{\&}\left(1 + \tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\aleph_{\&}\left(1 + \tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}\tilde{H}_{\&}$$

Signified the CPLDFWPoA operators with $\tilde{\eta}\left(\tilde{H}_{\&}\right)=\sum_{i\neq\&=1}^{\ni} S\left(\tilde{H}_i,\tilde{H}_{\&}\right)$, and $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)=1-D\left(\tilde{H}_i,\tilde{H}_{\&}\right)$, thus

4. $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)\in[0,1]$.
5. $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)=S\left(\tilde{H}_{\&},\tilde{H}_i\right)$.
6. When $S\left(\tilde{H}_i,\tilde{H}_{\&}\right)\geq S\left(\tilde{H}_k,\tilde{H}_l\right)$, then $D\left(\tilde{H}_i,\tilde{H}_{\&}\right)\leq D\left(\tilde{H}_k,\tilde{H}_l\right)$.

Where $\sum_{\&=1}^{\ni}\aleph_{\&}=1$, called weight vector.

Theorem 4.5. *Let* $\tilde{H}_{\&}=\left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right),\\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right)\end{array}\right),\&=1,2,\cdots,\ni.$ *be a group of CPLDF information. Then, using the information in Def. (6), we evaluate the information in Def. (9), such as*

$$CPLDFWPoA\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right)$$

$$=\left(\begin{array}{c}\left(\begin{array}{c}\left(1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}},1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right),\\ \left(\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}},\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right)\end{array}\right),\\ \left(\begin{array}{c}\left(1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}},1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right),\\ \left(\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}},\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right)\end{array}\right)\end{array}\right)$$

Property 4.6. Let $\tilde{H}_{\&}=\left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right),\\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right)\end{array}\right),\&=1,2,\cdots,\ni.$, be a group of CPLDF information.

1. If $\tilde{H}_{\&}=\tilde{H},\&=1,2,\cdots,\ni$, thus $CPLDFWPoA\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right)=\tilde{H}$, called the idempotency.
2. If $\tilde{H}_{\&}\leq\tilde{H}'_{\&},\&=1,2,\cdots,\ni$, thus $CPLDFWPoA\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right)\leq CPLDFWPoA\left(\tilde{H}'_1,\tilde{H}'_2,\cdots,\tilde{H}'_{\ni}\right)$, called the monotonicity.
3. If $\tilde{H}_{-}=min\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right)$, and $\tilde{H}_{+}=max\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right),\&=1,2,\cdots,\ni$, thus $\tilde{H}_{-}\leq CPLDFWPoA\left(\tilde{H}_1,\tilde{H}_2,\cdots,\tilde{H}_{\ni}\right)\leq\tilde{H}_{+}$, called the boundedness.

Definition 4.7. Let $\tilde{H}_{\&} = \left(\begin{matrix} \left(\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{matrix}\right), \& = 1, 2, \cdots, \ni.$, be a group of CPLDF information. Then

$$CPLDFPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) = \tilde{H}_1^{\frac{(1+\tilde{\eta}(\tilde{H}_1))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}} \otimes \tilde{H}_2^{\frac{(1+\tilde{\eta}(\tilde{H}_2))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}} \otimes \cdots \otimes \tilde{H}_{\ni}^{\frac{(1+\tilde{\eta}(\tilde{H}_{\ni}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}} = \otimes_{\&=1}^{\ni}\tilde{H}_{\&}^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}$$

Signified the CPLDFPoG operators with $\tilde{\eta}\left(\tilde{H}_{\&}\right) = \sum_{i\neq\&=1}^{\ni} S\left(\tilde{H}_i, \tilde{H}_{\&}\right)$, and $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) = 1 - D\left(\tilde{H}_i, \tilde{H}_{\&}\right)$, thus

1. $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) \in [0, 1]$.
2. $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) = S\left(\tilde{H}_{\&}, \tilde{H}_i\right)$.
3. When $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) \geq S\left(\tilde{H}_k, \tilde{H}_l\right)$, then $D\left(\tilde{H}_i, \tilde{H}_{\&}\right) \leq D\left(\tilde{H}_k, \tilde{H}_l\right)$.

Theorem 4.8. *Let* $\tilde{H}_{\&} = \left(\begin{matrix} \left(\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{matrix}\right), \& = 1, 2, \cdots, \ni.$, *be a group of CPLDF information. Then, using the information in Def. (6), we evaluate the information in Def. (10), such as*

$$CPLDFPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right)$$

$$= \left(\begin{matrix} \left(\begin{matrix} \left(\prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, \prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right), \\ \left(1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, 1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right)\end{matrix}\right), \\ \left(\begin{matrix} \left(\prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, \prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right), \\ \left(1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, 1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right)\end{matrix}\right)\end{matrix}\right)$$

Property 4.9. Let $\tilde{H}_{\&} = \left(\begin{matrix} \left(\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{matrix}\right), \& = 1, 2, \cdots, \ni.$, be a group of CPLDF information.

1. If $\tilde{H}_{\&} = \tilde{H}, \& = 1, 2, \cdots, \ni$, thus $CPLDFPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) = \tilde{H}$, called the idempotency.
2. If $\tilde{H}_{\&} \leq \tilde{H}'_{\&}, \& = 1, 2, \cdots, \ni$, thus $CPLDFPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) \leq CPLDFPoG\left(\tilde{H}'_1, \tilde{H}'_2, \cdots, \tilde{H}'_{\ni}\right)$, called the monotonicity.

3. If $\tilde{H}_{-} = min\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right)$, and $\tilde{H}_{+} = max\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right), \& = 1, 2, \cdots, \ni$, thus $\tilde{H}_{-} \leq CPLDFPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) \leq \tilde{H}_{+}$, called the boundedness.

Definition 4.10. Let $\tilde{H}_{\&} = \left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{array}\right), \& = 1, 2, \cdots, \ni.$, be a group of CPLDF information. Then

$$CPLDFWPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right) = \tilde{H}_1^{\frac{\aleph_1(1+\tilde{\eta}(\tilde{H}_1))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}} \otimes \tilde{H}_2^{\frac{\aleph_2(1+\tilde{\eta}(\tilde{H}_2))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}} \otimes \cdots \otimes \tilde{H}_{\ni}^{\frac{\aleph_{\ni}(1+\tilde{\eta}(\tilde{H}_{\ni}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}$$
$$= \otimes_{\&=1}^{\ni}\tilde{H}_{\&}^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}$$

Signified the CPLDFWPoG operators with $\tilde{\eta}\left(\tilde{H}_{\&}\right) = \sum_{i\neq\&=1}^{\ni} S\left(\tilde{H}_i, \tilde{H}_{\&}\right)$, and $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) = 1 - D\left(\tilde{H}_i, \tilde{H}_{\&}\right)$, thus

4. $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) \in [0, 1]$.
5. $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) = S\left(\tilde{H}_{\&}, \tilde{H}_i\right)$.
6. When $S\left(\tilde{H}_i, \tilde{H}_{\&}\right) \geq S\left(\tilde{H}_k, \tilde{H}_l\right)$, then $D\left(\tilde{H}_i, \tilde{H}_{\&}\right) \leq D\left(\tilde{H}_k, \tilde{H}_l\right)$.

Where $\sum_{\&=1}^{\ni}\aleph_{\&} = 1$, called weight vector.

Theorem 4.11. *Let* $\tilde{H}_{\&} = \left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{array}\right), \& = 1, 2, \cdots, \ni.$, *be a group of CPLDF information. Then, using the information in Def. (6), we evaluate the information in Def. (11), such as*

$$CPLDFWPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\ni}\right)$$
$$= \left(\begin{array}{c}\left(\begin{array}{c}\left(\prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, \prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right), \\ \left(1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, 1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right)\end{array}\right), \\ \left(\begin{array}{c}\left(\prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, \prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right), \\ \left(1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}\aleph_{\&}(1+\tilde{\eta}(\tilde{H}_{\&}))}}, 1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_{\&}))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_{\&}))}}\right)\end{array}\right)\end{array}\right)$$

Property 4.12. Let $\tilde{H}_{\&} = \begin{pmatrix} \left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right), \\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}_{\&}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right), \left(\frac{1-\varepsilon_{ip}{}_{\&}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right) \end{pmatrix}, \& = 1, 2, \cdots, \backepsilon.$, be a group of CPLDF information.

1. If $\tilde{H}_{\&} = \tilde{H}, \& = 1, 2, \cdots, \backepsilon$, thus $CPLDFWPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\backepsilon}\right) = \tilde{H}$, called the idempotency.

2. If $\tilde{H}_{\&} \leq \tilde{H}'_{\&}, \& = 1, 2, \cdots, \backepsilon$, thus $CPLDFWPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\backepsilon}\right) \leq CPLDFWPoG\left(\tilde{H}'_1, \tilde{H}'_2, \cdots, \tilde{H}'_{\backepsilon}\right)$, called the monotonicity.

3. If $\tilde{H}_{-} = min\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\backepsilon}\right)$, and $\tilde{H}_{+} = max\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\backepsilon}\right), \& = 1, 2, \cdots, \backepsilon$, thus $\tilde{H}_{-} \leq CPLDFWPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_{\backepsilon}\right) \leq \tilde{H}_{+}$, called the boundedness.

5 CPLDF MABAC Techniques

In this section, we analyze the MABAC technique for designed operators, called CPLDFPoA operator and CPLDFPoG operator to deliberate the consistency of the suggested theory. The graphical interpretation of the proposed application is given in the form of Figure 2.

For this, we have a group of alternatives $\tilde{H}_1, \tilde{H}_2, ..., \tilde{H}_{\backepsilon}$ with $A_1, A_2, ..., A_n$, called attributes for each alternative with the same order of weighted information, such as $\aleph_{\&} \in [0, 1]$ with $\sum_{\&=1}^{\backepsilon} \aleph_{\&} = 1$, thus, we design a matrix by putting their information in the form of CPLDFSs, such as

$$\tilde{H} = \left\{\left(\tau, \left(\mathcal{F}_{rp}^{\omega}(\tau), \mathcal{F}_{ip}^{\omega}(\tau)\right), \left(\Finv_{rp}^{\omega}(\tau), \Finv_{ip}^{\omega}(\tau)\right), \left(\zeta_{rp}^{\omega}(\tau), \zeta_{ip}^{\omega}(\tau)\right), \left(\Gamma_{rp}^{\omega}(\tau), \Gamma_{ip}^{\omega}(\tau)\right)\right) : \tau \in \mathbb{X}\right\}$$

In addition, we define the truth and parameter function according to their real and imaginary parts, such as

$$\mathcal{F}_{rp}^{\omega}(\tau) = \mathbb{L}_{rp}^{1}\left(\Finv_{rp}^{\omega}(\tau)\right), \left(\mathcal{F}_{ip}^{\omega}(\tau) = \mathbb{L}_{ip}^{1}\left(\Finv_{ip}^{\omega}(\tau)\right)\right)$$

and

$$\zeta_{rp}^{\omega}(\tau) = \mathbb{L}_{rp}^{2}\left(\Gamma_{rp}^{\omega}(\tau)\right), \left(\zeta_{ip}^{\omega}(\tau) = \mathbb{L}_{ip}^{2}\left(\Gamma_{ip}^{\omega}(\tau)\right)\right)$$

Then

$$\mathbb{L}_{rp}^{2}\left(\Gamma_{rp}^{\omega}(\tau)\right) * \mathbb{L}_{rp}^{1}\left(\Finv_{rp}^{\omega}(\tau)\right) + \Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau) \leq 1$$
$$\Rightarrow \Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau)\left(1 + \mathbb{L}_{rp}^{1}\mathbb{L}_{rp}^{2}\right) \leq 1 \Rightarrow \Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau) \leq \frac{1}{1 + \mathbb{L}_{rp}^{1}\mathbb{L}_{rp}^{2}}$$

form of the proposed technique..png form of the proposed technique.bb

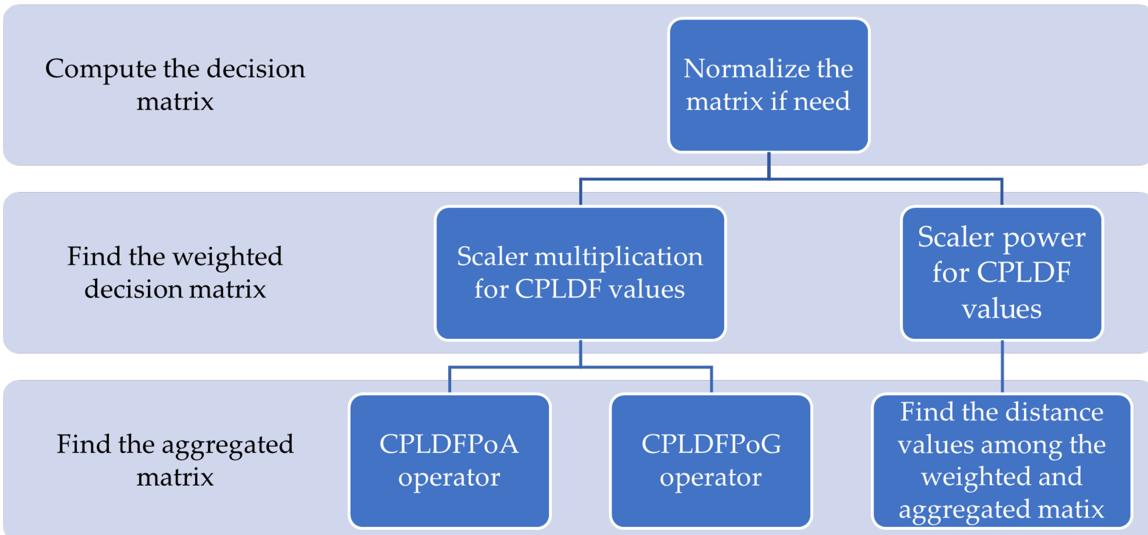

Figure 2: Graphical form of the proposed technique.

Thus, we have the following final shape, such as

$$\tilde{H}_{\&}=\begin{pmatrix}\left(\left(\mathbb{L}_{rp}^{1\&}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right),\mathbb{L}_{ip}^{1\&}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1\&}}\right),\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{1\&}}\right)\right)\right),\\ \left(\left(\mathbb{L}_{rp}^{2\&}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right),\mathbb{L}_{ip}^{2\&}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2\&}}\right),\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{2\&}}\right)\right)\right)\end{pmatrix},\&=1,2,\cdots,\ni$$

After constructing the decision matrix, we goal to design the procedure of the decision-making model for evaluating numerous genuine life problems. Therefore, we will follow the following technique for evaluating any type of problem, such as

Step 1: Construction of matrix: We focus on designing a matrix, where the value of the matrix must be the form of CPLDFNs, such as

$$DM=\left[\tilde{H}_{i\times\&}\right]_{n\times\ni}=\begin{bmatrix}\tilde{H}_{11} & \tilde{H}_{12} & \cdots & \tilde{H}_{1\ni}\\ \tilde{H}_{21} & \tilde{H}_{22} & \cdots & \tilde{H}_{2\ni}\\ \vdots & \vdots & \cdots & \vdots\\ \tilde{H}_{n1} & \tilde{H}_{n2} & \cdots & \tilde{H}_{n\ni}\end{bmatrix}$$

After the construction of the complex propositional linear Diophantine fuzzy matrix, we goal to normalize the data.

Step 2: Unvarying the matrix: We goal to normalize the data, if Cost types of data occurrences, such as

$$\tilde{H}=\left[\begin{array}{l}\left(\begin{array}{c}\left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right),\\ \left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right),\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right)\end{array}\right) benefit\\ \left(\begin{array}{c}\left(\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right),\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right),\mathbb{L}_{ip}^{1_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{1_{\&}}}\right)\right)\right),\\ \left(\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right),\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right),\mathbb{L}_{ip}^{2_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{ip}^{2_{\&}}}\right)\right)\right)\end{array}\right) cost\end{array}\right.$$

In another case, we goal to go to the next step.

Step 3: Weighted matrix construction: We goal to develop the weighted matrix, such as

$$\tilde{\eta}_{\Theta}\tilde{H}_{\&}$$
$$=\left(\begin{array}{c}\left(\left(1-\left(1-\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right),\left(\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},\left(\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right),\\ \left(\left(1-\left(1-\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right),\left(\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},\left(\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right)\end{array}\right)$$

$$\left(\tilde{H}_{\&}\right)^{\tilde{\eta}_{\Theta}}$$
$$=\left(\begin{array}{c}\left(\left(\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},\left(\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right),\left(1-\left(1-\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right),\\ \left(\left(\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},\left(\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right),\left(1-\left(1-\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}},1-\left(1-\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\tilde{\eta}_{\Theta}}\right)\right)\end{array}\right)$$

After evaluating the weighted decision matrix, we goal to address the aggregated matrix.

Step 4: Aggregation matrix construction: We goal to construct the aggregated values matrix by using the CPLDFPoA operator and CPLDFPoG operator, such as

$$CPLDFPoA\left(\tilde{H}_{1},\tilde{H}_{2},\cdots,\tilde{H}_{\ni}\right)$$
$$=\left(\begin{array}{c}\left(\begin{array}{c}\left(1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}},1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{1_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}\right),\\ \left(\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}},\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{1_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}\right)\end{array}\right),\\ \left(\begin{array}{c}\left(1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}},1-\prod_{\&=1}^{\ni}\left(1-\mathbb{L}_{rp}^{2_{\&}}\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}\right),\\ \left(\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{rp_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}},\prod_{\&=1}^{\ni}\left(\left(\frac{1-\varepsilon_{ip_{\&}}^{\omega}}{1+\mathbb{L}_{rp}^{2_{\&}}}\right)\right)^{\frac{\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}{\sum_{\&=1}^{\ni}\left(1+\tilde{\eta}\left(\tilde{H}_{\&}\right)\right)}}\right)\end{array}\right)\end{array}\right)$$

and

$$CPLDFPoG\left(\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_\ni\right)$$
$$= \left(\begin{array}{c}\left(\begin{array}{c}\left(\prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{1_\&}\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{1_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}, \prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{1_\&}\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{rp}^{1_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}\right), \\ \left(1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{1_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}, 1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{rp}^{1_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}\right)\end{array}\right), \\ \left(\begin{array}{c}\left(\prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{2_\&}\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{2_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}, \prod_{\&=1}^{\ni}\left(\mathbb{L}_{rp}^{2_\&}\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{rp}^{2_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}\right), \\ \left(1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{2_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}, 1-\prod_{\&=1}^{\ni}\left(1-\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{rp}^{2_\&}}\right)\right)^{\frac{(1+\tilde{\eta}(\tilde{H}_\&))}{\sum_{\&=1}^{\ni}(1+\tilde{\eta}(\tilde{H}_\&))}}\right)\end{array}\right)\end{array}\right)$$

To assess the values of the aggregated matrix, we will find the distance values among the information of weighted value and aggregated values.

Step 5: Distance matrix construction: We goal to design the values by distance function, such as

$$\tilde{H}_{\&k} = \begin{cases} D\left(\tilde{H}_\&, \tilde{H}_k\right) & if \tilde{H}_\& > \tilde{H}_k \\ 0 & if \tilde{H}_\& = \tilde{H}_k \\ -D\left(\tilde{H}_\&, \tilde{H}_k\right) & if \tilde{H}_\& < \tilde{H}_k \end{cases}$$

where

$$\begin{aligned} D\left(\tilde{H}_\&, \tilde{H}_k\right) =& \frac{1}{8}\left(\left|\mathbb{L}_{rp}^{1_\&}\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{1_\&}}\right) - \mathbb{L}_{rp}^{1_k}\left(\frac{1-\varepsilon_{rp}{}_k^\omega}{1+\mathbb{L}_{rp}^{1_k}}\right)\right| + \left|\mathbb{L}_{ip}^{1_\&}\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{ip}^{1_\&}}\right) - \mathbb{L}_{ip}^{1_k}\left(\frac{1-\varepsilon_{ip}{}_k^\omega}{1+\mathbb{L}_{ip}^{1_k}}\right)\right|\right. \\ &+ \left|\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{1_\&}}\right) - \left(\frac{1-\varepsilon_{rp}{}_k^\omega}{1+\mathbb{L}_{rp}^{1_k}}\right)\right| + \left|\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{ip}^{1_\&}}\right) - \left(\frac{1-\varepsilon_{ip}{}_k^\omega}{1+\mathbb{L}_{ip}^{1_k}}\right)\right| \\ &+ \left|\mathbb{L}_{rp}^{2_\&}\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{2_\&}}\right) - \mathbb{L}_{rp}^{2_k}\left(\frac{1-\varepsilon_{rp}{}_k^\omega}{1+\mathbb{L}_{rp}^{2_k}}\right)\right| + \left|\mathbb{L}_{ip}^{2_\&}\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{ip}^{2_\&}}\right) - \mathbb{L}_{ip}^{2_k}\left(\frac{1-\varepsilon_{ip}{}_k^\omega}{1+\mathbb{L}_{ip}^{2_k}}\right)\right| \\ &\left.+ \left|\left(\frac{1-\varepsilon_{rp}{}_\&^\omega}{1+\mathbb{L}_{rp}^{2_\&}}\right) - \left(\frac{1-\varepsilon_{rp}{}_k^\omega}{1+\mathbb{L}_{rp}^{2_k}}\right)\right| + \left|\left(\frac{1-\varepsilon_{ip}{}_\&^\omega}{1+\mathbb{L}_{ip}^{2_\&}}\right) - \left(\frac{1-\varepsilon_{ip}{}_k^\omega}{1+\mathbb{L}_{ip}^{2_k}}\right)\right|\right) \end{aligned}$$

Step 6: Appraisal matrix: We goal to address the appraisal information, such as

$$S_\& = \frac{1}{\ni}\sum_{k=1}^{\ni} D\left(\tilde{H}_\&, \tilde{H}_k\right)$$

Step 8: Ranking matrix: Calculate the ranking data according to the appraisal function for addressing the best one amid the group of a finite number of values.

6 CPLDF MABAC Deep Learning for Diagnosis of Alzheimers Disease

In this section, we goal to address the problem of the CPLDF MABAC deep learning model for diagnosis of Alzheimers disease for initiated techniques. Alzheimers disease is an unpredictable and progressive neurodegenerative disorder that initially affects memory thinking and behavior. The analysis of Alzheimers disease has been done by different scholars according to the information of crisp data, but to analyze the best one among the collection of data, we needed a soft and valuable technique that can help us in the evaluation of the procedure of decision-making models. Some key features of Alzheimers disease are memory loss, cognitive decline, behavioral changes, disorientation, and physical symptoms. In this article, we design the procedure of a multi-attributive border approximation area comparison deep learning algorithm for the diagnosis of Alzheimers Disease. Application point of view, we target data collection for diagnosing Alzheimers disease involves collecting a brief set of data from different sources, thus with the help of the above model, we aim to select the major means the best and worst ones among the collecting five, such as

1. Cognitive Assessments.
2. Neuroimaging Results.
3. Genetic Information.
4. Biomarkers.
5. Clinical History and Physical Examination.

Once selected, this information can be integrated into machine learning or deep learning models to analyze patterns and support diagnostic decision-making. Further, we have some attributes for the above alternatives, such as

1. Memory Loss.
2. Cognitive Decline.
3. Behavioral Changes.
4. Disorientation.
5. Physical symptom.

Therefore, to evaluate the above problems, we have a group of alternatives $\tilde{H}_1, \tilde{H}_2, \cdots, \tilde{H}_\ni$ with $A_1, A_2, \cdots, A_n$, called attributes for each alternative with the same order of weighted information, such as $\aleph_\& = \in [0,1]$ with $\sum_{\&=1}^{\ni} \aleph_\& = 1$, thus, we design a matrix by putting their information in the form of CPLDFSs, such as

$$\tilde{H} = \left\{ \left(\tau, \left(\mathcal{F}_{rp}^{\omega}(\tau), \mathcal{F}_{ip}^{\omega}(\tau) \right), \left(\text{Ⅎ}_{rp}^{\omega}(\tau), \text{Ⅎ}_{ip}^{\omega}(\tau) \right), \left(\zeta_{rp}^{\omega}(\tau), \zeta_{ip}^{\omega}(\tau) \right), \left(\Gamma_{rp}^{\omega}(\tau), \Gamma_{ip}^{\omega}(\tau) \right) \right) : \tau \in \mathbb{X} \right\}$$

In addition, we define the truth and parameter function according to their real and imaginary parts, such as

$$\mathcal{F}_{rp}^{\omega}(\tau) = \mathbb{L}_{rp}^{1} \left(\text{Ⅎ}_{rp}^{\omega}(\tau) \right), \left(\mathcal{F}_{ip}^{\omega}(\tau) = \mathbb{L}_{ip}^{1} \left(\text{Ⅎ}_{ip}^{\omega}(\tau) \right) \right)$$

and

$$\zeta_{rp}^{\omega}(\tau) = \mathbb{L}_{rp}^{2} \left(\Gamma_{rp}^{\omega}(\tau) \right), \left(\zeta_{ip}^{\omega}(\tau) = \mathbb{L}_{ip}^{2} \left(\Gamma_{ip}^{\omega}(\tau) \right) \right)$$

Then

$$\mathbb{L}_{rp}^{2} \left(\Gamma_{rp}^{\omega}(\tau) \right) * \mathbb{L}_{rp}^{1} \left(\text{Ⅎ}_{rp}^{\omega}(\tau) \right) + \Gamma_{rp}^{\omega}(\tau) * \text{Ⅎ}_{rp}^{\omega}(\tau) \leq 1$$
$$\Rightarrow \Gamma_{rp}^{\omega}(\tau) * \text{Ⅎ}_{rp}^{\omega}(\tau) \left(1 + \mathbb{L}_{rp}^{1} \mathbb{L}_{rp}^{2} \right) \leq 1 \Rightarrow \Gamma_{rp}^{\omega}(\tau) * \text{Ⅎ}_{rp}^{\omega}(\tau) \leq \frac{1}{1 + \mathbb{L}_{rp}^{1} \mathbb{L}_{rp}^{2}}$$

Similarly, we have imaginary parts, such as

$$\Gamma_{ip}^{\omega}(\tau) * \Finv_{ip}^{\omega}(\tau) \leq \frac{1}{1+\mathbb{L}_{ip}^{1}\mathbb{L}_{ip}^{2}}$$

thus

$$\begin{aligned}\varepsilon_{rp}^{\omega}(\tau) &= 1-\left(\zeta_{rp}^{\omega}(\tau) * \mathcal{F}_{rp}^{\omega}(\tau) * +\Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau)\right) = 1-\left(\mathbb{L}_{rp}^{2}\left(\Gamma_{rp}^{\omega}(\tau)\right) * \mathbb{L}_{rp}^{1}\left(\Finv_{rp}^{\omega}(\tau)\right)+\Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau)\right) \\ &= 1-\left(\Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau)\left(1+\mathbb{L}_{rp}^{1}\mathbb{L}_{rp}^{2}\right)\right)\end{aligned}$$

then

$$1-\varepsilon_{rp}^{\omega}(\tau) = \Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau)\left(1+\mathbb{L}_{rp}^{1}\mathbb{L}_{rp}^{2}\right)$$

and

$$\Gamma_{rp}^{\omega}(\tau) * \Finv_{rp}^{\omega}(\tau) = \frac{1-\varepsilon_{rp}^{\omega}(\tau)}{\left(1+\mathbb{L}_{rp}^{1}\mathbb{L}_{rp}^{2}\right)}$$

Similarly, we have

$$\Gamma_{ip}^{\omega}(\tau) * \Finv_{ip}^{\omega}(\tau) = \frac{1-\varepsilon_{ip}^{\omega}(\tau)}{\left(1+\mathbb{L}_{ip}^{1}\mathbb{L}_{ip}^{2}\right)}$$

But if we use the condition of IFSs, thus we have

$$\begin{gathered}\mathbb{L}_{rp}^{1}\left(\Finv_{rp}^{\omega}(\tau)\right)+\Finv_{rp}^{\omega}(\tau) \leq 1 \\ \Rightarrow \Finv_{rp}^{\omega}(\tau)\left(1+\mathbb{L}_{rp}^{1}\right) \leq 1 \Rightarrow \Finv_{rp}^{\omega}(\tau) \leq \frac{1}{\left(1+\mathbb{L}_{rp}^{1}\right)}\end{gathered}$$

Similarly, we have imaginary parts, such as

$$\Finv_{ip}^{\omega}(\tau) \leq \frac{1}{\left(1+\mathbb{L}_{ip}^{1}\right)}$$

Thus, we have the condition of refusal function in IFSs, such as

$$\varepsilon_{rp}^{\omega}(\tau) = 1-\left(\mathcal{F}_{rp}^{\omega}(\tau)+\Finv_{rp}^{\omega}(\tau)\right) = 1-\left(\mathbb{L}_{rp}^{1}\left(\Finv_{rp}^{\omega}(\tau)\right)+\Finv_{rp}^{\omega}(\tau)\right) = 1-\left(\Finv_{rp}^{\omega}(\tau)\left(1+\mathbb{L}_{rp}^{1}\right)\right)$$

then

$$1-\varepsilon_{rp}^{\omega}(\tau) = \Finv_{rp}^{\omega}(\tau)\left(1+\mathbb{L}_{rp}^{1}\right)$$

and

$$\Finv_{rp}^{\omega}(\tau) = \frac{1-\varepsilon_{rp}^{\omega}(\tau)}{\left(1+\mathbb{L}_{rp}^{1}\right)}, \left(\zeta_{rp}^{\omega}(\tau) = \frac{1-\varepsilon_{rp}^{\omega}(\tau)}{\left(1+\mathbb{L}_{rp}^{2}\right)}\right)$$

Similarly, we have

$$\Finv_{ip}^{\omega}(\tau) = \frac{1-\varepsilon_{ip}^{\omega}(\tau)}{\left(1+\mathbb{L}_{rp}^{1}\right)}, \left(\zeta_{ip}^{\omega}(\tau) = \frac{1-\varepsilon_{ip}^{\omega}(\tau)}{\left(1+\mathbb{L}_{ip}^{2}\right)}\right)$$

If $\varepsilon^{\omega}_{rp}(\tau) = \varepsilon^{\omega}_{ip}(\tau) = 0$, thus $\Finv^{\omega}_{rp}(\tau) = \frac{1}{(1+\mathbb{L}^{1}_{rp})}$ and $\Finv^{\omega}_{ip}(\tau) = \frac{1}{(1+\mathbb{L}^{1}_{ip})}$. Then

$$\mathcal{F}^{\omega}_{rp}(\tau) = \mathbb{L}^{1}_{rp}\left(\frac{1}{\left(1+\mathbb{L}^{1}_{rp}\right)}\right), \left(\mathcal{F}^{\omega}_{ip}(\tau) = \mathbb{L}^{1}_{ip}\left(\frac{1}{\left(1+\mathbb{L}^{1}_{ip}\right)}\right)\right)$$

and

$$\zeta^{\omega}_{rp}(\tau) = \mathbb{L}^{2}_{rp}\left(\frac{1}{\left(1+\mathbb{L}^{2}_{rp}\right)}\right), \left(\zeta^{\omega}_{ip}(\tau) = \mathbb{L}^{2}_{ip}\left(\frac{1}{\left(1+\mathbb{L}^{2}_{ip}\right)}\right)\right)$$

Then

$$\tilde{H} = \left\{\left(\begin{array}{l}\tau, \left(\left(\mathbb{L}^{1}_{rp}\left(\frac{1}{1+\mathbb{L}^{1}_{rp}}\right), \mathbb{L}^{1}_{ip}\left(\frac{1}{1+\mathbb{L}^{1}_{ip}}\right)\right), \left(\frac{1}{1+\mathbb{L}^{1}_{rp}}, \frac{1}{1+\mathbb{L}^{1}_{rp}}\right)\right), \\ \quad \left(\left(\mathbb{L}^{2}_{rp}\left(\frac{1}{1+\mathbb{L}^{2}_{rp}}\right), \mathbb{L}^{2}_{ip}\left(\frac{1}{1+\mathbb{L}^{2}_{ip}}\right)\right), \left(\frac{1}{1+\mathbb{L}^{2}_{rp}}, \frac{1}{1+\mathbb{L}^{2}_{rp}}\right)\right)\end{array}\right) : \tau \in \mathbb{X}\right\}$$

Thus, we have the following final shape, such as

$$\tilde{H}_{\&} = \left(\begin{array}{l}\left(\left(\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \mathbb{L}^{1_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{ip}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right)\right)\right), \\ \left(\left(\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \mathbb{L}^{2_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{ip}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right)\right)\right)\end{array}\right), \& = 1, 2, \cdots, \ni.$$

Therefore, we will follow the following technique for evaluating any type of problem, such as
Step 1: Construction of matrix: We focus on designing a matrix, where the value of the matrix must be the form of CPLDFNs, see Table 1.

Table 1: CPLDF information decision matrix.

	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$	$\boldsymbol{A_4}$	$\boldsymbol{A_5}$
$\tilde{H}_1$	$((4,3),(4,2))$	$((5,1),(5,3))$	$((6,2),(6,3))$	$((7,4),(7,4))$	$((1,3),(8,5))$
$\tilde{H}_2$	$((1,3),(2,6))$	$((2,2),(3,5))$	$((3,4),(4,4))$	$((4,4),(5,3))$	$((1,1),(8,5))$
$\tilde{H}_3$	$((3,3),(1,5))$	$((4,2),(4,4))$	$((5,3),(3,3))$	$((7,4),(5,2))$	$((1,3),(6,1))$
$\tilde{H}_4$	$((6,4),(1,2))$	$((5,1),(2,1))$	$((4,2),(3,2))$	$((3,3),(1,2))$	$((1,3),(2,1))$
$\tilde{H}_5$	$((1,5),(2,5))$	$((2,4),(3,4))$	$((3,3),(4,3))$	$((1,2),(1,2))$	$((1,3),(2,1))$

Step 2: Unvarying the matrix: We goal to normalize the data, if Cost types of data occurrences, such as

$$\tilde{H} = \left[\begin{array}{l}\left(\begin{array}{l}\left(\left(\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \mathbb{L}^{1_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{ip}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{ip}}\right)\right)\right), \\ \left(\left(\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \mathbb{L}^{2_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{ip}}\right)\right), \left(\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{ip}}\right)\right)\right)\end{array}\right) benefit \\ \left(\begin{array}{l}\left(\left(\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{ip}}\right)\right), \left(\mathbb{L}^{1_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{rp}}\right), \mathbb{L}^{1_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{1_{\&}}_{ip}}\right)\right)\right), \\ \left(\left(\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{ip}}\right)\right), \left(\mathbb{L}^{2_{\&}}_{rp}\left(\frac{1-\varepsilon_{rp}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{rp}}\right), \mathbb{L}^{2_{\&}}_{ip}\left(\frac{1-\varepsilon_{ip}{}^{\omega}_{\&}}{1+\mathbb{L}^{2_{\&}}_{ip}}\right)\right)\right)\end{array}\right) cost\end{array}\right.$$

In another case, we goal to go to the next step. So here we have benefit types of data in Table 1, so we will go to the next step.

Step 3: Weighted matrix construction: We goal to develop the weighted matrix, where $\tilde{\eta}_{\Theta} = 2$, see Table 2.

Table 2: CPLDF weighted information matrix.

	A_1	A_2	A_3	A_4	A_5
$\tilde{H}_1$	$\begin{pmatrix}(0.96,0.9375),\\(0.36,0.4375),\\(0.64,0.4444),\\(0.04.0.111)\end{pmatrix}$	$\begin{pmatrix}(0.9722,0.75),\\(0.3056,0.75),\\(0.6944,0.5625),\\(0.0278,0.0625)\end{pmatrix}$	$\begin{pmatrix}(0.9796,0.8889),\\(0.2653,0.5556),\\(0.7347,0.5625),\\(0.0204,0.0625)\end{pmatrix}$	$\begin{pmatrix}(0.9844,0.96),\\(0.2344,0.36),\\(0.7656,0.64),\\(0.0156,0.04)\end{pmatrix}$	$\begin{pmatrix}(0.75,0.9375),\\(0.75,0.4375),\\(0.7901,0.6944),\\(0.0123,0.0278)\end{pmatrix}$
$\tilde{H}_2$	$\begin{pmatrix}(0.75,0.9375),\\(0.75,0.4375),\\(0.4444,0.7347),\\(0.1111,0.0204)\end{pmatrix}$	$\begin{pmatrix}(0.8889,0.8889),\\(0.5556,0.5556),\\(0.5625,0.6944),\\(0.0625,0.0278)\end{pmatrix}$	$\begin{pmatrix}(0.9375,0.96),\\(0.4375,0.36),\\(0.64,0.64),\\(0.04,0.04)\end{pmatrix}$	$\begin{pmatrix}(0.96,0.96),\\(0.36,0.36),\\(0.6944,0.5625),\\(0.0278,0.0625)\end{pmatrix}$	$\begin{pmatrix}(0.75,0.75),\\(0.75,0.75),\\(0.7901,0.6944),\\(0.0123,0.0278)\end{pmatrix}$
$\tilde{H}_3$	$\begin{pmatrix}(0.9375,0.9375),\\(0.4375,0.4375),\\(0.25,0.6944),\\(0.25,0.0278)\end{pmatrix}$	$\begin{pmatrix}(0.96,0.8889),\\(0.36,0.5556),\\(0.64,0.64),\\(0.04,0.04)\end{pmatrix}$	$\begin{pmatrix}(0.9722,0.9375),\\(0.3056,0.4375),\\(0.5625,0.5625),\\(0.0625,0.0625)\end{pmatrix}$	$\begin{pmatrix}(0.9844,0.96),\\(0.2344,0.36),\\(0.6944,0.4444),\\(0.0278,0.1111)\end{pmatrix}$	$\begin{pmatrix}(0.75,0.9375),\\(0.75,0.4375),\\(0.7347,0.25),\\(0.0204,0.25)\end{pmatrix}$
$\tilde{H}_4$	$\begin{pmatrix}(0.9796,0.96),\\(0.2653,0.36),\\(0.25,0.4444),\\(0.25,0.1111)\end{pmatrix}$	$\begin{pmatrix}(0.9722,0.75),\\(0.3056,0.75),\\(0.4444,0.25),\\(0.111,0.25)\end{pmatrix}$	$\begin{pmatrix}(0.96,0.8889),\\(0.36,0.5556),\\(0.5625,0.4444),\\(0.0625,0.1111)\end{pmatrix}$	$\begin{pmatrix}(0.9375,0.9375),\\(0.4375,0.4375),\\(0.25,0.4444),\\(0.25,0.1111)\end{pmatrix}$	$\begin{pmatrix}(0.75,0.9375),\\(0.75,0.4375),\\(0.4444,0.25),\\(0.1111,0.25)\end{pmatrix}$
$\tilde{H}_5$	$\begin{pmatrix}(0.75,0.9722),\\(0.75,0.3056),\\(0.4444,0.6944),\\(0.111,0.0278)\end{pmatrix}$	$\begin{pmatrix}(0.8889,0.96),\\(0.5556,0.36),\\(0.5625,0.64),\\(0.0625,0.04)\end{pmatrix}$	$\begin{pmatrix}(0.9375,0.9375),\\(0.4375,0.4375),\\(0.64,0.5625),\\(0.04,0.0625)\end{pmatrix}$	$\begin{pmatrix}(0.75,0.8889),\\(0.75,0.5556),\\(0.25,0.4444),\\(0.25,0.1111)\end{pmatrix}$	$\begin{pmatrix}(0.75,0.9375),\\(0.75,0.4375),\\(0.4444,0.25),\\(0.1111,0.25)\end{pmatrix}$

Step 4: Aggregation matrix construction: We goal to construct the aggregated values matrix by using the CPLDFPoA operator and CPLDFPoG operator, see Table 3.

Table 3: CPLDF aggregated information matrix.

	CPLDFPoA	***CPLDFPoG***	**Weighted vector obtained with the help of power operators**
$\tilde{H}_1$	$\begin{pmatrix}(0.9618,0.9154),\\(0.4214,0.5317),\\(0.7226,0.5735),\\(0.0214,0.0548)\end{pmatrix}$	$\begin{pmatrix}(0.9261,0.8913),\\(0.3458,0.4916),\\(0.7296,0.5883),\\(0.0233,0.0615)\end{pmatrix}$	0.2011,0.199,0.2044,0.2024,0.1931
$\tilde{H}_2$	$\begin{pmatrix}(0.8891,0.9231),\\(0.6002,0.5177),\\(0.6139,0.6623),\\(0.0397,0.0331)\end{pmatrix}$	$\begin{pmatrix}(0.8534,0.8965),\\(0.5462,0.4716),\\(0.6442,0.67),\\(0.0515,0.0358)\end{pmatrix}$	0.1991,0.2036,0.2932,0.1995,0.1946
$\tilde{H}_3$	$\begin{pmatrix}(0.9525,0.9358),\\(0.4499,0.4497),\\(0.5409,0.4916),\\(0.0513,0.0712)\end{pmatrix}$	$\begin{pmatrix}(0.9186,0.9618),\\(0.3813,0.4416),\\(0.605,0.5442),\\(0.084,0.1005)\end{pmatrix}$	0.1962,0.204,0.2065,0.2026,0.1907
$\tilde{H}_4$	$\begin{pmatrix}(0.9491,0.9156),\\(0.4579,0.531),\\(0.3692,0.3546),\\(0.1374,0.1528)\end{pmatrix}$	$\begin{pmatrix}(0.9164,0.8918),\\(0.3936,0.491),\\(0.4022,0.375),\\(0.1612,0.1685)\end{pmatrix}$	0.2014,0.1973,0.2024,0.2037,0.1953
$\tilde{H}_5$	$\begin{pmatrix}(0.8389,0.9457),\\(0.67,0.4247),\\(0.4475,0.4893),\\(0.0947,0.0716)\end{pmatrix}$	$\begin{pmatrix}(0.8114,0.9391),\\(0.6341,0.4099),\\(0.4848,0.5436),\\(0.1176,0.1018)\end{pmatrix}$	0.2027,0.2022,0.1988,0.1968,0.1996

Step 5: Distance matrix construction: We goal to design the values by distance function, see Table 4.

Table 4: CPLDF distance values.

	CPLDFPoA	***CPLDFPoG***
$\tilde{H}_1$	0.0583,0.0704,0.032,0.0695,0.1102	0.056,0.0691,0.0294,0.0609,0.115
$\tilde{H}_2$	0.0887,0.0286,0.0577,0.0909,0.1169	0.0902,0.0327,0.0522,0.0845,0.1187
$\tilde{H}_3$	0.0972,0.0675,0.0363,0.0782,0.1452	0.1036,0.0569,0.0324,0.0723,0.148
$\tilde{H}_4$	0.1002,0.1079,0.07,0.0638,0.1137	0.0948,0.1054,0.0663,0.0665,0.1151
$\tilde{H}_5$	0.0729,0.0716,0.0852,0.0992,0.0784	0.0733,0.0647,0.0804,0.1061,0.0869

Step 6: Appraisal matrix: We goal to address the appraisal information, see Table 5.

Table 5: CPLDF ranking values.

	CPLDFPoA	***CPLDFPoG***
$\tilde{H}_1$	0.0681	0.0654
$\tilde{H}_2$	0.0765	0.0757
$\tilde{H}_3$	0.0849	0.0826
$\tilde{H}_4$	0.0911	0.0896
$\tilde{H}_5$	0.0815	0.0823

Step 8: Ranking matrix: Calculate the ranking data according to the appraisal function for addressing the best one amid the group of a finite number of values, see Table 6.

Table 6: CPLDF ranking values.

Methods	**Ranking values**	**Best idea**
CPLDFPoA operator	$\tilde{H}_4 > \tilde{H}_5 > \tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1$	$\tilde{H}_4$
CPLDFPoG operator	$\tilde{H}_4 > \tilde{H}_5 > \tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1$	$\tilde{H}_4$

According to the data in Table 6, the most preferable decision is $\tilde{H}_4$, called the Biomarkers for the MABAC model based on both operators. The simple representation of the data in Table 5 is available in the form of Figure 3.

form of data in Table 5..png form of data in Table 5.bb

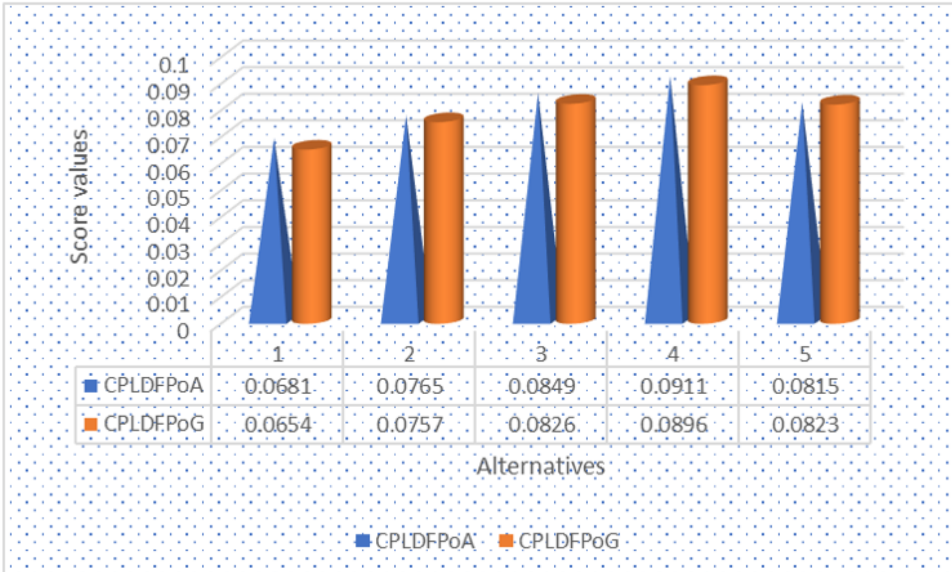

Figure 3: Graphical form of data in Table 5.

In addition, we will consider the data in Table 1 and, will evaluate it with the help of operators without the MABAC technique. Thus, the aggregated values matrix by using the CPLDFPoA operator and CPLDFPoG operator, see Table 7

Table 7: CPLDF aggregated matrix.

	CPLDFPoA	***CPLDFPoG***
$\tilde{H}_1$	$\begin{pmatrix}(0.8046,0.7091),\\(0.2393,0.3157),\\(0.8501,0.7573),\\(0.1462,0.2341)\end{pmatrix}$	$\begin{pmatrix}(0.7607,0.6843),\\(0.1954,0.2909),\\(0.8538,0.7659),\\(0.1499,0.2427)\end{pmatrix}$
$\tilde{H}_2$	$\begin{pmatrix}(0.6669,0.7227),\\(0.3677,0.3055),\\(0.7835,0.8138),\\(0.1993,0.1819)\end{pmatrix}$	$\begin{pmatrix}(0.6323,0.6945),\\(0.3331,0.2773),\\(0.8007,0.8181),\\(0.2165,0.1862)\end{pmatrix}$
$\tilde{H}_3$	$\begin{pmatrix}(0.7821,0.7466),\\(0.2583,0.2582),\\(0.7355,0.7011),\\(0.2266,0.2669)\end{pmatrix}$	$\begin{pmatrix}(0.7417,0.7418),\\(0.2179,0.2534),\\(0.7734,0.7331),\\(0.2645,0.2989)\end{pmatrix}$
$\tilde{H}_4$	$\begin{pmatrix}(0.7744,0.7095),\\(0.2637,0.3151),\\(0.6077,0.5955),\\(0.3706,0.3909)\end{pmatrix}$	$\begin{pmatrix}(0.7363,0.6849),\\(0.2256,0.2905),\\(0.6294,0.6091),\\(0.3923,0.4045)\end{pmatrix}$
$\tilde{H}_5$	$\begin{pmatrix}(0.5986,0.7671),\\(0.4256,0.2415),\\(0.669,0.6995),\\(0.3077,0.2675)\end{pmatrix}$	$\begin{pmatrix}(0.5744,0.7585),\\(0.4014,0.2329),\\(0.6923,0.7325),\\(0.331,0.3005)\end{pmatrix}$

Score value matrix: We goal to address the Score information, see Table 8.

Table 8: CPLDF ranking values.

	CPLDFPoA	***CPLDFPoG***
$\tilde{H}_1$	0.5464	0.5464
$\tilde{H}_2$	0.4832	0.4832
$\tilde{H}_3$	0.4888	0.4888
$\tilde{H}_4$	0.3367	0.3367
$\tilde{H}_5$	0.373	0.373

Ranking matrix: Calculate the ranking data according to the Score function for addressing the best one amid the group of a finite number of values, see Table 9.

Table 9: CPLDF ranking values.

Methods	**Ranking values**	**Best idea**
CPLDFPoA operator	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$
CPLDFPoG operator	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$

According to the data in Table 9, the most preferable decision is $\tilde{H}_3$, called the Genetic Information for both operators. The sensitivity of the proposed information for different values of parameters $\tilde{\eta}_\Theta$ is described in Table 10.

Table 10: Representation of the sensitive analysis.

$\tilde{\eta}_\Theta$	**Ranking values**	**Best idea**
2	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$
4	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$
6	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$
8	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$
10	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$
12	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$	$\tilde{H}_3$

According to the data in Table 10, the most preferable decision is $\tilde{H}_3$, called the Genetic Information for both operators for different values of parameters, anyhow, the proposed model is stable for all possible values of parameters, and the best value is $\tilde{H}_3$ for all values of the parameter. The simple representation of the data in Table 8 is available in the form of Figure 4.

form of data in Table 8..png form of data in Table 8.bb

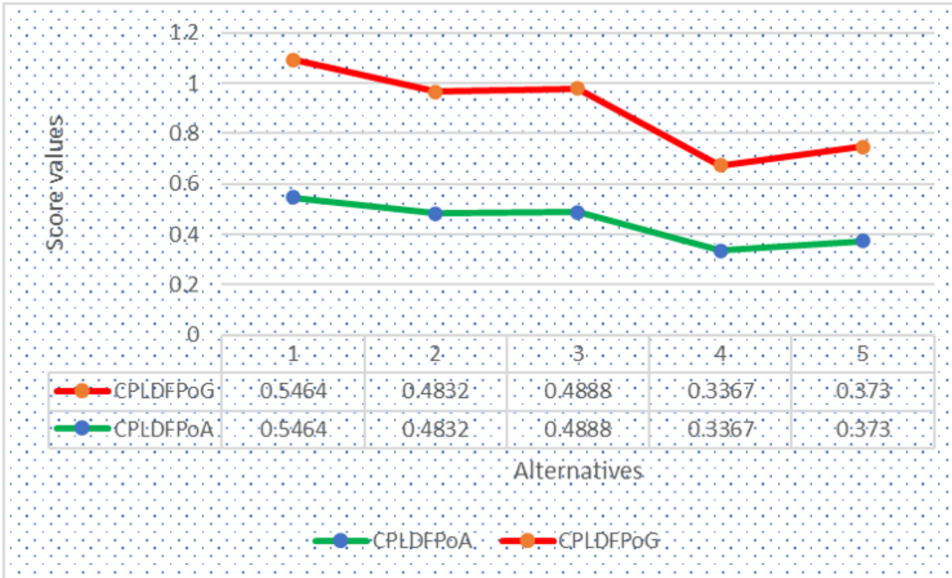

Figure 4: Graphical form of data in Table 8.

Additionally, we will compare the proposed ranking data with the ranking information of various existing techniques to discuss the efficiency of the invented theory.

7 Comparative Analysis

In this section, we scrutinize and deliberate the supremacy and validity of the designed technique and models by comparing their ranking values with the ranking values of various models. For this, we goal to collect various necessary techniques based on fuzzy models and their extensions, then we will evaluate the data in Table 1 with the help of considered information, such as Pamucar and Cirovic [21] invented the (multi-attributive border approximation area comparison) MABAC technique for classical set theory. Further, Yager [22] evaluated the power averaging (PoA) technique. In 2009, Xu and Yager [23] introduced the power geometric (PoG) technique for classical set theory. Jiang et al. [24] derived the power operators for IFSs. Wei and Lu [25] examined the power operators for PFSs. Garg et al. [26] initiated the power operators for Cq-ROFSs. Liu et al. [27] derived the power Dombi operators for CPFSs. Rani and Garg [28] evaluated the power operators for CIFSs. Ali [29] presented the power interaction operator for CIFSs. Ali et al. [30] described the power operators for complex intuitionistic fuzzy soft sets. Thus, the final ranking values are illustrated in Table 11.

Table 11: CPLDF comparative model.

Methods	**Score values**	**Ranking values**
Pamucar and Cirovic [21]	0.0,0.0,0.0,0.0,0.0	No
Yager [22]	0.0,0.0,0.0,0.0,0.0	No
Xu and Yager [23]	0.0,0.0,0.0,0.0,0.0	No
Jiang et al. [24]	0.0,0.0,0.0,0.0,0.0	No
Wei and Lu [25]	0.0,0.0,0.0,0.0,0.0	No
Garg et al. [26]	0.0,0.0,0.0,0.0,0.0	No
Liu et al. [27]	0.0,0.0,0.0,0.0,0.0	No
Rani and Garg [28]	0.0,0.0,0.0,0.0,0.0	No
Ali [29]	0.0,0.0,0.0,0.0,0.0	No
Ali et al. [30]	0.0,0.0,0.0,0.0,0.0	No
CPLDFPoA-MABAC	0.0681,0.0765,0.0849,0.0911,0.0815	$\tilde{H}_4 > \tilde{H}_3 > \tilde{H}_5 > \tilde{H}_2 > \tilde{H}_1$
CPLDFPoG-MABAC	0.0654,0.0757,0.0826,0.0896,0.0823	$\tilde{H}_4 > \tilde{H}_3 > \tilde{H}_5 > \tilde{H}_2 > \tilde{H}_1$
CPLDFPoA	0.5464,0.4832,0.4888,0.3367,0.373	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$
CPLDFPoG	0.5464,0.4832,0.4888,0.3367,0.373	$\tilde{H}_3 > \tilde{H}_2 > \tilde{H}_1 > \tilde{H}_5 > \tilde{H}_4$

According to the data in Table 6, the most preferable decision is $\tilde{H}_4$, called the Biomarkers for the MABAC model based on both operators. But, according to the data in Table 11, the most preferable decision is $\tilde{H}_3$, called the Genetic Information for both operators. In addition, the limitation of the existing models is described in Table 12.

Table 12: CPLDF theoretical comparison.

Methods	Truth value	Falsity value	Crisp function	Parameters for both function	Aggregation operators	Techniques/methods	Strong condition/not failed	Periodic function
Pamucar and Cirovic [21]	no	no	yes	no	Yes	yes	no	no
Yager [22]	no	no	yes	no	Yes	no	no	no
Xu and Yager [23]	yes	yes	yes	no	Yes	no	no	no
Jiang et al. [24]	yes	yes	yes	no	Yes	no	no	no
Wei and Lu [25]	yes	yes	yes	no	yes	no	no	no
Garg et al. [26]	yes	yes	yes	no	yes	no	no	yes
Liu et al. [27]	yes	yes	yes	no	yes	no	no	yes
Rani and Garg [28]	yes	yes	yes	no	yes	no	Yes	yes
Ali [29]	yes	yes	yes	no	yes	no	no	yes
Ali et al. [30]	yes	yes	yes	no	yes	no	no	yes
Proposed models	yes	yes	yes	Yes	yes	yes	Yes	yes

Finally, from the information in Table 12, we analyze that the existing techniques and models contain various limitations because of their features. Every point of view, we have discussed in Table 12, and from the data in Table 12, and Table 11, we concluded that the existing models are the special cases of the proposed theory. Hence, the designed techniques are more powerful and more reliable compared to existing models.

8 Conclusion

The complex propositional linear Diophantine fuzzy technique is a very powerful model for handling vague and uncertain data. The technique of complex propositional linear Diophantine fuzzy sets is the combination of numerous valuable ideas, where the key and major contributions of the designed techniques are followed, such as designing the procedure of a MABAC deep learning algorithm for the diagnosis of Alzheimers Disease. Further, we design the model of CPLDF information with their basic operational laws. In addition, we analyze the model of the CPLDFPoA operator, CPLDFWPoA operator, CPLDFPoG operator, and CPLDFWPoG operator, and also initiate their major properties. Moreover, we arrange relevant from different sources for diagnosing Alzheimers disease under the consideration of the designed technique. Lastly, we compare both (proposed and existing) ranking information to address the supremacy and strength of the designed models.

In the future, we will begin the model of complex propositional (p, q) Diophantine fuzzy sets with some new extensions. In addition, we will evaluate the model of operator, measures, and methods for designed models and discuss their application in decision-making, artificial intelligence, and data mining to improve the worth of fuzzy set theory.

Conflict of interest: About the publication of this manuscript, the authors declare that they have no conflict of interest.

Funding: No external funding has been received for this submission.

Ethical approval: The authors state that this is their original work and it is neither submitted nor under consideration in any other journal simultaneously.

Human and animal participants: This article does not contain any studies with human participants or animals performed by any of the authors.

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