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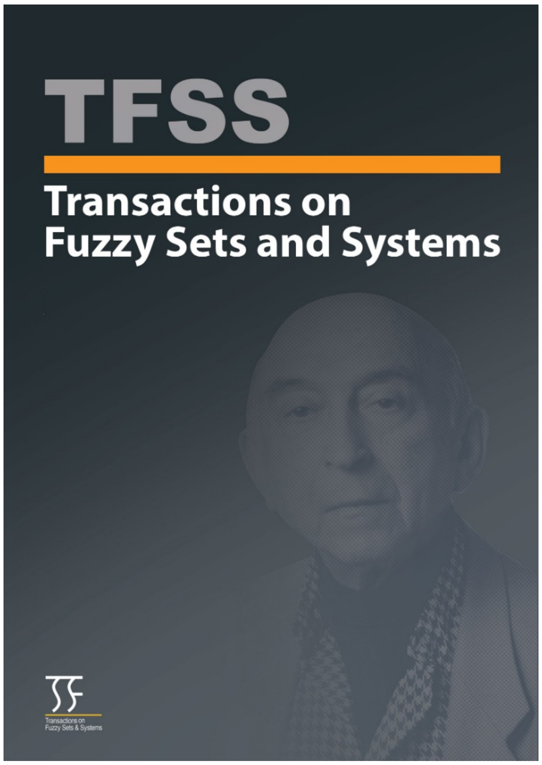

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An Investigation on the Fractional Transportation Problem via Weibull Distribution

Paraman Anukokila*, **Aravasa Gounder Vel Murugan**, **Bheeman Radhakrishnan**

Abstract. In this work, multi-objective fractional stochastic solid transportation problem uncertainties are represented using the Weibull distribution. This transformation converts the multi-objective fractional stochastic solid transportation problem into a goal programming problem with chance constraints, incorporating probabilistic constraints into its formulation. Goal programming and hyperbolic membership functions also assist in solving the fractional transportation problem. The proposed models serve as the basis for numerical examples and approaches to solving the problem under validated uncertainty. Furthermore, we conduct a sensitivity analysis to assess the impact of parameter changes on the proposed method.

AMS Subject Classification 2020: 90B06; 90C08; 90C29; 03E72
Keywords and Phrases: Chance constrained programming, Fractional transportation problem, Goal programming, Weibull distribution.

1 Introduction

Transportation systems are essential components of the economy and a necessary part of daily and social life. These systems aim to convey a uniform product from a supply location to a demand location at low costs. In the contemporary era of globalization, we see an increase in product transportation and a variety of methods for transporting these goods. In order to meet this demand, we must define the solid transportation problem. The STP is a critical research subject concerning both theoretical and practical dimensions. In the current STP, it is essential to optimize many goals, such as minimizing transportation costs, packing expenses, and transportation time. A solid transportation issue with a fractional objective function is a multi-objective fractional solid transportation problem, that optimizes the ratio of many functions. In numerous practical scenarios, individuals or groups extensively employ the performance metric to evaluate the financial dimensions of transportation companies and management contexts. These scenarios require individuals or groups to address the challenge of maintaining optimal ratios among critical parameters associated with the transportation of goods from specific suppliers to diverse demand centers using various modes of conveyance. Solving optimization problems with fractional objective functions is challenging and requires advanced optimization techniques or numerical approaches.

The transportation problem primarily consists of three parameters: cost coefficient, supply, and demand. Owing to unforeseen variables, these parameters are not consistently immutable. This inaccuracy arises from

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the absence of precise information. Arya et al. [1] developed a model for multi-choice stochastic transportation issues using extreme value distributions via binary variables. Stochastic programming tackles situations where random variables, rather than deterministic values, represent any of the optimization problem's parameters. In the current competitive market, suppose that the parameters adhere to random variables. The goal of the stochastic fractional transportation problem is to find an optimal transportation plan that minimizes the expected total cost, taking into account the uncertainty in the problem parameters. This means that instead of having fixed values for these parameters, there is a range of possible values with associated probabilities. In order to identify the best or nearly best solution given the stochastic nature of the problem, stochastic programming techniques, such as stochastic linear programming or scenario-based optimization, are often employed. The Weibull distribution is frequently used to compute the probability distribution of a continuous random variable. The Weibull distribution also finds applications in biological sciences, earth sciences, medicine, transportation planning, and other fields.

One mathematical programming tool used to address multi-objective optimization is the goal programming method. Fuzzy goal programming (FGP) has shown to be an effective way of managing future ambitions for decision-makers, even in the absence of exact information about future objectives. In other words, FGP is an extension of conventional goal programming used for addressing multi-objective choice problems with imprecisely defined model parameters. Several scholars have used the FGP technique to address multi-objective optimization problems. The aims of the FGP model formulation issue are converted into fuzzy objectives by giving an ambition level to each goal. Each fuzzy objective takes into account the attainment of the hyperbolic membership value. The special membership function hyperbolic is used to develop the stochastic solid transportation problem with the help of Lingo software.

1.1 Motivation of the study

- Motivated by Das and Lee [2], we extended their work to address a transportation problem with a fractional stochastic objective function using a Weibull distribution. The objectives of the fractional stochastic transportation problem are first converted into fuzzy goals by giving an ambition level to each target. We evaluate the attainment of the hyperbolic membership value for each aim to the greatest extent possible.

- The solution procedure to a deterministic model using chance constrained programming.

1.2 Novelty and Contribution

- In this paper, we propose a multi-objective fractional stochastic solid transportation problem by introduring probabilities constraints, we applied the Weibull distribution through the stochastic parameters.

- The chance-constrained programming and fractional goal programming approaches are used to handle the uncertainty.

- The hyperbolic membership function is used to aggregate the conflicting objectives. Both transportation cost and time are minimized to find the optimal solutions. As compared to the previous papers our proposed model shows greater domination.

Abbreviations in theis study	
MOFSSTP	Multi-Objective Fractional Stochastic Solid Transportation Problem
FTP	Fractional Transportation Problem
STP	Solid Transportation Problem
FGP	Fractional Goal Programming
CCP	Chance-Constrained Programming
CDF	Cumulative Distribution Function
PDF	Probability Density Function

2 Literature Review

This section presents an exposition of past research on the transportation problem and fractional transportation problem in stochastic solid environments. Mahapatra et al. [3] examined a stochastic transportation problem with a single source and a single destination after creating a stochastic variable for the demand using a joint cumulative distribution function. Schaible and Ibaraki [4], many researchers including those who have researched fractional programming. Ebrahimnejad [5] simplified fuzzy transportation problems with trapezoidal fuzzy numbers. Anukokila and Radhakrishnan [6] presented a fuzzy goal programming approach. Ojha et al. [7] introduced a stochastic discounted multi-objective stochastic linear time program in which the demand is a stochastic variable and is transformed into deterministic variables using the expected value criterion. However in real-world applications, supply and conveyance capacity are also stochastic with demand. Mahapatra, Prékopa and Williams [8, 9, 10] presented a multi-choice stochastic transportation problem with probabilistic limitations that involves extreme value distributions. Because the extreme value distribution is produced by taking the natural logarithm of the Weibull distribution, the Weibull and extreme value distributions are closely related. Holmberg and Tuy [11] proposed a branch-and-bound approach for solving the transportation problem with convex production costs and stochastic demand. Charnes and Cooper [12] presented a multi-objective transportation challenge that would minimize transportation time and cost and discussed the import of the problem. Sengupta et al. [13] presented green supply chain management. In [14, 15], researchers developed a transportation problem of this sort. Kataoka [16] presented a model of stochastic programming for a transportation problem with a single meaning. Chalam [15] produced a fuzzy goal programming method for stochastic total points while following financial limitations. Bhattacharya and Gupta [17, 18] introduced a fuzzy programming approach to solve a multi-objective transportation problem with distinct costs when a random variable has a normal distribution. Mahapatra et al. [3] presented a log-normal distribution-based multi-objective stochastic transportation problem. Gessesse and Mishra [19] addressed a linear fractional transportation problem with several objectives in a stochastic setting. When modeling a stochastic transportation problem or solid transportation problem, many researchers have employed a normal distribution to define the variables. Dambrosio et al. [20]. But for transportation problems or solid transportation problem models, stochastic variables have also been defined using different distributions [21, 22]. Also, the fuzzy theory has been presented as a means of characterizing uncertainty. Gao and Lee [23] examined a scenario-based multi-objective redistribution problem as a stochastic mixed-integer problem, in which the availability of the transportation network in catastrophic events is unpredictable, where the supply and demand at relief centers are uncertain. Agrawal and Ganesh [24] studied and presented a method for solving a multi-choice transportation problem in a stochastic setting using Newton's split difference interpolation. Gupta and Garg [25] perceived it as a capacitated stochastic transportation problem, where the uncertain parameters related to supply and demand restrictions are managed by the application of the maximum-likelihood estimation method and the chance-constrained programming method.

3 Formulation of Fractional Transportation Problem

Fractional transportation problem is usually applied to a particular class of optimization problems found in mathematical programming and operations research. The fractional transportation problem is the problem of minimizing "t" transportation valued objective function with transportation cost,

$$\begin{cases} Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}\mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}\mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}, t=1,2,...,\zeta \\ subject\ to: \\ \sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = a_{k_1},\ k_1=1,2,...,n_1, \\ \sum\limits_{k_1=1}^{n_1}\sum\limits_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = b_{k_2},\ k_2=1,2,...,n_2, \\ \sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} = c_{k_3},\ k_3=1,2,...,n_3, \\ \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,k_2,k_3, \end{cases} \tag{1}$$

where $(\mathcal{Z}^1(\mathcal{Y}), \mathcal{Z}^2(\mathcal{Y}), ..., \mathcal{Z}^\zeta(\mathcal{Y}))$ is a vector of t objective functions. $\mathcal{Y}_{k_1k_2k_3}$ represents the amount of the product to be shipped from k_1 source parameters to the k_2 destination parameter, and k_3 is the conveyance parameter. $\frac{\mathfrak{C}^t_{k_1k_2k_3}}{\mathfrak{D}^t_{k_1k_2k_3}}$, k_1 source parameters to the k_2 destination parameters and k_3 is the conveyance parameter. $a_{k_1}; (k_1 = 1,2,...,n_1), b_{k_2}; (k_2 = 1,\ 2,...,n_2)$ and $c_{k_3}; (k_3 = 1,2,...,n_3)$ and $\frac{\mathfrak{C}^t_{k_1k_2k_3}}{\mathfrak{D}^t_{k_1k_2k_3}}$ are the conveyance parameters, which are in the form of transportation problem values of source, destination and conveyance. Using real values for supply, demand, conveyance, and transportation cost, the suggested approach offers an easy means to identify the best way to solve a fuzzy FTP.

3.1 Chance Constrained Programming Model

Weibull [26], a Swedish physicist, suggested using it for modeling the stress distribution in order to break specimens. For chance-constrained programming (CCP), getting probability distributions for uncertain parameters and working out the right confidence levels are crucial. When there is ambiguity in the distribution's parameters, CCP with the Weibull distribution is a successful approach for determining dependability and risk. The stochastic Weibull distribution, a probability distribution, is used to model uncertainty in various events, such as the failure rate of system components. Using CCP with a three-parameter stochastic Weibull distribution is a successful method to make solid choices when there is unknown about the Weibull-distributed variables. The random variable $\mathfrak{g}$'s cumulative distribution function (cdf) and probability density function (pdf), as per a three-parameter Weibull distribution as

$$\mathfrak{f}(\mathfrak{g}) = \frac{\delta}{\vartheta}\left(\frac{\mathfrak{g}-\psi}{\vartheta}\right)^{\delta-1} exp\left\{-\left(\frac{\mathfrak{g}-\psi}{\vartheta}\right)^{\delta}\right\}, \tag{2}$$

$$F(g) = 1 - exp\left\{-\left(\frac{g-\psi}{\vartheta}\right)^{\delta}\right\}, \tag{3}$$

for, $\mathfrak{f}(\mathfrak{g}) \geq 0, \mathfrak{g} \geq 0$ or $\psi, \delta > 0, \vartheta > 0, -\infty < \psi < \infty$. Note that δ, ϑ, and ψ are the shape parameter, scale parameter, and the location parameter, respectively.

A closed form extraction of a probability distribution function's quantiles is the only way to get the deterministic constraints required by the suggested stochastic programming paradigm. In this paper, the chance-constrained programming model of the multi-objective fractional stochastic solid transportation problem is defined as follows:

$$\begin{cases} Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}, t = 1,2,...,\zeta \\ subject\ to: \\ P\left(\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \le q_{k_1}\right) \ge p_{q_{k_1}},\ k_1 = 1,2,...,n_1, \\ P\left(\sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \ge r_{k_2}\right) \ge p_{r_{k_2}},\ k_2 = 1,2,...,n_2, \\ P\left(\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} \le s_{k_3}\right) \ge p_{s_{k_3}},\ k_3 = 1,2,...,n_3, \\ \mathcal{Y}_{k_1k_2k_3} \ge 0, \forall\ k_1,k_2,k_3, \end{cases} \tag{4}$$

where the probabilities $p_{q_{k_1}}, p_{r_{k_2}}$, and $p_{s_{k_3}}$ are specified. It is assumed that the random variables q_{k_1}, r_{k_2}, and s_{k_3}, representing supply, demand, and conveyance capacity, adhere to the Weibull distribution, respectively. The Weibull distribution for q_{k_1} comprises three parameters: $\delta_{q_{k_1}}$(shape), $\vartheta_{q_{k_1}}$(scale), and $\psi_{q_{k_1}}$(location). Also, the parameters for the Weibull distribution for r_{k_2} and s_{k_3} are likewise specified. Constraint (4) represents the probabilistic limitation on the supply quality at source k_1, ensuring that with a certain probability $p_{q_{k_1}}$, the total shipments from source k_1 do not exceed q_{k_1}. Likewise, limitations (4) might be construed in terms of the demand at destination k_2 and the conveyance capacity of transportation mode k_3. Three cases are examined in which one random variable among $p_{q_{k_1}}, p_{r_{k_2}}$, and $p_{s_{k_3}}$ is uncertain, designated as Case I, Case II, and Case III. Case IV presents an environment in which all random variables represent uncertainty.

3.1.1 Case I

The equation (4) can therefore be rearranged as independent random variables $q_{k_1}(k_1 = 1,2,...,n_1)$ with three known parameters δ_{k_1}, ϑ_{k_1}, and ψ_{k_1}. The Weibull distribution has been proposed to apply to these factors.

$$P\left(q_{k_1} \ge \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3}\right) \ge p_{q_{k_1}},\ k_1 = 1,2,...,n_1. \tag{5}$$

$$\varphi_{q_{k_1}} = \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{k_3} \mathcal{Y}_{k_1k_2k_3}\ \forall\ k_2,k_3,\ k_1 = 1,2,...,n_1.$$

$$\int_{\infty}^{\varphi_{q_{k_1}}} \frac{\delta_{q_{k_1}}}{\vartheta_{q_{k_1}}}\left(\frac{q_{k_1}-\psi_{q_{k_1}}}{\vartheta_{q_{k_1}}}\right)^{\delta_{q_{k_1}}-1} exp\left\{-\left(\left(\frac{q_{k_1}-\psi_{q_{k_1}}}{\vartheta_{q_{k_1}}}\right)^{\delta_{q_{k_1}}}\right)\right\}dq_{k_1} \ge p_{q_{k_1}},\ k_1 = 1,2,...,n_1. \tag{6}$$

It is possible to further express equation (6) as $P(q_{k_1} \ge \varphi_{q_{k_1}}) \ge p_{q_{k_1}},\ k_1 = 1,2,...,n_1$

$$\int_{\psi_{q_{k_1}}}^{\varphi_{q_{k_1}}} \frac{\delta_{q_{k_1}}}{\vartheta_{q_{k_1}}}\left(\frac{q_{k_1}-\psi_{q_{k_1}}}{\vartheta_{q_{k_1}}}\right)^{\delta_{q_{k_1}}-1} exp\left\{-\left(\left(\frac{q_{k_1}-\psi_{q_{k_1}}}{\vartheta_{q_{k_1}}}\right)^{\delta_{q_{k_1}}}\right)\right\}dq_{k_1} \ge p_{q_{k_1}},\ k_1 = 1,2,...,n_1. \tag{7}$$

Integrating equation (7) gives

$$exp\left\{-\left(\left(\frac{\varphi_{q_{k_1}}-\psi_{q_{k_1}}}{\vartheta_{q_{k_1}}}\right)^{\delta_{q_{k_1}}}\right)\right\} \geq p_{q_{k_1}},\ k_1=1,2,...,n_1,$$

$$\left(\frac{\varphi_{q_{k_1}}-\psi_{q_{k_1}}}{\vartheta_{q_{k_1}}}\right) \leq (-lnq_{k_1})^{\frac{1}{\delta_{q_{k_1}}}},$$

$$\varphi_{q_{k_1}}-\psi_{q_{k_1}} \leq \vartheta_{q_{k_1}}\{-lnq_{k_1}\}^{\frac{1}{\delta_{q_{k_1}}}},$$

$$\varphi_{q_{k_1}} \leq \psi_{q_{k_1}}+\vartheta_{q_{k_1}}\{-ln(q_{k_1})\}^{\frac{1}{\delta_{q_{k_1}}}}.$$

The quantile of the Weibull distribution may be used to convert this into deterministic constraints in the following way.

$$\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{q_{k_1}}+\vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1=1,\ 2,...,n_1. \tag{8}$$

Hence, the multi-objective fractional solid transportation problem is given when the supply constraint is uncertain, as follows.

$$\begin{cases} \quad Minimize\ \mathcal{Z}^t(\mathcal{Y})=\dfrac{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}},\ t=1,\ 2,...,\zeta \\ subject\ to: \\ \quad \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{q_{k_1}}+\vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1=1,\ 2,...,n_1, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \geq r_{k_2},\ k_2=1,2,...,n_2, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} \leq s_{k_3},\ k_3=1,2,...,n_3, \\ \quad\quad \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,\ k_2,\ k_3. \end{cases} \tag{9}$$

3.1.2 Case II

The equation (4) is presented, with three known parameters, δ_{k_2}, ϑ_{k_2}, and ψ_{k_2} respectively. It is assumed that r_{k_2} $(k_2=1,\ 2,...,n_2)$ are independent random variables that follow the Weibull distribution.

$$\int_{-\infty}^{\varphi_{r_{k_2}}}\frac{\delta_{r_{k_2}}}{\vartheta_{r_{k_2}}}\left(\frac{r_{k_2}-\psi_{r_{k_2}}}{\vartheta_{r_{k_2}}}\right)^{\delta_{r_{k_2}}-1}exp\left\{-\left(\left(\frac{r_{k_2}-\psi_{r_{k_2}}}{\vartheta_{r_{k_2}}}\right)^{\delta_{r_{k_2}}}\right)\right\}dr_{k_2} \geq p_{r_{k_2}},\ k_2=1,2,...,n_2. \tag{10}$$

It is possible to further express equation (10) as $(r_{k_2} \geq \varphi_{r_{k_2}}) \geq r_{k_2},\ k_2=1,\ 2,...,n_2$

$$\int_{\psi_{r_{k_2}}}^{\varphi_{r_{k_2}}}\frac{\delta_{r_{k_2}}}{\vartheta_{r_{k_2}}}\left(\frac{r_{k_2}-\psi_{r_{k_2}}}{\vartheta_{r_{k_2}}}\right)^{\delta_{r_{k_2}}-1}exp\left\{-\left(\left(\frac{r_{k_2}-\psi_{r_{k_2}}}{\vartheta_{r_{k_2}}}\right)^{\delta_{r_{k_2}}}\right)\right\}dr_{k_2} \geq p_{r_{k_2}},\ k_2=1,2,...,n_2. \tag{11}$$

Integrating equation (11) gives

$$1 - exp \quad \left\{ - \left(\left(\frac{\varphi_{r_{k_2}} - \psi_{r_{k_2}}}{\vartheta_{r_{k_2}}} \right)^{\delta_{r_{k_2}}} \right) \right\} \geq p_{r_{k_2}},\ k_2 = 1, 2, ..., n_2.$$

The quantile of the Weibull distribution may be used to convert this into deterministic constraints in the following way.

$$\sum_{k_2=1}^{n_2} \sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1 k_2 k_3} \geq [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}} \{-ln(1 - p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1, 2, ..., n_2. \tag{12}$$

Therefore, in the case of an unknown demand constraint, the multi-objective fractional solid transportation issue is provided as follows.

$$\begin{cases} \quad Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum\limits_{k_1=1}^{n_1} \sum\limits_{k_2=1}^{n_2} \sum\limits_{k_3=1}^{n_3} \mathfrak{C}^t_{k_1 k_2 k_3} \mathcal{Y}_{k_1 k_2 k_3}}{\sum\limits_{k_1=1}^{n_1} \sum\limits_{k_2=1}^{n_2} \sum\limits_{k_3=1}^{n_3} \mathfrak{D}^t_{k_1 k_2 k_3} \mathcal{Y}_{k_1 k_2 k_3}},\ t = 1, 2, ..., \zeta \\ subject\ to: \\ \quad \sum\limits_{k_2=1}^{n_2} \sum\limits_{k_3=1}^{n_3} \mathcal{Y}_{k_1 k_2 k_3} \leq q_{k_1},\ k_1 = 1, 2, ..., n_1, \\ \quad \sum\limits_{k_1=1}^{n_1} \sum\limits_{k_3=1}^{n_3} \mathcal{Y}_{k_1 k_2 k_3} \geq [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}} \{-ln(1 - p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1, 2, ..., n_2, \\ \quad \sum\limits_{k_1=1}^{n_1} \sum\limits_{k_2=1}^{n_2} \mathcal{Y}_{k_1 k_2 k_3} \leq s_{k_3},\ k_3 = 1, 2, ..., n_3, \\ \qquad\qquad \mathcal{Y}_{k_1 k_2 k_3} \geq 0, \forall\ k_1, k_2, k_3. \end{cases} \tag{13}$$

3.1.3 Case III

The equation (4) is presented, with three known parameters, $\delta_{s_{k_3}}$, $\vartheta_{s_{k_3}}$, and $\psi_{s_{k_3}}$respectively. It is assumed that s_{k_3} $(k_3 = 1, 2, ..., n_3)$ are independent random variables that follow the Weibull distribution.

$$\int_{\infty}^{\varphi_{s_{k_3}}} \frac{\delta_{s_{k_3}}}{\vartheta_{s_{k_3}}} \left(\frac{s_{k_3} - \psi_{s_{k_3}}}{\vartheta_{s_{k_3}}} \right)^{\delta_{s_{k_3}} - 1} exp \left\{ - \left(\left(\frac{s_{k_3} - \psi_{s_{k_3}}}{\vartheta_{s_{k_3}}} \right)^{\delta_{s_{k_3}}} \right) \right\} ds_{k_3} \geq p_{s_{k_3}},\ k_3 = 1, 2, ..., n_3 \tag{14}$$

It is possible to further express equation (14) as $(s_{k_3} \geq \varphi_{s_{k_3}}) \geq s_{k_3},\ k_3 = 1, 2, ..., n_3.$

$$\int_{\psi_{s_{k_3}}}^{\varphi_{s_{k_3}}} \frac{\delta_{s_{k_3}}}{\vartheta_{s_{k_3}}} \left(\frac{s_{k_3} - \psi_{s_{k_3}}}{\vartheta_{s_{k_3}}} \right)^{\delta_{s_{k_3}} - 1} exp \left\{ - \left(\left(\frac{s_{k_3} - \psi_{s_{k_3}}}{\vartheta_{s_{k_3}}} \right)^{\delta_{s_{k_3}}} \right) \right\} ds_{k_3} \geq p_{s_{k_3}},\ k_3 = 1, 2, ..., n_3 \tag{15}$$

Integrating equation (15) gives

$$exp \left\{ - \left(\left(\frac{\varphi_{s_{k_3}} - \psi_{s_{k_3}}}{\vartheta_{s_{k_3}}} \right)^{\delta_{s_{k_3}}} \right) \right\} \geq p_{s_{k_3}},\ k_3 = 1, 2, ..., n_3.$$

The quantile of the Weibull distribution may be used to convert this into deterministic constraints in the following way.

$$\sum_{k_2=1}^{n_2} \sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1 k_2 k_3} \leq [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}} \{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3 = 1, 2, ..., n_3. \tag{16}$$

Consequently, when the conveyance constraint is unclear, the multi-objective fractional solid transportation problem is presented as follows.

$$\begin{cases} Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}},\ t=1,2,...,\zeta \\ subject\ to: \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \geq q_{k_1},\ k_1=1,2,...,n_1, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \geq r_{k_2},\ k_2=1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3=1,2,...,n_3, \\ \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,k_2,k_3. \end{cases} \tag{17}$$

3.1.4 Case IV

It is assumed that $\delta_i(i = 1,2,3,...,m)$, $\vartheta_j(j = 1,2,3,...,n)$, and $\psi_k(k = 1,2,3,...,l)$ are independent random variables using the Weibull distribution. By combining the derivations, the multi-objective fractional stochastic solid transportation problem (MOFSSTP) is a deterministic model for Cases I, II, and III.

Now let's consider Fractional stochastic multi-objective solid transportation problem of the type

$$\begin{cases} Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}},\ t=1,2,...,\zeta \\ subject\ to: \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{q_{k_1}} + \vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1=1,2,...,n_1, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}}\{-ln(1-p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2=1,2,...,n_2, \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3=1,2,...,n_3, \\ \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,k_2,k_3. \end{cases} \tag{18}$$

This portion presents the deterministic mathematical programming models for the multi-objective fractional stochastic solid transportation problem (MOFSSTP) using the quantile of the Weibull distributions. In a real-world situation, it makes sense that while certain components of supply, demand, and transportation capacity could be known with certainty, others would not. Thus, multi-objective fractional stochastic solid transportation problem (MOFSSTP) deterministic may be modified as needed, depending on the circumstances. This deterministic mathematical programming may be solved for optimality using the commercially available solvers.

3.2 Fractional Stochastic Solid Transportation Problem

For MOFSSTP, the objective value and constraints hold significant importance. Our primary aim is to minimize the total cost of transportation. In real-life situations, uncertainties in dimensions like cost, time, supply, demand, and transportation capacity provide obstacles for decision-makers in achieving optimal solutions. Fuzzy and stochastic variables can effectively address this situation. This model treats cost as a fuzzy variable and constraints as random variables. The unpredictability in supply, demand, and conveyance capacity limits may arise or not, depending on the specific circumstances of the demand management process. Therefore, we develop three MOFSSTP models based on the uncertainty of constraints. We construct a MOFSSTP model with k_1 sources, k_2 destinations and k_3 conveyance capacity as described below.

3.2.1 Model I

Modeling for deterministic MOFSSTP may be transformed into a stochastic model when supply is uncertain, but demand and conveyance capacity restrictions remain certain.

$$\begin{cases} Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\sum_{\mathfrak{k}=1}^{n_3} \mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\sum_{\mathfrak{k}=1}^{n_3} \mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}},\ t=1,2,...,\zeta \\ subject\ to: \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{q_{k_1}} + \vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1=1,2,...,n_1, \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = [ln\psi_{q_{k_1}} + \vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1=1,2,...,n_1, \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{q_{k_1}} + \vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1=1,2,...,n_1, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \geq r_{k_2},\ k_2=1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = r_{k_2},\ k_2=1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} \leq r_{k_2},\ k_2=1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} \geq s_{k_3},\ k_3=1,2,...,n_3, \\ \sum_{k_1=1}^{n_1}\sum_{K_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} = s_{k_3},\ k_3=1,2,...,n_3, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} \geq s_{k_3},\ k_3=1,2,...,n_3, \\ \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,k_2,k_3. \end{cases} \tag{19}$$

3.2.2 Model II

Modeling for deterministic MOFSSTP may be transformed into a stochastic model when demand is uncertain, but supply and conveyance capacity restrictions remain certain.

$$\begin{cases} Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}},\ t=1,2,...,\zeta \end{cases}$$

$$\begin{cases} subject\ to: \\ \quad \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \geq q_{k_1},\ k_1 = 1,2,...,n_1, \\ \quad \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} = q_{k_1},\ k_1 = 1,2,...,n_1, \\ \quad \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \leq q_{k_1},\ k_1 = 1,2,...,n_1, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}}\{-ln(1-p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1,2,...,n_2, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} = [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}}\{-ln(1-p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1,2,...,n_2, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}}\{-ln(1-p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1,2,...,n_2, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} \geq s_{k_3},\ k_3 = 1,2,...,n_3, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} = s_{k_3},\ k_3 = 1,2,...,n_3, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} \leq s_{k_3},\ k_3 = 1,2,...,n_3, \\ \qquad\qquad \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1, k_2, k_3. \end{cases} \tag{20}$$

3.2.3 Model III

Modeling for deterministic MOFSSTP may be transformed into a stochastic model when conveyance capacity is uncertain, but supply and demand restrictions remain certain.

$$\begin{cases} \qquad Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{D}^t_{\mathfrak{L}_{\mathfrak{k}_1\mathfrak{k}_2\mathfrak{k}_3}}\mathcal{Y}_{k_1k_2k_3}},\ t = 1,2,...,\zeta \\ subject\ to: \\ \quad \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \geq q_{k_1},\ k_1 = 1,2,...,n_1, \\ \quad \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} = q_{k_1},\ i = 1,2,...,n_1, \\ \quad \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \leq q_{k_1},\ k_1 = 1,2,...,n_1, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \geq r_{k_2},\ k_2 = 1,2,...,n_2, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} = r_{k_2},\ k_2 = 1,2,...,n_2, \\ \quad \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \leq r_{k_2},\ k_2 = 1,2,...,n_2, \end{cases}$$

$$\begin{cases} \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3 = 1,2,...,n_3, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} = [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3 = 1,2,...,n_3, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3 = 1,2,...,n_3, \\ \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,k_2,k_3. \end{cases} \tag{21}$$

3.2.4 Model IV

Modeling for deterministic MOFSSTP may be transformed into a stochastic model when supply and demand restrictions remain uncertain.

$$\begin{cases} Minimize\ \mathcal{Z}^t(\mathcal{Y}) = \dfrac{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}},\ t = 1,2,...,\zeta \\ subject\ to: \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{q_{k_1}} + \vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1 = 1,2,...,n_1, \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} = [ln\psi_{q_{k_1}} + \vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1 = 1,2,...,n_1, \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{q_{k_1}} + \vartheta_{q_{k_1}}\{-ln(p_{q_{k_1}})\}]^{\frac{1}{\delta_{q_{k_1}}}},\ k_1 = 1,2,...,n_1, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}}\{-ln(1-p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} = [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}}\{-ln(1-p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3} \mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{r_{k_2}} + \vartheta_{r_{k_2}}\{-ln(1-p_{r_{k_2}})\}]^{\frac{1}{\delta_{r_{k_2}}}},\ k_2 = 1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} \geq [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3 = 1,2,...,n_3, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} = [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3 = 1,2,...,n_3, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2} \mathcal{Y}_{k_1k_2k_3} \leq [ln\psi_{s_{k_3}} + \vartheta_{s_{k_3}}\{-ln(p_{s_{k_3}})\}]^{\frac{1}{\delta_{s_{k_3}}}},\ k_3 = 1,2,...,n_3, \\ \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,k_2,k_3. \end{cases} \tag{22}$$

3.3 Hyperbolic Membership Function

Hyperbolic membership function can be defined as,

$$\mu_t^H(Z^t(y)) = \frac{1}{2}tanh\Bigg[\frac{U_t + L_t}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{\mathfrak{k}=1}^{l}\Big[F_R^t, F_C^t\Big]x_{ij\mathfrak{k}}\Bigg]\alpha_t + \frac{1}{2}.$$

where $\alpha_t = \frac{6}{U_t - L_t}$. This membership function has following properties.

(a) $\mu_t^H(Y^t(y))$ is strictly monotonically decreasing function with respect to $(Z^t(y))$;

(b) $\mu_t^H(Z^t(y))=\frac{1}{2}$ iff $(Z^t(y))=\frac{1}{2}(U^t+L^t)$;

(c) $\mu_t^H(Z^t(y))$ is strictly convex for $Z^t(y) \geq \frac{1}{2}(U_t+L_t)$;

(d) $\mu_t^H(Z^t(y))$ satisfies $0 < \mu_t^H(Z^t(y)) < 1$ for $L_t < F^t(y) < 1$ for $L^t < Z^t(y) < U_t$;

The fuzzy approach relies heavily on the membership function, which enables it to evaluate unexpected and ambiguous topics. The membership function of a fuzzy set represents a distinct, subjective human perspective. In addition to non-linear membership functions, fuzzy mathematical programming may also make use of hyperbolic membership functions. Over one subset of the objective function value, the hyperbolic function is convex, whereas over the other subset, it is concave. The finest compromise and effective solutions for a multi-objective fractional transportation problem are found by fuzzy programming with hyperbolic membership functions. It expresses objective functions in a fuzzy context. The attributes listed below apply to this membership function, where the parameter α_t is defined as $\alpha_t=\frac{6}{U_t-L_t}$. Next, the fuzzy model's matching crisp model can be expressed as follows:

$$\begin{cases} Minimize\ \Phi \\ subject\ to: \\ \Phi \leq \frac{1}{2}tanh\left[\left[\frac{U_t+L_t}{2} - \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\frac{\mathfrak{C}^t_{k_1k_2k_3}}{\mathfrak{D}^t_{k_1k_2k_3}}\mathcal{Y}_{k_1k_2k_3}\right]\alpha_t\right]+\frac{1}{2}. \\ \sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3}=a_{k_1},\ k_1=1,2,...,n_1, \\ \sum_{k_1=1}^{n_1}\sum_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3}=b_{k_2},\ k_2=1,2,...,n_2, \\ \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3}=c_{k_3},\ k_3=1,2,...,n_3, \\ \qquad \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,\ k_2,\ k_3. \end{cases} \tag{23}$$

3.4 Goal Programming

A fuzzy set theory-based approach to goal formulation is referred to as fuzzy goal programming. The membership functions are then used to characterize the fuzzy goals. Once both positive and negative deviational variables are included, and each is assigned the greatest membership value, these membership functions are transformed into fuzzy flexible membership objectives. Minimizing the differences between aspiration levels G_t1 and G_{t2} and goal $\mathcal{Z}^t(\mathcal{Y})$achievement is the primary goal. The following is a mathematical formulation of goal programming:

$$\begin{cases} \quad Minimize \ \mathcal{Z}^t(\mathcal{Y}) = \frac{\sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}[\mathfrak{C}^t_{k_1k_2k_3}]\mathcal{Y}_{k_1k_2k_3}-G_{t1}}{\sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}[\mathfrak{D}^t_{k_1k_2k_3}]\mathcal{Y}_{k_1k_2k_3}-G_{t2}} \\ subject\ to: \\ \quad \sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = a_{k_1},\ k_1 = 1,2,...,n_1, \\ \quad \sum\limits_{k_1=1}^{n_1}\sum\limits_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = b_{k_2},\ k_2 = 1,2,...,n_2, \\ \quad \sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} = c_{k_3},\ k_3 = 1,2,...,n_3, \\ \quad \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,\ k_2,\ k_3, \end{cases} \tag{24}$$

where G_t is the aspiration level and $\mathcal{Y}_{k_1k_2k_3}$ is the linear function of the t^{th} objective. Let the function $\mathcal{Y}_{k_1k_2k_3} = D_t^+ - D_t^- + G_t$ be used to solve the objective programming. The achievement function can thus be expressed as follows:

$$\begin{cases} Minimize \ \sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}(D_t^+ - D_t^-) \\ subject\ to: \\ \quad \frac{[\mathfrak{C}^t_{k_1k_2k_3}]\mathcal{Y}_{k_1k_2k_3}-G_{t1}}{[\mathfrak{D}^t_{k_1k_2k_3}]\mathcal{Y}_{k_1k_2k_3}-G_{t2}} = D_t^+ - D_t^-, t = 1,2,...,\zeta, \\ \quad X \in F, (F\ is\ a\ feasible\ set) \\ \quad D_t^+ - D_t^- \geq 0,\ t = 1,2,...,\zeta. \end{cases} \tag{25}$$

3.5 Min-max Approach

Among the several methods that have been developed for goal programming are preemptive goal programming and min-max goal programming. The following model is produced by converting Zimmermann [27] minmax technique to fuzzy goal programming.

$$\begin{cases} \quad Minimize \ \mathcal{Z}^t(\mathcal{Y}) = \frac{\sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}\mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}\mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}, t = 1,2,...,\zeta \\ subject\ to: \\ \quad \sum\limits_{k_2=1}^{n_2}\sum\limits_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = a_{k_1},\ k_1 = 1,2,...,n_1, \\ \quad \sum\limits_{k_1=1}^{n_1}\sum\limits_{k_3=1}^{n_3}\mathcal{Y}_{k_1k_2k_3} = b_{k_2},\ k_2 = 1,2,...,n_2, \\ \quad \sum\limits_{k_1=1}^{n_1}\sum\limits_{k_2=1}^{n_2}\mathcal{Y}_{k_1k_2k_3} = c_{k_3},\ k_3 = 1,2,...,n_3, \\ \quad \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1, k_2, k_3. \end{cases} \tag{26}$$

where there is satisfaction of the equilibrium condition $\sum\limits_{k_1=1}^{n_1} a_{k_1} = \sum\limits_{k_2=1}^{n_2} b_{k_2} = \sum\limits_{k_3=1}^{n_3} c_{k_3}$.

4 Numerical Example

This section explains the problem by taking the example of a third party logistics company that has four (supply) suppliers, q_1, q_2, q_3, and q_4 and three destination places such as hospitals, pharmacies and medical labs are expressed as r_1, r_2, and r_3. Suppliers q_1, q_2, q_3,and q_4. supply Covid-19 (covaxin), pandemic virus (oseltamivir), viral fever (paracetamol), and heart attack (loading dose) medicines to the destination. The two conveyances of the transport are represented by the numbers s_1 and s_2 which has distinct loading powers. The Weibull distribution has been used to preserve the medicine from its leakage and expiretion date. Reducing the total expense and duration of transportation is the ultimate objective of this challenge. Everyone has shown that the optimal compromise solutions for multi-objective fractional stochastic solid transportation problem-solving models may be achieved through the use of FGP. All mathematical programming models' deterministic equivalents were solved using the Lingo solver. The likelihood that the desired quantity of produce is accessible for the provider q_1 is p_{q_1}. In the same way, for suppliers q_2, q_3, and q_4, respectively, probabilities p_{q_2}, p_{q_3}, and p_{q_4} are defined. It stems from incorrect forecasts, fluctuations in demand, or unexpected delivery delays. The likelihood that the estimated demand is needed for medicine hall. r_1 is therefore p_{r_1}. Similar to this, market halls r_2, r_3, respectively, have defined probabilities p_{r_2}, p_{r_3}. Similarly, the amount of conveyance capacity is unknown due to blockages and traffic congestion, p_{s_1} and p_{s_2} are the odds that the capacity of two conveyances is available. The decision makers might select these probabilities based on insights or forecasts.

$$\begin{cases} Minimize\ \mathcal{Z}^1 = \dfrac{\sum\limits_{k_1=1}^{4}\sum\limits_{k_2=1}^{3}\sum\limits_{k_3=1}^{2} \mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum\limits_{k_1=1}^{4}\sum\limits_{k_2=1}^{3}\sum\limits_{k_3=1}^{2} \mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}, \\ Minimize\ \mathcal{Z}^2 = \dfrac{\sum\limits_{k_1=1}^{4}\sum\limits_{k_2=1}^{3}\sum\limits_{k_3=1}^{2} \mathfrak{C}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}{\sum\limits_{k_1=1}^{4}\sum\limits_{k_2=1}^{3}\sum\limits_{k_3=1}^{2} \mathfrak{D}^t_{k_1k_2k_3}\mathcal{Y}_{k_1k_2k_3}}, \\ subject\ to: \\ P(\sum\limits_{k_2=1}^{3}\sum\limits_{k_3=1}^{2} \mathcal{Y}_{k_1k_2k_3}) \leq p_{q_{k_1}},\ k_1 = 1,2,3,4 \\ P(\sum\limits_{k_1=1}^{4}\sum\limits_{k_3=1}^{2} \mathcal{Y}_{k_1k_2k_3}) \geq p_{r_{k_2}},\ k_2 = 1,2,3 \\ P(\sum\limits_{k_1=1}^{4}\sum\limits_{k_2=1}^{3} \mathcal{Y}_{k_1k_2k_3}) \leq p_{s_{k_3}},\ k_3 = 1,2 \\ \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1,k_2,k_3. \\ and\ constraint (4) \end{cases} \quad (27)$$

Table 1(a,b) and 3(a,b) present the transportation costs $c^1_{k_1k_21}$ and $c^1_{k_1k_22}$ for two conveyance, respectively. Table 2(a,b) and 4(a,b) present the transportation costs $c^2_{k_1k_21}$ and $c^2_{k_1k_22}$ for two conveyance, respectively.

For the numerical experiments in the following sections, the nominal values of uncertain are given as $q_1 = 8.24, q_2 = 9.28, q_3 = 9.73, q_4 = 10.73, r_1 = 21.16, r_2 = 20.95, r_3 = 20.54, s_1 = 22.00$ and $s_2 = 22.44$.

Also, random probabilities are provided as $p_{q_1} = 0.96, p_{q_2} = 0.94, p_{q_3} = 0.93$, $p_{q_4} = 0.91, p_{r_1} = 0.54, p_{r_2} = 0.53, p_{r_3} = 0.51, p_{s_1} = 0.42$ and $P_{s_2} = 0.40$. The differences in the parameters of the Weibull distribution are seen as $\vartheta_{q_{k_1}} = \vartheta_{r_{k_2}} = \vartheta_{s_{k_3}} = 2.5$ and $\delta_{q_{k_1}} = \delta_{r_{k_2}} = \delta_{s_{k_3}} = 2.5$ and $\psi_{q_{k_1}} = \psi_{r_{k_2}} = \psi_{s_{k_3}} = 20.65$ given that it is expected that the constants q_{k_1}, r_{k_2}, and s_{k_3} all have Weibull distributions. It is simple to transform the probabilistic restrictions into their deterministic kinds by applying equations (4). For the reason of

Table 1: Transportation Cost

$\mathbf{c^1_{k_1k_21}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	1	2	3
q_2	4	5	6
q_3	9	8	7
q_4	11	10	12

Table 1(a). Transportation cost for conveyance e_2(*i.e.*, $c^1_{k_1k_21}$)

$\mathbf{c^1_{k_1k_22}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	4	5	6
q_2	8	2	3
q_3	10	1	2
q_4	3	4	5

Table 1(b). Transportation cost for conveyance e_2(*i.e.*, $c^1_{k_1k_22}$)

Table 2: Transportation Time

$\mathbf{c^2_{k_1k_21}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	6 hour	5 hour	3 hour
q_2	5 hour	1 hour	2 hour
q_3	4 hour	2 hour	6 hour
q_4	4 hour	3 hour	2 hour

Table 2(a). Transportation time for conveyance e_2(*i.e.*, c^2_{ij1})

$\mathbf{c^2_{k_1k_22}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	5 hour	2 hour	2 hour
q_2	7 hour	3 hour	4 hour
q_3	5 hour	4 hour	2 hour
q_4	3 hour	7 hour	3 hour

Table 2(b). Transportation time for conveyance e_2(*i.e.*, $c^2_{k_1k_22}$)

Table 3: Transportation Cost

$\mathbf{c^1_{k_1k_21}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	2	2	3
q_2	3	5	6
q_3	8	9	1
q_4	4	5	1

Table 3(a). Transportation cost for conveyance e_2(*i.e.*, $c^1_{k_1k_21}$)

$\mathbf{c^1_{k_1k_22}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	3	17	14
q_2	8	9	16
q_3	24	23	27
q_4	12	13	14

Table 3(b). Transportation cost for conveyance e_2(*i.e.*, $c^2_{k_1k_21}$)

Table 4: Transportation Time

$\mathbf{c^2_{k_1k_21}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	1 hour	3 hour	4 hour
q_2	2 hour	4 hour	6 hour
q_3	2 hour	2 hour	6 hour
q_4	5 hour	4 hour	2 hour

Table 4(a). Transportation time for conveyance e_2($i.e.$, $c^1_{k_1k_22}$)

$\mathbf{c^2_{k_1k_22}}$	$\mathbf{r_1}$	$\mathbf{r_2}$	$\mathbf{r_3}$
q_1	2 hour	5 hour	6 hour
q_2	1 hour	7 hour	3 hour
q_3	5 hour	5 hour	8 hour
q_4	4 hour	4 hour	2 hour

Table 4(b). Transportation time for conveyance e_2($i.e.$, $c^2_{k_1k_22}$)

conciseness, the full formulation for this numerical example has been left out.

$$\begin{cases} Minimize\ \mathcal{Z}^1 \\ = \frac{11x_{111}+21x-121+22x_{131}+13x_{211}+10x_{221}+19x_{231}+18x_{311}+25x_{321}+15x_{331}+23x_{411}+14x_{421}+21x_{431}+}{10x_{111}+9x_{121}+8x_{131}+16x_{211}+27x_{221}+21x_{231}+23x_{311}+31x_{321}+4x_{331}+12x_{411}+16x_{421}+13x_{431}+} \\ \quad \frac{13x_{112}+6x_{122}+20x_{132}+11x_{212}+22x_{222}+15x_{232}+12x_{312}+23x_{322}+14x_{332}+15x_{412}+24x_{422}+21x_{432}}{32x_{112}+17x_{122}+14x_{132}+8x_{212}+9x_{222}+16x_{232}+24x_{312}+23x_{322}+27x_{332}+12x_{412}+13x_{422}+14x_{432}} \\ Minimize\ \mathcal{Z}^2 \\ = \frac{6x_{111}+5x_{121}+3x_{131}+5x_{211}+9x_{221}+2x_{231}+10x_{311}+8x_{321}+6x_{331}+4x_{411}+7x_{421}+2x_{431}+}{1x_{111}+3x_{121}+4x_{131}+9x_{211}+8x_{221}+11x_{231}+2x_{311}+2x_{321}+6x_{331}+5x_{411}+4x_{421}+8x_{431}+} \\ \quad \frac{5x_{112}+2x_{122}+2x_{132}+7x_{212}+3x_{222}+4x_{232}+5x_{312}+4x_{322}+2x_{332}+8x_{412}+7x_{422}+3x_{432}}{2x_{112}+5x_{122}+6x_{132}+8x_{212}+7x_{222}+9x_{232}+5x_{312}+5x_{322}+8x_{332}+4x_{412}+4x_{422}+2x_{432}} \\ \quad subject\ \ to: \\ x_{111}+x_{112}+x_{121}+x_{122}+x_{131}+x_{132} \leq 8.2448; \\ x_{211}+x_{212}+x_{221}+x_{222}+x_{231}+x_{232} \leq 9.2846; \\ x_{311}+x_{312}+x_{321}+x_{322}+x_{231}+x_{232} \leq 9.7314; \\ x_{411}+x_{412}+x_{421}+x_{422}+x_{431}+x_{432} \leq 10.5305; \\ x_{111}+x_{211}+x_{311}+x_{411}+x_{112}+x_{212}+x_{312}+x_{412} \geq 21.1630; \\ x_{121}+x_{221}+x_{321}+x_{421}+x_{122}+x_{222}+x_{322}+x_{422} \geq 20.9545; \\ x_{131}+x_{231}+x_{331}+x_{431}+x_{132}+x_{232}+x_{332}+x_{432} \geq 20.5401; \\ x_{111}+x_{211}+x_{311}+x_{411}+x_{121}+x_{221}+x_{321}+x_{421}+x_{131}+x_{231}+x_{331}+x_{431} \leq 22.0086; \\ x_{112}+x_{212}+x_{312}+x_{412}+x_{122}+x_{222}+x_{322}+x_{422}+x_{132}+x_{232}+x_{332}+x_{432} \leq 22.4403; \\ \qquad\qquad \mathcal{Y}_{k_1k_2k_3} \geq 0, \forall\ k_1, k_2, k_3 \end{cases} \tag{28}$$

(a) Every single-objective transportation problem has an answer that is

$$[X_{111} = 8, X_{131} = 21, X_{211} = 9, X_{411} = 1, X_{412} = 4, X_{421} = 21],$$

$$[X_{111} = 4, X_{131} = 2, X_{211} = 1, X_{221} = 8, X_{311} = 3, X_{321} = 6, X_{421} = 7, X_{112} = 4, X_{412} = 10].$$

(b) The values of the objective function are

$$Z^1 = 587, Z^2 = 96.$$

(c) The following can be used to express the upper and lower limits of each objective function;

$$L = 96, U = 587.$$

(d) The following is the model

$$\begin{cases} Minimize\ \Phi \\ subject\ to: \\ \Phi \leq \frac{1}{2}tanh\left[\left[\frac{U_t+L_t}{2} - \sum_{k_1=1}^{n_1}\sum_{k_2=1}^{n_2}\sum_{k_3=1}^{n_3}\frac{\mathfrak{C}^t_{k_1k_2k_3}}{\mathfrak{D}^t_{k_1k_2k_3}}\mathcal{Y}_{k_1k_2k_3}\right]\alpha_t\right] + \frac{1}{2} + D_t^- - D_t^+ = 1. \end{cases} \tag{29}$$

After applying the linear interactive global optimization (LINGO) program to solve the problem, an ideal compromise was found to be as follows.
$Z^1 = 587,\ Z^2 = 96.$
$D^+ = [0.5]; D^- = [0]$ and Minimize Φ=0.1.

5 Comparison and Discussion

This section presents optimal solutions that account for changes in probability on uncertain parameters such as supply, demand and conveyance capacity in the MOFSSTP. Using the Lingo 18.0 software for the resolution and listing of the mixed integer problem. We derive the ideal answer as follows: $X_{111} = 8, X_{131} = 21, X_{211} = 9, X_{411} = 1, X_{412} = 4, X_{421} = 21, X_{111} = 4, X_{131} = 2, X_{211} = 1, X_{221} = 8, X_{311} = 3, X_{321} = 6, X_{421} = 7, X_{112} = 4, X_{412} = 10$ and all other decision variables are zero. The objective functions least cost and time are $Z_1 = 587$ and $Z_2 = 96$, respectively. Das and Lee [2], using lingo once more, resolved and listed the mixed integer solution. The following is the ideal answer: $X_{111} = 1.09, X_{121} = 0.22, X_{131} = 0.25, X_{112} = 0.25, X_{122} = 18.42, X_{132} = 0.24, X_{211} = 18.52, X_{221} = 0.24, X_{231} = 0.25, X_{212} = 0.25, X_{222} = 0.22$, all other decision variables are zero. The objective function's minimum cost and time are $Z_1 = 589.85$ and $Z_2 = 112$, respectively. The results boldly mark the minimal objective values. After obtaining the ideal answer, the decision maker determines the cost and time for each route, as well as the specific supply and demand for the presented problem. We cannot directly address the proposed issue without employing the MOFSSTP.

This section presents optimal solutions that account for changes in probability on uncertain parameters such as supply, demand and conveyance capacity in the MOFSSTP. Using the Schrage [28], Lingo 18.0 software for the resolution and listing of the mixed integer problem. We derive the ideal answer as follows: $X_{111} = 8, X_{131} = 21, X_{211} = 9, X_{411} = 1, X_{412} = 4, X_{421} = 21, X_{111} = 4, X_{131} = 2, X_{211} = 1, X_{221} = 8, X_{311} = 3, X_{321} = 6, X_{421} = 7, X_{112} = 4, X_{412} = 10$ and all other decision variables are zero. The objective functions least cost and time are $Z_1 = 587$ and $Z_2 = 96$, which are given in figure 1 respectively. Das and Lee [2], using lingo once more, i resolved and listed the mixed integer solution. The following is the ideal answer: $X_{111} = 1.09, X_{121} = 0.22, X_{131} = 0.25, X_{112} = 0.25, X_{122} = 18.42, X_{132} = 0.24, X_{211} = 18.52, X_{221} = 0.24, X_{231} = 0.25, X_{212} = 0.25, X_{222} = 0.22$, all other decision variables are zero. The objective function's minimum cost and time are $Z_1 = 589.85$ and $Z_2 = 112$, which are given in table (5) respectively. The results boldly mark the minimal objective values. After obtaining the ideal answer, the decision maker determines the cost and time for each route, as well as the specific supply and demand for the presented problem. We cannot directly address the proposed issue without employing the MOFSSTP.

Table 5: Compromise solution of the Proposed method of Existing method of MCSFTP

	z_1	z_2	Hyperbolic Membership Function
Proposed Approach	587	96	0.5, 0
Existing Approach	589.87	112	-

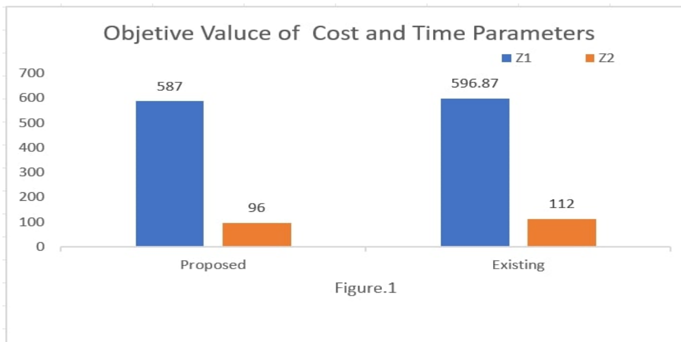

Figure 1: Optimal transportation cost and time in relation to the probability of demand availability

5.1 Analysis and Discussions of Sensitivity

This section exhibits a sensitivity analysis of the most effective solutions in the MOFSSTP with regard to variations in probability on the three unknown factors (supply, demand, and conveyance capacity). For the sensitivity analysis, we simply changed the probability on q_{k_1}, r_{k_2}, and s_{k_3}. Otherwise, we employed the identical test problems. By changing the probability for one parameter while keeping the other two probabilities at 0.49, we were able to solve Case IV of MOFSSTP for the test problem. For every stochastic setting, the best solutions' transportation costs and times were found and are shown in Table 6, Table 7 and Table 8. The minimal objective values are marked by the results in bold.

There are several intriguing trends in the sensitivity with regard to the likelihood of q_{k_1}. The transportation cost and time related to the likelihood for q_{k_1} are illustrated in Figures 2(a,b), respectively.

The lowest transportation cost was noted in Figure 2(a) for $p_{q_{k_1}} = 0.69$. When $(0 \geq p_{q_{k_1}} < 0.69)$, the cost of transportation either stays the same or steadily rises. A significant increase in transportation costs occurs when $p_{q_{k_1}} > 0.69$. The shortest transportation time was attained when $p_{q_{k_1}} = 0.7$ in Figure 2(b). There is no change in the transportation time for $(0 \geq p_{q_{k_1}} < 0.7)$. The transportation time rises when $p_{q_{k_1}} = 0.69$. The results of this sensitivity analysis show that the test problem's two objective functions are very responsive to changes in the probability for q_{k_1}. A decision-maker can select a suitable probability for the supply availability with the aid of this analysis.

Table 7 summarizes the results of the sensitivity analysis for the probability for r_{k_2}. Figure 3(a,b) displays the graphical depictions of the transportation cost and time in relation to the likelihood for r_{k_2}. As can be seen in Figure 3(a), the transportation cost progressively rises as the probability for r_{k_2} increases. Be aware that the cost of transportation is affected by changes in the likelihood of demand requirements. It can be seen from Figure 3(b) that for $0 \geq p_q k_1 < 0.7$, the transportation time is constant. The transportation time drops to the lowest value when $p_{r_{k_2}} = 0.7$, and it stays there for $0 \geq r_{k_2} \geq 0.69$. If decision-makers possess a comprehensive comprehension of the probability sensitivity patterns for the unknown parameters, they could be better equipped to construct the transportation network.

A sensitivity study of the conveyance capacity reveals that while there was no effect on the transportation time, there was a considerable impact on the transportation cost. This is so because the speed of a trans-

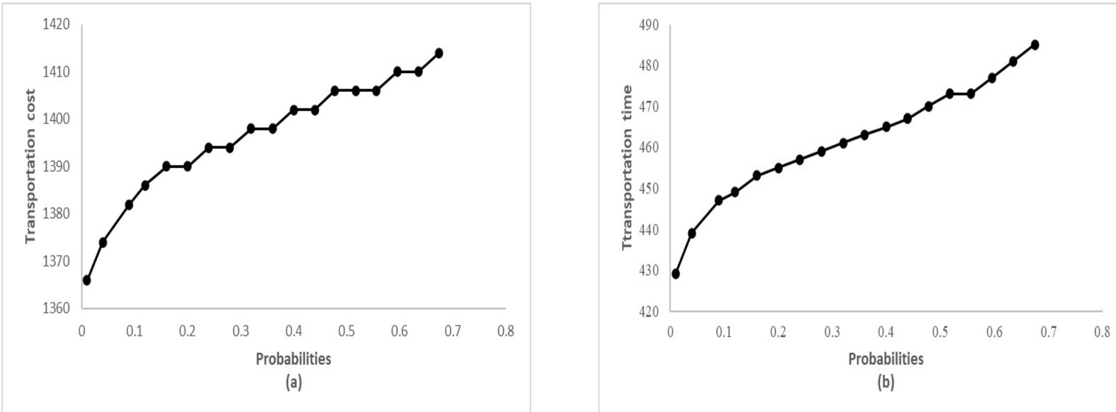

Figure 2: Sensitivity analysis of the best possible transportation time (b) and cost (a) in relation to the likelihood of demand availability

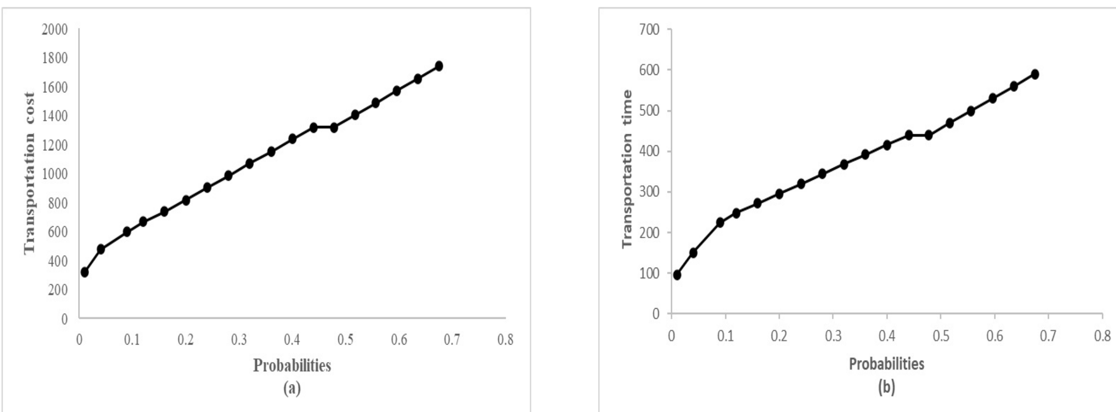

Figure 3: Analysis of the sensitivity of the coast (a) and optimal transit time (b) to the probability of demand availability

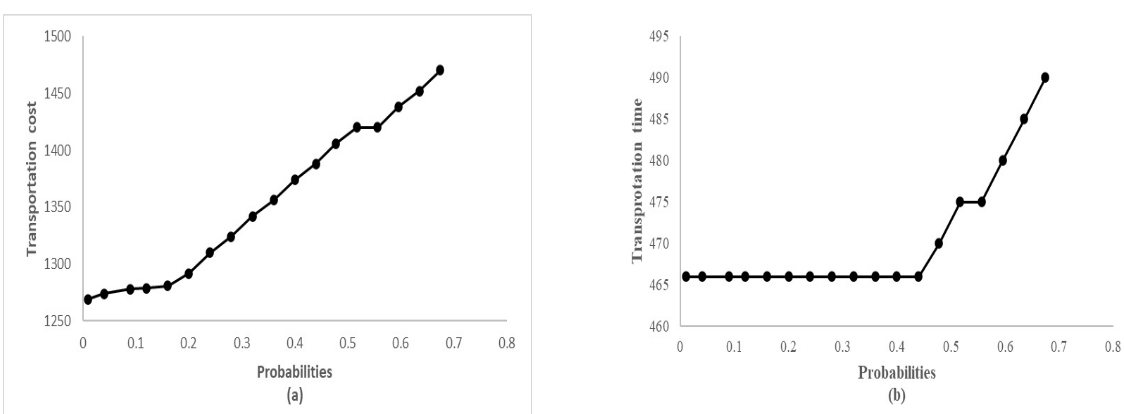

Figure 4: Analysis of the sensitivity of the cost (a) and optimal transit time (b) to the probability of demand availability

Table 6: Sensitivity analysis concerning the probability for q_{k_1}

Probability for $q_{k_1}(P_{q_{k_1}})$	**Probability for** $r_{k_2}(P_{r_{k_2}})$	**Probability for** $s_{k_3}(P_{s_{k_3}})$	**Transportation Cost**	**Transportation Time(h)**
0.01	0.49	0.49	1366	429
0.04	0.49	0.49	1374	439
0.09	0.49	0.49	1382	447
0.12	0.49	0.49	1386	449
0.16	0.49	0.49	1390	453
0.20	0.49	0.49	1390	455
0.24	0.49	0.49	1394	457
0.28	0.49	0.49	1394	459
0.32	0.49	0.49	1398	461
0.36	0.49	0.49	1398	463
0.40	0.49	0.49	1402	465
0.44	0.49	0.49	1402	467
0.47	0.49	0.49	1406	470
0.51	0.49	0.49	1406	473
0.55	0.49	0.49	1406	473
0.59	0.49	0.49	1410	477
0.63	0.49	0.49	1410	481
0.67	0.49	0.49	1414	485

Table 7: Analysis of sensitivity with regard to chances for r_{k_2}

Probability for $q_{k_1}(P_{q_{k_1}})$	**Probability for** $r_{k_2}(P_{r_{k_2}})$	**Probability for** $s_{k_3}(P_{s_{k_3}})$	**Transportation Cost**	**Transportation Time(h)**
0.49	0.01	0.49	318	96
0.49	0.04	0.49	477	151
0.49	0.09	0.49	599	224
0.49	0.12	0.49	668	248
0.49	0.16	0.49	737	272
0.49	0.20	0.49	816	296
0.49	0.24	0.49	902	320
0.49	0.28	0.49	984	344
0.49	0.32	0.49	1070	368
0.49	0.36	0.49	1152	392
0.49	0.40	0.49	1238	416
0.49	0.44	0.49	1320	440
0.49	0.47	0.49	1320	440
0.49	0.51	0.49	1405	470
0.49	0.55	0.49	1488	500
0.49	0.59	0.49	1574	560
0.49	0.63	0.49	1656	560
0.49	0.67	0.49	1742	590

Table 8: Sensitivity analysis of s_{k_3} with respect to probability

Probability for $q_{k_1}(P_{q_{k_1}})$	**Probability for** $r_{k_2}(P_{r_{k_2}})$	**Probability for** $s_{k_3}(P_{s_{k_3}})$	**Transportation Cost**	**Transportation Time(h)**
0.49	0.49	0.01	1269	466
0.49	0.49	0.04	1274	466
0.49	0.49	0.09	1278	466
0.49	0.49	0.12	1279	466
0.49	0.49	0.16	1281	466
0.49	0.49	0.20	1292	466
0.49	0.49	0.24	1310	466
0.49	0.49	0.28	1324	466
0.49	0.49	0.32	1342	466
0.49	0.49	0.36	1356	466
0.49	0.49	0.40	1374	466
0.49	0.49	0.44	1388	466
0.49	0.49	0.47	1406	470
0.49	0.49	0.51	1420	475
0.49	0.49	0.55	1420	475
0.49	0.49	0.59	1438	480
0.49	0.49	0.63	1452	485
0.49	0.49	0.67	1470	490

portation conveyance is independent of its capacity. Table 8 shows that while the transportation duration is constant throughout all experiments, the cost of transportation decreases progressively as the likelihood of a conveyance capacity increases for s_{k_3}. The sensitivity against the likelihood for the conveyance capacity for s_{k_3} is shown in Figure 4(a,b), correspondingly. The transportation cost is seen to grow progressively with the probability for s_{k_3} in Figure 4(a). It is demonstrated that for $0 \geq p_{q_{k_1}} < 0.45$, the transit time in Figure 4(b) stays constant. The distance traveled falls to the shortest when $p_{s_{k_3}} = 0.45$, and it stays at that point for $0 \geq r_{k_2} \geq 0.7$. Considering the test problem, the conveyance limits have a significant impact on the transportation cost.

The study advised MOFSSTP models, it is stated and provide the best results in an uncertain setting. In situations with a great deal of uncertainty, it is insignificant to take the more cautious course of action. As a result, it is seen that the STP becomes increasingly questionable as the probability for q_{k_1}, r_{k_2}, and s_{k_3} drop. It makes sense to select more conservative solutions when the optimization problem has more uncertainty. When the uncertainty caused by the probability for q_{k_1}, r_{k_2}, and s_{k_3} is modeled with MOFSSTP, the most conservative solutions are adopted. The sensitivity analysis, however, demonstrates how important it is to comprehend how sensitive different objectives are to rising levels of uncertainty. It gives a decision-maker information about which probability levels are appropriate for unclear parameters.

6 Conclusion

In multi-objective optimization, a hyperbolic membership function and a fractional fuzzy goal programming technique are shown to be effective methods for managing the problem's intrinsic uncertainty. The proposed models and techniques are not only theoretically sound but also practically applicable, by using computational results. The sensitivity analysis provides insights into the robustness of the proposed model. Overall, this research contributes to the field by offering a comprehensive framework that combines fractional programming, fuzzy goal programming, and stochastic modeling to tackle a practical transportation problem with inherent uncertainties. Stochastic optimization approaches, like simulation-based optimization, robust optimization, and stochastic programming, are frequently used to solve this challenge in order to discover dependable, strong solutions that function well under uncertainty in the future achieve global solution. The optimization challenge is solved with the Lingo software.

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