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A Novel Method of Decision-Making Based on Intuitionistic Fuzzy Set Theory

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(This paper is dedicated to Professor "John N. Mordeson" on the occasion of his 91st birthday.)

Abstract. Atanassov's intuitionistic fuzzy set is more adept at representing and managing uncertainty. Within intuitionistic fuzzy set theory, intuitionistic fuzzy measure is a significant field of study. In order to address decision making, we present a novel similarity metric between intuitionistic fuzzy sets in this study. First, based on the minimum and maximum levels of similarity, we suggest a new similarity metric between intuitionistic fuzzy values. It is capable of overcoming the limitations of current approaches to gauging the degree of resemblance between fuzzy intuitionistic sets. It is also possible to show some aspects of the suggested similarity measure between intuitionistic fuzzy sets by taking into account the modal operators and their different extensions. Finally, we apply the proposed similarity measure between intuitionistic fuzzy sets to deal with a real life problem. The suggested action can provide a precise outcome. The application section examines a real-world issue of choosing the best course of action among n options based on m criteria. A fictitious case study is created along with the method's algorithm.

AMS Subject Classification 2020: 90C70; 03F55
Keywords and Phrases: Intuitionistic fuzzy sets, Modal operators, Measure of similarity, Decision making, Optimal solution.

1 Introduction

In 1965, L.A. Zadeh [1] created and introduced the idea of a fuzzy set. Eighteen years later, in 1983, Atanassov [2] introduced the concept of intuitionistic fuzzy sets as an extension of fuzzy sets. The fundamental distinction between these two ideas is that, in intuitionistic fuzzy set theory, hesitation margin is taken into account in addition to both membership function and non-membership function. In fuzzy set theory, only the membership function is taken into account. Scholars and researchers [3, 4, 5, 6, 7, 8] are exerting great effort to advance and refine this field.
The notion of modal operators were first introduced by Atanassov [9] in 1986. Modal operators ($\Box, \Diamond$) defined over the set of all intuitionistic fuzzy sets that convert every intuitionistic fuzzy set into a fuzzy set. Atanassov [9] also introduced the operators ($\boxplus, \boxtimes$) in intuitionistic fuzzy set. More relations and properties on these operators are regorously studied in [10, 11, 12, 3, 4, 5]. The second extension of the operators $\boxplus$ and $\boxtimes$ are introduced by K. Dencheva [13].
There are circumstances in which fuzzy set theory is not the best fit and should be replaced with intuitionistic fuzzy set theory. intuitionistic fuzzy set theory has been researched as a helpful resource for decision-making

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issues, logic programming, etc. In this work, we establish a similarity measure between two intuitionistic fuzzy sets A and B of a set E and apply it to a problem involving decision-making. The issue under consideration is choosing the best course of action from n options based on m criteria in cases when the information at hand is intuitionistic fuzzy.

Recently, there has been a lot of focus on measures of similarity between Intuitionistic Fuzzy Sets as a crucial tool for image processing, machine learning, pattern detection, and decision making [14, 15]. Numerous measurements of similarity have been put forth. Some of them are derived from the widely used distance measures.

The first study was carried out by Szmidt and Kacprzyk [16] extending the well-known distances measures, such as the Hamming distance and the Euclidian distance, to IFS environment and comparing them with the approaches used for ordinary fuzzy sets. Therefore, several new distance measures were proposed and applied to pattern recognition. Grzegorzewski [17] also extended the Hamming distance, the Euclidean distance, and their normalized counterparts to IFS environment. Hung and Yang [18] extended the Hausdorff distance to Intuitionistic Fuzzy Sets and proposed three similarity measures.

On the other hand, instead of extending the well-known measures, some studies defined new similarity measures for Intuitionistic Fuzzy Sets. Dengfeng and Chuntian [19] suggested a new similarity measure for IFSs based on the membership degree and the nonmembership degree. Ye [15] conducted a similar comparative study of the existing similarity measures between Intuitionistic Fuzzy Sets and proposed a cosine similarity measure and a weighted cosine similarity measure. Xu and Chen [20] introduced a series of distance and similarity measures, which are various combinations and generalizations of the weighted Hamming distance, the weighted Euclidean distance, and the weighted Hausdorff distance. Xu and Yager [21] developed a similarity measure between Intuitionistic Fuzzy Sets and applied the developed similarity measure for consensus analysis in group decision making based on intuitionistic fuzzy preference relations.

Zeng and Guo [22] investigated the relationship among the normalized distance, the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets. It was also showed that the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets could be induced by the normalized distance of interval-valued fuzzy sets based on their axiomatic definitions. Moreover, Zhang and Yu [23] presented a new distance (or similarity) measure based on interval comparison, where the Intuitionistic Fuzzy Sets were, respectively, transformed into the symmetric triangular fuzzy numbers. Comparison with the widely used methods indicated that the proposed method contained more information, with much less loss of information. Li et al. [24] introduced an axiomatic definition of the similarity measure of Intuitionistic Fuzzy Sets. The relationship between the entropy and the similarity measure of IFS was investigated in detail. It was proved that the similarity measure and the entropy of IFS can be transformed into each other based on their axiomatic definitions.

Several writers have recently discussed the use of various similarity measures in image processing, pattern recognition, medical diagnosis, and decision making. Song et al.[25] presented some applications to pattern recognition and presented a new similarity metric for intuitionistic fuzzy sets. Ejegwa et al.[26] represented Thao et al.'s correlation coefficient of Intuitionistic fuzzy sets for medical diagnostic analysis on some selected patients. Based on Spearman's correlation coefficient, Ejegwa et al. [27] identified medical emergencies in 2024 using novel intuitionistic fuzzy correlation measurements. Recently tendency coefficient based on weighted distance measure for intuitionistic fuzzy sets was discussed by Anum et al. [28]. Additionally, Ejegwa et al. [29] presented a novel approach to calculating the distance between intuitionistic fuzzy sets and discussed about how to use it in the admissions process. Zhou et al.[30] provided a detailed discussion of the generalised similarity operator for intuitionistic fuzzy sets and how to apply it using the multiple criteria decision making technique and the recognition principle. In a paper pertaining to the intuitionistic fuzzy sets approach, Nwokoro et al.[31] also made predictions regarding maternal outcomes.

Therefore, we propose a novel method for decision-making based on intuitionistic fuzzy set theory. The proposed similarity measure depends on membership degree, and hesitation margin. This paper proves that the proposed measures satisfy the properties of the axiomatic definition for similarity measures. In addition, several numerical examples are provided to establish some relations. The final section presents the suggested similarity measure's use for decision-making.

2 Preliminary Concepts

Throughout this study, intuitionistic fuzzy set and fuzzy set are denoted by IFS and FS respectively.

Definition 2.1. [9] *Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A, \nu_A : x \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*
Furthermore, we have $\pi_A(x)= 1\text{-}\mu_A(x)\text{-}\nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ that is $\pi_A : x \to [0,1]$ and $0 \leq \pi_A(x) \leq 1$ for every $x \in X$.
$\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 2.2. [9] *Let X be a nonempty set. If A is an IFS drawn from X, then the modal operators which are also termed as necessity and possibility operators can be defined as*

1. $\Box A= \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$

2. $\Diamond A= \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle : x \in X\}$

For a proper IFS, $\Box A \subset A \subset \Diamond A$ and $\Box A \neq A \neq \Diamond A$.

Definition 2.3. [9] *Let X be a nonempty set. If A is an IFS drawn from X, then,*

1. $\boxplus A= \{\langle x, \frac{\mu_A(x)}{2}, \frac{\nu_A(x)+1}{2}\rangle : x \in X\}$

2. $\boxtimes A= \{\langle x, \frac{\mu_A(x)+1}{2}, \frac{\nu_A(x)}{2}\rangle : x \in X\}$

For a proper IFS, $\boxplus A \subset A \subset \boxtimes A$ and $\boxplus A \neq A \neq \boxtimes A$.

Definition 2.4. [32] *Let $\alpha \in [0,1]$ and let A be an IFS. Then the first extension of the operators $\boxplus$ and $\boxtimes$ can be defined as*

1. $\boxplus_\alpha A= \{\langle x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha\rangle : x \in X\}$

2. $\boxtimes_\alpha A= \{\langle x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x)\rangle : x \in X\}$.

Definition 2.5. [13] *Let $\alpha, \beta, \alpha + \beta \in [0, 1]$ and let A be an IFS. Then the second extension of the operators $\boxplus$ and $\boxtimes$ can be defined as*

1. $\boxplus_{\alpha,\beta} A = \{\langle x, \alpha\mu_A(x), \alpha\nu_A(x) + \beta\rangle : x \in X\}$

2. $\boxtimes_{\alpha,\beta} A = \{\langle x, \alpha\mu_A(x) + \beta, \alpha\nu_A(x)\rangle : x \in X\}$.

Definition 2.6. [33] *Let us consider two IFSs A and B of a fixed set E. The similarity measure between A and B denoted by $s(A, B)$ is defind by an interval $[e_{AB}, e'_{AB}]$, where*

$$e_{AB} = \max \min_{x \in E} \{\mu_A(x), \mu_B(x)\}$$

$$e'_{AB} = \max \min_{x \in E} \{\mu_A(x) + \pi_A(x), \mu_B(x) + \pi_B(x)\}$$

Here e_{AB} indicates the minimum amount of similarity and e'_{AB} indicates the maximum amount of similarity between A and B.
It can be noted that

1. $s(A, B) \subseteq [0, 1]$.

2. $s(A, B) = s(B, A)$.

3. *If $\pi_A(x) = 0$ and $\pi_B(x) = 0, \forall x \in E$, then $e_{AB} = e'_{AB}$.*

Moreover it may be mentioned that $e_{AB} \neq e'_{AB}$ for $A = B$.
.

Proposition 2.7. [33] *Let A and B be two IFSs and $s(A, B) = [e_{AB}, e'_{AB}]$, then*

1. $s(\Box A, \Box B) = e_{AB}$,
2. $s(\Diamond A, \Diamond B) = e'_{AB}$.

3 Measure of Similarity between Intuitionistic Fuzzy Sets

This section provides an example-based explanation of Definition 2.6, leading to some intriguing findings.

Example 3.1. Consider two IFSs A and B of $E = \{x_1, x_2, x_3, x_4\}$ given by the following table:

x	μ_A	ν_A	μ_B	ν_B
x_1	0.65	0.26	0.72	0.18
x_2	0.32	0.46	0.56	0.38
x_3	0.80	0.12	0.48	0.42
x_4	0.70	0.25	0.83	0.12

Using Definition 2.6, we have $e_{AB} = 0.70$, $e'_{AB} = 0.75$ and hence similarity measure between A and B is [0.70, 0.75].

Theorem 3.2. *Let A and B be two IFSs and $s(A, B) = [e_{AB}, e'_{AB}]$, then*

1. $s(\boxplus A, \boxplus B) = [\frac{1}{2}e_{AB},\ \frac{1}{2}e'_{AB}\ ]$,
2. $s(\boxtimes A, \boxtimes B) = [\frac{1}{2}e_{AB} + \frac{1}{2},\ \frac{1}{2}e'_{AB} + \frac{1}{2}]$.

Proof. 1. L.H.S $= \max\min_{x\in E}\{\frac{\mu_A(x)}{2}, \frac{\mu_B(x)}{2}\}, \max\min_{x\in E}\{\frac{\mu_A(x)}{2} + \frac{\pi_A(x)}{2}, \frac{\mu_B(x)}{2} + \frac{\pi_B(x)}{2}\}$
$= \max\min_{x\in E} \frac{1}{2}\{\mu_A(x), \mu_B(x)\}, \max\min_{x\in E} \frac{1}{2}\{\mu_A(x) + \pi_A(x), \mu_B(x) + \pi_B(x)\}$
$= \frac{1}{2}\max\min_{x\in E}\{\mu_A(x), \mu_B(x)\}, \frac{1}{2}\max\min_{x\in E}\{\mu_A(x) + \pi_A(x), \mu_B(x) + \pi_B(x)\}$
$= [e_{AB}, e'_{AB}]$
Similarly the other statement can be proved.

□

Theorem 3.3. *Let $\alpha \in [0, 1]$ and let A & B be two IFSs. If $s(A, B) = [e_{AB}, e'_{AB}]$, then*

1. $s(\boxplus_\alpha A, \boxplus_\alpha B) = [\alpha e_{AB},\ \alpha e'_{AB}]$,
2. $s(\boxtimes_\alpha A, \boxtimes_\alpha B) = [\alpha e_{AB} + 1 - \alpha,\ \alpha e'_{AB} + 1 - \alpha\ ]$.

Proof. 1. L.H.S $= \max\min_{x\in E}\{\alpha\mu_A(x), \alpha\mu_B(x)\}, \max\min_{x\in E}\{\alpha\mu_A(x) + \alpha\pi_A(x), \alpha\mu_B(x) + \alpha\pi_B(x)\}$
$= \alpha\max\min_{x\in E}\{\mu_A(x), \mu_B(x)\}, \alpha\max\min_{x\in E}\{\mu_A(x) + \pi_A(x), \mu_B(x) + \pi_B(x)\}$
$= [\alpha e_{AB}, \alpha e'_{AB}]$
Similarly the other statement can be proved.
□

Theorem 3.4. *Let A & B be two IFSs with $\alpha, \beta \in [0, 1]$ and $\alpha + \beta = 1$. If $s(A, B) = [e_{AB}, e'_{AB}]$, then*

1. $s(\boxplus_{\alpha,\beta} A, \boxplus_{\alpha,\beta} B) = [\alpha e_{AB},\ \alpha e'_{AB}]$,
2. $s(\boxtimes_{\alpha,\beta} A, \boxtimes_{\alpha,\beta} B) = [\alpha e_{AB} + \beta,\ \alpha e'_{AB} + \beta\ ]$.

Proof. Similar to the Theorem 3.3 □
The above theorem is not true for $\alpha, \beta \in [0, 1]$ and $\alpha + \beta < 1$.
If we consider the example 3.1 with $\alpha = 0.7$ and $\beta = 0.1$ then it is found that $s(\boxplus_{\alpha,\beta} A, \boxplus_{\alpha,\beta} B) = [0.49,\ 0.725]$ $\neq [\alpha e_{AB},\ \alpha e'_{AB}]$ and $s(\boxtimes_{\alpha,\beta} A, \boxtimes_{\alpha,\beta} B) = [0.59,\ 0.825] \neq [\alpha e_{AB} + \beta,\ \alpha e'_{AB} + \beta\ ]$.

Example 3.5. Consider the IFSs A and B of E as in example 3.1. To find $s(\Box A, \Box B)$ and $s(\Diamond A, \Diamond B)$ we have to construct the new tables as

x	μ_A	$1-\mu_A$	μ_B	$1-\mu_B$
x_1	0.65	0.35	0.72	0.28
x_2	0.32	0.68	0.56	0.44
x_3	0.80	0.20	0.48	0.52
x_4	0.70	0.30	0.83	0.17

Hence $s(\Box A, \Box B) = 0.70 = e_{AB}$.
And

x	$1-\nu_A$	ν_A	$1-\nu_B$	ν_B
x_1	0.74	0.26	0.82	0.18
x_2	0.54	0.46	0.62	0.38
x_3	0.88	0.12	0.58	0.42
x_4	0.75	0.25	0.88	0.12

Hence $s(\Diamond A, \Diamond B) = 0.75 = e'_{AB}$.

Example 3.6. Consider the IFSs A and B of E as in example 3.1. To find $s(\boxplus A, \boxplus B)$ and $s(\boxtimes A, \boxtimes B)$ we have to construct the new tables as

x	$\frac{\mu_A(x)}{2}$	$\frac{\nu_A(x)+1}{2}$	$\frac{\mu_B(x)}{2}$	$\frac{\nu_B(x)+1}{2}$
x_1	0.325	0.63	0.36	0.59
x_2	0.16	0.73	0.28	0.69
x_3	0.40	0.56	0.24	0.71
x_4	0.35	0.625	0.415	0.56

Hence $s(\boxplus A, \boxplus B) = [0.35,\ 0.375] = [\frac{e_{AB}}{2}, \frac{e'_{AB}}{2}]$.
And

x	$\frac{\mu_A(x)+1}{2}$	$\frac{\nu_A(x)}{2}$	$\frac{\mu_B(x)+1}{2}$	$\frac{\nu_B(x)}{2}$
x_1	0.825	0.13	0.86	0.09
x_2	0.66	0.23	0.78	0.19
x_3	0.90	0.06	0.74	0.21
x_4	0.85	0.125	0.915	0.06

Hence $s(\boxtimes A, \boxtimes B) = [0.85,\ 0.875] = [\frac{e_{AB}}{2} + \frac{1}{2}, \frac{e'_{AB}}{2} + \frac{1}{2}]$.

Example 3.7. Consider the IFSs A and B of E as in example 3.1. To find $s(\boxplus_\alpha A, \boxplus_\alpha B)$ we construct the table with $\alpha = 0.7$.

x	$\alpha\mu_A(x)$	$\alpha\nu_A(x)+1-\alpha$	$\alpha\mu_B(x)$	$\alpha\nu_B(x)+1-\alpha$
x_1	0.455	0.482	0.504	0.426
x_2	0.224	0.622	0.392	0.566
x_3	0.56	0.384	0.336	0.594
x_4	0.49	0.475	0.581	0.384

Hence $s(\boxplus_\alpha A, \boxplus_\alpha B) = [0.49,\ 0.525] = [\alpha e_{AB}, \alpha e'_{AB}]$.
In a similar manner, we create the table that follows to locate $s(\boxtimes_\alpha A, \boxtimes_\alpha B)$.

x	$\alpha\mu_A(x)+1-\alpha$	$\alpha\nu_A(x)$	$\alpha\mu_B(x)+1-\alpha$	$\alpha\nu_B(x)$
x_1	0.755	0.182	0.804	0.126
x_2	0.524	0.322	0.692	0.266
x_3	0.86	0.084	0.636	0.294
x_4	0.79	0.175	0.881	0.084

Hence $s(\boxtimes_\alpha A, \boxtimes_\alpha B) = [0.79,\ 0.825] = [\alpha e_{AB} + 1 - \alpha, \alpha e'_{AB} + 1 - \alpha]$.

Example 3.8. Consider the IFSs A and B of E as in example 3.1. To find $s(\boxplus_{\alpha,\beta}A, \boxplus_{\alpha,\beta}B)$ we construct the table taking $\alpha = 0.7$ and $\beta = 0.3$ with $\alpha, \beta \in [0,1]$ and $\alpha + \beta = 1$.

x	$\alpha\mu_A(x)$	$\alpha\nu_A(x) + \beta$	$\alpha\mu_B(x)$	$\alpha\nu_B(x) + \beta$
x_1	0.455	0.482	0.504	0.426
x_2	0.224	0.622	0.392	0.566
x_3	0.56	0.384	0.336	0.594
x_4	0.49	0.475	0.581	0.384

Hence $s(\boxplus_{\alpha,\beta}A, \boxplus_{\alpha,\beta}B) = [0.49,\ 0.525] = [\alpha e_{AB}, \alpha e^{'}_{AB}]$.
In a similar manner, we create the table that follows to locate $s(\boxtimes_{\alpha,\beta}A, \boxtimes_{\alpha,\beta}B)$.

x	$\alpha\mu_A(x) + \beta$	$\alpha\nu_A(x)$	$\alpha\mu_B(x) + \beta$	$\alpha\nu_B(x)$
x_1	0.755	0.182	0.804	0.126
x_2	0.524	0.322	0.692	0.266
x_3	0.86	0.084	0.636	0.294
x_4	0.79	0.175	0.881	0.084

Hence $s(\boxtimes_{\alpha,\beta}A, \boxtimes_{\alpha,\beta}B) = [0.79,\ 0.825] = [\alpha e_{AB} + \beta, \alpha e^{'}_{AB} + \beta]$.
The measure of similarity has been thoroughly explored and defined in intuitionistic fuzzy set theory by numerous authors [33, 34, 35].
Chen [36] defined a similarity measure between two fuzzy sets A and B of X using the vector approach as follows:

$$s(A,B) = \frac{\overline{A}.\overline{B}}{\overline{A}^2 \vee \overline{B}^2} \tag{1}$$

Where, $\overline{A}$ is the vector $\langle \mu_A(x_1), \mu_A(x_2), ...\rangle$, $\overline{B}$ is the vector $\langle \mu_B(x_1), \mu_B(x_2), ...\rangle$ and $X = \{x_1, x_2, x_3, ...\}$, the symbol ”.” stands for scalar product of two vectors.
De et al.[33] also provide an analogous definition for the similarity measurement between two IFSs A and B of E.

$$s(A,B) = \frac{\sum_{x\in E} \overline{A}_x.\overline{B}_x}{\sum_{x\in E}(\overline{A}_x^2) \vee \sum_{x\in E}(\overline{B}_x^2)} \tag{2}$$

Where $\overline{A}_x$ is the vector $[\mu_A(x), \pi_A(x)]$ and $\overline{B}_x$ is the vector $[\mu_B(x), \pi_B(x)] \forall x \in E$.
Clearly,

1. $s(A,B) \in [0,1]$.
2. $s(A,B) = s(B,A)$.
3. $e_{AB} = e^{'}_{AB}$ if $A = B$.
4. If $\pi_A(x) = 0$ and $\pi_B(x) = 0, \forall x \in E$, then $s(A,B)$ becomes equal to the measure of similarity defined by Chen [4].

In this section, a new kind of similarity measure between two intuitionistic fuzzy sets are defined.

Definition 3.9. *Let us consider two IFSs A and B of a fixed set E. Similarity measure $s(A,B)$ between A and B is defined by*

$$s(A,B) = \frac{e_{AB}}{e'_{AB}} = \frac{\max\min_{x\in E}\{\mu_A(x), \mu_B(x)\}}{\max\min_{x\in E}\{\mu_A(x)+\pi_A(x), \mu_B(x)+\pi_B(x)\}} \tag{3}$$

The larger the value of $s(A,B)$, the more the similarity between the intuitionistic fuzzy sets.
Now let's look at example 3.1. It may be demonstrated that, for equation (2), the value of similarity measure $s(A,B) = 0.9254$, while, by Definition 3.9, similarity measure $s(A,B) = 0.9333$. Therefore, Definition 3.9 is more suited to offer the optimal solution.

Theorem 3.10. *For any two IFSs A and B of a fixed set E, the following statements are true:*

1. $0 \leq s(A,B) \leq 1$.
2. $s(A,B) = s(B,A)$.
3. *If $\pi_A(x) = 0$ and $\pi_B(x) = 0, \forall x \in E$, then $s(A,B)$ becomes equal to 1.*

Proof. Obvious. $\square$
In the above theorem, $e_{AB} \neq e'_{AB}$ if $A = B$.

4 Application for Decision Making

This section describes a procedure for determining, given n possibilities, the most efficient course of action based on m criteria. Suppose that there are n actions A, B, C,...where each action depends upon all of the m criteria x_1, x_2, x_3,... .
A criterion-value $\langle \mu_A, \nu_A \rangle$ consists of the membership value and the non-membership value of the alternative A. The indeterministic or hesitation part is the remaining amount $\pi_A = 1 - \mu_A - \nu_A$. Here $\langle \mu_A, \nu_A \rangle$ are the IFSs of the set A under all criteria.
For two IFSs A and B of E, A is said to dominate B if $s(S,A) \geq s(S,B)$. It is clear that the super IFS S dominates all.

4.1 Algorithm

The steps of algorithm of this method are as follows:
First step: Construct the criteria-matrix using the standard and available alternatives.
Second step: Calculate $s(S,X) = \frac{e_{SX}}{e'_{SX}}$.
Third step: Find all the similarity measures like $s(S,X)$, where $X = A, B, C, D$ and E.
Fourth step: If $s(S,X)$ has more than one value, choose that one corresponding to which the indeterministic part is greatest.
Fifth step: Choose the optimal action.

4.2 A Case-Study

Here, we look at how a student might be selected for a desirable engineering branch based on a few different factors. Let S be the standard alternative and A, B, C, D, and E, are the available alternatives or the desirable engineering branches as Computer Science, Electronics, Biotechnology, Chemical and Mechanical Engineering. Moreover, the criteria are

1. Cut-off marks in entrance test (x_1),
2. Students' choice (x_2),
3. Availability of subjects or branches (x_3),
4. Availability of seats (x_4).

Here, we create a case study using hypothetical information. The criteria-matrix is displayed as follows.

x	S $\langle\mu_S,\nu_S\rangle$	A $\langle\mu_A,\nu_A\rangle$	B $\langle\mu_B,\nu_B\rangle$	C $\langle\mu_C,\nu_C\rangle$	D $\langle\mu_D,\nu_D\rangle$	E $\langle\mu_E,\nu_E\rangle$
x_1	$\langle 0.9, 0.05\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.76, 0.2\rangle$	$\langle 0.86, 0.1\rangle$	$\langle 0.9, 0.02\rangle$	$\langle 0.75, 0.2\rangle$
x_2	$\langle 0.8, 0.1\rangle$	$\langle 0.75, 0.22\rangle$	$\langle 0.83, 0.14\rangle$	$\langle 0.78, 0.18\rangle$	$\langle 0.79, 0.15\rangle$	$\langle 0.79, 0.15\rangle$
x_3	$\langle 0.85, 0.05\rangle$	$\langle 0.81, 0.12\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.81, 0.14\rangle$	$\langle 0.83, 0.13\rangle$
x_4	$\langle 0.88, 0.05\rangle$	$\langle 0.65, 0.25\rangle$	$\langle 0.61, 0.24\rangle$	$\langle 0.68, 0.3\rangle$	$\langle 0.57, 0.28\rangle$	$\langle 0.67, 0.28\rangle$

Hence we get,
$s(S,A) = \frac{e_{SA}}{e'_{SA}} = \frac{\max\{0.70,0.75,0.81,0.65\}}{\max\{0.80,0.78,0.88,0.75\}} = \frac{0.81}{0.88} = 0.92045.$
$s(S,B) = \frac{e_{SB}}{e'_{SB}} = \frac{\max\{0.76,0.80,0.80,0.61\}}{\max\{0.80,0.86,0.90,0.76\}} = \frac{0.80}{0.90} = 0.88889.$
$s(S,C) = \frac{e_{SC}}{e'_{SC}} = \frac{\max\{0.86,0.78,0.70,0.68\}}{\max\{0.90,0.82,0.80,0.70\}} = \frac{0.86}{0.90} = \mathbf{0.95556}.$
$s(S,D) = \frac{e_{SD}}{e'_{SD}} = \frac{\max\{0.90,0.79,0.81,0.57\}}{\max\{0.95,0.85,0.86,0.72\}} = \frac{0.90}{0.95} = 0.94737.$
$s(S,E) = \frac{e_{SE}}{e'_{SE}} = \frac{\max\{0.75,0.79,0.83,0.67\}}{\max\{0.80,0.85,0.87,0.72\}} = \frac{0.83}{0.87} = 0.95402.$
This indicates that the best alternative is C i.e., Biotechnology is the optimal solution.

5 Conclusion

In order to determine the similarity measure between intuitionistic fuzzy sets, we describe a model or method for intuitionistic fuzzy sets in this study. The primary characteristic of this model is that the hesitation margin has also been taken into account and computed.We looked at a multi-criteria decision-making problem where the data were intuitionistic fuzzy rather than crisp. We accomplish this by comparing each of the criterion value sets with the super intuitionistic fuzzy set S. The best effective course of action is determined to be the criteria value set that most closely resembles S. The similarity measuring method is the name of the procedure. In addition to determining the best course of action, the method assists in creating a panel that reveals the second, third, and so on ideal actions. The proposed similarity measure shows great capacity for determining intuitionistic fuzzy sets. It has been illustrated that the proposed similarity measure performs as well as or better than previous measures. Further research will be focused on its applications in other practical fields.

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References

[1] Zadeh LA. Fuzzy sets . *Information and Control*. 2018; 8: 338-353. DOI: https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Atanassov KT. Intuitionistic fuzzy sets. *Int. J. Bioautomation*, 2016, 20(S1), S1-S6.

[3] Bhattacharya J. Some special operations and related results on Intuitionistic fuzzy sets. *International Journal of Scientific Research in Mathematical and Statistical Sciences*. 2021; 8(4): 10-13. DOI: https://doi.org/10.26438/ijsrmss/v8i4.1013

[4] Bhattacharya J. Some results on certain properties of Intuitionistic fuzzy sets. *Journal of Fuzzy Extension and Applications*. 2021; 2(4): 377-387. DOI: https://doi.org/10.22105/jfea.2021.290202.1155

[5] Citil M, Tugrul F. Some new equalities on the intuitionistic fuzzy modal operators. *Sakarya University Journal of Science*. 2018; 22(6): 1524-1531. DOI: https://doi.org/10.16984/saufenbilder.338899

[6] Ejegwa PA, Akowe SO, Otene PM, Ikyule JM. An overview on intuitionistic fuzzy sets. *Int. Journal of Scientific and Technology Research*. 2014; 3(3): 142-145.

[7] Eker E, Tagrul F, Citil M. New equalities on the intuitionistic fuzzy operators and operations. *Notes on Intuitionistic Fuzzy Sets*. 2015; 21(4): 124-128.

[8] Tarsuslu S, Citil M, Dimirbas E, Aydin M. Some modal operators with intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*. 2017; 23(5): 20-28.

[9] Atanassov KT. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*. 1986; 20(1): 87-96. DOI: https://doi.org/10.1016/S0165-0114(86)80034-3

[10] Atanassov KT. New operations defined over Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*. 1994; 61(2): 137-142. DOI: https://doi.org/10.1016/0165-0114(94)90229-1

[11] Atanassov KT. Type-1 fuzzy sets and intuitionistic fuzzy sets. *Algorithms*. 2017; 10(3): 106. DOI: https://doi.org/10.3390/a10030106

[12] Bhattacharya J. A few more on Intuitionistic fuzzy set. *Journal of Fuzzy set Valued Analysis*. 2016; 3: 214-222. DOI: https://doi:10.5899/2016/jfsva-00322

[13] Dencheva K. Extension of intuitionistic fuzzy modal operators $\boxplus$ and $\boxtimes$. *Proceedings of the Second Int. IEEE Symposium: Intelligent Systems, Varna*. 2004; (3): 21-22. DOI: http://dx.doi.org/10.1109/IS.2004.1344843

[14] Xia M, Xu Z. Some new similarity measures for intuitionistic fuzzy values and their application in group decision making. *Journal of Systems Science and Systems Engineering*. 2010; 19(4): 430-452. DOI: https://doi.org/10.1007/s11518-010-5151-9

[15] Ye J. Cosine similarity measures for intuitionistic fuzzy sets and their applications. *Mathematical and Computer Modelling*. 2011; 53(1-2): 91-97. DOI: https://doi.org/10.1016/j.mcm.2010.07.022

[16] Szmidt E, Kacprzyk J. Distances between intuitionistic fuzzy sets. *Fuzzy Sets and Systems*. 2000; 114(3): 505-518. DOI: https://doi.org/10.1016/S0165-0114(98)00244-9

[17] Grzegorzewski P. Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric. *Fuzzy Sets and Systems*. 2004; 148(2): 319-328. DOI: https://doi.org/10.1016/j.fss.2003.08.005

[18] Hung WL, Yang MS. Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. *Pattern Recognition Letters*. 2004; 25(14): 1603-1611. DOI: https://doi.org/10.1016/j.patrec.2004.06.006

[19] Dengfeng L, Chuntian C. New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. *Pattern Recognition Letters*. 2002; 23(1-3): 221225. DOI: https://doi.org/10.1016/S0167-8655(01)00110-6

[20] Xu ZS, Chen J. An overview of distance and similarity measures of intuitionistic fuzzy sets. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*. 2008; 16(04): 529-555. DOI: https://doi.org/10.1142/S0218488508005406

[21] Xu ZS, Yager RR. Intuitionistic and interval-valued intuitionistic fuzzy preference relations and their measures of similarity for the evaluation of agreement within a group. *Fuzzy Optimization and Decision Making*. 2009; 8: 123-139. DOI: https://doi.org/10.1007/s10700-009-9056-3

[22] Zeng W, Guo P. Normalized distance, similarity measure, inclusion measure and entropy of interval-valued fuzzy sets and their relationship. *Information Sciences*. 2008; 178(5): 1334-1342. DOI: https://doi.org/10.1016/j.ins.2007.10.007

[23] Zhang H, Yu L. New distance measures between intuitionistic fuzzy sets and interval-valued fuzzy sets. *Information Sciences*. 2013; 245: 181-196. DOI: https://doi.org/10.1016/j.ins.2013.04.040

[24] Li J, Deng G, Li H, Zeng W. The relationship between similarity measure and entropy of intuitionistic fuzzy sets. *Information Sciences*. 2012; 188: 314-321. DOI: https://doi.org/10.1016/j.ins.2011.11.021

[25] Song Y, Wang X, Lei L, Xue A. A new similarity measure between intuitionistic fuzzy sets and its application to pattern recognition. *Abstract and Applied Analysis*. 2014; 2014(1):1-11. DOI: https://doi.org/10.1155/2014/384241

[26] Ejegwa PA, Onyeke IC. Medical diagnostic analysis on some selected patients based on modified Thao et. al.'s correlation coefficient of intuitionistic fuzzy sets via an algorithmic approach. *Journal of Fuzzy Extension and Applications*. 2020; 1(2): 122-132. DOI: https://doi.org/10.22105/jfea.2020.250108.1014

[27] Ejegwa PA, Kausar N, Agba JA, Ugwuh F, Ozbilge E, Ozbilge E. Determination of medical emergency via new intuitionistic fuzzy correlation measures based on Spearman's correlation coefficient. *AIMS Mathematics*. 2024; 9(6): 15639-15670. DOI: https://doi.org/10.3934/math.2024755

[28] Anum MT, Zhang H, Ejegwa PA, Feng Y. Tendency coefficient-based weighted distance measure for intuitionistic fuzzy sets with applications. In: *2024 12th Int. Conf. on Intelligent Control and Information Processing (ICICIP)*. 2024. p.54-61. DOI: https://10.1109/ICICIP60808.2024.10477789

[29] Ejegwa PA, Anum MT, Isife KI. A new method of distance measure between intuitionistic fuzzy sets and its applications in admission procedure. *Journal of Uncertain Systems*. 2024; 17(2): 2440005. DOI: https://doi.org/10.1142/S1752890924400051

[30] Zhou Y, Ejegwa PA, Johney SE. Generalized similarity operator for intuitionistic fuzzy sets and its applications based on recognition principle and multiple criteria decision making technique. *International Journal of Computational Intelligence Systems*. 2023; 16(1): 85. DOI: https://doi.org/10.1007/s44196-023-00245-2

[31] Nwokoro CO, Inyang UG, Eyoh IJ, Ejegwa PA. Intuitionistic fuzzy approach for predicting maternal outcomes. *Fuzzy Optimization, Decision-making and Operations Research*. 2023; 399-421. DOI: http://dx.doi.org/10.1007/978-3-031-35668-1-18

[32] Atanassov KT. Some operators in Intuitionistic fuzzy sets. *First Int. Conf. on IFS, Sofia, NIFS*. 1997; 3(4): 28-33. http://ifigenia.org/wiki/issue:nifs/3/4/28-33

[33] De SK, Biswas R, Roy AR. Multicriteria decision making using intuitionistic fuzzy set theory. *The Journal of Fuzzy Mathematics*. 1998; 6(4): 837-842.

[34] Kozae AM, Elshenawy A, Omran M. Intuitionistic fuzzy set and its Application in selecting specialization: a case study for engineering students. *Int. Journal of Mathematical Analysis and Applications*. 2015; 2(6): 74-78.

[35] Szmidt E, Kacprzyk J. Intuitionistic fuzzy sets in group decision making. *Notes on Intuitionistic Fuzzy Sets*. 1996; 2(1): 11-14.

[36] Chen SM. A new approach to handling fuzzy decision making problems. *IEEE Transactions on Systems, Man, and Cybernetics*. 1988; 18(6): 1012-1016. DOI: https://doi.org/10.1109/21.23100

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