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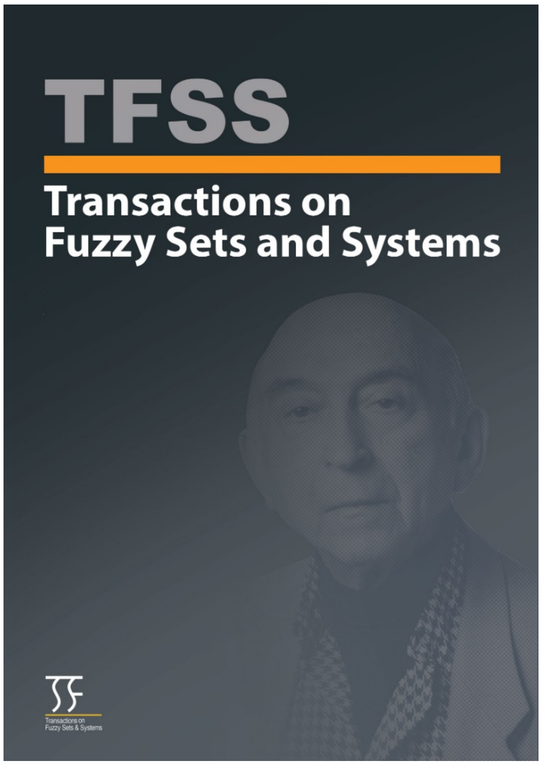

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Generalized Interval-Valued Neutrosophic Set and Its Application

Anjan Mukherjee*, **Rakhal Das**

(This article is dedicated to Prof. Witold Pedrycz in recognition of his pioneering contributions to the field of Granular Computing.)

Abstract. In this work, we introduce the idea of generalized interval-valued neutrosophic sets. After providing the fundamental definitions of operations related to these sets, we establish several key properties and explore the relationships between generalized interval-valued neutrosophic sets and other related concepts. Finally, we extend the notion of generalized neutrosophic topological spaces to incorporate generalized interval-valued neutrosophic topological spaces. We define the concept of generalized interval-valued neutrosophic g-continuous function between two generalized interval-valued neutrosophic topological spaces. Lastly, an application has been shown in decision making problem.

AMS Subject Classification 2020: 54D10; 20F38; 54A10.
Keywords and Phrases: Neutrosophic Set, Neutrosophic Topology, Interval-Valued Neutrosophic set, Generalized Neutrosophic set, Generalized Interval-Valued Neutrosophic Set, Generalized Interval-Valued Neutrosophic Topological Space.

1 Introduction

Neutrosophic sets, which generalize both fuzzy sets and intuitionistic fuzzy sets (IFsets), have been created to effectively represent various types of uncertainty, imprecision, incompleteness, and inconsistency that are often encountered in real-world scenarios. These sets extend traditional fuzzy set theory by allowing a more nuanced representation of information. In particular, interval neutrosophic sets (IN sets) were introduced to overcome the limitations of representing information with a single specific value. Instead, IN sets provide a framework for dealing with ranges of values within the real unit interval, offering a more flexible and comprehensive approach to handle uncertainties. This extension enables the modeling of more complex situations where the degree of membership, indeterminacy, and non-membership of elements can vary within intervals, rather than being confined to precise numerical values. The introduction of IN sets allows for improved handling of data that exhibit variability and uncertainty across a spectrum of possible values. This makes them particularly useful in applications where traditional fuzzy and intuitionistic fuzzy sets may fall short in representing the full extent of uncertainty and variability.

Neutrosophy has paved the way for a broad range of new mathematical theories, extending both classical and fuzzy set concepts, including neutrosophic set theory. The concept of fuzzy sets was first introduced by Zadeh [1] in 1965, where each element is associated with a degree of membership, providing a means to

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handle uncertainty by representing partial truths. Building on this, Atanassov [2] introduced the IFset in 1983. This set extends fuzzy set theory by incorporating both the degree of membership and the degree of non-membership for each element, offering a more nuanced approach to represent uncertainty where the sum of membership and non-membership degrees is less than or equal to one. This addition allows for a more flexible representation of uncertainty and partiality. Further extending these concepts, neutrosophic sets were developed ([3], [4]) to address even broader aspects of uncertainty and imprecision. Neutrosophic sets introduce three independent degrees: membership, indeterminacy, and non-membership. This framework allows for a more comprehensive representation of uncertainty, capturing not only how well an element belongs to a set but also the degree of indeterminacy regarding its membership. Despite these theoretical advancements, practical applications require a clear definition of neutrosophic sets and their associated operators. It is crucial from both scientific and engineering perspectives to ensure that these concepts are well-defined and operationalized, enabling their effective use in real-world scenarios where complex and varied forms of uncertainty are encountered.

In this work, we present set-theoretic operators for a specific type of neutrosophic set (NS) known as the Generalized Interval-Valued Neutrosophic Set (GIVN set). The GIVN set is designed to represent various forms of uncertainty, imprecision, incompleteness, and inconsistency that are commonly encountered in real-world situations. This extends the traditional frameworks provided by NSs ([5], [3], [4]), interval-valued NSs [6], and generalized NSs [7]. We begin by defining the GIVN set, establishing its fundamental properties and operations. We then explore the relationships between GIVN sets and other related set types to highlight the distinctiveness and applicability of GIVN sets in different contexts. Building on the foundational concepts, we extend the ideas of neutrosophic topological spaces [5] and generalized neutrosophic topological spaces [7] to introduce the concept of GIVN topological spaces. Within these spaces, we define and analyze the g-closure and g-interior operators, providing insights into their properties and implications. Furthermore, we define the notion of a GIVN g-continuous function between two GIVN topological spaces, expanding the theoretical framework for functions in this context. To illustrate the practical application of our theoretical developments, we demonstrate how these concepts can be applied to decision-making problems. Additionally, we review relevant studies and literature ([8], [9], [10], [11], [12], [13], [14], [15], [16]) to situate our contributions within the broader research landscape and highlight the significance of our research work.

2 Terminologies

In this section, we recall some fundamental concepts and definitions that will be essential for our work. We start with the basic notion of NSs and their generalizations, which provide the foundation for our theoretical framework.

Definition 2.1. ([3], [4]) Suppose Z be a fixed non-empty set. A NS G is described as:

$$N = \{\langle x, T_G(k), I_G(k), F_G(k)\rangle : k \in Z\}$$

where $T_G(k)$, $I_G(k)$, and $F_G(k)$ describe the degree of membership function (DMF), the degree of indeterminacy (DIF), and the degree of non-membership function (DNMF) of each $k \in Z$ to the set G, respectively. Here, $T_G, I_G, F_G : Z \to]0^-, 1^+[$ and the following condition holds:

$$0^- \leq T_G(k) + I_G(k) + F_G(k) \leq 3^+$$

This definition introduces the NS as a framework for dealing with uncertainty and imprecision in data. The degrees $T_G(k)$, $I_G(k)$, and $F_G(k)$ provide a way to quantify the membership, indeterminacy, and non-membership of elements with respect to a set, extending classical set theory to handle more complex information scenarios.

Definition 2.2. [7] Suppose Z be a fixed non-empty set. A generalized NS (GNS) G is described as:

$$N = \{\langle x, T_G(k), I_G(k), F_G(k)\rangle : k \in Z\}$$

where $T_G(k)$, $I_G(k)$, and $F_G(k)$ represent the DMF, the DIF, and the DNMF, respectively, of each $k \in Z$ to the set G. The functions satisfy the following conditions:

$$T_G(k) \wedge I_G(k) \wedge F_G(k) \leq 0.5$$

$$T_G, I_G, F_G : Z \to \left]0^-, 1^+\right[$$

$$0^- \leq T_G(k) + I_G(k) + F_G(k) \leq 3^+$$

The condition $T_G(k) \wedge I_G(k) \wedge F_G(k) \leq 0.5$ ensures that the combined DMF, DIF, and DNMF, not exceed a certain threshold, allowing for a more nuanced representation of uncertainty.

Definition 2.3. [6] Suppose Z be the non-empty fixed set. An IVNS G in Z is of the form
$G = \{< k, T_G(k), I_G(k), F_G(k) >: k \in Z\}$
where $T_G(k) = \left[T^l_G(k), T^r_G(k)\right], I_G(k) = \left[I^l_G(k), I^r_G(k)\right]$, and $F_G(k) = \left[F^l_G(k), F^r_G(k)\right]$

Which represents the DMF, the DIF, and the DNMF, for each part $k \in Z$ into the set A. where for each element $k \in Z, T_G(k) \in Int[0,1], I_G(k) \in Int[0,1], F_G(k) \in Int[0,1]$, where $Int([0,1])$ denotes the set of all closed sub intervals of [0, 1]. This definition extends the concept of NSs to handle ranges of values rather than precise single values, allowing for a more flexible and comprehensive representation of uncertainty.

Definition 2.4. [6] The complement of an interval-valued NS (IVNS) A in Z is
$A^c = \{< k, T_{A^c}(k), I_{A^c}(k), F_{A^c}(k) >: k \in Z\}$,
where $T_{A^c}(k) = [F^l_G(k), F^r_G(k)], I_{A^c}(k) = [1 - I^r_G(k), 1 - I^l_G(k)]$, and $F_{A^c}(k) = [T^l_G(k), T^r_G(k)]$.

The complement of an IVNS G is obtained by swapping the roles of membership and non-membership degrees while adjusting the indeterminacy degrees accordingly.

3 Generalized interval valued Neutrosophic set

In this section, we introduce the concept of GIVN sets and examine some of their fundamental properties. These sets extend the traditional NSs by incorporating interval-valued memberships, indeterminacies, and non-memberships.

Definition 3.1. Suppose Z be a non-empty set. A GIVN Set G is an entity with the structure

$$G = \{< k, T_G(k), I_G(k), F_G(k) >: k \in Z\}$$

Where, $T_G(k) = \left[T^l_G(k), T^r_G(k)\right], I_G(k) = \left[I^l_G(k), I^r_G(k)\right]$, and $F_G(k) = \left[F^l_G(k), F^r_G(k)\right]$

are the true DMF, DIF and falsity DMF for each point $k \in Z, T_G(k), I_G(k), F_G(k) \in int[0,1]$ and satisfy the condition $\operatorname{Sup} T_G(k) \wedge \operatorname{Sup} I_G(k) \wedge \operatorname{Sup} F_G(k) \leq 0.5$.

This definition introduces a framework where the membership, indeterminacy, and non-membership degrees are not single values but intervals. The condition involving the supremum ensures that the combined DMF, DIF, and DNMF, remains within a bound, providing a balanced representation of uncertainty.

Example 3.2. Suppose $Z = \{a, b, c, d\}$ and $A = \left\{< k, \left[T^l_G(k), T^r_G(k)\right], \left[I^l_G(k), I^r_G(k)\right], \left[F^l_G(k), F^r_G(k)\right] > /k :\in Z\right\}$ given by

Z	$\left[T_G^l(k), T_G^r(k)\right]$	$\left[I_G^l(k), I_G^r(k)\right]$	$\left[F_G^l(k), F_G^r(k)\right]$	$\operatorname{Sup} T_G(k) \wedge \operatorname{Sup} I_G(k) \wedge \operatorname{Sup} F_G(k)$
a	$[0.3, 0.6]$	$[0.2, 0.5]$	$[0.1, 0.3]$	$0.6 \wedge 0.5 \wedge 0.3 = 0.3$
b	$[0.3, 0.5]$	$[0.4, 0.6]$	$[0.2, 0.3]$	$0.5 \wedge 0.6 \wedge 0.3 = 0.3$
c	$[0.2, 0.4]$	$[0.2, 0.5]$	$[0.2, 0.4]$	$0.4 \wedge 0.5 \wedge 0.4 = 0.4$
d	$[0.1, 0.3]$	$[0.2, 0.3]$	$[0.1, 0.5]$	$0.3 \wedge 0.3 \wedge 0.5 = 0.3$

In this example, G is a valid GIVN Set on Z. Each element of Z is associated with intervals for the true DMF, DIF, and falsity DMF, and the supremum condition is satisfied for all elements. This example illustrates how GIVN sets can be used to model complex scenarios where uncertainty is represented by intervals rather than precise values.

Definition 3.3. Suppose Z be a non-empty set. For a GIVN set G, we define its complement A^c as follows:

Suppose the GIVN Set G is given by:

$$A = \left\{ \langle k, \left[T_G^l(k), T_G^r(k)\right], \left[I_G^l(k), I_G^r(k)\right], \left[F_G^l(k), F_G^r(k)\right] \rangle : k \in Z \right\},$$

where $T_G(k)$, $I_G(k)$, and $F_G(k)$ are the DMF, the DIF, and the DNMF for each element $k \in Z$, respectively. Each function is represented as an interval within $[0, 1]$, such that:

$$T_G(k) = \left[T_G^l(k), T_G^r(k)\right], \quad I_G(k) = \left[I_G^l(k), I_G^r(k)\right], \quad F_G(k) = \left[F_G^l(k), F_G^r(k)\right].$$

The complement A^c of G is then defined as:

$$A^c = \left\{ \langle k, \left[F_G^l(k), F_G^r(k)\right], \left[1 - I_G^r(k), 1 - I_G^l(k)\right], \left[T_G^l(k), T_G^r(k)\right] \rangle : k \in Z \right\}.$$

Here, $\left[F_G^l(k), F_G^r(k)\right]$ presents the new degree of membership for the complement, $\left[1 - I_G^r(k), 1 - I_G^l(k)\right]$ presents the new degree of indeterminacy, and $\left[T_G^l(k), T_G^r(k)\right]$ represents the new degree of non-membership. The condition to ensure valid complement values is:

$$\operatorname{Sup} T_G(k) \wedge \operatorname{Sup} I_G(k) \wedge \operatorname{Sup} F_G(k) \leq 0.5.$$

The maximum value of a GIVN Set is $1_G = \langle [1, 1], [0, 0], [0, 0] \rangle$ and the minimum value is $0_G = \langle [0, 0], [1, 1], [1, 1] \rangle$.

Example 3.4. Consider a GIVN Set G defined on $Z = \{a, b, c, d\}$ as follows:

$$A = \left\{ \begin{array}{l} \langle a, [0.3, 0.6], [0.2, 0.5], [0.1, 0.3] \rangle, \\ \langle b, [0.3, 0.5], [0.4, 0.6], [0.2, 0.3] \rangle, \\ \langle c, [0.2, 0.4], [0.2, 0.5], [0.2, 0.4] \rangle, \\ \langle d, [0.1, 0.3], [0.2, 0.3], [0.1, 0.5] \rangle \end{array} \right\}.$$

To find the complement A^c of G, we apply the formula provided in the definition:

$$A^c = \left\{ \begin{array}{l} \langle a, [0.1, 0.3], [0.5, 0.8], [0.1, 0.3] \rangle, \\ \langle b, [0.2, 0.3], [0.4, 0.6], [0.3, 0.5] \rangle, \\ \langle c, [0.2, 0.4], [0.5, 0.8], [0.2, 0.4] \rangle, \\ \langle d, [0.1, 0.5], [0.7, 0.8], [0.1, 0.3] \rangle \end{array} \right\}.$$

In this example: - For element a, the true membership function $T_G(a)$ is $[0.3, 0.6]$. Its complement $F_G(a)$ is $[0.1, 0.3]$. The degree of indeterminacy $I_G(a)$ is complemented to $[0.5, 0.8]$. - This process is repeated for each element in Z to obtain the full complement set.

This example illustrates how to compute the complement of a GIVN Set and highlights how the complement values reflect the inverse of the original GIVN Set values while maintaining the required conditions.

Definition 3.5. A GIVN set G is contained in the other GIVNS B, $G \subseteq B$ if and only if $T_G^l(k) \leq T_B^l(k), T_G^r(k) \leq T_B^r(k), I_G^l(k) \geq I_B^l(k), I_G^r(k) \geq I_B^r(k)$ and $F_G^l(k) \geq F_B^l(k), F_G^r(k) \geq F_B^r(k)$ for all $k \in Z$.

Definition 3.6. The union of two GIVN sets G and B is a GIVN set denoted as $C = G \cup B$ whose truth DMF, DIF and the DNMF are related to those of G and B by

$$\begin{aligned}
T_C^l(k) &= \max\left\{T_G^l(k), T_B^l(k)\right\} \\
T_C^r(k) &= \max\left\{T_G^r(k), T_B^r(k)\right\} \\
I_C^l(k) &= \min\left\{I_G^l(k), I_B^l(k)\right\} \\
I_C^r(k) &= \min\left\{I_G^r(k), I_B^r(k)\right\} \\
F_C^l(k) &= \min\left\{F_G^l(k), F_B^l(k)\right\} \\
F_C^r(k) &= \min\left\{F_G^r(k), F_B^r(k)\right\} \text{ for all } k \in Z
\end{aligned}$$

Note: $G \cup B$ is the smallest GIVN Set containing both the sets G and B .

Definition 3.7. The intersection of two GIVN Sets G and B is a GIVN Set denoted as $D = G \cap B$ whose truth DMF, DIF and the DNMF are related to those of G and B by

$$\begin{aligned}
T_D^l(k) &= \min\left\{T_G^l(k), T_B^l(k)\right\} \\
T_D^r(k) &= \min\left\{T_G^r(k), T_B^r(k)\right\} \\
I_D^l(k) &= \max\left\{I_G^l(k), I_B^l(k)\right\} \\
I_D^r(k) &= \max\left\{I_G^r(k), I_B^r(k)\right\} \text{ and} \\
F_D^l(k) &= \max\left\{F_G^l(k), F_B^l(k)\right\} \\
F_D^r(k) &= \max\left\{F_G^r(k), F_B^r(k)\right\} \text{ for all } k \in Z
\end{aligned}$$

Note: $G \cap B$ s the largest GIVN Set contained in both the sets G and B.

Example 3.8. Suppose G and B be two GIVN Sets defined as
$G = \left\{< \frac{[0.3,0.5][0.2,0.4][0.1,0.2]}{G}, \frac{[0.3,0.4][0.5,0.7][0.2,0.3]}{b}, \frac{[0.1,0.3][0.2,0.4][0.1,0.4]}{c} >\right\}$.
Then $B = \{< \frac{[0.2,0.4][0.1,0.4][0.2,0.3]}{G}, \frac{[0.2,0.5][0.4,0.6][0.1,0.2]}{b}, \frac{[0.3,0.4][0.3,0.5][0.1,0.3]}{c} >\Big\}$
Here $G \cup B = \left\{< \frac{[0.3,0.5][0.1,0.4][0.1,0.2]}{G}, \frac{[0.3,0.5][0.4,0.6][0.1,0.2]}{b}, \frac{[0.3,0.4][0.2,0.4][0.1,0.3]}{c} >\right\}$.
$G \cap B = \left\{< \frac{[0.2,04][0.2,0.4][0.2,0.3]}{G}, \frac{[0.2,0.4][0.5,0.7][0.2,0.3]}{b}, \frac{[0.1,0.3][0.3,0.5][0.1,0.4]}{c} >\right\}$.
Here $G \cup B$ and $G \cap B$ are both the GIVN Sets.

3.1 GIVN topological Spaces

Here, we extend the ideas of generalized neutrosophic topological spaces, as introduced in [7], to encompass GIVN topological spaces. This extension involves defining and analyzing the new structure of GIVN topological spaces, including their fundamental properties and relationships.

Definition 3.9. A generalized interval valued neutrosophic topology (GIVNT) on a nonempty set Z is defined as a family τ of GIVN sets in Z satisfying the following:
a) $0_Z, 1_Z \in \tau$.

b) $Q_1 \cap Q_2 \in \tau$ for every $Q_1, Q_2 \in \tau$.
c) $\cup Q_i \in \tau, \forall \{Q_i : i \in J\} \in \tau$.

In this context, (Z, τ) is referred to as a GIVNT space. The elements of τ are known as GIVN open sets.

Example 3.10. Suppose $Z = \{k\}$ and $A = \{< k, [0.3, 0.5], [0.4, 0.5], [0.1, 0.4] >\}$,
$B = \{< k, [0.2, 0.4], [0.5, 0.6], [0.6, 0.8] >\}$, $C = \{\langle k, [0.2, 0.4], [0.4, 0.5], [0.5, 0.8] >\}$
$D = \{< k, [0.2, 0.4], [0.4, 0.5], [0.6, 0.8] >\}$

Here, $A \cup B = \{< k, [0.3, 0.5], [0.4, 0.5], [0.1, 0.4]\rangle\} = A$

$$A \cup C = \{\langle k, [0.3, 0.5], [0.4, 0.5], [0.1, 0.4]\rangle\} = A$$
$$A \cup D = \{\langle k, [0.3, 0.5], [0.4, 0.5], [0.1, 0.4]\rangle\} = A$$

Also, $A \cup B \cup C = A$ and $A \cup B \cup C \cup D = A$

$$B \cup C = \{\langle k, [0.2, 0.4], [0.4, 0.5], [0.5, 0.8]\rangle\} = C$$
$$B \cup D = \{\langle k, [0.2, 0.4], [0.4, 0.5], [0.6, 0.8]\rangle\} = D$$

Also, $B \cup C \cup D = \{\langle k, [0.2, 0.4], [0.4, 0.5], [0.5, 0.8]\rangle\} = C$ and
$C \cup D = \{< k, [0.2, 0.4], [0.4, 0.5], [0.5, 0.8] >\} = C$

$$A \cap B = \{\langle k, [0.2, 0.4], [0.4, 0.5], [0.5, 0.8]\rangle\} = C$$

$B \cap C = B, B \cap D = B, C \cap D = D, A \cap B \cap C = B, A \cap B \cap C \cap D = B, B \cap C \cap D = B, A \cap C = C, A \cap D = D$.
Thus the family $\tau = \{0_Z, 1_Z, A, B, C, D\}$ GIVN Sets in Z is GIVNT Space on Z.

Remark: Suppose (Z, τ) be a GIVN topological space. Then the closure and interior of $V \in \tau$ is defined as
$\text{int}(V) = \cup\{P : P$ is GIVN open set in Z and $V \supseteq P\}$ andcl $(V) = \cap\{D : D$ is GIVN closed set in Z and $V \subseteq D\}$

Definition 3.11. Suppose (Z, τ) be a GIVN topological space and let A be a subset of Z. Then, A is said to be a GIVN g-closed set if $cl(A) \subseteq G$ where $A \subseteq G$ and G is a GIVN-open set. The complement of a GIVN g-closed set is called a GIVN g-open set.

Definition 3.12. Suppose (Z, τ) be a GIVN topological space. Then for any GIVN set A, the interior of A and closure of A operators are defined as $g - \text{int}(A) = \text{U}\{B : B$ is GIVNg open set in Z and $A \supseteq B\}$ and $g - \text{cl}(A) = \cap\{D : D$ is GIVNg - closed set in Z and $A \subseteq D\}$.

Let us denoted the g-closure and g-interior of $V \in \tau$ as g-int and g-cl.

Note that every GIVN open set is GIVN g-open set

Proposition 3.13. *Suppose (Z, τ) be a GIVN topological space. Let A and B be any two GIVN sets in (Z, τ). Then the following holds*

1. $A \subseteq g - \text{cl}(A)$.
2. $g - \text{int}(A) \subseteq A$.
3. $A \subseteq B \Rightarrow g - cl(A) \subseteq g - cl(B)$
4. $A \subseteq B \Rightarrow g - \text{int}(A) \subseteq g - \text{int}(B)$

5. $g-cl(A\cup B)=g-cl(A)\cup g-cl(B)$
6. $g-\operatorname{int}(A\cap B)=g-\operatorname{int}(A)\cap g-\operatorname{int}(B)$
7. $(g-\mathrm{cl}(A))^{c}=g-\operatorname{int}(A^{c})$
8. $(g-\operatorname{int}(A))^{c}=g-\mathrm{cl}(A^{c})$

Proof.

Properties (i) and (ii) follow directly from the definitions of g-closure and g-interior. Specifically, a set A is always contained within its g-closure, and the g-interior of a set is always a subset of the set itself.

(iii) $A\subseteq B$

$$\begin{aligned} g-cl(B) &= \cap\{D : D \text{ is GIVNg - closed set in } Z \text{ and } B\subseteq D\} \\ &\supseteq \cap\{D : D \text{ is GIVN } g- \text{ closed set in } Z \text{ and } A\subseteq D\}\supseteq g-cl(A)\end{aligned}$$

Similarly (iv).

(v) $g-\mathrm{cl}(A\cup B)=\cap\{D : D$ is GIVNg - closed set in Z and $(A\cup B)\subseteq D\}$
$=\cap\{D : D$ is GIVNg - closed set in Z and $A\subseteq D\}\cup\{\cap$
$\{D : D$ is GIVN $g-$ closed set in Z and $B\subseteq D\}\}$
$=\mathrm{gcl}(A)\cup\mathrm{gcl}(B)$
(vi) Similar to (v)
Proof (vii) $g-cl(A)=\cap\{D : D$ is GIVNg - closed set in Z and $A\subseteq D\}$

$$\begin{aligned}(g-cl(A))^{c} &= \cup\{D^{c} : D^{c} \text{ is GIVN } g- \text{ open set in } Z \text{ and } A^{c}\supseteq D^{c}\} \\ &= \cup\{P : P \text{ is GIVN } g- \text{ open set in } Z \text{ and } A^{c}\supseteq P\}=g-\operatorname{int}(A^{c})\end{aligned}$$

Similarly for (viii). □

Proposition 3.14. *Suppose (Z,τ) be a GIVN topological space. If W is a GIVN g-closed set in (Z,τ) and $V\subseteq G\subseteq g-cl(V)$ then G is also GIVN g-closed.*

Proof. Suppose A be a GIVN open set in (Z,τ) such that $G\subseteq A$ since $V\subseteq$, therefore $V\subseteq$ A but $g-cl(G)\subseteq g-cl(V)$. Since $g-cl(G)\subseteq g-cl(V)\subseteq A$.

Hence G is a GIVN g-closed set.
□

Proposition 3.15. *Suppose (Z,τ) be a GIVN topological space and A be a GIVN open set in (Z,τ). Then A is a GIVNg- open set ifand only if $V\subseteq g-\operatorname{int}(A)$ whenever V is a GIVN closed set and $V\subseteq A$.*

Proof. The proof is obvious. □

Proposition 3.16. *If $g-\operatorname{int}(A)\subseteq V\subseteq A$ and if A is a GIVN g-open set then W is also a GIVN g- open set.*

Proof.

We have $A^{c}\subseteq W^{c}\subseteq(g-\operatorname{int}(A))^{c}=g-\mathrm{cl}(A^{c})$.
Since A is a GIVN g-open set. Thus A^{c} is GIVN g-closed set.

By Proposition-2 W^{c} is a GIVN g-closed set $\Rightarrow$ V is a GIVN g-open set. □

Definition 3.17. Suppose A and B be any two non-empty sets and $f : A \to B$ be a function then notion of pre image of GIVN sets is

If $W = \{(p, T_w(p), I_w(p), F_w(p) : p \in B\}$ is a GIVN set in B then the pre-image of W is defined by $f^{-1}(V) = \left\{< h, f^{-1}\left(T_W(h)\right), f^{-1}\left(I_W(h)\right), f^{-1}\left(F_W(h)\right) >: h \in A\right\}$.

Example 3.18. Suppose $f : (A, \tau_1) \to (B, \tau_2)$ be a function between two GIVN topological spaces.

Suppose,
$W = \{\langle a, [0.2, 0.4], [0.3, 0.5], [0.2, 0.3]\rangle, < b, [0.3, 0.5], [0.5, 0.8], [0.2, 0.4]\rangle, < c, [0.3, 0.4], [0.5, 0.7], [0.3, 0.5] >\}$ be a GIVN set in B.

Let $f(a) = b, f(b) = a$ and $f(c) = c$ then $f^{-1}(V) == \{< a, [0.3, 0.5], [0.5, 0.8], [0.2, 0.4] >, < b, [0.2, 0.4], [0.3, 0.5], [0.2, 0.3] >, < c, [0.3, 0.4], [0.5, 0.7], [0.3, 0.5] >\}$

Remark 3.19. From the above results we may the claim the following remark:

1. Every GIVN open set is GIVN g-open set but every GIVN g-open set is not GIVN- open set.
2. Every GIVN set is IVN set but every IVN set is not GIVN set.

Definition 3.20. Suppose (A, τ_1) and (B, τ_2) be two GIVN topological spaces, and let $f : (A, \tau_1) \to (B, \tau_2)$ be a function.

The function f is said to be a GIVN g-continuous function if the preimage of every GIVN g-closed set in (B, τ_2) is a GIVN g-closed set in (A, τ_1).

Similarly, f is GIVN g-continuous if the preimage of every GIVN g-open set in (B, τ_2) is a GIVN g-open set in (A, τ_1).

In other words, a function f is considered GIVN g-continuous if it preserves the g-closed and g-open structures of sets between the topological spaces (A, τ_1) and (B, τ_2). This means that the inverse image of a GIVN g-closed set under f is always a GIVN g-closed set, and similarly, the inverse image of a GIVN g-open set under f is a GIVN g-open set. This concept generalizes the notion of continuity in traditional topological spaces to the context of GIVN topological spaces.

Theorem 3.21. *Suppose (A, τ_1) and (B, τ_2) be GIVN topological spaces, and let $f : (A, \tau_1) \to (B, \tau_2)$ be a GIVN g-continuous function. Then for every GIVN set W in (A, τ_1), the following inclusion holds:*

$$f(g - \mathrm{cl}(V)) \subseteq \mathrm{cl}(f(V)).$$

Proof. Suppose W be a GIVN set in (A, τ_1).

By definition, $g - \mathrm{cl}(V)$ denotes the g-closure of W in (A, τ_1). The g-closure of W is the intersection of all GIVN g-closed sets that contain W.

Since f is a GIVN g-continuous function, the preimage of every GIVN g-closed set in (B, τ_2) under f is a GIVN g-closed set in (A, τ_1). Consequently, $f^{-1}\left(\mathrm{cl}(f(V))\right)$ is a GIVN g-closed set in (A, τ_1) because $\mathrm{cl}(f(V))$ is a GIVN g-closed set in (B, τ_2).

By the definition of g-closure, we have:

$$V \subseteq f^{-1}\left(\mathrm{cl}(f(V))\right).$$

Since $g - \mathrm{cl}(V)$ is the largest GIVN g-closed set containing W, it follows that:

$$g - \mathrm{cl}(V) \subseteq f^{-1}\left(\mathrm{cl}(f(V))\right).$$

Applying the function f to both sides of this inclusion, we get:

$$f(g-\mathrm{cl}(V)) \subseteq f\left(f^{-1}\left(\mathrm{cl}(f(V))\right)\right).$$

Because $f\left(f^{-1}\left(\mathrm{cl}(f(V))\right)\right) \subseteq \mathrm{cl}(f(V))$ (as f maps any set to a subset of its closure), we conclude:

$$f(g-\mathrm{cl}(V)) \subseteq \mathrm{cl}(f(V)).$$

□

Theorem 3.22. *Suppose (A,τ_1) and (B,τ_2) be two GIVN topological spaces. If $f:(A,\tau_1)\to(B,\tau_2)$ is GIVN continuous, then f is also a GIVN g-continuous function.*

Proof. Suppose W be a GIVN open set in (B,τ_2).

Since f is a GIVN continuous function, the preimage of W under f, denoted $f^{-1}(V)$, must be a GIVN open set in (A,τ_1).

By definition, every GIVN open set is also a GIVN g-open set. Therefore, if $f^{-1}(V)$ is a GIVN open set in (A,τ_1), it is also a GIVN g-open set.

Thus, for any GIVN open set W in (B,τ_2), the preimage $f^{-1}(V)$ is a GIVN g-open set in (A,τ_1). This shows that f is GIVN g-continuous.

Note, however, that the converse of this theorem is not necessarily true. Specifically, if $f:(A,\tau_1)\to(B,\tau_2)$ is a GIVN g-continuous function, it does not guarantee that f is GIVN continuous. The GIVN g-continuity is a more specific condition than general GIVN continuity. □

Remark 3.23. Now it is natural to raise the question- " Under what condition the converse of the theorem 4.13 will be true?"

In our next paper we will introduce a special GIVN topological space in which every GIVN g-open set is also GIVN -open set.

Theorem 3.24. *Suppose (A,τ_1) and (B,τ_2) be two GIVN topological spaces. Let $f:(A,\tau_1)\to(B,\tau_2)$ is GIVN g-continuous then for every GIVN set W in B, then $g-\mathrm{cl}\left(f^{-1}(V) \subseteq f^{-1}(cl(V)\right)$.*

Proof.

Suppose W be a GIVN set in (B,τ_2). Let $K=f^{-1}(V)$ then $f(K)=ff^{-1}(V)\subseteq W$ by theorem 4.11, $f\left(g-cl\left(f^{-1}(V)\right)\right)\subseteq cl\left(ff^{-1}(V)\right)$

Then $g-cl\left(f^{-1}(V)\subseteq f^{-1}(cl(V)\right)$. □

3.1.1 Application

In order to study more efficiently the decision method process we introduce here GIVN Sets.

Let $T=\left[T^l,T^r\right], I=\left[I^l,I^r\right], F=\left[F^l,F^r\right]$ and $\left(\left[T_1^l,T_1^r\right],\left[I_1^l,I_1^r\right],\left[F_1^l,F_1^r\right]\right)$ and $\left(\left[T_2^l,T_2^r\right],\ \left[I_2^l,I_2^r\right],\left[F_2^l,F_2^r\right]\right)$ be two GIVN sets where $\sup T\wedge\sup I\wedge\sup F\leq 0.5$

The sum $\left(\left[T_1^l,T_1^r\right],\left[I_1^l,I_1^r\right],\left[F_1^l,F_1^r\right]\right)+\left(\left[T_2^l,T_2^r\right],\left[I_2^l,I_2^r\right],\left[F_2^l,F_2^r\right]\right)$
$=\left(\left[T_1^l+T_2^l,T_1^r+T_2^r\right],\left[I_1^l+I_2^l,I_1^r+I_2^r\right],\left[F_1^l+F_2^l,F_1^r+F_2^r\right]\right)$

If k be a positive integer then k $\left(\left[T_1^l,T_1^r\right],\left[I_1^l,I_1^r\right],\left[F_1^l,F_1^r\right]\right)$
$=\left(\left[kT_1^l,kT_1^r\right],\left[kI_1^l,kI_1^r\right],\left[kF_1^l,kF_1^r\right]\right)$

The summation and scalar product of (1) and (2) need not be closed operation. It may happen that $\sup\left(\left[T_1^l+T_2^l,T_1^r+T_2^r\right]\right)\wedge\sup\left(\left[I_1^l+I_2^l,I_1^r+I_2^r\right]\right)\wedge\sup\left(\left[F_1^l+F_2^l,F_1^r+F_2^r\right]\right)\nsubseteq 0.5$

and

$$\sup\left(\left[kT_i^l, kT_i^r\right]\right) \wedge \sup\left(\left[kI_i^l, kI_i^r\right]\right) \wedge \sup\left(\left[kF_i^l, kF_i^r\right]\right) \not\subseteq 0.5$$

Let $i = 1, 2, 3, \ldots, k$ and $n = n_1 + n_2 + n_3 + \cdots + n_k$ then the mean value of
$\left[T_1^l(k), T_1^r(k)\right], \left[I_1^l(k), I_1^r(k)\right], \left[F_1^l(k), F_1^r(k)\right]$
$\left[T_2^l(k), T_2^r(k)\right], \left[I_2^l(k), I_2^r(k)\right], \left[F_2^l(k), F_2^r(k)\right], \ldots,$
$\left[T_k^l(k), T_k^r(k)\right], \left[I_k^l(k), I_k^r(k)\right], \left[F_k^l(k), F_k^r(k)\right]$ is

$$\frac{1}{G}\left\{\left(n_1\left(\left[T_1^l, T_1^r\right], \left[I_1^l, I_1^r\right], \left[F_1^l, F_1^r\right]\right) + n_2\left(\left[T_2^l, T_2^r\right], \left[I_2^l, I_2^r\right], \left[F_2^l, F_2^r\right]\right) + \cdots + n_k\left(\left[T_k^l, T_k^r\right]\right.\right.\right.$$
$$\left.\left.\left.\left[I_k^l, I_k^r\right], \left[F_k^l, F_k^r\right]\right)\right)\right\} \ldots\ldots (3)$$

Example 3.25. A company is tasked with selecting one candidate from a pool of six applicants: $k_1, k_2, k_3, k_4, k_5, k_6$. The desired qualifications for the new candidate are represented by the set $\mathrm{P} = \{p_1, p_2, p_3, p_4\}$, where:

- p_1 denotes "Fast",
- p_2 denotes "Young",
- p_3 denotes "Intelligent", and
- p_4 denotes "Experienced".

The committee has evaluated each of the six candidates based on these qualifications using GIVN sets. This approach allows for the representation of uncertain, imprecise, incomplete, and inconsistent information regarding how well each candidate meets the desired qualifications. Consequently, the decision-making process was structured into a tabular matrix, which reflects the evaluations of each candidate. Based on this matrix, the best decision for the company in selecting the optimal candidate can be determined.

	P_1	P_2	P_3
k_1	$[0.4, 0.6], [0.2, 0.5], [0.3, 0.5]$	$[0.3, 0.7], [0.2, 0.4], [0.1, 0.3]$	$[0.2, 0.4], [0.4, 0.6], [0.3, 0.5]$
k_2	$[0.3, 0.6], [0.2, 0.4], [0.1, 0.3]$	$[0.4, 0.7], [0.3, 0.5], [0.1, 0.2]$	$[0.2, 0.5], [0.3, 0.5], [0.3, 0.4]$
k_3	$[0.5, 0.7], [0.2, 0.4], [0.2, 0.3]$	$[0.4, 0.6], [0.3, 0.5], [0.1, 0.3]$	$[0.3, 0.5], [0.2, 0.3], [0.1, 0.4]$
k_4	$[0.3, 0.5], [0.2, 0.4], [0.1, 0.2]$	$[0.5, 0.8], [0.1, 0.3], [0.2, 0.3]$	$[0.3, 0.6], [0.2, 0.5], [0.1, 0.3]$
k_5	$[0.7, 0.8], [0.1, 0.3], [0.1, 0.2]$	$[0.4, 0.6], [0.2, 0.3], [0.1, 0.3]$	$[0.3, 0.5], [0.2, 0.5], [0.2, 0.3]$
k_6	$[0.2, 0.4], [0.5, 0.7], [0.4, 0.6]$	$[0.4, 0.6], [0.2, 0.4], [0.1, 0.4]$	$[0.5, 0.8], [0.1, 0.4], [0.2, 0.4]$

	P_4
k_1	$[0.2, 0.5], [0.3, 0.7], [0.1, 0.5]$
k_2	$[0.2, 0.4], [0.4, 0.6], [0.3, 0.5]$
k_3	$[0.4, 0.7], [0.3, 0.5], [0.1, 0.4]$
k_4	$[0.2, 0.4], [0.3, 0.5], [0.2, 0.4]$
k_5	$[0.4, 0.6], [0.2, 0.5], [0.1, 0.2]$
k_6	$[0.3, 0.6], [0.2, 0.4], [0.4, 0.5]$

The choice value for each candidate is determined by the mean value of the GIVN sets to which they belong. Thus, according to equation (3), the choice value for k_1 is calculated as follows:

$$\frac{1}{4}[<[0.4,0.6],[0.2,0.5],[0.3,0.5]>+<[0.3,0.7],[0.2,0.4],[0.1,0.3]>+<$$
$$[0.2,0.4],[0.4,0.6],[0.3,0.5]>+<[0.2,0.5],[0.3,0.7],[0.1,0.5]>]$$
$$=\frac{1}{4}[<[1.1,2.6],[1.1,2.2],[0.8,1.8]>]$$
$$=<[0.275,0.65],[0.275,0.55],[0.2,0.45]>$$

In the same way one finds the choice values of k_2, k_3, k_4, k_5 and k_6 as
Choice value of k_2 is $<[0.275,0.55],[0.3,0.5],[0.2,0.35]>$
Choice value of $k_3 =<[0.4,0.625],[0.25,0.425],[0.125,0.35]>$
Choice value of $k_4 =<[0.325,0.575],[0.175,0.425],[0.15,0.3]>$
Choice value of $k_5 =<[0.4,0.625],[0.175,0.4],[0.125,0.25]>$
Choice value of $k_6 =<[0.35,0.6],[0.25,0.475],[0.275,0.475]>$
Mean average of $k_1 =<0.4625,0.4125,0.325>$
Mean average of $k_2 =<0.4125,0.4,0.275>$
Mean average of $k_3 =<0.5125,0.3375,0.2375>$
Mean average of $k_4 =<0.45,0.3,0.225>$
Mean average of $k_5 =<0.5375,0.2875,0.1875>$
Mean average of $k_6 =<0.475,0.3625,0.375>$

In this case, the expert may apply an optimistic criterion by selecting the candidate with the highest average truth degree or by using a criterion based on the lowest average falsity degree. Based on these criteria, the expert selects candidate k_5 for the company.

4 Conclusion

In this work, we introduced a novel parametric decision-making method that combines hybrid approaches to enhance decision processes in complex scenarios. The method was illustrated through examples focused on selecting a new candidate for a company, showcasing how the hybrid approach can effectively integrate different criteria and preferences. The findings of this study indicate that hybrid methods, particularly within fuzzy environments, offer substantial improvements over traditional decision-making techniques. The results suggest that such methods can be highly effective across various scenarios, demonstrating their versatility and robustness. Furthermore, we have developed and presented the concept of GIVN sets. These sets represent an advancement in the representation of uncertainty by combining interval-valued membership, indeterminacy, and non-membership functions. We described key operators within the context of GIVN topological spaces, including the g-closure and g-interior operators. These operators help in understanding and manipulating the properties of GIVN sets within a topological framework. The distinction between GIVN sets and IVN sets is an important aspect of our findings. While every GIVN set is indeed a type of IVN set, the reverse is not necessarily true. This implies that GIVN sets offer a broader and more flexible framework for dealing with uncertainty compared to IVN sets. The generalization provided by GIVN sets allows for more nuanced modeling of real-world situations where uncertainty cannot always be precisely quantified by single values.

This paper underscores the potential of hybrid decision-making methods and advanced set theories in improving decision-making processes and modeling uncertainties. The hybrid approach demonstrated here is not limited to decision-making scenarios but can be applied to various fields where complex, uncertain, or imprecise information needs to be managed. The concepts introduced and the methods developed pave the way for future research and applications, highlighting their relevance and utility across different domains.

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