



# **Multi-objective Allocation of Distributed Generation Resources and Capacitor Banks Based on a Two-stage Fuzzy Method and $\epsilon$ -constrained Optimization**

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## **Abstract**

Proper operation of distributed generation resources (DGs) in power systems has considerable advantages, including decreasing losses, reducing congestion in feeders, improving voltage profile, and enhancing stability, reliability, and security. On the other hand, using capacitor banks helps improve voltage profile and power quality in distribution systems. The optimal allocation of capacitor banks (CBs) and DGs has a significant impact on the efficiency of the distribution systems. This paper presents a method for distribution system planning based on the optimal allocation of DGs and CBs. The main objectives of the proposed method are to improve the voltage profile, reduce investment and operation costs, and reduce renewable energy curtailment. The planning problem is solved through multi-objective scheduling based on a two-stage fuzzy method and the  $\epsilon$ -constrained optimization. The stochastic two-stage method is used to model uncertainty. The proposed method is implemented on an IEEE 33-bus test network in MATLAB and evaluated under three scenarios. It is proven that the voltage profile can be improved in the scenario of allocating capacitor banks based on lower investment costs compared to other scenarios. However, the voltage profile is improved more in the scenario of simultaneous allocation of capacitor banks and DGs by investing in more costs. In general, the proposed method properly improves the distribution system's performance in different aspects.

**Keywords:** Multi-Objective Optimization, Capacitor Bank Allocation, DG Allocation, Two-Stage Method.

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## 1. INTRODUCTION

Most distribution systems are passive radial networks with unidirectional power flow. Upon the introduction of DGs to distribution networks, these systems are converted to active systems with bidirectional power flow [1]. DGs bring benefits, such as reducing losses, decreasing congestion in feeders, improving stability, voltage profile, and power quality, peak-shaving, and reducing investment, operation, and reliability costs. In addition, the power systems become more secure [2]. Studies have shown that power losses in a distribution system comprise power system losses [3]. Operationally speaking, real power loss directly affects the power system's efficiency. However, to preserve voltage in the allowed range and free transmission capacity, the reactive power flow in the feeders should be limited to a certain value [4]. With optimal operation management of DG units, the reactive power flow can be controlled, and the losses can be reduced [5]. However, the main problem with using DG units is finding the optimal capacity and location, considering the operation constraints. The application of DG units in distribution systems leads to the inverse power flow direction, resulting in high losses and overloading [6]. In addition, the high penetration level of DG units in the system reduces the balance between consumption and generation, which increases losses [7]. In distribution systems, power loss is an essential factor for increasing system efficiency. To this end, optimal allocation of capacitor banks (CBs), network configuration, and DG allocation are well-known methods [8].

Proper planning of the penetration level of DGs affects reducing active and reactive losses [9]. DG resource owners tend to sell more energy. On the other hand, a distribution system operator (DSO) seeks to absorb more local power generation capacity to control the network and curb power losses, operation costs, and emission of destructive pollutants. However, improper determination and measurement of DG resources aggravate power losses, which is undesired for DSOs [10].

With the growth of demand and updates in distribution and power networks, investment in energy has increased significantly. On the other hand, requirements for higher power quality have motivated investors to consider improving power quality and reliability and reducing harmonic current and voltage distortions more accurately [11]. A common strategy to improve power quality is to install CBs to compensate for reactive power, correct the power factor, reduce energy loss, and preserve the bus voltage. CBs are costly components whose capacity, location, and switching influence stability and power quality significantly [12]. Therefore, their proper allocation is crucial for maximizing energy loss reduction and improving losses. Various parameters like voltage constraints, load changes, and other parameters have complicated the configuration of capacitances and their switching.

Numerous studies have addressed the optimal allocation of DG units and CBs. In [13], a method has been presented for distribution system planning, considering flexibility requirements and adjustment laws. In this study, mixed integer linear

programming has been used based on local resources and passive elements in the distribution network. In [14], a method has been presented to design active distribution systems, considering active network management and optimization for load curtailment. In this paper, the researchers have presented an optimal load curtailment model for distribution planning. The proposed method seeks to determine the optimal type, location, and capacity of the feeders, DGs, CBs, static compensators, and regulators. In [15], a robust two-stage optimization method has been presented for planning active distribution networks coupled with urban transportation networks. Load uncertainty, renewable energy resources, and traffic demand have been considered in this study. In [16], a method has been presented for planning distribution networks considering uncertainty, prolonging investment, hydrogen storage devices, and wind resources. The authors in [17] have focused on the effect of load changes on the planning of a distribution system and optimal allocation of DGs, considering the reduction of energy losses. In [18], distribution system planning based on MICP in a radial distribution network has been presented for optimal allocation. The proposed model is based on hybrid integer conical programming. In [19], the planning of distribution systems and the optimal allocation of DGs have been studied in a radial distribution network based on the stability index under load growth. In [20], a model has been presented for the optimization of distribution systems, the allocation of DG resources, and the configuration of distribution networks based

on the BF-SD algorithm considering the phase imbalance. The authors in [21] have studied the multi-objective optimal power flow with DG resource allocation using TLBO and MIPSO algorithms. In [22], a method based on environmental, technical, and economic goals has been presented for the optimal allocation of DG resources in distribution networks.

Ref. [23] has contributed to designing, modeling, and scheduling DG resources, including wind and solar systems, using a particle swarm algorithm. DGs are used to supply the output load during peak hours of the day and night. Probability distribution functions are used, and outputs are expressed as probability density distribution functions instead of absolute numbers. In [24], microgrid optimal scheduling considering normal and emergency operations has been investigated. A chance-constrained model is developed to handle normal operation and emergency conditions of the microgrid, including DG outage and unwanted islanding. Storage purchase from the upstream network is also considered. In addition, the uncertainty of loads and renewable resources is included in the model. Ref. [25] has been dedicated to comparing network reliability indicators before and after the introduction of DGs and analyzing their impact on improving network reliability. The improvement of indicators based on customer satisfaction, including the reduction of SAIFI and SAIDI, is evaluated. More precisely, the improvement of the most important index based on load and energy, i.e., energy not supplied (ENS), is investigated. In order to reduce the distribution system loss, the simultaneous optimal placement of DG

resources and capacitors in radial distribution systems has been studied in [26] in which the crow search algorithm has been used for the optimization. Ref. [27], an optimization approach based on an arithmetic optimization algorithm (AOA) is proposed for specifying the optimal allocation of distribution generations/generators (DGs) and capacitor banks (CBs) in radial distribution systems. The AOA is a new population-based meta-heuristic algorithm that is essentially based on using basic arithmetic operators in mathematics. The proposed approach is employed to specify the optimum placement, capacity, and power factor of DGs and CBs to decrease the distribution systems' total power loss and voltage deviation. In Ref. [28] introduces the Energy Valley Optimizer, a novel tool designed for the strategic placement of distributed generation units and capacitor banks. This placement is crucial not only for optimizing energy loss and enhancing bus voltage stability but also for promoting sustainable energy use and reducing environmental impact over the long term. By minimizing energy loss and voltage fluctuations, the optimizer contributes to a more sustainable and resilient energy system. Ref. [29] proposes a two-stage procedure to enhance the distribution system performance by determining the optimal sizes and locations of distributed generations (DGs) and capacitors considering single and multi-objective functions. In stage-1, two voltage sensitivity factors (VSFs) based on voltage deviation (VD), and voltage stability index (VSI) are proposed to reduce the search space (SS) by selecting the candidate buses for DGs and capacitors placement. In stage-2, the chaotic bat algorithm (CBA) is applied to

find the optimal sizes and locations of DGs and capacitors, according to different objective functions (OFs) and system constraints. The considered OFs are real power loss reduction, total VD minimization, and total VSI maximization. The multi-OF, which aims to optimize these objectives simultaneously, is also considered.

The present paper proposes a novel method to optimally allocate the DG resources and capacitor banks. Multiple objectives are considered, including reducing operation and investment costs, improving voltage profile and reliability, and reducing curtailment costs. Moreover, uncertainties are included in the proposed method using stochastic modeling. The multi-objective  $\epsilon$ -constrained optimization method is used to solve the planning program. In addition, fuzzy decision-making is used to select the best solution among non-dominated solutions.

This paper is organized as follows. Section 2 describes the problem formulation, including objective functions and constraints. Section 3 describes the solution approach, including the optimization method, uncertainty modeling, and fuzzy decision-making. Section 4 describes the results and discussion. Finally, Section 5 is dedicated to conclusion.

## 2. PROBLEM FORMULATION

This section presents the cost function of optimal DG and CB allocation. In this modeling, the objectives are to minimize the operation and investment costs, improve reliability and voltage profile, and reduce power curtailment. In general, distribution network companies are responsible for

satisfying the consumers' requirements and managing CBs and DGs in the distribution system. The objectives of interest are described below.

### A. Objective functions

The first objective function is to minimize CB and DG costs. Costs include the DG investment cost ( $C_{DG}$ ), which is given in Eq. (1) [27]:

$$C_{DG} = \sum_{i=1}^{NDG} P_{DGi} Inv_{DG} \quad (1)$$

in which  $P_{dgi}$  is the capacity of the DG units in MW,  $INV_{dg}$  is the investment cost of the unit in \$/MW, and  $Ndg$  is the number of units installed on the system. The investment cost of the CB ( $C_{cap}$ ) is calculated as follows [27]:

$$C_{cap} = \sum_{j=1}^{Ncap} Q_{capj} Inv_{cap} \quad (2)$$

in which  $Q_{capj}$  is the capacity of the capacitor bank in MVar,  $INV_{cap}$  is the investment cost of the CBs in \$/MW, and  $Ncap$  is the number of banks installed in the system.

Along with investment costs, operation and maintenance costs are also considered. Here, the maintenance and operation costs include fuel cost and annual maintenance cost. The operation cost ( $C_{oDG}$ ) for DG units is calculated by

$$C_{ODG} = \left[ \sum_{i=1}^{Ndg} P_{DGi} * O_{DG} * Oh \right] * \sum_{y=1}^T \left( \frac{1 + inf}{1 + int} \right)^y \quad (3)$$

in which  $O_{DG}$  represents the operation and maintenance cost of the DG unit in \$/MWh, and  $Oh$  represents the total number of operation hours in a year, which is 8760. Also,  $T$  denotes the number of years of the planning period,  $y$  denotes the year of operation,  $Inf$  denotes the inflation rate, and  $Int$  denotes the interest rate used to convert these costs to the current value. For the CBs, the cost only includes the maintenance cost ( $C_{mcap}$ ) that is considered with the parameter  $MC_{cap}$  [27].

$$C_{Mcap} = [MCcap] * \sum_{y=1}^T \left( \frac{1 + inf}{1 + int} \right)^y \quad (4)$$

The second objective function, which is considered in the allocation problem, is reliability improvement. The following steps are taken to calculate reliability. The majority of distribution systems are radial. The annual failure rate (AFR), average outage time (AOT), and annual outage time (UOT) are the main parameters of reliability [28].

$$AFR = \sum_{j=1}^{Nsec} \lambda_j \quad (5)$$

$$AOT = \sum_{j=1}^{Nsec} \lambda_j * g_j \quad (6)$$

$$UOT = \frac{AOT}{AFR} = \frac{\sum_{j=1}^{Nsec} \lambda_j * g_j}{\sum_{j=1}^{Nsec} \lambda_j} \quad (7)$$

in which  $\lambda_j$  and  $g_j$  are the AFR and the AOT of the  $j^{\text{th}}$  feeder, respectively. The current passing through the feeder sections has two active (IA) and reactive (IR) components. Optimal installation of DG resources and CBs partially supports the power demand,

which reduces the magnitude of the current passing through the feeder sections. This reduction minimizes the costs. The allocation of CBs and DGs improves reliability. Before optimal allocation, each section of the feeder has an uncompensated failure rate of  $\lambda_{uncompj}$ . After allocation, if the feeder's current component is completely compensated, the failure rate reaches 75% of  $\lambda_{uncompj}$  [28], which is represented by  $\lambda_{compj}$ . If the feeder's current is not completely compensated, its failure rate is defined linearly by compensation percentage. The new failure rate after allocation is defined as

$$\lambda_{jnew} = \beta_j * (\lambda_j^{uncomp} - \lambda_j^{comp}) + \lambda_j^{comp} \quad (8)$$

in which  $\beta$  is the compensation factor calculated by

$$\beta_j^{AR} = \left| \frac{I_j^{new}}{I_j^{old}} \right| = \left| \frac{\sqrt{(I_j^{Anew})^2 + (I_j^{Rnew})^2}}{\sqrt{(I_j^{Aold})^2 + (I_j^{Rold})^2}} \right| \quad (9)$$

in which A represents the active part, R represents the reactive part, and  $\beta_j^{AR}$  represents the factor in which the active and reactive parts of the current are considered. Also, new indicates after allocation, and old indicates before allocation. The absolute value of the current is used in the calculations. ECOST is used for reliability calculations. Thus,  $C_{rel}$  is determined by

$$C_{rel} = \sum_{b=2}^n L_i^{avg} C_i^{int} \lambda_j^{new} * \sum_{y=1}^T \left( \frac{1+inf}{1+int} \right)^y \quad (10)$$

in which  $L_i^{avg}$  is the average load connected to bus i in KW and  $C_i^{int}$  is the load disconnection cost in \$/kW. Here, the total outage time of  $L_i^{avg}$  is related to the outage of each section of the feeder with  $\lambda_j = \lambda_j^{new}$ .

Therefore, the first objective function considered in the planning is calculated as follows:

$$Cost_{total} = C_{dg} + C_{cap} + C_{ODG} + C_{Mcap} + C_{rel} \quad (11)$$

$$F_1 = Ecost = \sum_{m=1}^M Cost_{Total}^m \quad (12)$$

in which M is the number of scenarios and  $Cost_{total}^m$  is the corresponding cost for the m<sup>th</sup> scenario. This objective function is formulated as follows:

$$f_{volt} = \frac{1}{T} \sum_{y=1}^T \frac{1}{N_{Bus}} \sum_{j=1}^{N_{Bus}} |V_i - V_{Ref}| \quad (13)$$

in which  $N_{bus}$  is the number of network buses and  $V_i$  is the real per unit voltage of the bus. It should be noted that  $V_{ref}$  is also 1 pu. The average voltage of the buses should be close to 1 pu so that the voltage profile can be improved. Therefore, the second objective function based on stochastic modeling is calculated as follows:

$$F_2 = E f_{volt} = \sum_{m=1}^M f_{volt}^m \quad (14)$$

in which the third objective function, which is related to the power curtailment cost, is given as:

$$F_{3,i} = Pen_i(P_{DG_i}^{rated} - P_{DG_i}) \quad (15)$$

in which  $i$  is the index of the buses,  $n$  is the number of network buses, and  $j$  is the index of the buses. In addition,  $P_{DG_i}$  and  $P_{DG_i}^{rated}$  are the injected active power and the rated PV output at bus  $i$ , respectively. Also,  $Pen_i$  is the penalty factor of bus  $i$  for power curtailment.

The third objective function based on stochastic modeling is calculated as follows:

$$F_3 = F_{DGCurt} = \sum_{m=1}^M \sum_{i=1}^I F_{3,i} \quad (16)$$

### B. Constraints

The constraints of the allocation problem for the DG units and the CBs are as follows [30]:

$$P_{DG}^{Min} \leq \sum_{i=1}^{Ndg} P_{dgi} \leq P_{DG}^{Max} \quad (17)$$

$$Q_{cap}^{Min} \leq \sum_{j=1}^{Ncap} Q_{capj} \leq Q_{cap}^{Max} \quad (18)$$

$$P_{DG}^{Min} = 0.1 * \sum_{b=2}^{Nbus} P_{loadb} \quad (19)$$

$$Q_{cap}^{Min} = 0.1 * \sum_{b=2}^{Nbus} Q_{loadb} \quad (20)$$

$$P_{DG}^{Max} = 0.6 * \sum_{b=2}^{Nbus} P_{lb} \quad (21)$$

$$Q_{cap}^{Max} = 0.6 * \sum_{b=2}^{Nbus} Q_{lb} \quad (22)$$

in which  $P_{DG}^{min}$  and  $P_{DG}^{max}$  are the minimum and maximum power generation of the DGs,

respectively. Also,  $Q_{cap}^{min}$  and  $Q_{cap}^{max}$  are the minimum and maximum reactive power of the CBs, respectively. Also,  $P_{lb}$  and  $Q_{lb}$  are the active and reactive load power in bus  $b$ , respectively.

Another constraint that should be considered is the power flow constraint as follows:

$$P_{Gb} - P_{Lb} - V_i \sum_{j \in Ni} V_j [G_{bj} \cos(\theta_{bj}) + B_{bj} \sin(\theta_{bj})] = 0 \quad (23)$$

$$Q_{Gb} - Q_{Lb} - V_b \sum_{j \in Ni} V_j [G_{bj} \sin(\theta_{bj}) - B_{bj} \cos(\theta_{bj})] = 0 \quad (24)$$

in which  $P_{Gb}$  is the active power generated on bus  $b$ ,  $P_{Lb}$  is the active power on bus  $b$ ,  $V_b$  is the voltage of bus  $b$ ,  $V_j$  is the voltage of bus  $j$ ,  $\theta_{bj}$  is the angle between bus  $b$  and  $j$ ,  $Q_{Gb}$  is the reactive power generation on bus  $b$ , and  $Q_{Lb}$  is the reactive power on bus  $b$ .

The line capacity constraint is as follows:

$$S_{ij}^{min} \leq S_{ij} \leq S_{ij}^{max} \quad (25)$$

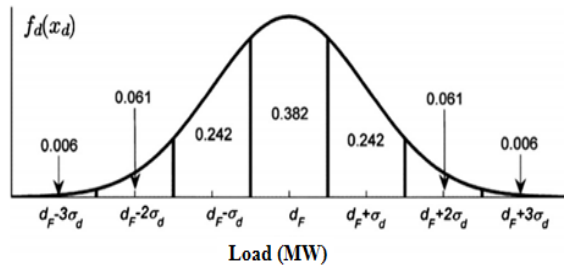
in which  $S_{ij}$  is the apparent power of line  $ij$ , and  $S_{ij}^{min}$  and  $S_{ij}^{max}$  are the minimum and maximum apparent power, respectively.

The multi-scenario balance should be satisfied. The stochastic model has a higher number of energy balance constraints than certain methods.

### 3. SOLUTION APPROACH

This paper considers demand uncertainty as stochastic scheduling. Then, the load is modeled. The power system operator should

predict the demand at each hour of the upcoming hours. Since the prediction process has errors, the demand prediction is not accurate. The demand prediction uncertainty is usually modeled with a normal distribution density function. To create a limited number of demands at each hour, a distribution curve is developed like Fig. 1.



**Fig. 1. 7-interval Gaussian distribution approximation.**

In the proposed method, the demand error is assumed to be of normal distribution, and the weights  $P_d$  are determined using the area under the upper and lower constraint curves of each interval [31]:

$$P_d = \frac{1}{\sigma_d \sqrt{2\pi}} \int_{l_j}^{u_j} e^{-(x-d_f)^2} / 2\sigma_d^2 dx \quad (26)$$

in which  $u_j$  and  $l_j$  are the upper and lower demand constraints in each interval and  $d_f$  and  $\sigma_d$  are the predicted demand and std of the normal PDF, respectively. Stochastic modeling is carried out using the Markov process. Then, multi-objective modeling is carried out.

The  $\epsilon$ -constrained method is a proper technique for solving multi-objective functions [32]. In the  $\epsilon$ -constrained method, ranges of  $N-1$  objective functions are required for  $N$  objective functions. The pay-off table is the most common technique to

obtain these ranges. The pay-off concept, which shows the relationship between all possible events or acts and the values associated with the consequences, can help decision-making concerning competitive bid determinations. In this method,  $N-1$  functions are used as additional constraints. A comprehensive description of the payment table and the ranges of the objective functions is given in [33]. Despite the advantages of the  $\epsilon$ -constrained method, it has two major problems: first, the range of the objective functions in the efficient set might not be optimal; second, the optimal solutions that are generated by the  $\epsilon$ -constrained method might not be dominant. The lexicographic method is suggested to overcome the first problem, and the  $\epsilon$ -constrained technique is suggested to solve the second problem, according to [32]. The importance of the objective function for generating Pareto solutions is modeled using the hybrid aggregated-weighted  $\epsilon$ -constrained method. In previous studies, efforts have been made to implement lexicographic optimization and the hybrid aggregated-weighted  $\epsilon$ -constrained method for solving multi-objective optimal allocation problems. In this study, the lexicographic optimization and the hybrid aggregated-weighted  $\epsilon$ -constrained method are used to solve the stochastic optimal multi-objective allocation problem. The modified  $\epsilon$ -constrained method is given below.

The hybrid aggregated-weighted  $\epsilon$ -constrained method can be modeled by:

$$\begin{aligned} & \text{Min /Max } F_1(x) \\ & + \frac{dir_1 r_1}{W_1} \sum_{n=2}^N \frac{w_n S_n^{nk}}{r_n} \end{aligned} \quad (27)$$



This equation is constrained by

$$\begin{aligned} F_n(x) - \text{dir}_n S_n^{nk} - e_n^{nk} &= 0 \\ S_n^{nk} &\in R^+, \\ nk &= 0, \quad a, \dots, q_n, n = 2, \dots, N \end{aligned} \quad (28)$$

$$\begin{aligned} e_n^{nk} &= \frac{F_n^{\min}(1 + \text{dir}_n)}{2} \\ &\quad - \frac{F_n^{\max}(-1 + \text{dir}_n)}{\text{dir}_n r_n nk} \\ &\quad + \frac{q_n}{q_n} \end{aligned} \quad (29)$$

$$nk = 0, 1, \dots, q_n, \quad n = 2, \dots, N$$

in which  $\text{dir}_n$  is the direction of objective function  $n$ , and  $\text{dir}_n$  is  $-1$  when the objective function  $n$  is minimized and  $+1$  when the objective function is maximized. The main relationship should be optimized to achieve non-dominated optimal Pareto solutions. These relationships and the optimization are based on the method presented in [34].

Fuzzy decision-making could be employed by the system operator to select one of the optimal Pareto solutions. The fuzzy decision-making can select the best solution among non-dominated solutions. In this technique, the linear membership function is calculated for each objective function in the optimal Pareto solution.

The linear membership for the  $n^{\text{th}}$  objective function that should be minimized or maximized is defined by the following equations:

$$\mu_n^r = \begin{cases} 0 & F_n^r \leq F_n^{\min} \\ \frac{F_n^{\max} - F_n^r}{F_n^r - F_n^{\min}} & F_n^{\min} \leq F_n^r \leq F_n^{\max} \\ 1 & F_n^{\max} \leq F_n^r \end{cases} \quad (30)$$

$$\mu_n^r = \begin{cases} 1 & F_n^r \leq F_n^{\min} \\ \frac{F_n^{\max} - F_n^r}{F_n^r - F_n^{\min}} & F_n^{\min} \leq F_n^r \leq F_n^{\max} \\ 0 & F_n^{\max} \leq F_n^r \end{cases} \quad (31)$$

The total membership of  $r^{\text{th}}$  optimal Pareto solution based on independent membership functions of  $\mu_n^r$  is calculated as follows:

$$\mu^r = \frac{\sum_{n=1}^N W_n \mu_n^r}{\sum_{n=1}^N W_n} \quad (32)$$

The system operator can select the value of  $w_n$  to prioritize the objective functions depending on the system conditions. The best optimal Pareto solution is selected by the fuzzy decider by selecting the maximum value for  $\mu^r$ . There are several solution methods to determine the Pareto function. The three goals of Pareto optimization can be defined and measured as follows: the distance from the set of non-dominant results to the Pareto front should be minimized, proper distribution of the obtained solutions should be established, and the size of the obtained non-dominant front should be maximized.

The problem of optimal stochastic planning multi-objective optimal allocation is solved using the stochastic planning method to consider the demand uncertainty. According to stochastic two-stage planning, the decision-making variables are divided into two groups, including fixed subsets, e.g., here-and-now sets, and the expectation and observation variables. The optimal values of the expectation and observation variables depend on different scenarios, and their optimal values are obtained after the scenarios are realized. The optimal values of

the here-and-now variables are fixed for all scenarios.

The here-and-now variables are obtained in the first stage, and the expectation and observation variables are obtained in the second stage. The first stage of the stochastic problem is implemented using load predictions to obtain the optimal values of the here-and-now variables. The Monte Carlo and the corresponding scenarios are obtained in parallel using the described method. Then, the second stochastic planning stage is implemented to obtain the optimal values of the expectation and observation variables. Since the stochastic planning method is used, three objective functions with expected values of the scenarios are used. The expected values of the expectation and observation variables depend on the scenarios and their weights, which are obtained by aggregating the corresponding values adopted from the scenarios. Finally, the Pareto fronts of the objective functions are obtained using the modified  $\epsilon$ -constrained optimization method, and fuzzy decision-making is used to attain the best compromise solution.

The procedure for solving the problem is that first the input information including load, capacitor banks and DG products is received. Then, the input variables of the hybrid constrained  $\epsilon$  optimization, i.e., its input settings, are obtained. After that, the objective functions are calculated. That is, the optimization method takes into account a starting point, for example, the positions and capacities for the capacitor bank and distributed generation, and based on them, the objective functions are calculated. It should be considered that by calculating the

objective functions, all variables are calculated probabilistically and the functions are calculated for all scenarios. Once the calculation procedure is completed, the variable optimization procedure is changed again and more favorable points are sought for the location and capacity of distributed generation units and capacitor banks, and the objective functions are calculated again. These steps are repeated until the termination criterion of the hybrid-constrained  $\epsilon$  optimization algorithm is met.

The flowchart of the proposed method is shown in Figure 2.

#### 4. RESULTS AND DISCUSSION

In this study, an IEEE 33-bus test system was used to analyze the performance of the proposed method. The baseline kV of this network was 12.66kV. In this network, the main breaker was located in the main feeder. To calculate the reliability of the reference bus, it was considered 1. The feeder section with maximum impedance had a maximum failure rate of 0.5 failures per year, and the feeder section with minimum impedance had a minimum failure rate of 0.1 failures per year [33]. The failure rates for other sections were calculated using a linear relationship between these two rates.

After allocating the DG resources and CBs, if section  $j$  was compensated completely, its failure rate was reduced to 75% of  $\lambda^{\text{uncomp}}$ ; otherwise,  $\lambda^{\text{new}}$  was calculated through descriptive relationships for partial compensation. The time required for fault detection and switching was half an hour, and the repair time was considered four hours [35].

Since allocating DG resources and CBs would affect the reliability of other components like transformers, all other components of the distribution network except feeder sections were considered to be reliable. Table 1 presents the economic factors of the DG resources. Table 2 presents the economic factors of the CBs. Table 3 reports the total economic parameters.

For all case studies, the modified  $\epsilon$ -constrained optimization method was applied to the objective function.

*Scenario 1:* Optimal planning of optimal allocation of DG resources

*Scenario 2:* Optimal planning of optimal allocation of CBs

*Scenario 3:* Optimal planning of optimal allocation of DGs and CBs simultaneously

### A. Scenario 1: Analyzing the Results of DG Allocation

In this section, only DG resource allocation and its impact on the system is evaluated. It is assumed that three DGs are installed. The purpose is to examine the problem with two and three objectives. In this case, the first objective function is to minimize the costs, which is considered the main objective function. Also, the side objectives include improving the voltage profile and reducing generation curtailment; the weight factors for  $F_1$ ,  $F_2$ , and  $F_3$  are selected as 1, 1, and 1. However, the decider (system operator) might require different weights to obtain the desired solutions. Fig. 3 and Fig. 4 show the two objective solutions of the optimization output. As it can be seen in Fig. 3, by installing DG resources and increased investment costs, the power curtailment

decreases. It can be inferred from Fig. 4 that by increasing the investment cost on the set of solutions, the voltage profile improves. As mentioned, the voltage profile improvement objective function is obtained by minimizing the difference between the voltage of each bus from 1 pu, which is a very small number. The figures show the average real voltage for  $f_{\text{voltage}}$  to help understand the voltage profile improvement and closeness to 1 pu. Also, the voltage at the output of the objective function refers to average voltage profile of all buses.

Fig. 5 shows the output set of the Pareto solutions of the three-objective solution. The best solution among the solutions in Fig. 5 is selected by the fuzzy method based on the operator's decision about compromising among different objectives, which is given in Table 4.

**Table 1. Economic parameters of DGs [27].**

Parameter	Value
Investment cost (\$/MW)	3180000
Operation cost (\$/MWh)	36

**Table 2. Economic parameters of CBs [27].**

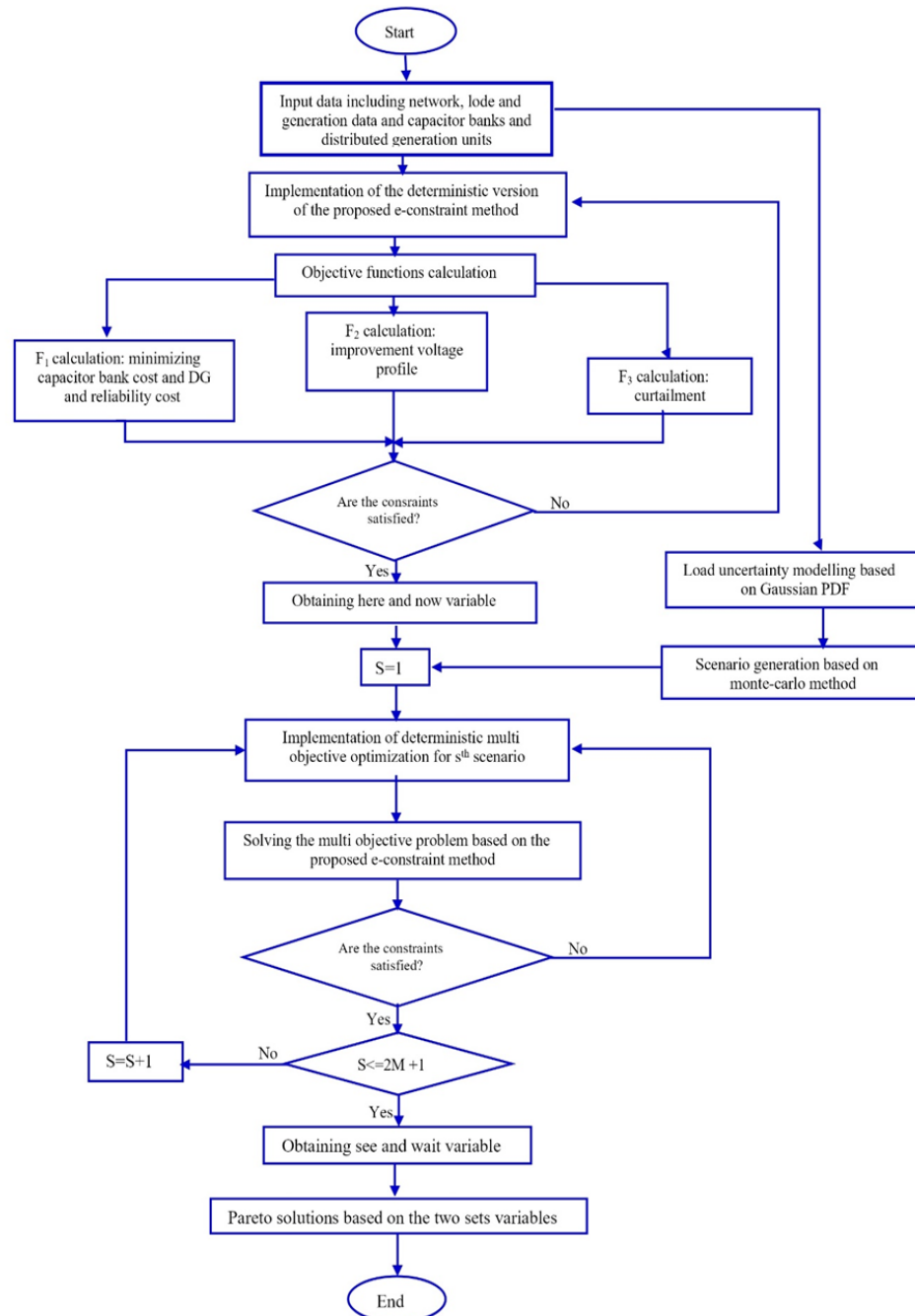
Parameter	Value
Investment cost (\$/MVar)	4000
Operation cost (\$/Year)	10% of investment cost

**Table 3. Total economic parameters [27].**

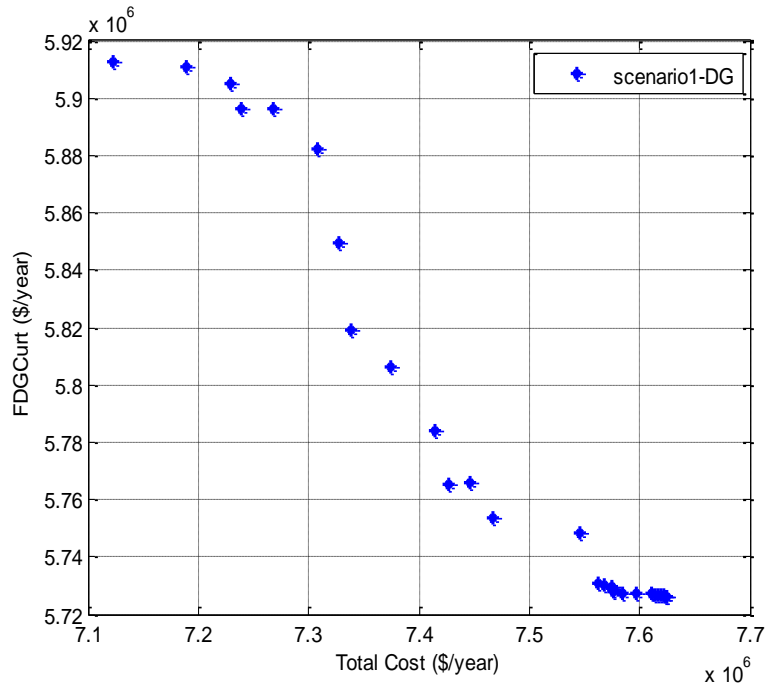
Parameter	Value
Inflation rate (%)	9
Interest rate (%)	12.5
Ks (\$/MWh)	49

According to Table 4, buses 4, 11, and 31 represent the locations of DG output with the given capacities. Table 4 also shows the

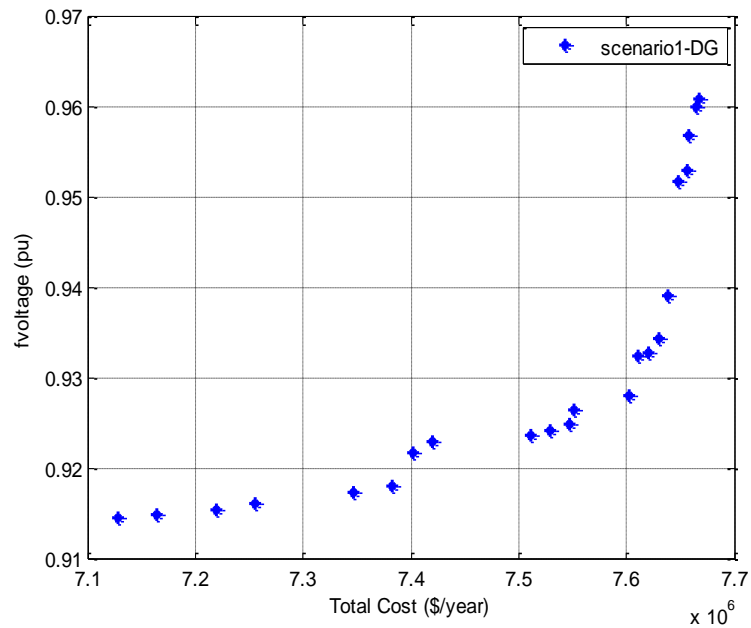
reliability cost ( $5.0604 \times 10^5$ ), which has improved by 21% compared to the base scenario.



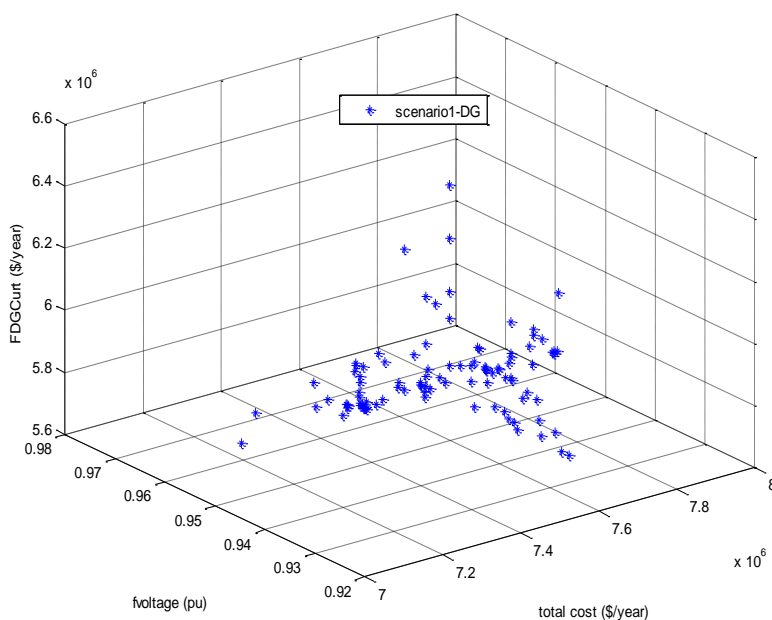
**Fig. 2. The flowchart of the proposed method.**



**Fig. 3. The set of Pareto fronts of the two-objective solution of the first scenario (objectives: cost and power curtailment).**



**Fig. 4. The set of Pareto fronts of the two-objective solution of the first scenario (objectives: cost and voltage profile).**



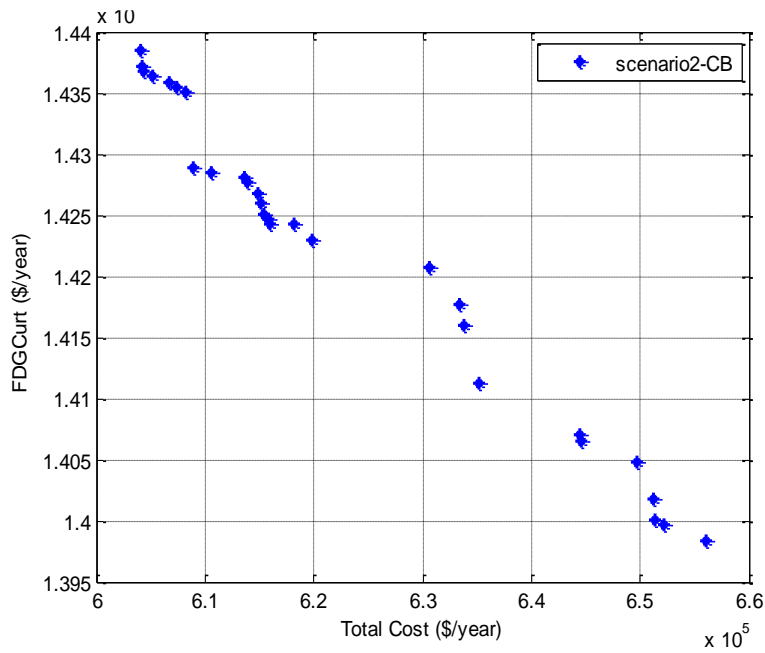
**Fig. 5.** The set of Pareto outputs of three-objective solution in the first scenario.

**Table 4.** The outputs of the fuzzy method for selecting the best solution of three objective solutions for the first scenario.

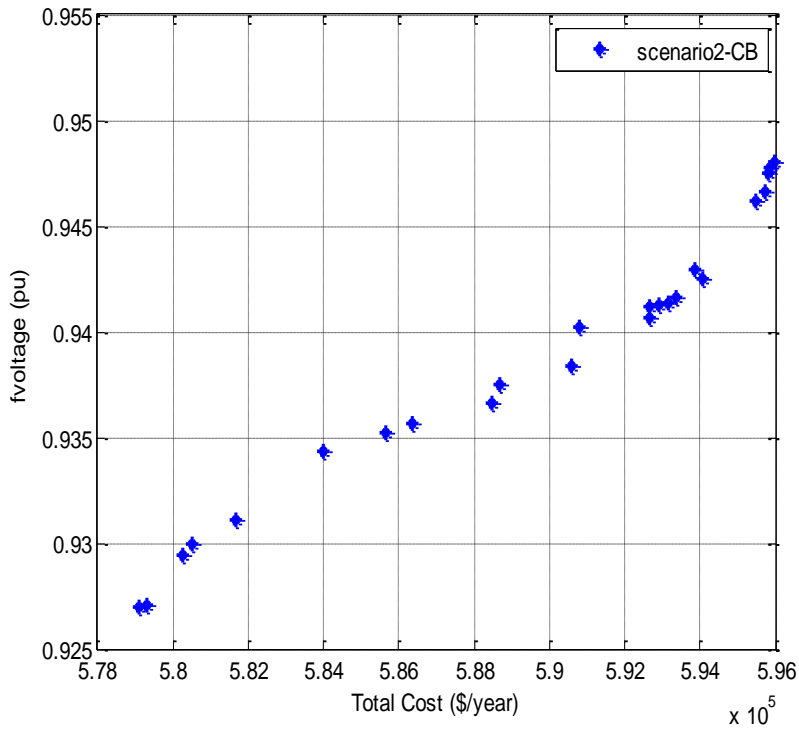
Parameter	Value
Location of DG1	Bus 4
Capacity of DG1 (MW)	0.7526
Location of DG2	Bus 11
Capacity of DG2 (MW)	0.7397
Location of DG3	Bus 31
Capacity of DG3 (MW)	0.7413
Investment cost of DGs (\$/year $\times 10^5$ )	7.9702
Operation cost of DGs (\$/year $\times 10^6$ )	6.1079
Reliability cost (\$/year $\times 10^5$ )	5.0604
Total cost (\$/year $\times 10^6$ )	7.411
Power curtailment cost (\$/year $\times 10^6$ )	5.887

## B. Scenario 2: Analyzing the results of CB Allocation

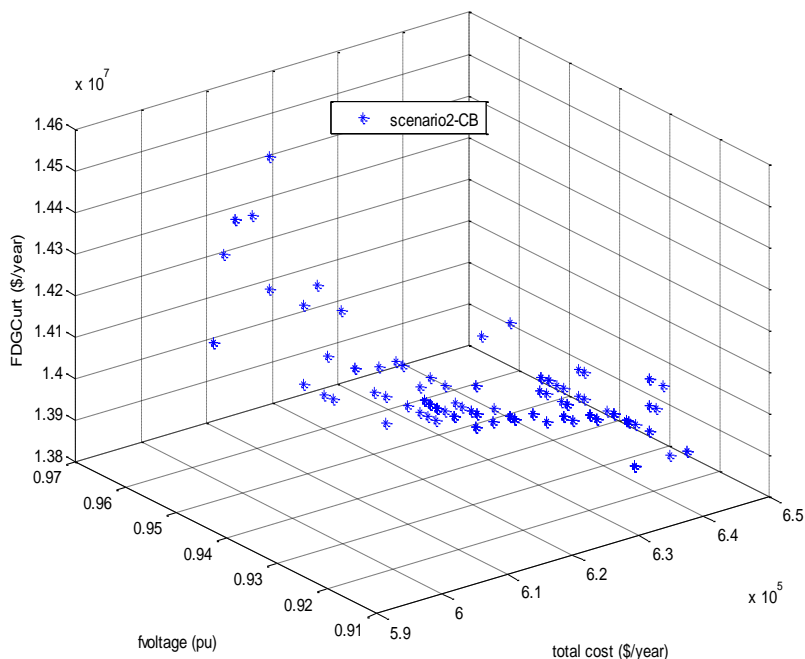
This section examines the allocation of CBs and its impact on the test system. It is assumed that three CBs are installed. The purpose is to examine the problem with two and three objectives. To this end, in the modified  $\epsilon$ -constrained method, the first objective function, which is cost minimization, is considered the main objective function. The other objectives include improving the voltage profile and power curtailment, and the weighting factors for all objective functions are set to 1. Figs. 6 and 7 show the two objective solutions of the optimization output for CB allocation. Also, it can be inferred that the desired voltage profile can be achieved by less investment cost in CB allocation.



**Fig. 6. The set of Pareto fronts of the two-objective solution of the second scenario (objectives: cost and power curtailment).**



**Fig. 7. The set of Pareto fronts of two objective solutions of the second Scenario (objectives: cost and voltage profile).**



**Fig. 8.** The set of Pareto fronts of the three-objective solution in the second scenario.

Fig. 8 shows the output set of the Pareto solutions of the three-objective solution. The output of the fuzzy method for operator decision is given in Table 5. This solution is selected among the solutions given in Fig. 8. According to Table 5, buses 12, 26, and 29 are the output locations of the CBs with given capacities. The reliability cost ( $5.998 \times 10^5$ ) is also shown in this table, which has improved by 6% compared to the base case. This improvement arises from voltage profile enhancement and the reduction of its impacts on the power supply. It should be mentioned that CB allocation does not result in much difference compared to the base case, which is due to the dependency of CBs on voltage, and the impact of these CBs is more obvious in the voltage profile. Considering the lower costs, the CBs help the voltage profile; by installing them with DGs, simultaneously, more objectives are realized, which are discussed in the next section.

**Table 5.** The results of the fuzzy method for selecting the best solution of the three-objective solutions for the second scenario.

Parameter	Value
Location of CB1	Bus 12
Capacity of CB1 (MVar)	0.4472
Location of CB2	Bus 26
Capacity of CB2 (MVar)	0.4490
Location of CB3	Bus 29
Capacity of CB3 (MVar)	0.4674
Investment cost of CBs (\$/year $\times 10^3$ )	5.418
Operation cost of CBs (\$/year $\times 10^3$ )	4.572
Reliability cost(\$/year $\times 10^5$ )	5.998
Total cost(\$/year $\times 10^5$ )	6.098
Power curtailment cost (\$/year $\times 10^7$ )	1.401

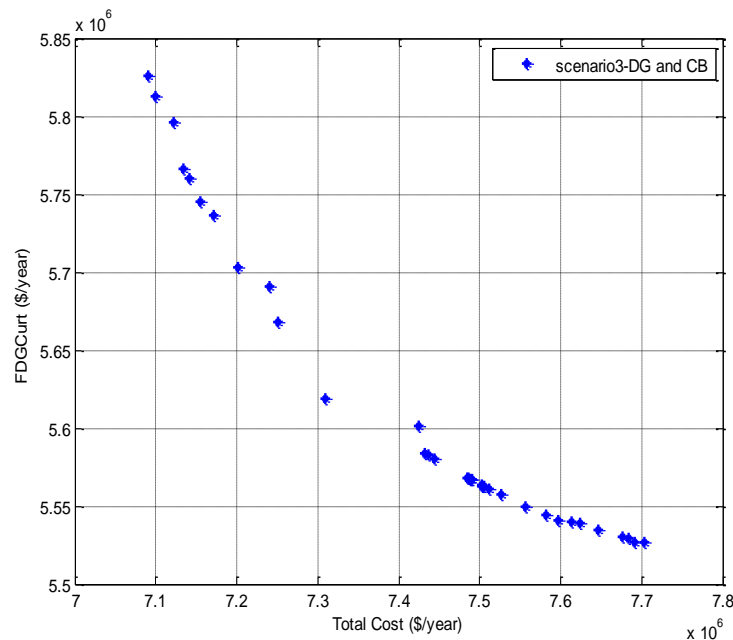


### C. Scenario 3: Analyzing results of Allocating DGs and CBs

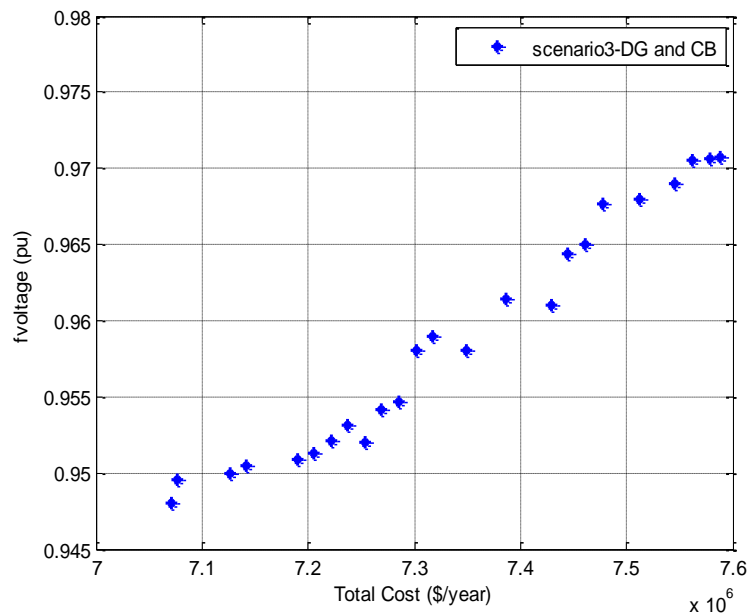
This section reports the results of the simultaneous allocation of CBs and DGs. It is assumed that three DGs and three CBs can be installed. In this scenario, like the two previous scenarios, the problem is examined with two and three objectives. To this end, in the modified  $\epsilon$ -constrained method, the first objective function, which is cost minimization, is considered the main one. Also, side objectives include improving voltage profile and power curtailment. The weighting factors for all objective functions are 1. Figs. 9 and 10 depict the set of two objective solutions for the simultaneous allocation of CBs and DGs. According to these figures, more desired solutions are obtained compared to the first and second scenarios, and the voltage profile is improved properly.

Fig. 11 shows the set of the three-objective Pareto fronts for the third scenario. The output of the fuzzy method for operator decision is given in Table 6. This solution is selected among the solutions presented in Fig. 1, based on which the best solution for operating among the set of solutions is obtained using the fuzzy method.

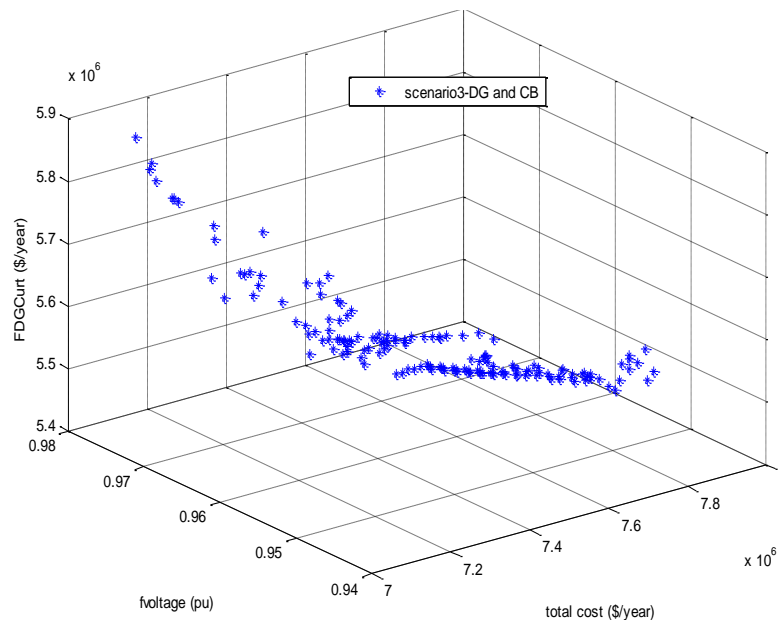
According to Table 6, buses 12, 25, and 30 are the output locations of the CBs, and buses 12, 24, and 30 are the optimal locations of the DGs. The reliability cost ( $4.53 \times 10^5$ ) is also shown in this table, which has improved by 30% compared to the base case, and it is better than the first and second scenarios. Also, considering all figures, it can be inferred that the best voltage profile occurred in the third scenario, which is due to the optimal simultaneous allocation of CBs and DGs. Also, the power curtailment cost of the network in this scenario is better than that in the two other Scenarios.



**Fig. 9. The set of Pareto fronts of two objective solutions in the third scenario (objectives: cost and power curtailment).**



**Fig. 10.** The set of Pareto fronts of two objective solutions in the third scenario (objectives: cost and voltage profile).



**Fig. 11.** Pareto fronts of three-objective solutions in the third scenario.

Compared to reference [26], this paper proposed a new multi-objective method for simultaneous optimization of DG and CBs placement under uncertainty. In [26], loss reduction and voltage profile enhancement

are the objective functions. In this paper, voltage profile improvement, investment and operation costs reduction, and renewable energy curtailment reduction are the objective functions. The renewable energy

curtailment reduction is rarely discussed in the literature. Moreover, this paper used two-stage stochastic fuzzy modeling compared to [26]. The results proved the higher efficiency and accuracy of the proposed method by considering the uncertainty.

**Table 6. The results of the fuzzy method for selecting the best solution among the three-objective solutions for the third scenario.**

Parameter	Value
Location of DG1	Bus 12
Capacity of DG1 (MW)	0.7415
Location of DG2	Bus 24
Capacity of DG2 (MW)	0.7383
Location of DG3	Bus 30
Capacity of DG3 (MW)	0.7422
Location of CB1	Bus 12
Capacity of CB1 (MVar)	0.3450
Location of CB2	Bus 25
Capacity of CB2 (MVar)	0.2615
Location of CB3	Bus 30
Capacity of CB3 (MVar)	0.4422
Investment cost of DGs (\$/year $\times 10^5$ )	7.0776
Operation cost of DGs(\$/year $\times 10^6$ )	6.3428
Investment cost of CBs(\$/year $\times 10^3$ )	4.644
Operation cost of CBs(\$/year $\times 10^3$ )	3.919
Reliability cost(\$/year $\times 10^5$ )	4.531
Total cost(\$/year $\times 10^6$ )	7.512
Power curtailment cost(\$/year $\times 10^6$ )	5.495

## 5. CONCLUSION

This paper proposed a novel method for the optimal allocation of DG resources and CBs. Reducing operation and investment costs, improving voltage profile and reliability, and reducing curtailment costs were considered objective functions. The multi-objective  $\epsilon$ -constrained optimization method was used to solve the problem. The proposed method was implemented on an IEEE 33 bus test system in three scenarios. The proposed method improved the performance of the system in different terms, including voltage profile and reliability. Moreover, it was concluded that the voltage profile improvement could be achieved in the scenario of capacitor banks' allocation by lower investment costs compared to other cases. As a result of the use of DG and capacitor banks, less voltage deviation and lower curtailment costs were obtained. Also, the reliability of the network would be better due to equipping the network with DGs and capacitor banks, and ultimately more desirable goals could be achieved.

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