Signal Processing and Renewable Energy
March 2018, (pp.9-14)
ISSN: 2588-7327
eISSN: 2588-7335

A New Algorithm to Estimate the Direction of Non-coherent Narrowband Signals with High Resolution and Low Sample Size

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Abstract
This paper introduces a new algorithm named C2-ESPRIT to estimate the direction of narrow band signals which are not coherent. The new technique estimates direction of arrival (DOA) by dividing the received signals into two sub-array and performing sub-array processing where a new diagonal matrix is obtained. As the elements of this matrix are associated with the main angles of arrival signals, it can be used to estimate the DOA of signals. The main idea in this paper is how the signal's sub-array are obtained and a processing method is used. Unlike the two algorithms named ESPRIT and C-SPRIT, where the result of processing the signals were a diagonal matrix and diagonal conjugated matrix, in C2-ESPRIT it may appears in the form of the square conjugate diagonal matrix. The advantages of this method compared to ESPRIT and C-SPRIT, are the reduction of the variance, high resolution and requirement of fewer samples in the receiver. MATLAB is used to compare and verify new C2-ESPRIT algorithm efficiency with three known algorithms: Root-Music, ESPRIT and C-SPRIT.

Keywords: Coherent signal, DOA estimation, Narrowband signal, Non-coherent, Subspace.

1. INTRODUCTION

Estimates direction of arrival (DOA) is one of the most important and widely used branches in signal processing field and applications such as sonar, radar, and mobile communication systems. The most important processing method for DOA estimation is subspace method where the eigenvalues and eigenvectors of covariance matrix are computed and then both orthogonal subspace of noise and signal are obtained. MUSIC [1], [2] ESPRIT [3], Root MUSIC [4] and C-ESPRIT [5], [6] are the well known algorithms based on the subspace method. We assume noise variance is constant however there are algorithms where the variance changes with time [7]. In [7] time-frequency techniques with ESPRIT algorithm are used to estimate DOA of signals. The proposed algorithm in this paper is based on subspace methods and estimation errors in angles which will be less than the Root-Music, ESPRIT, and C-SPRIT as well.

The proposed method is based on using the received signals with its all arrays element and using the complex conjugate signal of second and third element. In This technique our process is

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like ESPRIT and C-SPRIT methods but we obtain a new diagonal matrix. Diagonal elements of this matrix have relation with main direction of arrival signal. Later we will see that the new diagonal matrix is a square diagonal matrix from the C-SPRIT method. Accordingly, we don't need to use the complex conjugate of the signal in two elements. In proposed algorithm is used for two array sensor that is the reason we call it C2-ESPRIT. Furthermore it is seen in simulation results where different level of SNR, different sample numbers, different number of elements and different angles and distance between antennas are considered, the error estimated in the proposed algorithm is lower than other algorithms, i.e. Root-Music, ESPRIT and C-SPRIT. It is shown that the variance of the probability density function of the random variable in estimation of angles using the proposed algorithm is always less than those three mentioned algorithms.

The rest of this paper is organized as follows. In the Section 2, we explain the signal model and assumptions. In Section 3, we propose the algorithm C2-ESPRIT and the simulation results are presented in Section 4. Finally in Section 5, the conclusion of this paper is presented.

2. SIGNAL MODEL AND ASSUMPTIONS

Let's consider an M-sensor linear array where the sensors are uniformly spaced. Assume the number of signal sources k is either known or can be estimated. K narrow band signals have the same frequency f_0 from different directions $\theta_1, \theta_2, \dots, \theta_K$. In order to minimize the frequency interference of receiver signals we consider distance between elements as $d = \frac{\lambda}{2}$ where $\lambda = \frac{c}{f_0}$ is signal wavelength and c is the speed of propagation of signals. $s_k(t)$ is k-th narrow band signal and can be considered as follows [8]:

$$s_k(t) = u_k(t) e^{j(2\pi f_0 t + \varphi_k(t))} \tag{1}$$

where $u_k(t)$ and $\varphi_k(t)$ are respectively amplitude and phase of signal which changes slowly with time. Slowly changing over the time means that $u_k(t) \approx u_k(t-\tau)$, $\varphi_k(t) \approx \varphi_k(t-\tau)$ for any arbitrary delay time τ. Therefore, simply we can show that the effect of k-th signal time delay in equation (1) will appear as a phase changing, i.e.:

$$s_k(t-\tau) \approx s_k(t) e^{-j2\pi f_0 \tau} \tag{2}$$

We consider the first element of the received signal as the reference so for i-th element of the signal without noise we have:

$$\begin{aligned} x_i(t) &= \sum_{k=1}^{K} s_k\left(t - \tau_i(\theta_k)\right) \\ &= \sum_{k=1}^{K} s_k(t) e^{-j2\pi f_0 \tau_i(\theta_k)} \quad i = 1, \dots, M \end{aligned} \tag{3}$$

where $x_1(t)$ andare received signal without $x_i(t)$ noise of the first element and i-th element respectively, at time t, and $\tau_i(\theta_k)$ is the time delay between $x_1(t)$ andth signal is received -$k$when $x_i(t)$ with direction of θ_k and can be obtained as follows:

$$\tau_i(\theta_k) = \frac{(i-1) d\cos(\theta_k)}{c} \quad i = 1, 2, \dots, N \tag{4}$$

The received signal element i at time t considering the effect of noise on elements, have the form of equation 5:

$$\begin{aligned} y_i(t) &= x_i(t) + w_i(t) \\ &= \sum_{k=1}^{K} s_k(t) e^{-j2\pi f_0 \tau_i(\theta_k)} + w_i(t) \end{aligned} \tag{5}$$

Where $y_i(t)$ and $w_i(t)$ are respectively the i-th element of the signal and noise. Considering M sensors to receive signals, the matrix form of (5) is:

$$\begin{aligned} & y(t) = As(t) + w(t) \\ & y(t) = [y_1(t), y_2(t), \dots, y_M(t)]^T \\ & A = [a(\theta_1), a(\theta_2), \dots, a(\theta_K)] \\ & S(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T \\ & w(t) = [w_1(t), w_2(t), \dots, w_M(t)]^T \end{aligned} \tag{6}$$

The transpose operator in (6) is T and $a(\theta_k)$ is defined as follows:

$$a(\theta_k) = [1, \vartheta_k, \dots, \vartheta_k^{M-1}]^T \quad k = 1, \dots, K \tag{7}$$

where $\vartheta_k^{i-1} = e^{-j2\pi f_0 \tau_i(\theta_k)}$, $i = 1, 2, \dots, M$.

In this paper the following assumptions have been considered:

1. The number of signals is less than the number of elements, M> K
2. Signals are stationary with zero mean and the following variance:

$$E\left\{s_i(t)s_j^*(t)\right\}=\left(r_{s_i}-r_{s_{ij}}\right)\delta_{ij}+r_{s_{ij}} \quad (8)$$

where $E(.)$ is the expectation value or statistical mean, $(.)^*$ and δ_{ij} are complex conjugate and Kronecker delta function respectively.

3. Additive noise for each element is Gaussian white noise with zero mean and the following variance:

$$E\left\{w_i(t)w_j^*(t)\right\}=\sigma_n^2\delta_{ij} \quad (9)$$

where σ_n^2 is the noise power of each element.

4. Noise signals are non-coherent.

By using the mentioned assumptions and considering the Eq. (6), the covariance matrix of signals can be acquired as follows:

$$R_Y=E\left\{y(t)y(t)^H\right\}=AR_sA^H+\sigma_n^2 I_{\mathrm{M}} \quad (10)$$

where I_m is the identity matrix with size m × m, $(.)^H$ refers to the Hermitian operator and $R_s=E\{s(t)s(t)^H\}$ is covariance matrix of source signal respectively.

In practice R_Y is estimated with sampling of the received signal vector $y(t)$ at the time $t_j=1,2,\dots,N$ and then K signal obtained as follow:

$$\hat{R}_Y=\frac{1}{N}\sum_{j=1}^{N}y(t_j)y(t_j)^H \quad j=1,\dots,N \quad (11)$$

Because the calculation of covariance matrix in (10) requires expectation value, so we use mean time instead of statistical average according to (11).

3. PROPOSED C2-ESPRIT ALGORITHM

In this section we represent the proposed algorithm that is called C2-ESPRIT. The main idea is to use the complex conjugate of the signal in both the second and third element of array sensor (see Fig. 1).

As seen in Fig. 1, vectors of the first and the second sub-array according to (6) are as follows:

$$\begin{aligned} Y(t)&=\left[y_1(t),y_2(t),\dots,y_M(t)\right]^T\\ &=As(t)+N_1(t)\\ Z(t)&=\left[y_3^*(t),y_2^*(t),y_1(t),\dots,y_{M-2}(t)\right]^T\\ &=A\Phi^{*2}s(t)+N_2(t) \end{aligned} \quad (12)$$

where $y_i(t)$ is the received signal from *i-th* element so we have:

$$\begin{aligned} &N_1(t)=[w_1(t),w_2(t),\dots,w_M(t)]^T\\ &N_2(t)=\left[w_3^*(t),w_2^*(t),w_1(t),\dots w_{M-2}(t)\right]^T\\ &\Phi=diag(\vartheta_1,\vartheta_2,\dots,\vartheta_k)\\ &\vartheta_k=e^{-j\pi\cos(\theta_k)} \end{aligned} \quad (13)$$

Two vectors that are related to sub-array in (12) are defined with the matrix G as bellow:

$$\begin{aligned} G&=\begin{bmatrix}Y(t)\\ Z(t)\end{bmatrix}=\begin{bmatrix}A\\ A\Phi^{*2}\end{bmatrix}s(t)+\begin{bmatrix}w_1(t)\\ w_2(t)\end{bmatrix}\\ &=\bar{A}s(t)+W_G(t) \end{aligned} \quad (14)$$

where $\bar{A}=[A \quad A\Phi^{*2}]^T$. For covariance matrix G in (14) we have:

$$R_{GG}=\bar{A}R_s\bar{A}^H+\sigma_n^2 I_{2M} \quad (15)$$

By splitting R_{GG} into the eigenvalues and eigenvectors of equation 15 we obtain:

$$R_{GG}=G_s\Lambda_sG_s^H+G_n\Lambda_nG_n^H \quad (16)$$

where Λ_s and Λ_n are eigenvalues of subspace signal and noise subspace respectively also G_s and G_n are eigenvectors of signal subspace and noise subspace as well.

Because the signals are non-coherent, therefore R_{GG} covariance matrix is full rank of order K:

$$span\left\{\bar{A}\right\}=span\left\{G_s\right\} \quad (17)$$

where span {.} is defined as the space of matrix's column. The relation between G_s and $\bar{A}$ can be expressed as follows:

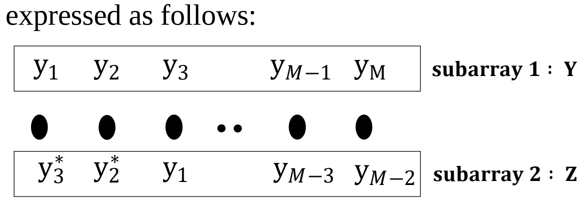

Fig. 1. Choosing sub-array in proposed algorithm, C2-ESPRIT.

$$G_s = \bar{A}T \quad (18)$$

where T is a matrix with non-zero determinant of $K \times 2M$ and:

$$G_s = \begin{bmatrix} G_Y \\ G_Z \end{bmatrix} = \begin{bmatrix} AT \\ A\Phi^{*2}T \end{bmatrix} \quad (19)$$

G_Y and G_Z are sub-matrices of G_s and both are with size $M \times K$. We define a matrix with $M \times 2K$ dimension as follows:

$$G_{sYZ} = [G_Y \quad G_Z] \quad (20)$$

We also define a new matrix F as follows in order to find the null space matrix which defined in (20):

$$G_{sYZ}F = 0 \quad (21)$$

Equation 21 can be rewritten as follows:

$$[G_Y \quad G_Z]\begin{bmatrix} F_Y \\ F_Z \end{bmatrix} = [AT \quad A\Phi^{*2}T]\begin{bmatrix} F_Y \\ F_Z \end{bmatrix} \quad (22)$$
$$= ATF_Y + A\Phi^{*2}TF_Z = 0$$

where $F = \begin{bmatrix} F_Y \\ F_Z \end{bmatrix}$, then we have:

$$\Phi^{*2} = -TF_YF_Z^{-1}T^{-1} \quad (23)$$

Suppose $\Psi = -F_YF_Z^{-1}$, then according to (23) we have:

$$\Phi^{*2} = T\Psi T^{-1} \quad (24)$$

Obviously, the eigenvalues of Ψ matrix are Φ^{*2} matrix and the angles of signals or direction of arrival (DOA) can be estimated as follows:

$$\theta_k = \cos^{-1}\left[\frac{-arg(\Phi = eign(\Psi))}{2\pi}\right] \quad (25)$$

where, $eign(.)$ is eigenvalues and arg is angle of the complex number. We can obtain Ψ matrix according to (22) as follows:

$$[G_Y \quad G_Z]\begin{bmatrix} F_Y \\ F_Z \end{bmatrix} = G_YF_Y + G_ZF_Z = 0 \quad (26)$$

In this case, we can write:

$$\Psi = -F_YF_Z^{-1} = (G_Y^HG_Y)^{-1}G_Y^HG_Z \quad (27)$$

4. SIMULATION RESULTS

In this section, the error estimation of the proposed algorithm is compared with ESPRIT and C-SPRIT algorithms in terms of signal-to-noise ratio (SNR), sample size and the distance between elements. For this purpose, MATLAB software is used.

The root mean square error (RMSE) is defined as follows:

$$[RMSE] = \sqrt{\sum_{i=1}^{K}\left(\hat{\theta}_i - \theta_i\right)^2 \Big/ K} \quad (28)$$

here θ_i and $\hat{\theta}_i$ are respectively angle and the estimation angle of *i*-th signal.

Assume that the antenna receives two uncorrelated sinusoid signals with angles of arrival 50 and 60 degrees. Fig. 2 shows the DOA error estimation in terms of different SNR for the proposed algorithm, ESPRIT and C-SPRIT for N = 100 and M = 6 and d = λ / 2 = 0.5. As it is seen in Fig. 2, increasing SNR is equivalent to decreasing the DOA error estimation for either the proposed method or ESPRIT and C-SPRIT. However, the error for C2-ESPRIT is less than other algorithms.

Fig. 3 shows of the DOA error estimation for different number of samples. All setting are the same as before except SNR = 0 dB. As it is seen, by changing the number of samples, the error in the proposed algorithm is lower than other algorithms.

Fig. 4 shows the performance comparison of the three methods where the difference angles between the two sinusoid signals is varying. In this case, we choose SNR = 10 dB and N = 20. As it is seen, the error in the proposed algorithm is lower than the other algorithms.

Fig. 5, compares the error estimation angles of the three mentioned methods where the sensors distance are changing. We assume SNR = 10 dB and N = 20.

All algorithms are working well for $d = \frac{\lambda=1}{2} = 0.5$ and all algorithms for further distances do not have good performance. Also it can be seen that the angle estimation error of the proposed method is lower than other algorithms for d = 0.5.

Fig. 6 shows angles estimation error of the proposed method and ESPRIT angles and C-SPRIT with the same assumptions explained before and for $d = 0.5$ and $N = 20$ in terms of changing the number of elements.

As seen in Fig. 6, by changing the number of elements, error in the proposed method will be less than other algorithms. It is indicated that the efficiency of new method is better than ESPRIT and C-SPRIT methods. It also clear that when number of antenna elements is increased, the going down trend in the proposed algorithm is more than ESPRIT and C-SPRIT algorithms.

Fig. 7 to 10 show the histograms (equal of probability density functions) of the proposed method and Root-Music and ESPRIT and C-SPRIT for two uncorrelated signals with angles of 50 and 60 degrees with the 500 number of repetition and with assuming SNR = 5 dB and N = 50 and M = 6.

Because the Gaussian white noise with zero mean is considered, therefore, probability density function of the received signal and also density function of angle estimation in all methods are considered to have Gaussian distribution.

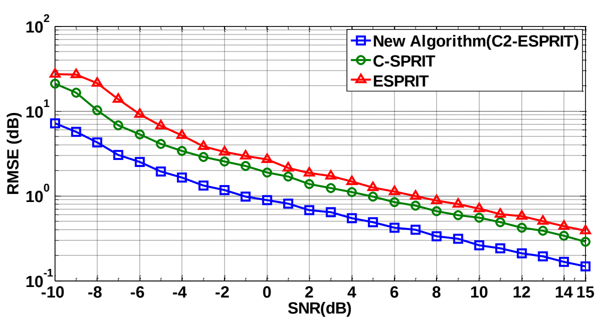

Fig. 2. Comparing the DOA error estimation of the proposed algorithm, ESPRIT and C-SPRIT for different SNR.

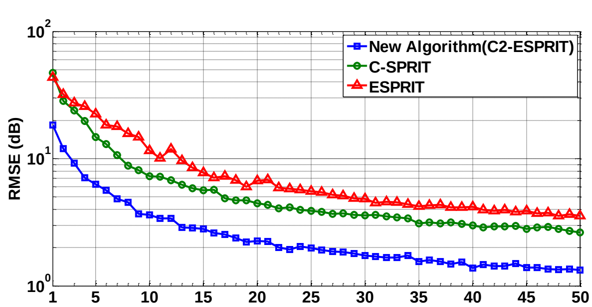

Fig. 3. Comparing error of angle estimation for proposed algorithm, ESPRIT and C-ESPRIT algorithms in terms of the number of samples.

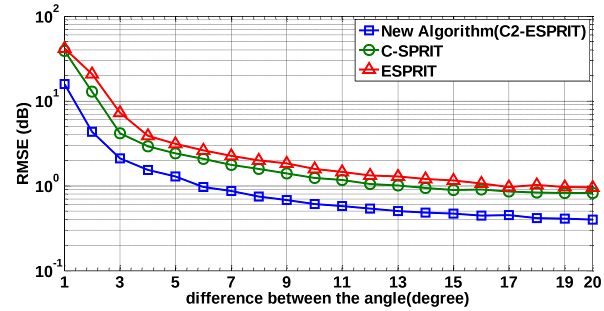

Fig. 4. Comparing error of angle estimation for proposed algorithm and ESPRIT and C-ESPRIT algorithm in terms of difference of angle of arrival signals.

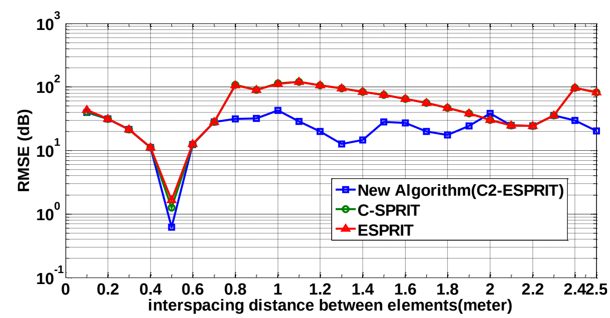

Fig. 5. Angles estimation error for proposed algorithm and ESPRIT and C-ESPRIT algorithm in terms of different distance between elements.

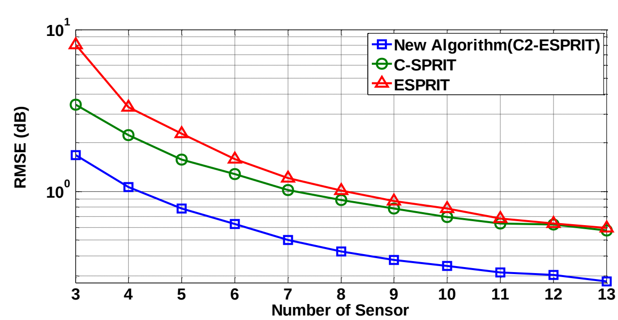

Fig. 6. Comparing error of angle estimation for proposed algorithm and ESPRIT and C-ESPRIT algorithm in terms of number of elements.

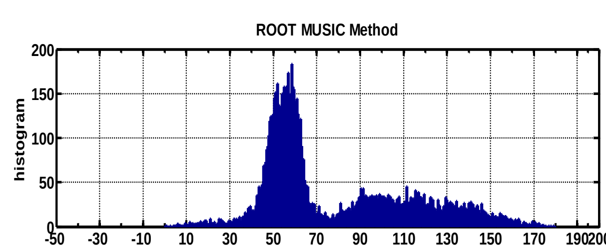

Fig. 7. Histogram (≡PDF) of Root-Music algorithm.

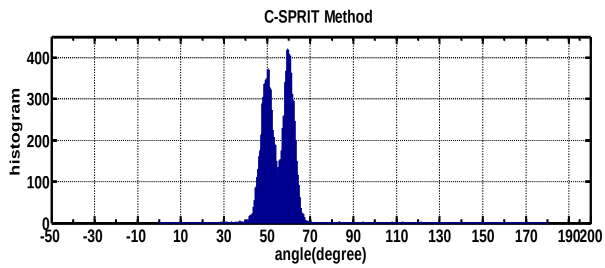

Fig. 8. Histogram (≡PDF) of C-SPRIT algorithm.

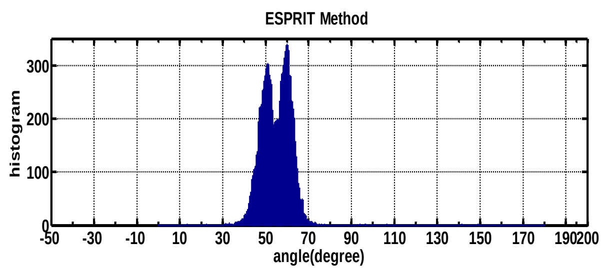

Fig. 9. Histogram (≡PDF) of E-SPRIT algorithm.

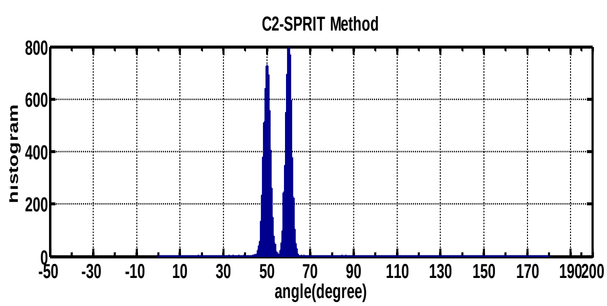

Fig. 10. Histogram (≡PDF) of the proposed algorithm.

It is clear that the variance in the proposed method is lower than Root-Music and ESPRIT and C-SPRIT. Root-Music and ESPRIT and C-SPRIT method do not have efficiency for low samples but the proposed method estimates the angle of the signal well.

5. CONCLUSION

In this paper, we proposed an algorithm called C2-ESPRIT to estimate the K non-coherent narrowband signal. The basic idea is processing the received signals in all array elements and using the complex conjugate on second and third elements. So, a new diagonal matrix like ESPRIT and C-SPRIT is obtained where the diagonal elements of the new matrix are associated with the main angles of signals. So for C2- ESPRIT, the sub-array signal should be chosen precisely in order to process and obtain the square diagonal matrix similar to the one used in C-SPRIT algorithm. This property results in, the variance of the probability density function of angle estimations even in low samples be low.

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