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Malmquist Productivity Index under Network Structure and Negative Data: An Application to Banking Industry

Seyed Ehsan Shojaie[1], Seyed Jafar Sadjadi[2*],Reza Tavakkoli-Moghaddam[3]

Abstract

In this paper, we present a comprehensive approach for evaluating efficiency in complex networks by integrating network data envelopment analysis (NDEA) with the Malmquist productivity index. The proposed method addresses the inherent challenge of accommodating negative data within the network efficiency evaluation framework, which is a common occurrence in real-world network operations. Through the introduction of a two-stage structure, the model not only effectively manages the presence of negative values, but also provides a robust and insightful assessment of network efficiency. A case study from banking sector is employed to demonstrate the efficacy of the proposed approach, showcasing its capacity to offer valuable and actionable insights for decision-making in complex network environments. The results highlight the practical applicability and importance of the extended approach in addressing the complexities associated with evaluating efficiency in diverse network settings

Keywords: *Network Data Envelopment Analysis; Malmquist Productivity Index; Negative Data; Production Possibility Set; Two-Stage Structure; Banking Industry*

Introduction

In many real-world situations, entities are interconnected and collaborate to achieve common goals. In such cases, a traditional data envelopment analysis (DEA) model that treats each entity as an isolated decision-making unit may not accurately capture the interactions and dependencies within the network. Network data envelopment analysis (NDEA) addresses this limitation by considering the interrelationships among entities, allowing for a more realistic and accurate analysis (Chen et al., 2009; Tone & Tsutsui, 2009). In other words, NDEA is a method used to evaluate the efficiency of complex systems. Unlike traditional data envelopment analysis, which analyzes the efficiency of individual decision-making units (DMUs), NDEA takes into account the interdependencies and interactions among sub-DMUs in a network, providing a more comprehensive and accurate measure of efficiency (Cook et al., 2010; Koronakos, 2019).

In a network structure, DMUs are interconnected, meaning that the efficiency of one sub-DMU can be influenced by the efficiency of other sub-DMUs in the network. NDEA considers these interconnected relationships and captures the systemic effects that result from resource sharing, collaboration, and coordination among sub-DMUs (Kao, 2014; Kao, 2016). By doing so, it provides a more realistic assessment of efficiency by considering the overall network performance rather than just individual unit performances. NDEA has been applied to various domains and industries, providing valuable insights into the performance and efficiency of network-based systems (Ratner et al., 2023). Here are some notable applications of NDEA:

- Supply Chain Management: NDEA is widely used to assess the efficiency and

1. PhD Candidate, Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
2*.School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran (Corresponding Author: sjsadjadi@iust.ac.ir)
3. School of Industrial Engineering, college of Engineering, University of Tehran, Tehran, Iran

effectiveness of supply chain networks. It helps identify bottlenecks, inefficiencies, and areas for improvement in the entire supply chain system. NDEA can also guide decision-making related to network configurations, supplier selection, inventory management, and logistics optimization (Chen & Yan; 2011; Mirhedayatian et al., 2014; Hosseinzadeh Lotfi et al., 2023).

- Financial Networks: NDEA has been applied to financial networks, such as banking systems or investment portfolios, to assess their efficiency and risk-resilience. It helps identify underperforming entities, evaluate diversification strategies, measure systemic risk, and optimize asset allocation within the network (Matthews, 2013; Tone et al., 2019; Peykani et al., 2021b; 2022e; Fukuyama et al., 2023).
- Transportation Networks: NDEA is employed to analyze the performance of transportation systems, such as freight networks or public transit systems. It aids in evaluating the effectiveness of routes, identifying congestion points, optimizing routing decisions, and guiding transportation policy-making (Zhao, 2011; Stefaniec et al., 2020; Kang et al., 2023).
- Telecommunication Networks: NDEA is used to evaluate the efficiency and effectiveness of communication networks, such as telecommunication infrastructure or internet service providers. It helps identify network congestion, measure service quality, optimize resource allocation, and guide decision-making related to network infrastructure investments (Du et al., 2019; Wang et al., 2022; Lafuente et al, 2024).
- Healthcare Networks: NDEA is used to evaluate the efficiency and performance of healthcare delivery networks, such as hospital networks or healthcare systems. It helps identify areas for improvement, measure resource utilization, optimize patient flow, and guide healthcare policy-making (Khushalani & Ozcan, 2017; Gong et al., 2019; Gavurova et al., 2021).

This approach allows for a more accurate assessment of performance by accounting for the complexities and interdependencies within the network. By considering the holistic view of the entire network and capturing the interdependencies between DMUs, NDEA enables decision-makers to identify inefficiencies, reallocating resources more effectively, and improving overall network performance. It should be noted that numerous variants and extensions of NDEA have been developed so far to accommodate different types of networks, such as two-stage, series, parallel, and mixed networks (Kao & Hwang, 2008; Despotis et al., 2016; Zhou et al., 2018; Lo Storto, 2020; Shi et al., 2021; Latifiee & Shafiee, 2023).

The goal of this paper is to introduce a new approach for evaluating variations in the efficiency of network DMUs across various time intervals. This approach is designed to effectively handle negative data within a two-stage network framework. To reach this goal, the network DEA approach based on possibility production set (PPS), Malmquist productivity index (MPI), and formulation of variant of radial measure (VRM) are applied. The remainder of this paper proceeds as follows. The preliminaries and research backgrounds of this paper will be presented in Section 2. Then, a new two-stage network data envelopment analysis model using variant of radial measure will be proposed in Section 3. In the subsequent, a novel Malmquist productivity index will be presented in Section 4. In the following, the efficacy of the proposed approach will be illustrated through a real case study in Section 5. Finally, conclusions and future research directions will be discussed in Section 6.

Preliminaries

In this section, first, we will present the conceptualization of the network data envelopment analysis approach that will be utilized in this research study. Then, we will explore the idea of expressing negative data using variant of radial measure.

Two-Stage Network DEA Model

Figure 1 visually represents a two-stage structure, consisting of a group of K homogeneous decision-making units $\text{DMU}_k\ (k=1,\ldots,K)$. Each decision-making unit has I inputs $x_{ik}\ (i=1,\ldots,I)$ in the first stage, G intermediate variables $z_{gk}\ (g=1,\ldots,G)$ that connect the first and second stages, and J outputs $y_{jk}\ (j=1,\ldots,J)$ in the second stage.

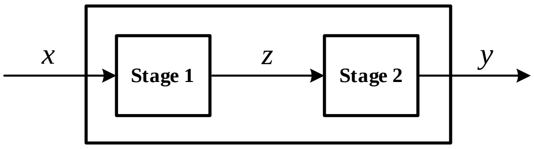

Figure 1. *Two-Stage Network Structure*

It should be noted that the performance measurement of the first stage of DMU under investigation is conducted using Model (1), which is based on the input-oriented DEA model assuming variable returns to scale.

$$\begin{aligned}
&\text{Min} \quad \Theta && (1)\\
&\text{S.t.} \quad \sum_{k=1}^{K} \lambda_k x_{ik} \le \Theta x_{ip}, \quad \forall i=1,\ldots,I\\
&\sum_{k=1}^{K} \lambda_k z_{gk} \ge z_{gp}, \quad \forall g=1,\ldots,G\\
&\sum_{k=1}^{K} \lambda_k = 1\\
&\lambda_k \ge 0, \quad \forall k=1,\ldots,K
\end{aligned}$$

Similarly, the performance measurement of the second stage of DMU under investigation is carried out using Model (2), which is based on the output-oriented DEA model assuming variable returns to scale.

$$\begin{aligned}
&\text{Max} \quad \Phi && (2)\\
&\text{S.t.} \quad \sum_{k=1}^{K} \mu_k z_{gk} \le z_{gp}, \quad \forall g=1,\ldots,G\\
&\sum_{k=1}^{K} \mu_k y_{jk} \ge \Phi y_{jp}, \quad \forall j=1,\ldots,J\\
&\sum_{k=1}^{K} \mu_k = 1\\
&\mu_k \ge 0, \quad \forall k=1,\ldots,K
\end{aligned}$$

Chen & Zhou (2004) introduced a novel approach in network DEA that combines the envelopment form of both the first and second stages. In fact, by merging Models (1) and (2), a network DEA model is obtained in the form of Model (3), which has the ability to evaluate the efficiency of each stage and the overall efficiency simultaneously.

$$\begin{aligned}
&\text{Min} \quad \Delta^1 \Theta - \Delta^2 \Phi && (3)\\
&\text{S.t.} \quad \sum_{k=1}^{K} \lambda_k x_{ik} \le \Theta x_{ip}, \quad \forall i=1,\ldots,I\\
&\sum_{k=1}^{K} \lambda_k z_{gk} \ge \tilde{z}_{gp}, \quad \forall g=1,\ldots,G\\
&\sum_{k=1}^{K} \lambda_k = 1\\
&\lambda_k \ge 0, \quad \forall k=1,\ldots,K\\
&\sum_{k=1}^{K} \mu_k z_{gk} \le \tilde{z}_{gp}, \quad \forall g=1,\ldots,G\\
&\sum_{k=1}^{K} \mu_k y_{jk} \ge \Phi y_{jp}, \quad \forall j=1,\ldots,J\\
&\sum_{k=1}^{K} \mu_k = 1\\
&\mu_k \ge 0, \quad \forall k=1,\ldots,K
\end{aligned}$$

In Model (3), it is important to clarify that the weights Δ^1 and Δ^2 are determined by the user and represent their preference towards the performance of the two stages. Additionally, the symbol "~" is used to denote unknown decision variables.

Variant of Radial Measure

Let's consider a situation where there are K similar decision-making units $\text{DMU}_k\ (k=1,\ldots,K)$ that transform I inputs $\alpha_{ik}\ (i=1,\ldots,I)$ into J outputs $\beta_{jk}\ (j=1,\ldots,J)$. Now, let's focus on a specific DMU, denoted as DMU_p, which is currently being evaluated. Cheng et al. (2013) presented a VRM model, referred to as Model (4), to assess the performance of decision-making units in situations where the presence of negative values can be observed.

$$\begin{aligned}
&\text{Max} \quad \delta && (4)\\
&\text{S.t.} \quad \sum_{k=1}^{K} \xi_k \alpha_{ik} \le \alpha_{ip} - \delta\left|\alpha_{ip}\right|, \quad \forall i=1,\ldots,I\\
&\sum_{k=1}^{K} \xi_k \beta_{jk} \ge \beta_{jp} + \delta\left|\beta_{jp}\right|, \quad \forall j=1,\ldots,J\\
&\sum_{k=1}^{K} \xi_k = 1
\end{aligned}$$

$$\xi_k, \delta \geq 0, \quad \forall k = 1,\ldots,K$$

Model (4) represents the envelopment form of the variant of radial measure, incorporating the assumption of variable returns to scale (VRS). It should be noted that in the VRM model, the variable δ represents the measure of inefficiency for a DMU_p.

The Proposed Two-Stage Network VRM Model

This section introduces a network VRM model as Model (5) that provides a powerful framework for assessing the performance of two-stage DMUs, even in scenarios where negative values are present. It should be explained that to propose Model (5), the concepts of Models (3) and (4) are combined.

$$\text{Max} \quad \Delta^1 \Psi^1_p + \Delta^2 \Psi^2_p \qquad (5)$$

$$\text{S.t.} \quad \sum_{k=1}^{K} \lambda_k x_{ik} \leq x_{ip} - \Psi^1_p |x_{ip}|, \quad \forall i = 1,\ldots,I$$

$$\sum_{k=1}^{K} \lambda_k z_{gk} \geq z_{gp} + \Psi^1_p |z_{gp}|, \quad \forall g = 1,\ldots,G$$

$$\sum_{k=1}^{K} \lambda_k = 1$$

$$\lambda_k \geq 0, \quad \forall k = 1,\ldots,K$$

$$\sum_{k=1}^{K} \mu_k z_{gk} \leq z_{gp} - \Psi^2_p |z_{gp}|, \quad \forall g = 1,\ldots,G$$

$$\sum_{k=1}^{K} \mu_k y_{jk} \geq y_{jp} + \Psi^2_p |y_{jp}|, \quad \forall j = 1,\ldots,J$$

$$\sum_{k=1}^{K} \mu_k = 1$$

$$\mu_k \geq 0, \quad \forall k = 1,\ldots,K$$

The proposed network DEA model addresses the challenge of assessing performance for two-stage DMUs, particularly in scenarios where negative values are present. Traditional network DEA models often struggle to handle negative values, as they are designed to evaluate efficiency based on positive inputs and outputs. However, the proposed Model (5) overcomes this limitation by incorporating an extended approach that allows for the inclusion of negative values in the assessment process. By integrating the envelopment form of both the first and second stages, the model provides a comprehensive evaluation of efficiency at each stage and the overall efficiency simultaneously. This capability is crucial in understanding the performance of two-stage DMUs, as it allows for a more accurate and holistic assessment. Researchers and practitioners can now gain insights into the efficiency of each stage and identify areas for improvement, even when negative values are present.

The inclusion of negative values in the assessment process opens up new possibilities for analyzing complex systems and real-world applications (Tavana et al., 2018). It enables a more nuanced understanding of performance, as negative values can represent various factors such as inefficiencies, losses, or deviations from optimal performance. This enhanced evaluation capability can lead to more informed decision-making and targeted interventions to improve overall efficiency. In general, with the ability to handle negative values, the network VRM model enables a more accurate and reliable assessment of performance, allowing decision-makers to gain valuable insights and make informed decisions in real-world settings.

The Malmquist productivity index is a widely-used tool for measuring and analyzing productivity changes over time (Färe & Grosskopf, 1992; Peykani et al., 2021a). It is particularly useful in the context of network structures and negative data, which are common in many industries and sectors. Thus, in this section, we will put forth an expanded version of the Malmquist productivity index that can effectively operate within a two-stage network structure and accommodate negative data. Accordingly, by utilizing the proposed network VRM model, $\Omega^t_p(x^t_p, z^t_p, y^t_p)$, $\Omega^{t+1}_p(x^{t+1}_p, z^{t+1}_p, y^{t+1}_p)$, $\Omega^t_p(x^{t+1}_p, z^{t+1}_p, y^{t+1}_p)$, and $\Omega^{t+1}_p(x^t_p, z^t_p, y^t_p)$ can be calculated from Models (6) to (9), respectively:

$$\Omega^t_p(x^t_p, z^t_p, y^t_p) = \text{Max} \quad \Delta^1 \Psi^1_p + \Delta^2 \Psi^2_p \qquad (6)$$

$$
\begin{aligned}
\text{S.t.}\quad & \sum_{k=1}^{K}\lambda_k x_{ik}^{t} \le x_{ip}^{t} - \Psi_p^1 \left|x_{ip}^{t}\right|, \quad \forall i=1,\dots,I\\
& \sum_{k=1}^{K}\lambda_k z_{gk}^{t} \ge z_{gp}^{t} + \Psi_p^1 \left|z_{gp}^{t}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\lambda_k = 1\\
& \lambda_k \ge 0, \quad \forall k=1,\dots,K\\
& \sum_{k=1}^{K}\mu_k z_{gk}^{t} \le z_{gp}^{t} - \Psi_p^2 \left|z_{gp}^{t}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\mu_k y_{jk}^{t} \ge y_{jp}^{t} + \Psi_p^2 \left|y_{jp}^{t}\right|, \quad \forall j=1,\dots,J\\
& \sum_{k=1}^{K}\mu_k = 1\\
& \mu_k \ge 0, \quad \forall k=1,\dots,K
\end{aligned}
$$

$$
\begin{aligned}
& \Omega_p^{t+1}(x_p^{t+1}, z_p^{t+1}, y_p^{t+1}) = \text{Max}\ \ \Delta^1\,\Psi_p^1 + \Delta^2\,\Psi_p^2 \qquad (7)\\
\text{S.t.}\quad & \sum_{k=1}^{K}\lambda_k x_{ik}^{t+1} \le x_{ip}^{t+1} - \Psi_p^1 \left|x_{ip}^{t+1}\right|, \quad \forall i=1,\dots,I\\
& \sum_{k=1}^{K}\lambda_k z_{gk}^{t+1} \ge z_{gp}^{t+1} + \Psi_p^1 \left|z_{gp}^{t+1}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\lambda_k = 1\\
& \lambda_k \ge 0, \quad \forall k=1,\dots,K\\
& \sum_{k=1}^{K}\mu_k z_{gk}^{t+1} \le z_{gp}^{t+1} - \Psi_p^2 \left|z_{gp}^{t+1}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\mu_k y_{jk}^{t+1} \ge y_{jp}^{t+1} + \Psi_p^2 \left|y_{jp}^{t+1}\right|, \quad \forall j=1,\dots,J\\
& \sum_{k=1}^{K}\mu_k = 1\\
& \mu_k \ge 0, \quad \forall k=1,\dots,K
\end{aligned}
$$

$$
\begin{aligned}
& \Omega_p^{t}(x_p^{t+1}, z_p^{t+1}, y_p^{t+1}) = \text{Max}\ \ \Delta^1\,\Psi_p^1 + \Delta^2\,\Psi_p^2 \qquad (9)\\
\text{S.t.}\quad & \sum_{k=1}^{K}\lambda_k x_{ik}^{t} \le x_{ip}^{t+1} - \Psi_p^1 \left|x_{ip}^{t+1}\right|, \quad \forall i=1,\dots,I\\
& \sum_{k=1}^{K}\lambda_k z_{gk}^{t} \ge z_{gp}^{t+1} + \Psi_p^1 \left|z_{gp}^{t+1}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\lambda_k = 1\\
& \lambda_k \ge 0, \quad \forall k=1,\dots,K\\
& \sum_{k=1}^{K}\mu_k z_{gk}^{t} \le z_{gp}^{t+1} - \Psi_p^2 \left|z_{gp}^{t+1}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\mu_k y_{jk}^{t} \ge y_{jp}^{t+1} + \Psi_p^2 \left|y_{jp}^{t+1}\right|, \quad \forall j=1,\dots,J\\
& \sum_{k=1}^{K}\mu_k = 1\\
& \mu_k \ge 0, \quad \forall k=1,\dots,K
\end{aligned}
$$

$$
\begin{aligned}
& \Omega_p^{t+1}(x_p^{t}, z_p^{t}, y_p^{t}) = \text{Max}\ \ \Delta^1\,\Psi_p^1 + \Delta^2\,\Psi_p^2 \qquad (8)\\
\text{S.t.}\quad & \sum_{k=1}^{K}\lambda_k x_{ik}^{t+1} \le x_{ip}^{t} - \Psi_p^1 \left|x_{ip}^{t}\right|, \quad \forall i=1,\dots,I\\
& \sum_{k=1}^{K}\lambda_k z_{gk}^{t+1} \ge z_{gp}^{t} + \Psi_p^1 \left|z_{gp}^{t}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\lambda_k = 1\\
& \lambda_k \ge 0, \quad \forall k=1,\dots,K\\
& \sum_{k=1}^{K}\mu_k z_{gk}^{t+1} \le z_{gp}^{t} - \Psi_p^2 \left|z_{gp}^{t}\right|, \quad \forall g=1,\dots,G\\
& \sum_{k=1}^{K}\mu_k y_{jk}^{t+1} \ge y_{jp}^{t} + \Psi_p^2 \left|y_{jp}^{t}\right|, \quad \forall j=1,\dots,J\\
& \sum_{k=1}^{K}\mu_k = 1\\
& \mu_k \ge 0, \quad \forall k=1,\dots,K
\end{aligned}
$$

Finally, the network Malmquist productivity index is derived by employing Equation (10):

$$
\text{MPI}_p^N = \sqrt{\frac{\Omega_p^{t}(x_p^{t+1}, z_p^{t+1}, y_p^{t+1}) * \Omega_p^{t+1}(x_p^{t+1}, z_p^{t+1}, y_p^{t+1})}{\Omega_p^{t}(x_p^{t}, z_p^{t}, y_p^{t}) * \Omega_p^{t+1}(x_p^{t}, z_p^{t}, y_p^{t})}} \qquad (10)
$$

It is important to clarify that the interpretation of the productivity change for the DMU being analyzed is determined by the value of the MPI, which can either be greater than, equal to, or less than one:

- $\text{MPI}_p^N > 1$: If the MPI is greater than one, it indicates an improvement in productivity over the analyzed period. This means that the DMU has experienced positive changes in efficiency and technology, resulting in increased productivity compared to the reference period.
- $\text{MPI}_p^N = 1$: When the MPI is equal to one, it suggests that there has been no change in productivity over the analyzed period. This means that the DMU's efficiency and technology have remained constant, resulting in the same level of productivity as the reference period.
- $\text{MPI}_p^N < 1$: If the MPI is less than one, it signifies a decline in productivity over the analyzed period. This indicates that the DMU has experienced negative

changes in efficiency and technology, resulting in decreased productivity compared to the reference period.

Real-Life Application: Banking Industry

In this section, we present the outcomes achieved through the implementation of the suggested methodology in order to assess 10 branches of the Detroit National Bank. Notably, every branch is composed of two distinct stages: the initial stage signifies the execution of "**Operations**", while the subsequent stage symbolizes the attainment of "**Profitability**". Also, it should be explained that two inputs including X (1): **Assets**, X (2): **Expenses**, three intermediate measures including Z (1); **Operating Cash Flows**; Z (2): **Deposits**; Z (3): **Loans**, and two outputs including Y (1): **Revenues**; Y (2): **Profit Margin**, are considered in this study. Accordingly, Tables (1) and (2) showcase the data sets pertaining to the ten branches of the Detroit National Bank over two consecutive time periods.

Table 1.
Data Set of 10 Bank Branches: First Period

Banks	Inputs		Intermediate Measures			Outputs	
	X (1)	X (2)	Z (1)	Z (2)	Z (3)	Y (1)	Y (2)
Bank 01	155,324,912	12,285,520	-179,401	6,908,469	9,819,005	2,052,938	-0.3889
Bank 02	21,480,033	12,806,491	339,144	135,620,246	22,374,600	4,726,509	-0.0802
Bank 03	248,386,983	9,968,648	-207,293	2,176,347	2,364,003	1,446,928	-0.2443
Bank 04	58,910,193	2,368,322	664,181	79,908,959	44,831,014	8,976,752	0.0926
Bank 05	25,901,426	2,339,403	-428,782	53,427,000	23,631,501	1,035,975	1.5561
Bank 06	7,939,028	2,635,965	411,495	175,572,374	13,255,184	3,183,641	-0.0961
Bank 07	87,572,735	3,494,479	-322,695	80,223,432	28,443,510	5,616,903	-0.0277
Bank 08	11,203,678	3,518,579	-65,656	36,960,223	21,591,987	2,731,982	-0.2142
Bank 09	127,917,698	9,745,164	105,947	4,301,475	8,474,633	2,655,259	-0.4758
Bank 10	1,468,462	5,736,594	-173,953	52,390,460	13,951,362	181,798	-3.6964

Table 2.
Data Set of 10 Bank Branches: Second Period

Banks	Inputs		Intermediate Measures			Outputs	
	X (1)	X (2)	Z (1)	Z (2)	Z (3)	Y (1)	Y (2)
Bank 01	394,156,140	32,568,000	-213,573	80,331,040	10,558,070	23,871,376	0.4861
Bank 02	24,976,782	13,770,420	403,743	159,553,230	23,552,210	5,626,796	-0.0944
Bank 03	279,086,498	45,582,750	-69,256	23,915,909	26,863,681	1,702,268	-0.2809
Bank 04	71,841,699	2,602,552	840,735	88,787,732	47,692,569	9,652,421	0.1144
Bank 05	29,771,754	2,961,270	-461,056	64,369,880	27,478,490	1,311,361	-1.8976
Bank 06	29,722,371	8,399,956	452,192	209,014,731	15,779,981	3,882,489	-0.1104
Bank 07	100,658,316	4,314,171	-370,914	84,445,718	30,259,053	5,975,428	-0.035
Bank 08	12,046,965	4,139,504	-74,609	43,482,615	23,469,551	3,035,536	-0.2711
Bank 09	234,466,294	30,827,960	1,287,741	13,616,235	52,078,561	32,780,986	0.5802
Bank 10	1,596,155	6,102,760	-207,086	60,218,920	16,808,870	199,778	-3.891

After gathering the necessary data, we present the outcomes of solving and executing Models (6) to (9), along with Equation (10) discussed in Section 4. These results are displayed in Tables (3) to (5) correspondingly. As shown in Table (3), when considering the first stage, Banks 03, 04, 05, 09, and 10 exhibited enhanced productivity throughout the analyzed period. Additionally, as indicated in Table (4), the productivity of all banks in the second stage, with the exception of Banks 06, 07, and 10, has witnessed a decline. Finally, according to the results provided in Table (5), it is evident that Banks 04, 05, 09, and 10 have experienced a notable enhancement in their overall productivity throughout the specified period.

Table 3.
The Experimental Results: First Stage

Banks	$\Omega_p^t(x_p^t, z_p^t, y_p^t)$	$\Omega_p^{t+1}(x_p^{t+1}, z_p^{t+1}, y_p^{t+1})$	$\Omega_p^t(x_p^{t+1}, z_p^{t+1}, y_p^{t+1})$	$\Omega_p^{t+1}(x_p^t, z_p^t, y_p^t)$	MPI_p^N
Bank 01	0.19512	0.16652	0.07142	0.21184	0.53640
Bank 02	1.00000	1.00000	1.01219	1.02972	0.99145
Bank 03	0.23568	0.25113	0.33117	0.08598	2.02579
Bank 04	1.00000	1.00000	1.21000	1.05587	1.07050
Bank 05	1.00000	1.00000	0.94284	0.06198	3.90022
Bank 06	1.00000	1.00000	1.12848	1.92590	0.76547
Bank 07	0.70352	0.77093	0.57517	0.82277	0.87524
Bank 08	1.00000	1.00000	1.05451	1.16390	0.95184
Bank 09	0.24984	1.00000	1.48423	0.16841	5.93926
Bank 10	1.00000	1.00000	1.15103	0.96262	1.09349

Table 4.
The Experimental Results: Second Stage

Banks	$\Omega_p^t(x_p^t, z_p^t, y_p^t)$	$\Omega_p^{t+1}(x_p^{t+1}, z_p^{t+1}, y_p^{t+1})$	$\Omega_p^t(x_p^{t+1}, z_p^{t+1}, y_p^{t+1})$	$\Omega_p^{t+1}(x_p^t, z_p^t, y_p^t)$	MPI_p^N
Bank 01	1.00000	1.00000	1.58282	2.38195	0.81517
Bank 02	0.97276	0.48482	1.04179	0.54856	0.97289
Bank 03	1.00000	1.00000	0.57133	4.10169	0.37322
Bank 04	1.00000	0.50940	1.00857	0.53497	0.97998
Bank 05	1.00000	1.00000	1.07865	1.59901	0.82133
Bank 06	0.99405	0.66908	1.01017	0.52775	1.13507
Bank 07	1.00000	1.00000	1.11098	0.97876	1.06541
Bank 08	0.75622	0.93725	0.76154	1.04652	0.94968
Bank 09	1.00000	1.00000	1.53887	3.47566	0.66540
Bank 10	0.51757	1.00000	0.60579	1.16094	1.00409

Table 5.
The Experimental Results: Overall

Banks	$\Omega_p^t(x_p^t, z_p^t, y_p^t)$	$\Omega_p^{t+1}(x_p^{t+1}, z_p^{t+1}, y_p^{t+1})$	$\Omega_p^t(x_p^{t+1}, z_p^{t+1}, y_p^{t+1})$	$\Omega_p^{t+1}(x_p^t, z_p^t, y_p^t)$	MPI_p^N
Bank 01	0.59756	0.58326	0.82712	1.29689	0.78899
Bank 02	0.98638	0.74241	1.02699	0.78914	0.98970
Bank 03	0.61784	0.62556	0.45125	2.09383	0.46713
Bank 04	1.00000	0.75470	1.10928	0.79542	1.02591
Bank 05	1.00000	1.00000	1.01075	0.83049	1.10320
Bank 06	0.99702	0.83454	1.06933	1.22682	0.85415
Bank 07	0.85176	0.88547	0.84308	0.90077	0.98640
Bank 08	0.87811	0.96863	0.90803	1.10521	0.95198
Bank 09	0.62492	1.00000	1.51155	1.82204	1.15218
Bank 10	0.75878	1.00000	0.87841	1.06178	1.04417

The experimental results highlight the benefits of utilizing NDEA in the presence of negative data and a two-stage structure. It allows for a comprehensive assessment of performance within a network, considering interdependencies between entities and capturing changes in efficiency over time. This information can support decision-making processes, resource allocation, and the identification of areas for improvement within the network.

Conclusions and Future Research Directions

Network data envelopment analysis is a powerful approach that allows for a comprehensive evaluation of interconnected entities within network-based systems. It considers the dependencies, interactions, and flows among DMUs by incorporating network structures, constraints, and interdependencies. NDEA has found applications in various domains, such as supply chain management, financial networks, transportation networks, telecommunication networks, and healthcare networks. The application of NDEA offers several benefits and advantages:

- Comprehensive Evaluation: NDEA provides a more comprehensive evaluation of interconnected entities within a network, considering their interactions, dependencies, and flows. It offers a holistic perspective on efficiency and performance, which may be missed by traditional DEA or other standalone efficiency assessment methods.
- Realistic Modeling: NDEA captures the interconnected nature of real-world systems, allowing for more accurate modeling, analysis, and decision-making. It enables organizations to better understand the dynamics of their networks, identify inefficiencies, and optimize operations.
- Network-Specific Insights: NDEA provides insights and measures specific to network structures and interactions. It helps in understanding the unique dynamics, strengths, and weaknesses of network-based systems, leading to targeted improvements and enhanced performance.

This paper proposes a novel approach for evaluating efficiency in two-stage networks through the integration of network data envelopment analysis and Malmquist productivity index. The proposed method addresses the challenge of incorporating negative data within the network efficiency evaluation framework. By applying a variant of radial measure, the proposed approach accommodates the presence of negative values while providing a comprehensive assessment of network efficiency.

The effectiveness of the proposed approach is demonstrated through a case study from banking industry, showcasing its capacity to provide valuable insights for decision-making in two-stage network environments. For future studies, the proposed approach can be further expanded to effectively handle situations involving uncertain data. This expansion will enable the utilization of well-known techniques in uncertain programming, including fuzzy optimization (Olfat et al., 2016; Peykani et al., 2018; 2019; Abbasi Bastami et al., 2021; Heidari et al., 2021; Peykani et al., 2022a; 2022b; Allahakbari et al., 2023; Fendereski et al., 2023), robust optimization (Shakouri et al., 2019; Shirazi, & Mohammadi, 2019; Peykani et al., 2020a; 2020b; 2022d; Toloo et al., 2022; Omrani et al., 2023), uncertain theory (Mohmmad Nejad & Ghaffari-Hadigheh, 2018; Jiang et al., 2021; Peykani et al., 2022c; Pourmahmoud & Bagheri, 2023; Peykani & Pishvaee, 2024), and stochastic optimization (Huang & Li, 2001; Kao & Liu, 2009; Olesen & Petersen, 2016; Banker, 2021; Khodadadipour et al., 2021; Wanke et al., 2023).

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Conflicts of Interest

The authors declare no conflict of interest.

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