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Stochastic Facilities location Model by Using Stochastic Programming

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Abstract. Finding the location for plans like factories or warehouses for any organization is an important and strategic decision. Costs of transportation which are the main part of the price of the goods, is the function of the location of these projects. to find the optimum location of these projects, there have been various methods proposed which are usually defined (not random). In reality and in dealing with real world conditions, taking into account influential and effective parameters leads to unexpected results. In this research, while introducing these defined algorithms, it been tried to random location model based on the existing models. Regarding this, by studying the models taken in random location we tried to propose an efficient and effective model. For this purpose, by using randomized planning and randomized constrained planning, we turn random model to defined model, which is solvable by using the last algorithm or standard planning methods.

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1. Introduction

According to geographical and economical history, before 60 AD, the fundamental assumption based on economical humans believes on how industries constructed and minimized their costs. In this way, researchers imagine that humans have capabilities to compete with other competitors; futurism ability and enjoying of information and data that is need to decision-making processes. Cornuejols et al (1990).

It is obvious that this type of assumptions were in contrast completely with real world circumstances. In the other side, however, the transportation costs were considered as the basic for minimum costs, which in this study the demands, were neglected as a crucial and important effective item to determine the geographical location. This caused the theories' analysis went to the exploring of available markets and start to compete for development and expanding the markets.

Leon Cooper (1978) according to this theory, not only economical employers decrease costs (such as transportation costs) but they also close to high ensured and expanded markets, try to maximize their incomes. S.Guha, A. Myerson, K. Munagala (2013) D. Shmoys et.al (1997). Finally, location theories with inspiration from systematic approaches in last of 70 AD decade, try to modulation simultaneously 2 proposed theories. In this given approach, however, activities and industries should locate in positions that differences between incomes and costs (benefits) conduct employers to maximum benefit.

Generally, location is an activity that identifies spatial and no special lands abilities and provides the possibility of selection suitable location for special matters. In recent years, location researches have introduced as a key components of successful achievements and survival of industries. In Stochastic facility location problem with facility location problem and stochastic positions of demand centers, try to allocate facilities to demand centers in optimum way in minimum cost and distance between them C.K.Y.Lin(2009). In fact, in real problems the exact estimation of parameters such as the demand volume of services or special item of facility, the location of facilities, the location of demand centers etc. is one of hardest challenges. In this type of model, researchers are capable to consider a wide range of models in responding to the volume and type of demands (it could encompass the capacity and locations of demands centers).

In this paper, after introducing the whole points of problem, the literatures review will be provided. Afterward, the introduced models with their objective functions and constraints will be explained. In next session, upon the review of other models and gap analysis in this field of study, an efficient model with considering all the circumstances of real environment will be proposed. In other words, in this part more clarifications about objective function, constraints and model character will be introduced respectively. Finally, a real problem will be solved, their results will be discussed, and the potential grounds for future studies will be suggested.

2. Literature Review

Peter Schultz et.al (2008) introduced a stochastic model with minimizing the cost of the single facility-product allocation and assign associated probabilistic demands and requests. In this study, a tow-step formulation is used. In the first stage, however, the cost of facilities is determined and in the next stage, the linear product-product cost function presented. Furthermore, in the initial step the objective function is the minimizing the probable of occurrence of alternatives with fulfillment of all demands distinctions with facilities. In this phase, all the alternatives are upon such parameter as destinations location, facilities locations, destinations amount of request and the capacity of each facilities. In the second phase, the objective function is the minimizing the cost of production and transportation of facilities to destinations with satisfying all request limitations and allocate facilities to demands, which have enough capacities and there are possibility to receive facilities.

Jian Zhou, Baoding Liu (2003) conducted a research upon location-allocation problem with pre-predicted data and introduced a model finally. In this way, they provided different criteria with regarding to stochastic-constrained programming and dependent-chance programming.

For solving this stochastic programming model, the simplex algorithm was used and genetic algorithm and stochastic simulation, also had greatest role in this efficient model. In this model, however, the main objective is minimizing the supplying cost of costumers with corresponding to location between facilities and costumers in relation to satisfying all demand distinctions. At final, a numerical example applied to show off the efficiency of proposed model. U. K. Bhattacharya (2012) has done a research for considering request with stochastic locations. In the model there are m facility with coordination (a_i, b_i) which each of them has the probable of location $f(a_i, b_i)$. In fact, (x, y) consider as new position of new facility and W_i shows the positive allocated weight to current facilities. Moreover, the proposed model shows as follow:

$$minimize[maxE\{W_i(|x-a_i|-|y-b_i|)=f(a_i,B_i)\}]$$

St:

$$C_{j1}x + C_{j2}x \leqslant C_{j_3}x$$

The C_1, C_2 and C_3 are fixed coefficient, which used to provide linear limitations and therefore, the marginal probability of each facility location is estimated as bellow:

$$E[f_1(x,y)] = W_i \int_{-\infty}^{+\infty} \left\{ |x-a_i| \int_{-\infty}^{+\infty} f(a_i,b_i)db_1 \right\} da_1$$
$$+ W_i \int_{-\infty}^{+\infty} \left\{ |y-b_i| \int_{-\infty}^{+\infty} f(a_i,b_i)da_i \right\} db_i = W_i \int_{-\infty}^{+\infty} |x-a_i| f_{1(a_i)} da_i$$
$$+ W - i \int_{-\infty}^{+\infty} |x-b_i| f_{2(b_i)} db_i = E[f_i(x)] + E[f_i(y)]$$

In the next phase, a numerical example and simulation will be proposed to provide evidence related to the efficiency of the model. Kennan Viswanath et.al (2010) introduced a stochastic model for allocating facility to locations with random demand centers. In this model, however, an interval distance used to consider as stochastic location for demands points and facilities. In this way, with regarding to corresponding costs between demands points and facilities, a model was introduced to minimize the total cost. Qian Wang et.al (2002) provided a model with random demands locations and fixed facility location.in their model; assume that customer prefers the closest facility for the first choice.

The objective function in the model is the minimizing total cost of customer transportations and cost expectation. In addition, some constrains related to the number of facilities, which should be available and upper limitation for ideal expectation time were considered simultaneously. To solve such this problem researcher applied three Meta heuristic methods, which include: Fastest climb, branch, bound, and tabu search. Finally, upon the obtained numerical consequences, the performance of the fastest climb method was not appropriate like the branch and bound and tabu search methods.

For instance, the obtained solution in some cases was far away around 20 percent from optimum solution. Mousavi and Akhavan Niaki (2013) conducted a research on confined facility location, which demands and customer positions were not deterministic points. In this study, however, demand obtained from fuzzy method, positions had normal distribution and the distances between facilities and customers had Euclidean distribution. In addition, two types of Euclidean's phrases and Squared Euclidean were considered to assess expectation distance between facilities and customers. Moreover, for solving the considered problem, also, Meta heuristic algorithms such as simplex algorithm, fuzzy simulation and modified genetic algorithm were applied. Finally, to show the efficiency of proposed model a numerical example solved and results provided. S. Guha et.al (2013) introduced facility locations that contain stochastic inventories and demands.

In this model, all demands have Poisson distribution and considered as independent points. The fundamental assumption in the proposed model is that the coefficient a defined to be as an obstacle toward zero inventories. In the other words, $(1-a)$ to be considered as an acceptance interval. The main objective in this model is making decision toward locations of facilities that level of availability and allocated demands satisfy the $(1-a)$ Confidence level and therefor it minimize the total costs of reordering.

p → the number of ranks

$$\lambda_{ij} = \begin{cases} 1 & if\ the\ facility\ i\ is\ avaviable \\ 0 & O,W \end{cases}$$

In other word, λ_{ij} estimate the demand value of position j. Therefore, with these assumptions stochastic location model will be as bellow:

$$min \sum_{i\in f}(f_i + c_i v_i)y_i + \sum_{i\in f}\sum_{j\in D}\lambda_i c_{ij} x_{ij}$$

st:

$$x_{ij} \leqslant y_i \qquad i \in f, j \in D \tag{1}$$

$$\sum_{i\in f} x_{ij} = 1, \qquad j \in D \tag{2}$$

$$p\,(an\ arbitary\ request\ arriving\ an\ facility\ with\ inventory\ level\ v_i\ is\ lost) \tag{3}$$

The objective function assigned for minimizing the total cost of variable and constant costs, which is related to the amount of demand for all points. The first constraint that is a binary limit defines as satisfaction of demands by facilities. The second constraint considered the fulfillment of all demands by offering facilities and the third constraint provided the ideal confidential level of demand satisfactory, which is determined by programmer. In advance, the model will re-write with allocation of Poisson distribution function to constraint num. 3 and then the stochastic facility location model will reshape. Finally, the numerical example will solve by using the new model.

3. Proposed Model

In stochastic facility location problem, there are a set of facilities allocate to demand point (customers or demand destinations). Ordinary, the objective function is the minimizing the total transportation cost. There are not any restrictions for facility service volume, which considered with probabilistic rule in the model.

The Main Components

The objective function is to minimize the overall cost and in particular, in this problem the costs contain constant cost of facilities and variable costs. It should be mentioned that the variable costs is related to satisfying the allocated facilities to demands points with corresponding to distance between facilities and demands sectors. The important constraints satisfy the demand points contain a limitation for insurance of satisfying demands (in direction to accelerate finding integer solution) and the probabilistic of supplying the volume of demands with considering stochastic characteristics. In matrix format, that brings bellow (x_i, y_i) shows the location of facilities, (a_j, b_j) define the customer positions and Z provides the possible way for satisfying the demands for each facility that can see as follow:

$$(x,y) = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \dots & \dots \\ x_m & y_m \end{pmatrix}, (a,b) = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ \dots & \dots \\ a_n & b_n \end{pmatrix}, Z = \begin{pmatrix} Z_{11} & Z_{12} & \dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots & Z_{2m} \\ & \dots & \dots & \\ Z_{n1} & Z_{n2} & \dots & Z_{nm} \end{pmatrix}$$

For modeling, at first it should be necessary to define variables and parameters adequately that include:

Indexes:

i : index for each facility $\quad i = 1, ..., m$
j : index for each demand $\quad j = 1, ..., n$

Parameters:

f_i: The constant cost to facility with index i
y_i: The alternatives for allocating facility with index i
x_{ij}: The binary variable to allocate facility with index i to j demand
β: The constant number in cost constraint related to distance

Variables:

C_{ij}: The cost of supplying each part of demand of facility i for demand j

d_j: The volume of demand of customer j
Q_j: The capacity of facility j
a_i: The probability of whether the randomized demand of facility i could be satisfied
(x_i, y_i): The decision variable that shows the location of facility
(a_j, b_j): The decision variable that shows the location of customers
Z_{ij}: The satisfied volume for customer j by facility i after receiving the stochastic demand

Modeling of stochastic location problem

The produced model with considering variables, objective function and constraints is:

$$Minimize\ Z = \sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij}$$

$st:$

$$\sum_{j=1} x_{ij} = 1 \qquad i = 1, ..., m \tag{1}$$

$$\sum_{i=1} x_{ij} \leqslant y_i \qquad j = 1, ..., n \tag{2}$$

$$prob\left(\sum_{j=1}^{n} d_j.x_{ij} \leqslant Q_i y_i\right) \geqslant a_i \qquad i = 1, ..., m \tag{3}$$

$$Z_{ij} \geqslant 0 \qquad i = 1, ..., m \qquad j = 1, ..., n \tag{4}$$

$$\sum_{i=1} z_{ij} \leqslant Q_i \qquad j = 1, ..., n \tag{5}$$

$$\sum_{j=1} z_{ij} = d_j \qquad i = 1, ..., m \tag{6}$$

$$c_{ij} = \beta\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \qquad i = 1, ..., m \qquad j = 1, ..., n \tag{7}$$

$$x_{ij} = 0, 1, z_{ij} \geqslant 0, \qquad j = 1, ..., n \qquad i = 1, ..., m$$

The objective function is to minimize the overall cost that is consisting of constant costs of facilities and variable costs related to satisfying demands points of allocated for each facility. For more explanation about constraints, however, it should be necessary to provide extra clarification. Constraint number1 ensure that each demand will be satisfied by a facility. Constraint number 2 represent that demands from special resource should be smaller than the number of resources. Constraint number 3 express that the probability of supplying the stochastic demands should be proportional with the pre-determinate number (this number will be pointed out by decision maker which it is 90%, 85% and 80% in this study). Constraint number 4 ensure that facility i supply the demand value for customer j. Constraint number 5 give satisfied values for customer j by facility i should be smaller than the capacity of each facility. Constraint number 6 express that facilities and finally, the Constraint number 7 must satisfy all the demands is related to costs of each facility in proportion with distance between facilities and demands points.

Different demands distribution

According to given explanation in previous sessions, the distributions, which are considered in stochastic locations, are generally Normal distribution or Poisson distribution. Therefore, in this step some explanations will be given about the Normal distribution and its effect on objective function and their constraints.

Demands with normal distribution

First, assume that the overall demands allocated to the facility adapt to Normal distribution with known mean and variance.

$$\sum_{j=1}^{n} = u_j \ x_{ij}, \sum_{j=1}^{n} = \sigma_j^2 \ x_{ij}$$

Thus, the probabilistic constraint could rewrite as follow:

$$\frac{Q_i y_i - \sum_{j=1}^{n} \mu_j \ x_{ij}}{\sqrt{\sum_{j=1}^{n} \sigma_j^2 x_{ij}}} \geqslant z_{ai} \qquad i = 1, ..., m$$

In given function Z_{ai} shows the critical values of standard Normal distribution with probability $prob(Z \leqslant Z_{ai})$. Additionally, for solving the proposed model it could be possible to use stochastic programming and Meta heuristic algorithms.

4. Numerical Example

The proposed model was applied in iron and steel industry in Mashhad city in north-west of Iran in 2013.In this case, five types of facilities were located in Chena ran, Manzel-Abad, Rashtkhoar, Kabkan and Industry Park. Moreover, 15 demand centers or points were considered from Industry Park, Santo Road and Golbahar. In addition, the values of demand have mean equal 100 and variance 10. The other complementary information is classified in table 1.

Table 1.Information, which need to numerical example

Parameter name	value	Parameter name	value	Parameter name	value	Parameter name	value
f_1	800000	α_2	0.85	d_2	109	d_{10}	91
f_2	800000	α_4	0.9	d_3	102	d_{11}	104
f_3	800000	Q_1	280	d_4	100	d_{12}	107
f_4	1000000	Q_2	350	d_5	109	d_{13}	93
f_5	1000000	Q_3	330	d_6	104	d_{14}	107
α_1	0.9	Q_4	325	d_7	108	d_{15}	109
α_3	0.9	Q_5	300	d_8	92		
α_5	0.85	d_1	95	d_9	90		

According to above data on table1 and solved model. The average amount of total costs in all scenarios over planning horizon is shown in Figure 1 and the obtained results express that there is 12.04 percent decrease in total cost, which could count as an outstanding number in this particular industry. After review the outlets more precisely, it could clear that all demands of demand centers were satisfied by one supplier. Although this result seems unreasonable firstly, with regarding this fact that the most of times the one facility is not unable to satisfy all the demands, therefore after supplying the demands from different facilities, this matter could be result in lower cost in overall. The final obtained results contain total cost trend and variable value are shown in figures 1 & 2 and table2.

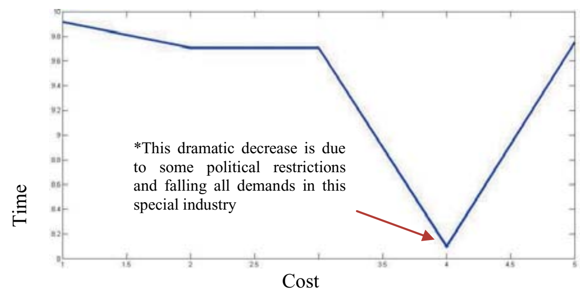

Figure 1. Average amount of total costs in all scenarios over planning horizon before applying the proposed model

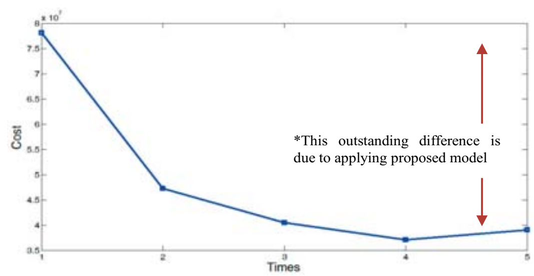

Figure 2. Average amount of total costs in all scenarios over planning horizon after applying the proposed model

Table 2. Obtained values from numerical example

Variable name	Obtained value	Variable name	Obtained value
z_{015}	95	z_{073}	108
z_{024}	6	z_{084}	92
z_{025}	103	z_{094}	38
z_{035}	102	z_{091}	90
z_{042}	100	z_{103}	91
z_{052}	109	z_{111}	104
z_{062}	71	z_{123}	107
z_{064}	33	z_{133}	46
z_{141}	107	z_{154}	109

Other values of Z are zero and thus these results show that the proposed model have a high level of efficiency.

5. Conclusions

One of the most important challenges of implementation in urban services locating projects that is exist in our country Iran is the conventional attitude toward the location of this centers. The majority of managers' act based on their experiences in locating projects and therefore they have not understated adequately enough the importance of science methods in this way. In reality cases, however, the estimation of fundamental parameters and their effective roles seems to a hard activity or in some cases, they considered as impossible matter. Thus, to dominate dynamic circumstances of real environment, applying the methods for considering uncertainty is proposed. In facility location problems, using the stochastic methods will be suitable tools to simulate somewhat the circumstances of real world. In the other word, in stochastic location problems with considering the amount of stochastic demands for existing facilities, researchers are capable more than before to provide the real situations.

Additionally, they could find appropriate solution if they assess the character of real environment and interactions between their components. In this study, however, a new model was presented based on real characters on iron and steel industry. In addition, the objective function and constraint of the model were adapted to real case as much as possible. In proposed model, also, the majority of constraints that affect real circumstances and allocated facilities to demand centers were considered as well. At least the efficiency of proposed model was measured. For future research in stochastic facility location problem, the authors suggest that the times period could be considered to satisfy stochastic demands. By using this type of problem solving method, the researchers are capable to enhance the efficiency of the model.

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