



Z-Cognitive Map: An Integrated Cognitive Maps and Z-Numbers Approach under Cognitive Information

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Abstract

Usually, in real-world engineering problems, there are different types of uncertainties about the studied variables, which can be due to the specific variables under investigation or interaction between them. Fuzzy cognitive maps, which addresses the cause-effect relation between variables, is one of the most common models for better understanding of the problems, especially when the quantitative data are not available or the nature of the problem is qualitative. One of the significant issues of automatic construction of fuzzy cognitive maps is that it does not consider the experts' uncertainties because of their cognitive attitudes such as age, experience, etc., which affects the quality and validity of modeling of the complex problems. Therefore, in this paper, a Z-Cognitive maps approach is proposed based on the distance-based automatic construction approach and Z-numbers to capture uncertainty and ambiguity of experts' opinions on the variables and causality relationships. The suggested approach can act as decision support for cognitive maps problems considering the experts' cognitive information, which is tested by a numerical example.

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1. Introduction

There are numerous events in the world which have complicated relations with variables and other events. Any occurrence in these events is under the influence of a series of factors, which also affects other phenomena. One of the essential tools for identifying and analyzing the factors affecting an event or object is the cognitive maps method. With the development of fuzzy logic and fuzzy systems, cognitive maps have developed and replaced by fuzzy cognitive maps (FCM). Fuzzy systems make the possibility of analyzing (what will happen in this case?) more specific and more precise (Song et al., 2011). Cognitive maps depict a picture of phenomena, which relates the facts, objects, and processes to values, policies, and goals, and it allows the prediction of mutual interactions and how complex events work. With the help of FCM, complex systems, and cause-effect relations between variables and related vital factors can be analyzed. Nowadays, FCM have become a dynamic tool for various sciences, including engineering, management, humanities, military sciences, medicine, geography, environment, and so on (Amirkhani et al., 2018; Alipour et al., 2017; Bottero et al., 2018; Kalantari and Khoshalhan, 2018; Martinez et al., 2018; Osoba and Kosko, 2017). FCM is established by the use of linguistic variables that are based on experts' comments on variables and their inter-relations (Gray et al., 2015). When employing FCM, it is important to know how reliable is the experts' views, since the classical FCM is unable to capture the unreliability of the expert's information due to different factors related to experts including, risk preference, age, knowledge background, experience, and as well as other factors (Ma et al., 2019; Wang et al., 2017; Ye et al., 2018). Z-numbers have

been proposed by Zadeh (2006) as a generalized version of the theory of uncertainty (Zadeh, 2011) and have the capability to provide a basis for computation with numbers that are not entirely reliable. A Z-number is an ordered pair of fuzzy numbers defined as $Z = (\tilde{A}, \tilde{R})$. The first component \tilde{A} , a limitation on the values, is a real-valued uncertain variable X. The second component \tilde{R} , is a measure of validity for the first component. In this paper, the Z-Cognitive maps approach is developed by integrating the Z-numbers and FCM to capture uncertainty and ambiguity of the variables and relationships based on the distance-based automatic construction approach.

2. Literature Review

IT Cognitive mapping was initially introduced by Axelrod (1976) in the field of political science and was then used for various applications. Considering limitations in the initial model, Kosko (1986) introduced the fuzzy concept in cognitive maps for the first time. According to his definition, an FCM is a graphic diagram guided by concepts such as rules, events, and the like, with the nodes and causality relationships between them. The structure of the fuzzy diagrams is intended to illustrate the cause and effect arguments, and their fuzzy character representations are vague levels of the relation between the concepts. Generally, FCM is a soft computational method for modeling systems which combines and uses neural networks and fuzzy logic at the same time. FCM Models can be developed by an expert or a group of experts. When expert people are not able to express their knowledge or, in cases where there is no expert, learning methods and data mining are developed. Papageorgiou (2012) studied, analyzed, and compared the learning algorithms of FCM. In general, the following methods can be considered in this regard: the distance-based method, the genetic algorithm-based method, the particle optimization algorithm-based method, and the neural networks-

based method. Papageorgiou et al., (2003) introduced a new method with an approach which increased the performance of FCM learning algorithms by combining expert knowledge and the knowledge derived from different data based on fuzzy rules. They showed that the adaptation of the Hebbian learning algorithm with evolutionary algorithms such as genetics and their combination improve learning performance in FCM. Carvalho and Tome (2001) presented the fuzzy rules approach in FCM. They introduced time in this model and provided several types of relationships between factors to cover the complexity of quantitative dynamic systems. Another approach adopted in FCM is the dynamic cognitive network (DCN), first proposed by Miao et al., (2001). DCN improves FCM by the quantization of concepts and considering nonlinear dynamic functions in each arc. Aguilar (2003) proposed a dynamic random FCM model for dynamical systems modeling, which covers the absence of a dynamic deductive mechanism in the FCM model and includes a nonlinear dynamic function in the deduction process. This model focuses on dynamic cause-effect relations. Kottas et al., (2007) introduced a Fuzzy Cognitive Network (FCN) model in FCM that improved the performance of the common FCM model through the feedback mechanism from the actual system and the previously acquired knowledge. The FCN is capable of exploring the conditions of the stable state and the relationship between the input values and their total weights, as well as the knowledge extracted in the fuzzy rules, which store the total data that can be used in controlling the functions. Wei and Yanchun (2008) presented a model that revealed the relationship between the nodes at the time and, based on their model; one can obtain an analysis of the behavior of the system over time. Song et al., (2011) suggested an approach to unifying fuzzy rules in cognitive maps and used this approach for classification and prediction. They converted the reasoning mechanism into the original FCM model in the fuzzy if-then rules. Rodriguez

et al., (2007) presented a model for FCM modeling which used four elements: the primary matrix of factors, the fuzzy matrix of factors, the matrix of the power of the relationship between factors, the final matrix of factors. Salmeron et al., (2012) presented a model for ranking scenarios in the space of uncertainty. For this purpose, they used the combination of the Delphi method, the Fuzzy Cognitive Maps, and TOPSIS. Papageorgiou and Salmeron (2013) investigated various applications of FCM in ten years. They showed that FCM have been used for solving multiple problems such as modeling, prediction, decision-making, classification, explanation, strategic planning, etc. in the fields of behavioral sciences, medicine, engineering, trade and management, system production, information systems, investment analysis, modeling systems, simulation, creation of knowledge base, etc. So far, many efforts have been made to use the FCM model in highly reliable environments. Salmeron (2010) proposed a grey FCM method and used the theory of grey systems, which is a useful mathematical theory for environments with a high degree of uncertainty and incomplete and discrete data. The results showed that this method would provide better human perceptions. Iakovidis and Papageorgiou (2011) introduced the intuitionistic model of FCM. They used the intuitive fuzzy set approach to apply the experts' uncertainties in decision making, which led the experts to explain the cause-effect communication between two concepts in FCM better. Najafi et al., (2017) used a combination of cognitive maps and Calculations With Words (FCM-CWW) to cover uncertainty as a decision support system in medicine. They used the second-order distance fuzzy function in the cognitive maps model. In a published book by Papageorgiou and Kontogianni (2013), FCM's various models were reviewed and, based on their characteristics and usage, the range of applicability of FCM was categorized into four different groups as described in Table 1.

Table 1.

Classification and Scope of Application of FCM

Application domain	Related research
Dynamic systems with uncertainties and/or time delays	Boutalis et al. (2009); Kottas (2010); Gray et al. (2015); Kheirandish et al (2017); Carvalho (2019)
Extremely uncertain environment	Salmeron et al. (2010); Iakovidis & Papageorgiou (2011); Salmeron & Papageorgiou (2012); Zhang et al. (2018); Haeri & Rezaei (2019)
Human decision making oriented	Salmeron (2009); Song et al. (2011); Papageorgiou & Froelich (2011); Rezaee et al. (2018)
Real-time systems and control	Alizadeh et al. (2008); Baykasoglu et al. (2011); Chen et al. (2012); Poczeta et al. (2018); Yang & Liu (2019)

The concept of Z-numbers is intended to provide a basis for computation with numbers that are not entirely reliable. Z-numbers have been proposed by Zadeh (2006) as a generalized version of the theory of uncertainty (Zadeh, 2011). A Z-number is an ordered pair of Fuzzy numbers defined as $Z = (\tilde{A}, \tilde{R})$. The first component \tilde{A} , a limitation on the values, is a real-valued uncertain variable X. The second component \tilde{R} is a measure of validity for the first component. Z-numbers can be used to model uncertain information in the real world. For example, in risk analysis the loss of severity of the fifth component is shallow, with the confidence of very likely, which can be written as a Z-number as follows: $Z = (\text{very low}; \text{very likely})$ (Zadeh, 2011). So far, the Z-numbers approach has been used to solve various issues such as multiple decision making (Gardashova et al., 2014; Aliev et al., 2016); supply chain management (Aboutorab et al., 2018); psychological (Aliev & Memmedova, 2015); earned value management (Salari et al., 2014); Social research (Yildiz & Kahraman, 2019). Based on this classification, environments with very high

uncertainty in variables and their internal relation, and decision-making based on experts (human-centered) are two main groups of FCM models. The proposed model of this paper is in these two categories and is presented for use in extremely uncertain environments and according to expert opinions. This developed model is a method for automatic construction of the theory of FCM based on the data created by the experts.

3. Method

The In this section, the concepts of the fuzzy sets, triangular fuzzy numbers, selected fuzzy membership function, the graded mean integration representation (GMIR) and multiplication on triangular fuzzy numbers which are used in this study are explained.

Definition 1. A fuzzy set \tilde{A} is defined on a universe X as presented below:

$$\tilde{A} = \{(x; \mu_{\tilde{A}}(x)) | x \in X\} \quad (1)$$

where $\mu_{\tilde{A}}: X \rightarrow [0, 1]$ is the membership function \tilde{A} . The membership value $\mu_{\tilde{A}}(x)$ describes the grade of membership of $x \in X$ in \tilde{A} (Aboutorab et al., 2018).

Definition 2. A triangular fuzzy number \tilde{A} can be determined by a triplet (a_1, a_2, a_3) where the membership can be determined by Eq. (2).

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \in (-\infty, a_1) \\ \frac{x - a_1}{a_2 - a_1}, & x \in [a_1, a_2] \\ \frac{a_3 - x}{a_3 - a_2}, & x \in [a_2, a_3] \\ 0, & x \in (a_3, +\infty) \end{cases} \quad (2)$$

Definition 3. Let X be the universe of discourse, which includes five linguistic variables describing the degree of security, $X = \{\text{Very Low; Low; Medium; High; Very High}\}$, presuming that only two

adjoining linguistic variables have the overlap of meanings (Kang et al., 2012). Moreover, let \tilde{A} be a fuzzy set of the universe of discourse X

$$\left\{ \begin{array}{l} f_{Very\ Low}(x) = -4x + 1; 0 \leq x \leq 0.25 \\ f_{Low}(x) = \begin{cases} 4x, & 0 \leq x \leq 0.25, \\ -4x + 2; & 0.25 \leq x \leq 0.5 \end{cases} \\ f_{Medium}(x) = \begin{cases} 4x - 1, & 0.25 \leq x \leq 0.5, \\ -4x + 3; & 0.5 \leq x \leq 0.75 \end{cases} \\ f_{High}(x) = \begin{cases} 4x - 2, & 0.5 \leq x \leq 0.75, \\ -4x + 4; & 0.75 \leq x \leq 1 \end{cases} \\ f_{Very\ High}(x) = 4x - 3; 0.75 \leq x \leq 1 \end{array} \right. \quad (3)$$

subjectively defined as follow:

Where $f_{Very\ Low}$; f_{Low} ; f_{Medium} ; f_{High} and $f_{Very\ High}$ are the membership functions of the fuzzy sets and are shown in Figure. 1.

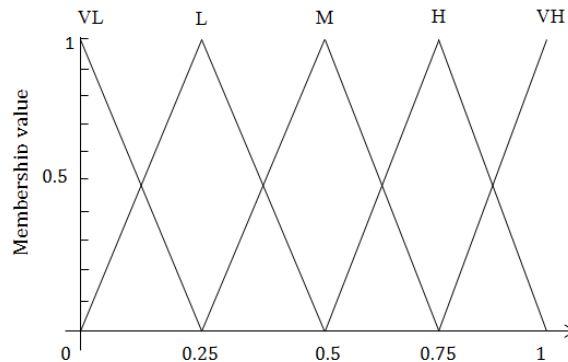


Fig. 1. Membership Function of Criteria

Definition 4. Let $\tilde{A} = a_1(a_1, a_2, a_3)$ and $\tilde{R} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. The graded mean integration representation of triangular fuzzy number \tilde{A} and \tilde{R} (Figure. 2) can be obtained as follows:

$$P(\tilde{A}) = \frac{1}{6} (a_1 + 4 \times a_2 + a_3) \quad (4)$$

$$P(\tilde{R}) = \frac{1}{6} (b_1 + 4 \times b_2 + b_3) \quad (5)$$

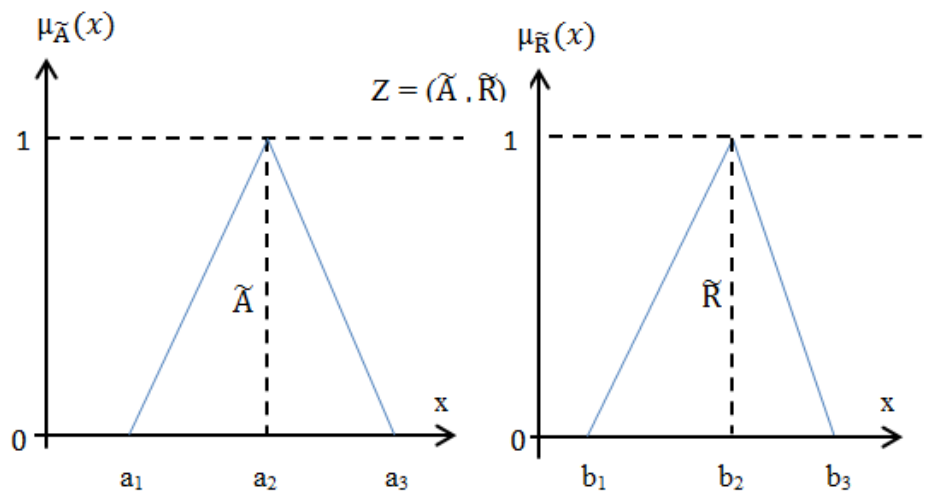


Fig. 2. A simple Z-number

The canonical representation of operation on triangular fuzzy numbers, which are based on the graded mean integration representation method (Kang et al., 2012), is used to obtain the weight of each criterion.

Definition 5. The canonical representation of the multiplication operation on triangular fuzzy numbers \tilde{A} and \tilde{R} can be defined as (Chou, 2003):

$$P(\tilde{A} * \tilde{R}) = P(\tilde{A}) \times P(\tilde{R}) = \frac{1}{6} (a_1 + 4 \times a_2 + a_3) \times \frac{1}{6} (b_1 + 4 \times b_2 + b_3) \quad (6)$$

In this part article, the of the Z-Cognitive maps methodology is proposed to overcome the ambiguity in the data and the experts' views. The proposed model can be used to model problems in extremely uncertain environments in

which experts point out their knowledge and information with ambiguity. The proposed model is developed based on of automatic construction of fuzzy cognitive maps based on the data created by the experts, previously presented by Rodriguez-Repiso et al., (2007). In the proposed method, coverage of ambiguity and uncertainty has been considered in the expert's opinion, and by applying a Z-numbers approach and combining it with a fuzzy cognitive model, we tried to improve the degree of certainty. The proposed model has the following steps:

Step 1. Selecting experts related to the subject:

As the first step in the proposed method, it is necessary to choose the relevant experts. The number of experts is directly related to the complexity of the issue under study. In general, to fully cover the dimensions of the problem, the number of experts related to the topic should be at least five experts. Each expert's comment can be of different importance. Determining the significance of each expert is the responsibility of the ultimate decision-maker. For calculations, the weight of each expert should be normalized after determining the importance of each of them.

Step 2. Identification of effective variables on the subject:

At this stage, by applying various techniques such as the brainstorming and Delphi, various dimensions of the subject are expanded, and then the related variables are identified. Since the problem is in a high ambiguity space, the number of variables can be relatively high. Variables that are more frequent among experts are selected as effective variables.

Step 3. Creating an initial matrix:

The initial matrix O is a $[N \times M]$ matrix in which N is the number of key factors that are also referred to as concepts or variables, and M is the number of people interviewed for data collection. The matrix O_{ij} element of this matrix represents the importance that expert (j) attaches to a particular notion

(i) as compared to other concepts/variables. To cover the ambiguity in the commentary of the experts, the level of importance is expressed by verbal expressions and the Z-numbers approach. Each element of $O_{i1}, O_{i2}, \dots, O_{im}$, which are verbally presented, is the vector V_i element associated with key factors belonging to the matrix row (i).

Step 4. Determination of the importance of variables:

At this stage, the significance of each variable is determined. To prevent the experts from influencing each other's views, determining the degree of importance is done independently by each expert, using the Z-numbers approach. The degree of the final significance of each of the variables is obtained by taking into account the weight of each expert through the weighted sum method.

Step 5. Obtaining a definite matrix:

Using Eq. (6), the initial matrix is converted to the definitive matrix. Then, the derived importance factor of variables and the importance factor of experts should be considered, and the final definite matrix is obtained from matrix elements. The numbers obtained in this matrix are in the range of zero to one.

5.1. Creation of the Strength of Relationships Matrix (SRM):

The Strength of Relationships Matrix is a $[N \times N]$ matrix. Rows and columns are related to the key factors of the matrix. Each element in the matrix indicates the relationship between factor (i) and factor (j); S_{ij} can accept values in $[-1, 1]$. Each key factor is represented as a numerical vector S_i , containing N elements for each concept shown in the map. There are three possible relationships between the two concepts (i) and (j):

- $S_{ij} > 0$ indicates the direct cause-effect relation (positive) between the concepts (i) and (j). That is, an increase in the value of the concept (i) increases the value of the concept (j).

- $S_{ij} < 0$ indicates the reverse cause-effect relation (negative) between the concepts (i) and (j). That is, an increase in the value of the concept (i) decreases the value of the concept (j).
- $S_{ij} = 0$ indicates that there is no relation between the concepts (i) and (j). Therefore, when determining the values, three parameters must be considered. The sign of S_{ij} indicates the relationship between the concepts (i) and (j). The power of S_{ij} shows how powerful concept (i) affects concept (j), and the path of causality, which indicates that the concept (i) causes (j) and vice versa.

5.2. Determining the Duality of Relationships

Since V_1 and V_2 are vectors associated with factors 1 and 2, and $X_1(V_j)$ and $X_2(V_j)$ are the degrees of membership of j in vectors V_1 and V_2 , these vectors have an increasing relationship (The direct link between concepts 1 and 2 and $S_{ij} > 0$). If for all or most elements associated with two vectors $X_1(V_j)$ is similar to $X_2(V_j)$ and vectors V_1 and V_2 have a decreasing relationship between concepts 1 and 2 exclusively, and if for all or most elements associated with two vectors $X_1(V_j)$ is similar to $(X_2(V_j) - 1)$, then $S_{ij} < 0$.

5.3. Determine the Strength of Relationships Matrix (SRM)

The proximity of the relationship between the two vectors V_1 and V_2 concerning the calculation of the similarity between these two vectors confirms the relation between concepts 1 and 2 related to these two vectors, which is represented by the element S_{12} . The proximity of the relationship between the two vectors is based on the concept of the distance between vectors. Different calculations are required for directly related vectors and those that have inverse relationships. If the vectors V_1 and V_2 are directly connected, then the closest relation between them for each j ($j = 1, \dots, m$) is

when $X_1(V_j) = X_2(V_j)$, and If d_j is the distance between the elements of j , the vectors V_1 and V_2 are as following:

$$d_j = |X_1(v_j) - X_2(v_j)| \quad (7)$$

Moreover, AD indicates the mean distance between vectors V_1 and V_2 :

$$AD = \frac{\sum_{j=1}^m |d_j|}{m} \quad (8)$$

The proximity or similarity of S between two vectors is shown with this equation:

$$S = 1 - AD \quad (9)$$

$S = 1$ confirms the complete similarity, and $S = 0$ indicates the maximum degree of non-similarity.

If V_1 and V_2 vectors have an inverse relation, then the method for calculating the similarity between them is similar to that of the previous one, with the exception that, in this case, the equation for calculating the distance between the corresponding elements has an inverse relationship with vectors V_1 and V_2 .

$$d_j = |X_1(v_j) - (1 - X_2(v_j))| \quad (10)$$

The remaining equations are similar in calculating the mean distance between the two vectors (AD) of equation (8) and their similarity (S) of equation (9).

In this case, $S = 1$ indicates a complete inverse similarity and $S = 0$ shows the non-complete inverse similarity between two vectors. Of course, at the time of studying the relations between numeric vectors, neither the complete similarity nor the complete non-similarity is expected.

For each pair of vectors V_1 and V_2 , the proposed method calculates the similarity between the two vectors twice, one of them being based on the direct relationship and the other being based on the inverse relationship. A higher degree of similarity confirms the duality of the relationship between the critical factors of success (i) and the key factors (j) of the positive (direct) or

the negative (inverted) and the strength of that relationship in defining the value of $\pm S_{ij}$ introduced in the SRM.

Step 6. Creating the Final Communication Matrix

Once the SRM matrix has been completed, some of the data contained therein can be misleading. Not all of the key factors presented in the matrix are related, and there is not always a causal relationship between them. An expert opinion is required to analyze the data and convert the SRM matrix to the final matrix, which only contains fuzzy numerical elements that represent cause-effect relationships among crucial factors. When analyzing data in an SRM matrix, two vectors can be mutually related to each other. Vectors can represent close mathematical relations and, at the same time, logically, two indicators/concepts can be completely unrelated to each other. These unusual relationships can be easily identified by experts.

Step 7. Graphical representation of the fuzzy cognitive map

Graphical representation of the final matrix of success, like an FCM, depicts a purposeful FCM to trace the critical success factors. In the ultimate display, each arrow of the factors (i) and (j) has a marked weight. This value represents the power of the director inverse cause-and-effect relationship between both elements and the value contained in the final matrix of success in the cell presented in the row (i) and column (j).

4. Findings

In this section, to illustrate the application of the proposed method, the problem of identifying and investigating the relationship between factors affecting the delay of construction projects is considered. The number of relevant experts is selected according to the complexity of the problem. In this case, in order to reduce the volume of calculations, the number of relevant experts is considered five people. The importance of experts' opinions that are

identified by the final decision maker is Respectively (0.2,0.1,0.2,0.3,0.2). Experts identified fifteen variables as factors affecting the delay of construction projects, among which the eight elements that had the highest frequency were selected as effective variables. These factors are shown in Table 2.

Table 2.

Effective Factors of Delay in Construction Projects

Variable number	Effective factors of delay in construction projects
1	Failure to accurately estimate the volume of work, equipment and project time
2	The technical weakness of the monitoring device in solving technical and operational problems of the workshop
3	Inadequate allocation and funding at an inappropriate time
4	Contractor's wrong choice and inappropriate pricing of contractors
5	Non-use of modern engineering contracts
6	Insufficient financial, procurement, and contractor performance
7	Lack of sufficient knowledge of the employer's experts about project management and the weakness of the consultant in providing the scheduling
8	Problems of coordination between different project components

In the following, the initial matrix is created, being a $[8 \times 5]$ numeric matrix in the issue of delay in construction projects. Experts use their linguistic variables and Z-numbers concepts to express their opinions on variables. In order to prevent the orientation of experts' views from each other, it would be better to question an expert exclusively. For example, for the third expert, the sixth variable is very important to other variables, and the level of expert's assurance is moderate.

Table 3.

Initial Matrix (the expert's opinion on variables)

Variable Number	Expert Number				
	1	2	3	4	5
1	(H,VH)	(H,M)	(L,M)	(H,M)	(L,H)
2	(VH,M)	(H,M)	(M,H)	(L,VH)	(VL,H)
.					
.					
8	(H,H)	(VH, H)	(M,VH)	(VH,L)	(VH,M)

The importance of the factors affecting the delay in construction projects, depending on the views of the experts and the significance of the considered for each of them is determined. In Table 3, comments are expressed in terms of the linguistic variable and with the Z-numbers approach. As shown in Figure 2, the Z-number has two parts (\tilde{A}, \tilde{R}) . To calculate the coefficient of the importance of the first variable, the linguistic variables are converted to crisp numbers, and we use the total balanced method of commenting by each expert using the relation number 10. Accordingly, the coefficient of the importance of the first variable consists of two parts (\tilde{A}, \tilde{R}) and forms as the following:

$$\text{Component } \tilde{A}_{\text{var 1}} = (0.2H + 0.3H + 0.2L + 0.1H + 0.2L) = ((0.2 \times 0.75) + (0.3 \times 0.75) + (0.2 \times 0.25) + (0.1 \times 0.75) + (0.2 \times 0.25)) = 0.55$$

$$\text{Component } \tilde{R}_{\text{var 1}} = (0.2VH + 0.3M + 0.2M + 0.1M + 0.2H) = ((0.2 \times 0.958) + (0.3 \times 0.5) + (0.2 \times 0.75) + (0.1 \times 0.5) + (0.2 \times 0.75)) = 0.641$$

The coefficient of importance of the variable is the number $1 = 0.55 \times 0.641 = 0.352$

Similarly, we compute the importance factor for other variables. The results of calculations are given in Table 4.

Table 4.

Variable Importance Coefficient

Variable Number	Z-number (A, R)		Final weight	Ranking
	A	R		
1	0.55	0.641	0.35	3
2	0.45	0.737	0.33	5
3	0.591	0.575	0.34	4
4	0.572	0.562	0.32	6
5	0.304	0.542	0.16	8
6	0.717	0.704	0.50	1
7	0.433	0.542	0.23	7
8	0.825	0.592	0.49	2

In the following, the definite matrix is obtained. First, using Eq. (10) the initial matrix is created and converted to the definite matrix (Table 5), and then the importance coefficient of the obtained variables and the importance coefficient of each expert should be considered and, by multiplying them in each matrix element, the final definite matrix (Table 6) is obtained. For example, the first expert will comment (H, VH) on the first variable, which means that this expert is considering a high degree for the first variable and expresses it with a high degree of certainty.

Table 5.

Definite Initial Matrix

Variable Number	Expert Number				
	1	2	3	4	5
1	0.718	0.375	0.125	0.375	0.187
2	0.479	0.375	0.375	0.239	0.031
.					
.					
.					
8	0.562	0.718	0.479	0.239	0.479

Table 6.

Final Crisp Matrix

Variable Number	Expert Number				
	1	2	3	4	5
1	0.503	0.394	0.088	0.131	0.1309
2	0.316	0.371	0.248	0.079	0.0205
.					
.					
.					
8	0.551	1.055	0.469	0.117	0.4694

Strength of relationships matrix in the issue of delay in construction projects is a $[8 \times 8]$ matrix. The rows and columns are related to the critical elements of the matrix, and each element in the matrix indicates the relationship between factors (i) and (j). Using Eqs. (1), (2), and (3), the corresponding calculations were made, and the results of the calculations are given in Table 7. Using expert opinions, meaningless communication is eliminated, and the orientation of the cause-effect of relations is determined. Results are presented in Table 9. For example, for the first variable we have:

$$AD \text{ var } 1 = (|0.186| + |0.022| + |-0.16| + |0.052| + |0.11|) / 5 = 0.106$$

$$S12 = 1 - 0.106 = 0.893$$

Table 7.

The Strength of Relationships Matrix

Variable number	1	2	3	4	5	6	7	8
1	-	0.893	0.815	0.823	0.789	0.786	0.843	0.711
2	0.893	-	0.858	0.756	0.809	0.735	0.86	0.674
.								
.								
.								
8	0.711	0.674	0.673	0.828	0.506	0.699	0.56	-

Table 8.

The Final Strength of Relationships Matrix

Variable number	1	2	3	4	5	6	7	8
1	-	0.893	-	-	-	0.786	0.843	-
2	0.893	-	-	-	-	-	-	0.674
3	-	-	-	-	-	0.754	-	0.673
4	-	-	-	-	0.678	0.64	-	0.828
5	-	-	-	0.678	-	-	-	-
6	0.786	-	0.754	0.64	-	-	-	-
7	0.843	-	-	-	-	-	-	0.56
8	-	0.674	0.673	0.828	-	-	0.56	-

In the end, by using Table 8, drawn the cognitive map for the issue of delay in construction projects. In the final display, each arrow of the factors (i) and (j) has a marked weight (figure 3). This value represents the power of the direct or inverse cause-and-effect relationship between both factors.

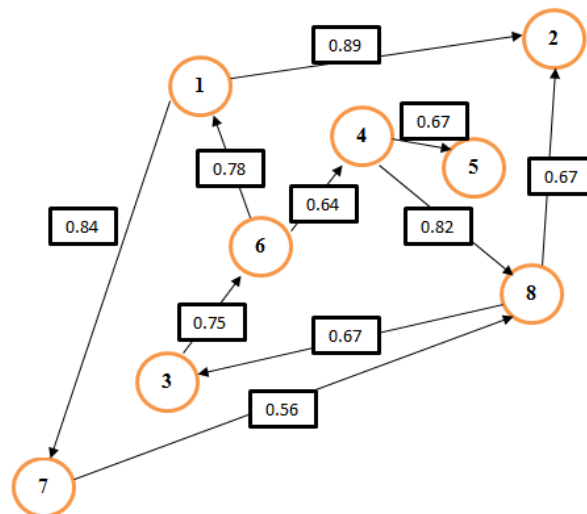


Fig. 3. Final Cognitive Map for Factors Affecting the Delay of Construction Projects

5. Discussion and Conclusions

The proposed Z-Cognitive maps method is based on automated and distance-based constructions, relying on linguistic variables and expert opinions. This method applies to problems with no quantitative data, and the construction of cognitive mapping should be done with the help of expert information. This method can act as a decision support method for the final decision maker. In addition to the advantages mentioned above, there are limitations such as the lack of a designed software package, the high volume of computation in the presence of a large number of experts, and the number of variables. The proposed method can be used in cases where decision-makers express their opinion with ambiguity and uncertainty. The role of the final decision maker in this method is vital because it can determine the coefficients of experts on the one hand and, on the other hand, when drawing the final map, it can be the final decision maker. In other words, the group of experts in this model acts as the decision maker support for final decision-maker. The proposed model helps the final decision maker to better understand the problem by using the expert group and by identifying the variables and cause-effect relations between them and then, by drawing the graphics model, it leads to a comprehensive understanding of the subject under study.

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