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Application of iterative Jacobi method for an anisotropic diffusion in image processing

M. Khanian [a,*] A. Davari [b]

[a] *Department of Mathematics, Khorasgan (Isfahan) Branch, Islamic Azad University, Isfahan, Iran.*

[b] *Department of Mathematics, Faculty of sciences, University of Isfahan, Isfahan, Iran.*

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Abstract

Image restoration has been an active research area. Different formulations are effective in high quality recovery. Partial Differential Equations (PDEs) have become an important tool in image processing and analysis. One of the earliest models based on PDEs is Perona-Malik model that is a kind of anisotropic diffusion (ANDI) filter. Anisotropic diffusion filter has become a valuable tool in different fields of image processing specially denoising. This filter can remove noises without degrading sharp details such as lines and edges. It is running by an iterative numerical method. Therefore, a fundamental feature of anisotropic diffusion procedure is the necessity to decide when to stop the iterations. This paper proposes the modified stopping criterion that from the viewpoints of complexity and speed is examined. Experiments show that it has acceptable speed without suffering from the problem of computational complexity.

Key words: Image restoration, Anisotropic diffusion, iterative numerical method.

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* Corresponding Author's E-mail:Mar.Khanian@gmail.com(M.Khanian), Tel:+983117934635

1 Introduction

Existence of noise in the image is a serious problem that it can be created in transmission or formation of the image. Therefore, denoising is one of the most important outlines of image processing. During the last decade, nonlinear diffusion filter has become a powerful and useful tool in different fields of image processing such as image restoration, multi-scale image analysis and etc. Denoising is an important preliminary step in many machine vision tasks like object detection and recognition. A significant feature of denoising models is to preserve important image features such as edges, lines that can easily be detected by the human visual system. PDE based models are designed for preserving sharp discontinuities in image while removing noise. One of the earliest and well known examples of PDE based models is Perona-Malik model. This filter is introduced in 1990 by Perona and Malik [1]. The Perona-Malik equation is given by:

$$\frac{\partial U}{\partial t} = div(g(||\nabla U||^2)\nabla U) = (gU_x)_x + (gU_y)_y; \quad t \geq 0 \qquad (1.1)$$

$$0 \leq x, y \leq N, 0 \leq U(x, y) \leq L$$

where L is the maximal gray level, $|\nabla U|$ is the gradient magnitude of the image and $g(|\nabla U|^2)$ is the diffusion coefficient. Here diffusion coefficient is a nonlinear function and it is based on the following equation: $g(s^2) = 1/(1 + s^2/\lambda^2)$; , $s = |\nabla U|$, $\lambda > 0$

where λ separates low contrast diffusion areas from high contrast diffusion areas. An iterative numerical method is applied for exploiting Perona-Malik filter. If we want to restore noisy images using iterative method which starts from the input image and produces a set of possible filtered images, the crucial question is: when to stop filtering in order to obtain the optimal restoration result? On the other hand, we need the stopping criterion. In this paper, one of the stopping criteria is improved in order to easy and fast running. This paper is organized as follows: Section 2 is a review of previous working on stopping criterion. Section 3 explains modified algorithm for stopping criterion. Section 4 is devoted to experimental results. Section 5 presents the discussion on proposed stopping criterion. The conclusion is given in Section 6.

2 Survey:

Finding an optimal stopping criterion has been studied in many previous works. Capuzzo, Dolcetta and Ferretti [2] found the optimal time by determining a minimum of a performance index. Their method needs a constant that is found by experimentation using a usual image with similar details and discontinuities. This is a rather vague requirement and they need some approximation to the constant. Other proposed methods for stopping criterion needed some approximation to statistics properties like variance and covariance [3,5]. One of the works in this area is proposed by Ilyevsky-Turkel [6]. In their study, Perona-Malik equation is solved by explicit and implicit (with multigrid) methods. These methods advanced the filter in time. They proposed the stopping criterion as well. Now, we propose a modification on their method for easier and faster running.

3 Modified algorithm:

We consider equation (1-1). We know that noise in the frequency domain is usually represented by high frequencies. The converse is not true in general, because all high frequencies are not noise. High frequencies are defined as frequencies from $\pm\frac{N}{2}$ to $\pm\frac{N}{4}$. Consider L_2^h the Euclidian norm of the image in the frequency domain when only high frequencies are considered and $(L_2^h)^n$ is the L_2^h norm of the restored image in step n. So we will have the following algorithm:

(1) Execute a few iterations of implicit method by Jacobi method until gaining a blurred image with no noise. Compute $(L_2^h)^b$ (high frequencies) norm of the obtained image.
(2) Solve Perona-Malik equation by explicit method and compute $(L_2^h)^n$ in each step n.
(3) Stop when $r = \frac{(L_2^h)^n}{(L_2^h)^b} \simeq 1.3$.

Proposed method is examined on different images with different variances of Gaussian noise(σ^2). All results are shown in Section 4. n is the number of iterations for restoring corrupted image and t is the time measuring in seconds. Peak signal to noise ratio (PSNR) and mean structural similarity (MSSIM) [7] are also computed. The $PSNR$ is calculated as:

$$PSNR = 20\log(\frac{MAX}{\sqrt{MSE}}),$$

$$with, \quad MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}|O_{ij} - R_{ij}|,$$

where O is an original non-corrupted image, R is a restored image and MAX is the maximum possible pixel value of the image. All pictures are in size of (512×512). But in regard to this point that the total time of proposed algorithm is suitable; therefore, it can be used for images with higher dimensions without considerable extra time.

4 Experimental Results:

This new stopping criterion is examined on some images with different levels of the Gaussian noise. Results of this research are presented.

a b

a: Corrupted Image, $\sigma^2 = 0.02$
b: Restored Image, n=18, t=7.91 PSNR=27.2058, MSSIM=0.72808781, $r \simeq 1.3$

c d

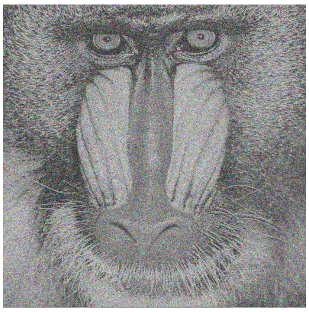
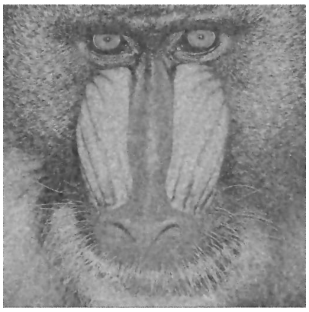
c: Corrupted Image, $\sigma^2 = 0.04$
d: Restored Image, n=30, t=12.87 PSNR=27.2008, MSSIM=0.68575411, $r \simeq 1.3$

e f

e: Corrupted Image, $\sigma^2 = 0.06$
f: Restored Image, n=38, t=16.07s, PSNR=27.1898, MSSIM=0.64536200, $r \simeq 1.3$

Discussion:

One of the important features of a good algorithm is high speed and low complexity. Multigrid method suffers from a problem of computational complexity. Therefore, it is not a fast method. It can be a serious problem specially, for images with high dimensions. Here, we use easy methods like explicit and implicit (with Jacobi method). If we employ a good software programming for running those methods, their running speed

g h

g: Corrupted Image, $\sigma^2 = 0.02$
h: Restored Image, n=14, t=6.41s, PSNR=26.8192, MSSIM=0.59451628, $r \simeq 1.3$

i j

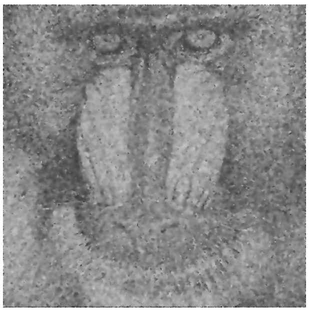
i: Corrupted Image, $\sigma^2 = 0.04$
j: Restored Image, n=20, t=8.5s, PSNR=26.8304, MSSIM=0.52137360, $r \simeq 1.3$

will become fast. Also, the proposed technique is computationally much simpler than multigrid method.

Conclusion

In this paper, we present an automatic method of tuning parameter choice (stopping criterion) for Perona-Malik model. With comparing to

k



l

k: Corrupted Image, $\sigma^2 = 0.1$
l: Restored Image, n=30, t=12.81s, PSNR=26.8591 , MSSIM=0.42642450, $r \simeq 1.3$

other available stopping criteria, it is not very complex and it does not need to extra information like statistics information about noise or image. Further, applied methods are computationally easier. Also, by using a good software programming it has a good running speed. Finally, from the point of denoising, the quality of obtained images is better than images which are obtained from multigrid method.

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