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Extend a ranking method of trapezoidal fuzzy numbers to all fuzzy numbers by weighting functions

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Abstract

Recently Abbasbandy and Hajjari (Computers and Mathematics with Applications 57 (2009) 413-419) have introduced a ranking method for the trapezoidal fuzzy numbers. This paper extends theirs method to all fuzzy numbers, which uses from a defuzzification of fuzzy numbers and a general weighting function. Extended method is interesting for ranking all fuzzy numbers, and it can be applied for solving and optimizing engineering and economics problems in a fuzzy environment.

Key words: Fuzzy numbers; Ranking; Weighting function.

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1 Introduction

Ranking fuzzy numbers plays an importance role in fuzzy decision making problems; therefore, deriving the final efficiency and powerful ranking are helpful to decision makers when solving fuzzy problems. Selecting a good ranking method can apply to choosing a desired criterion in a fuzzy environment. In recent years many ranking methods have been introduced by researchers; some of these ranking methods have been compared and reviewed by Bortolan and Degani [2]. Wang and Kerre [23,24] proposed some axioms as reasonable properties to determine the rationally of a fuzzy ranking method and systematically compared a wide array of existing fuzzy ranking methods. Almost all method, however have pitfalls in some aspect, such as inconsistency with human intuition, indiscrimination, and difficulty of interpretation. What seems to be clear is that there exists no uniquely best method for comparing fuzzy numbers and different methods may satisfy different criteria. Among the existing ranking methods of fuzzy numbers, a number of them are based on area measurements with the integral value of the membership function of fuzzy numbers. A commonly used ranking technique for fuzzy numbers is the centroid based ranking method. Some other methods use statistical techniques such as simulation and hypothesis and quadratic fuzzy regression.

In the following, we first introduce the developments of centroid- based fuzzy number ranking methods. Yager [28] proposed the centroid index ranking method with a weighting function. Chen and others have proposed a centroid index ranking method that calculates the distance between the centroid point of each fuzzy number and the original point to improve some of the ranking methods [4–6,26]. They also proposed a coefficient of variation (CV index) to improve Lee and Li's method [14]. Chu and Tsao [6] proposed a ranking method of fuzzy numbers by using the area between the centroid and original point. Chen and Chen [7] proposed a ranking index based on the centroid point and standard deviations to overcome some the drawbacks of previous centroid point indices. Lee [13] proposed a fuzzy number ranking method with user viewpoints. Yager and Filve [27] proposed a ranking method with parameterized valuation functions. Detyniecki and Yager [10] proposed a fuzzy number ranking

method with an α weighting function. Tran and Duckstein [21] proposed a weighting function that represents the decision maker's attitude. Lee and Li [14] introduced a ranking method for the normalized trapezoidal fuzzy numbers (NTFNs). Tang [22] showed Lee and Li's method is inconsistent. Liu and Han [25] proposed a method to rank fuzzy numbers with preference weighting function expectation. Cheng [5] has proposed the distance method for ranking fuzzy numbers. Goetschel and Voxman [12] introduced a method for ranking fuzzy numbers: their definition for ordering fuzzy numbers was motivated by the desire to give less importance to the lower levels of fuzzy numbers. Deng, Zhenfu and Qi [8] introduced the ranking of fuzzy numbers by an area method using the radius of gyration (ROG). Wang et al. [26] improved the correct centroid formula for ranking fuzzy numbers that justified them from the viewpoint of analytical geometry. Seidifar [20] has applied a weighting function and a weighted mean to ranking fuzzy numbers. Therefore, the essential subject of paper is extend a ranking method of trapezoidal fuzzy numbers to all fuzzy numbers and for any arbitrary weighting function.

The rest of this paper is organized as follows. In section 2, we recall some of the basic definitions and notions. In section 3, we introduce a extend method for ranking fuzzy numbers by the weighting mean and its properties are mentioned. The last section (Section 4)is devoted to discussion and conclusion.

2 Basic definitions and notions

Let $\mathbb{R}$ be the set of all real numbers. We assume a fuzzy number A that can be expressed for all $x \in \mathbb{R}$ in the form

$$A(x) = \begin{cases} A_L(x) \ , \ x \in [a,b], \\ 1 \qquad , \ x \in [b,c] \\ A_R(x) \ , \ x \in [c,d], \\ 0 \qquad , \ otherwise, \end{cases} \tag{I}$$

where a, b, c and d are real numbers such that $a < b \leq c < d$, A_L is real-valued function that is increasing and right continuous and A_R is a real-valued function that is decreasing and left continuous. Notice that (I) is an **L-R** fuzzy number with strictly monotone shape function, as proposed by Dubois and Prade in 1981. Each fuzzy number A described by (I) has the following γ-level sets$(\gamma - cuts)$
$[A]_\gamma = [A_L^{-1}(\alpha), A_R^{-1}(\gamma)] = [\underline{a}(\gamma), \overline{a}(\gamma)]$ for all $\gamma \in [0, 1]$. The family of fuzzy numbers will be denoted by $\mathcal{F}$.

Definition 2.1 *A fuzzy number $A = (a, b, c, d)$ is called a trapezoidal fuzzy number if its membership function $A(x)$ has the following form:*

$$A(x) = \begin{cases} \frac{x-a}{b-a} & , x \in [a, b], \\ 1 & , x \in [b, c] \\ \frac{d-x}{d-c} & , x \in [c, d], \\ 0 & , otherwise. \end{cases} \tag{II}$$

Definition 2.2 *[1] Let $A = (a, b, c, d)$ be a trapezoidal fuzzy number with $\gamma - cut[A]_\gamma[\underline{a}(\gamma), \overline{a}(\gamma)] = [a + (b - a)\gamma, d - (d - c)[A]_\gamma$. The magnitude of the trapezoidal fuzzy number A is defined as*

$$Mag(A) = \frac{1}{2}\int_0^1 (\underline{a}(\gamma) + \overline{a}(\gamma) + x_0 + y_0) f(\gamma) d\gamma, \tag{2.1}$$

where $x_0 = b, y_0 = c$ and the function $f(\gamma)$ is a non-negative and increasing function on $[0, 1]$ whit $f(0) = 0, f(1) = 14$ and $\int_0^1 f(\gamma) = \frac{1}{2}$. Function $f(\gamma)$ is considered as a weighting function.

Abbasbandy and Hajjari[1] have applied $Mag(A)$ for ranking of trapezoidal fuzzy numbers and a especial function $f(\gamma) = \gamma$. Therefore for the two trapezoidal fuzzy numbers A and B they defined

$$Mag(A) > Mag(B) \Leftrightarrow A \succ B$$

3 Extend ranking method of trapezoidal fuzzy numbers

In this section, we will extend the above method to all fuzzy numbers and for any weighting function.

Suppose $f = (\underline{f}, \overline{f}) : ([0,1],[0,1]) \to (\mathbb{R},\mathbb{R})$ is a weighting function such that the functions $\underline{f}, \overline{f}$ are non-negative, monotone increasing, and satisfy the following normalization condition

$$\int_0^1 \underline{f}(\gamma)d\gamma = 1, \int_0^1 \overline{f}(\gamma)d\gamma = 1.$$

Note that if $g = (\underline{g}, \overline{g}) : ([0,1],[0,1]) \to (\mathbb{R},\mathbb{R})$ is a function non-negative, and monotone increasing, then we can consider

$$\underline{f}(\gamma) = \frac{\underline{g}(\gamma)}{\int_0^1 \underline{g}(\gamma)d\gamma}, \overline{f}(\gamma) = \frac{\overline{g}(\gamma)}{\int_0^1 \overline{g}(\gamma)d\gamma}.$$

Let $f = (\underline{f}, \overline{f})$ is a weighting function. Then for any arbitrary fuzzy number $A \in \mathcal{F}$ with set $\alpha - cut\ [A]_\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$, we define the magnitude of the fuzzy number A as

$$Mag^f(A) = \frac{\int_0^1 [(\underline{a}(\gamma) + x_{01})\underline{f}(\gamma) + (\overline{a}(\gamma) + x_{02})\overline{f}(\gamma)]d\gamma}{2(\int_0^1 \underline{f}(\gamma)d\gamma + \int_0^1 \overline{f}(\gamma)d\gamma)}, \tag{3.1}$$

where

$$x_{01} = min\{x|A(x) \geq max(A(x))\}, \quad x_{02} = max\{x|A(x) \geq max(A(x))\}.$$

The function $f(\gamma) = (\underline{f}(\gamma), \overline{f}(\gamma))$ is considered as a weighting function. In actual applications, function $f(\gamma)$ can be chosen according to the actual situation. In this paper we can apply the different weighting functions for ranking fuzzy numbers. Obviously, the $Mag^f(A)$ of a fuzzy number A, synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. We state the properties of $Mag^f(A)$ by the following theorems.

Theorem 3.1 *Let $A \in \mathcal{F}$ be a fuzzy number with $[A]_\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$ and $f(\gamma) = (\underline{f}(\gamma), \overline{f}(\gamma))$ be a weighted function, and λ_1, λ_2 be real numbers. Then*

$$Mag^f(\lambda_1 A + \lambda_2) = \lambda_1 Mag^f(A) + \lambda_2. \tag{3.2}$$

Proof. Suppose A is a fuzzy number with $[A]_\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$. Then for the real numbers $\lambda_1 > 0$ and λ_2, we have
$[\lambda_1 A + \lambda_2]_\gamma = [\lambda_1 \underline{a}(\gamma) + \lambda_2, \lambda_1 \overline{a}(\gamma) + \lambda_2]$, and also for any $\gamma \in [0,1]$, we get

$$\begin{aligned}(\lambda_1 \underline{a}(\gamma) + \lambda_2) + (\lambda_1 x_{01} + \lambda_2) &= \lambda_1(\underline{a}(\gamma) + x_{01}) + 2\lambda_2,\\ (\lambda_1 \overline{a} + \lambda_2) + (\lambda_1 x_{02} + \lambda_2) \quad &= \lambda_1(\overline{a}(\gamma) + x_{02}) + 2\lambda_2,\end{aligned}$$

so

$$\begin{aligned}Mag^f(\lambda_1 A + \lambda_2) = & \\ & \frac{\int_0^1 [(\lambda_1(\underline{a}(\gamma)+x_{01})+2\lambda_2)\underline{f}(\gamma)+(\lambda_1(\overline{a}(\gamma)+x_{02})+2\lambda_2)\overline{f}(\gamma)]d\gamma}{2(\int_0^1 \underline{f}(\gamma)d\gamma+\int_0^1 \overline{f}(\gamma)d\gamma)}\\ &= \frac{\lambda_1 \int_0^1 [(\underline{a}(\gamma)+x_{01})\underline{f}(\gamma)+(\overline{a}(\gamma)+x_{02})\overline{f}(\gamma)]d\gamma+2\lambda_2(\int_0^1 \underline{f}(\gamma)d\gamma+\int_0^1 \overline{f}(\gamma)d\gamma)}{2(\int_0^1 \underline{f}(\gamma)d\gamma+\int_0^1 \overline{f}(\gamma)d\gamma)}\\ &= \lambda_1 Mag^f(A) + \lambda_2.\end{aligned}$$

Similarly, the theorem for $\lambda_1 < 0$ is hold. □

Theorem 3.2 *Let $A,\ B \in \mathcal{F}$ be two fuzzy numbers with $[A]_\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$, and $[B]_\gamma = [\underline{b}(\gamma), \overline{b}(\gamma)]$. And $f(\gamma) = (\underline{f}(\gamma), \overline{f}(\gamma))$ be a weighted function. Then*

$$Mag^f(A + B) = Mag^f(A) + Mag^f(B). \tag{3.3}$$

Proof. Suppose that

$$\begin{aligned}x_{01}^A &= \min\{x | A(x) \geq max(A(x))\},\\ x_{02}^A &= \max\{x | A(x) \geq max(A(x))\},\\ x_{01}^B &= \min\{x | B(x) \geq max(B(x))\},\\ x_{02}^B &= \max\{x | B(x) \geq max(B(x))\}.\end{aligned}$$

Then for any $\gamma \in [0,1]$, we have $[A + B]_\gamma = [\underline{a}(\gamma) + \underline{b}(\gamma), \overline{a}(\gamma) + \overline{b}(\gamma)]$,

and also

$$[\underline{a}(\gamma)+\underline{b}(\gamma),\overline{a}(\gamma)+\overline{b}(\gamma)]+[x_{01}^{A}+x_{01}^{B},x_{02}^{A}+x_{02}^{B}]$$
$$=[\underline{a}(\gamma)+x_{01}^{A}+\underline{b}(\gamma)+x_{01}^{B},\overline{a}(\gamma)+x_{02}^{A}+\overline{b}(\gamma)+x_{02}^{B}],$$
$$=[\underline{a}(\gamma)+x_{01}^{A},\overline{a}(\gamma)+x_{02}^{A}]+[\underline{b}(\gamma)+x_{01}^{B},\overline{b}(\gamma)+x_{02}^{B}],$$

thus with replacement in Eq.2, implies that

$$Mag^{f}(A+B)=Mag^{f}(A)+Mag^{f}(B).$$

□

Theorem 3.3 *Let $A\in\mathcal{F}$ is a symmetrical fuzzy numbers with $[A]_{\gamma}=[\underline{a}(\gamma),\overline{a}(\gamma)]$, and $\underline{f}(\gamma)=\overline{f}(\gamma)=f(\gamma)$ be a weighted function. Then we have $Mag^{f}(A)=k$, where $k=\frac{x_{01}+x_{02}}{2}$.*

Proof. Since fuzzy number A with $[A]_{\gamma}=[\underline{a}(\gamma),\overline{a}(\gamma)]$ is a fuzzy symmetrical fuzzy number, then for any $\gamma\in[0,1]$ we have $\frac{\underline{a}(\gamma)+\overline{a}(\gamma)}{2}=k$,and so

$$Mag^{f}(A)=\frac{\int_{0}^{1}[(\underline{a}(\gamma)+x_{01})f(\gamma)+(\overline{a}(\gamma)+x_{02})f(\gamma)]d\gamma}{2(\int_{0}^{1}f(\gamma)d\gamma+\int_{0}^{1}f(\gamma)d\gamma)}$$
$$=\frac{\int_{0}^{1}[(\underline{a}(\gamma)+\overline{a}(\gamma))f(\gamma)+(x_{01}+x_{02})f(\gamma)]d\gamma}{4}=\frac{2k+2k}{4}=k.$$

Definition 3.1 *[19] Let $A\in\mathcal{F}$ be a fuzzy number with $[A]_{\gamma}=[\underline{a}(\gamma),\overline{a}(\gamma)]$ and $f(\gamma)=(\underline{f}(\gamma),\overline{f}(\gamma))$ be a weighting function. Then the $f-$weighted mean of A is defined as*

$$\bar{M}_{f}(A)=\int_{0}^{1}\frac{\underline{f}(\gamma)\underline{a}(\gamma)+\overline{f}(\gamma)\overline{a}(\gamma)}{2}d\gamma. \tag{3.4}$$

Saeidifar [19] has shown that $\bar{M}_{f}(A)$ is a weighting mean of fuzzy number A, and it is also the nearest weighted point approximation to fuzzy number A which is unique. □

Theorem 3.4 *Let $A\in\mathcal{F}$ is a symmetrical fuzzy numbers with $[A]_{\gamma}=[\underline{a}(\gamma),\overline{a}(\gamma)]$, and $\underline{f}(\gamma)=\overline{f}(\gamma)=f(\gamma)$ be a weighted function. Then we have $Mag^{f}(A)=\bar{M}_{f}(A)=k$.*

Proof. The proof is similar to the proof of theorem 3. □

Proposition 3.1 *Let $A = (a, b, c, d)$ be a trapezoidal fuzzy number and let $f(\gamma) = (\underline{f}(\gamma), \overline{f}(\gamma))$ be a weighting function, then the following hold.*

(1) For $f(\gamma) = (1, 1)$, we have

$$Mag^f(A) = \frac{a + 3b + 3c + d}{8}. \tag{3.5}$$

(2) For $f(\gamma) = (2\gamma, 2\gamma)$,

$$Mag^f(A) = \frac{a + 5(b + c) + d}{12}, \tag{3.6}$$

(3) For $f(\gamma) = (n\gamma^{n-1}, n\gamma^{n-1}), n \in \mathcal{N}$ (natural numbers),

$$Mag^f(A) = \frac{a + (b + c)(2n + 1) + d}{4(n + 2)}, \tag{3.7}$$

(4) For $f(\gamma) = (m\gamma^{m-1}, n\gamma^{n-1}), m, n \in \mathcal{N}$,

$$Mag^f(A) = \frac{a + (2m + 1)b}{4(m + 1)} + \frac{d + (2n + 1)c}{4(n + 1)}. \tag{3.8}$$

Proof. The proof is simple.

Proposition 3.2 *Let $A = (a, b, c, d)$ be a trapezoidal fuzzy number and let $f(\gamma) = (m\gamma^{m-1}, n\gamma^{n-1}), m, n \in \mathcal{N}$ be a weighting function. Then, for $m, n \to \infty$*

$$Mag^f(A) = \frac{b + c}{2}$$

.

We now apply the $Mag^f(A)$ for ranking fuzzy numbers as the following definition.

Definition 3.2 *For two fuzzy numbers A , $B \in \mathcal{F}$, and the weighting function f, we define the ranking of A and B by $Mag^f(A)$, i.e.*

- $Mag^f(A) < Mag^f(B)$ *if and only if* $A \prec B$
- $Mag^f(A) = Mag^f(B)$ *if and only if* $A \sim B$
- $Mag^f(A) > Mag^f(B)$ *if and only if* $A \succ B$.

Then we formulate the order $\preceq$ and $\succeq$ as

$A \preceq B$ if and only if $A \prec B$ or $A \sim B$

$A \succeq B$ if and only if $A \succ B$ or $A \sim B$.

We consider the following reasonable properties for the ordering approaches (see [24]).

A_1 : For an arbitrary finite subset Γ of $\mathcal{F}$ and $A \in \Gamma$, $A \succeq A$.
A_2 : For an arbitrary finite subset Γ of $\mathcal{F}$ and $(A, B) \in \Gamma^2$, $A \succeq B$ and $B \succeq A$, we should have $A \sim B$.
A_3 : For an arbitrary finite subset Γ of $\mathcal{F}$ and $(A, B, C) \in \Gamma^3$, $A \succeq B$ and $B \succeq C$, we should have $A \succeq C$.
A_4 : For an arbitrary finite subset Γ of $\mathcal{F}$ and $(A, B) \in \Gamma^2$, $infsup(A) \geq infsup(B)$ we should have $A \succeq B$.
$A_4^{'}$: For an arbitrary finite subset Γ of $\mathcal{F}$ and $(A, B) \in \Gamma^2$, $infsup(A) > infsup(B)$ we should have $A \succ B$.
$A_5^{'}$: Let Γ and $\Gamma^{'}$ be two arbitrary finite subset of $\mathcal{F}$; also, A and B are in $\Gamma \cap \Gamma^{'}$. We obtain the ranking order $A \succ B$ by $Mag^f(.)$ on $\Gamma^{'}$ if and only if $A \succ B$ by $Mag^f(.)$ on Γ.
A_6 : Let A, B, $A+C$ and $B+C$ be elements of $\mathcal{F}$. If $A \succeq B$, then $A+C \succeq B+C$.
$A_6^{'}$: Let A, B, $A+C$ and $B+C$ be elements of $\mathcal{F}$. If $A > B$, then $A+C \succ B+C$.
A_7 : For an arbitrary finite subset Γ of $\mathcal{F}$ and $A \in \Gamma$, $Mag^f(A)$ must belong to its support.

Theorem 3.5 *The function* $Mag^f(.)$ *has the properties* $A_1, A_2, ..., A_7$.

Proof. It is easy to verify that the properties $A_1 - A_6$ hold. For the proof of A_7 we consider the fuzzy number $[A]_\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$, and the weighting function $f(\gamma) = (\underline{f}(\gamma), \overline{f}(\gamma))$. For all $\gamma \in [0, 1]$, we have $a \leq$

$\underline{a}(\gamma) \leq \overline{a}(\gamma) \leq d$; hence

$$\begin{aligned}\frac{\underline{f}(\gamma)(a+a)+\overline{f}(\gamma)(a+a)}{4} &\leq \frac{\underline{f}(\gamma)(\underline{a}(\gamma)+x_{01})+\overline{f}(\gamma)(\overline{a}(\gamma)+x_{02})}{4} \\ &\leq \frac{\underline{f}(\gamma)(d+d)+\overline{f}(\gamma)(d+d)}{4},\end{aligned}$$

so

$$\begin{aligned}\int_0^1 \frac{\underline{f}(\gamma)2a+\overline{f}(\gamma)2a}{4}d\gamma &\leq \int_0^1 \frac{\underline{f}(\gamma)(\underline{a}(\gamma)+x_{01})+\overline{f}(\gamma)(\overline{a}(\gamma)+x_{02})}{4}d\gamma \\ &\leq \int_0^1 \frac{\underline{f}(\gamma)2d+\overline{f}(\gamma)2d}{4}d\gamma,\end{aligned}$$

or

$$\begin{aligned}\frac{2a}{4}\int_0^1(\underline{f}(\gamma)+\overline{f}(\gamma))d\gamma &\leq \int_0^1 \frac{\underline{f}(\gamma)(\underline{a}(\gamma)+x_{01})+\overline{f}(\gamma)(\overline{a}(\gamma)+x_{02})}{4}d\gamma \\ &\leq \frac{2d}{4}\int_0^1(\underline{f}(\gamma)+\overline{f}(\gamma))d\gamma,\end{aligned}$$

and this implies that

$$a \leq Mag^f(A) \leq d.$$

□

Example 3.1 *Let A, B, C and D be four fuzzy numbers with the following membership functions (Fig.1) and $f(\gamma) = (f_1(\gamma), f_2(\gamma))$ be a weighting function.*

$$A(x) = \begin{cases} \frac{7-x}{4} & 3 \leq x \leq 7, \\ 0 & otherwise, \end{cases} \quad B(x) = \begin{cases} 1-\frac{(x-5)^2}{4} & 3 \leq x \leq 7, \\ 0 & otherwise, \end{cases}$$

$$C(x) = \begin{cases} \frac{x-3}{2} & 3 \leq x \leq 5, \\ \frac{7-x}{2} & 5 \leq x \leq 7, \\ 0 & otherwise, \end{cases} \quad D(x) = \begin{cases} \frac{x-3}{4} & 3 \leq x \leq 7, \\ 0 & otherwise. \end{cases}$$

Then we have
$[A]_\gamma = [\underline{b}(\gamma), \overline{b}(\gamma)] = [3, 7-4\gamma]$,
$[B]_\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)] = [5-2\sqrt{1-\gamma}, 5+2\sqrt{1-\gamma}]$,
$[C]_\gamma = [\underline{c}(\gamma), \overline{c}(\gamma)] = [3+2\gamma, 7-2\gamma]$,
$[D]_\gamma = [\underline{c}(\gamma), \overline{c}(\gamma)] = [3+4\gamma, 7],\ \gamma \in (0,1]$.

For the different weighting functions the results are given in the Table 1.

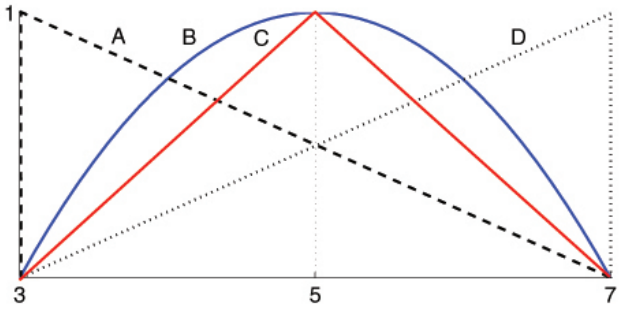

Fig. 1. Fuzzy numbers A, B, C and D from Example 1.

Table 1 : Comparative results of Example 1.

Fuzzy number	A	B	C	D
Abbasbandy and Hajjari method	3.333	not defined	5	6.5
result	$A \prec C \prec D$			
Extended method,$f(\gamma) = (1, 1)$	3.5	5	5	6.5
result	$A \prec B \sim C \prec D$			
Extended method,$f(\gamma) = (2\gamma, 2\gamma)$	3.333	5	5	6.667
Result	$A \prec B \sim C \prec D$			
Extended method,$f(\gamma) = (3\gamma^2, 2\gamma)$	3.333	5.038	5.042	6.75
Result	$A \prec B \prec C \prec D$			
Extended method,$f(\gamma) = (2\gamma, 3\gamma^2)$	3.25	4.962	3.958	6.667
Result	$A \prec C \prec B \prec D$			

This example shows that extended method is flexible and can solve his shortcoming by select a suitable weighting function, even it can rank symmetrical fuzzy numbers such that previously method (methods) cannot rank these fuzzy numbers, and this is an interesting property for extended metod. Also, one can see that with change the left and right weighting functions $(\underline{f}, \overline{f})$ is may that change the ordering of fuzzy numbers, specially for symmetrical fuzzy numbers(see Table 1).

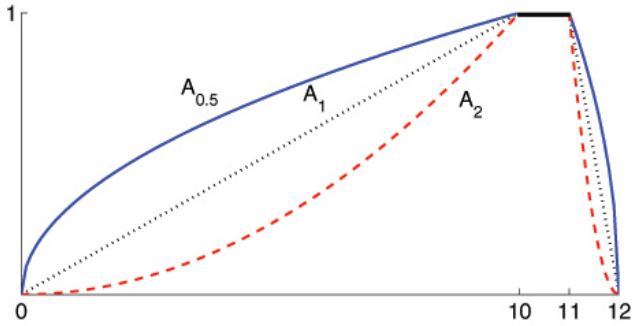

Fig. 2. Fuzzy numbers $A_{0.5}, A_1, A_2$ from Example 2 We obtain that

Example 3.2 *Consider fuzzy number A_n (Fig.2) as follows:*

$$A_n(x) = \begin{cases} (\frac{x}{10})^n & , \quad x \in [0,10], \\ 1 & , \quad x \in (10,11], \\ (12-x)^n & x \in (11,12], \\ 0 & , \quad otherwise. \end{cases}$$

$iff(\gamma) = (1,1)$ then $Mag^f(A_{0.5}) = 9, Mag^f(A_1) = 9.375, Mag^f(A_2) = 9.75$ and hence the ranking order is $A_{0.5} \prec A_1 \prec A_2$. If $f(\gamma) = (2\gamma, 2\gamma)$ then $Mag^f(A_{0.5}) = 9.375, Mag^f(A_1) = 9.75, Mag^f(A_2) = 10.05$, and hence $A_{0.5} \prec A_1 \prec A_2$. If $f(\gamma) = (3\gamma^2, 2\gamma)$ then $Mag^f(A_{0.5}) = 9.625$ $Mag^f(A_1) = 9.958$ and $Mag^f(A_2) = 10.193$. So $A_{0.5} \prec A_1 \prec A_2$.

The above examples show that this method is extended and improved of ranking method of trapezoidal fuzzy numbers [1].

Note that decision makers can select the other suitable functions for ranking fuzzy numbers. Therefore extended method of ranking fuzzy numbers is interesting and flexible.

Example 3.3 *Consider the following sets (Fig.3); see [20].*

- *Set 1:* $A = (0.3, 0.4, 0.7, 0.9), B = (0.3, 0.7, 0.9), C = (0.5, 0.7, 0.9)$,
- *Set 2:* $A = (0.3, 0.5, 0.7), B = (0.3, 0.5, 0.8, 0.9), C = (0.3, 0.5, 0.9)$,
- *Set 3:* $A = (0, 0.4, 0.7, 0.8), B = (0.2, 0.5, 0.9), C = (0.1, 0.6, 0.8)$.

To compare with other methods we refer the reader to Table 2.

Table 2 : Comparative results of Example 3.

Authors	*Fuzzy number*	*set 1*	*set 2*	*set 3*
Choobineh and Li	A	*0.458*	*0.333*	*0.50*
	B	*0.583*	*0.4167*	*0.5833*
	C	*0.667*	*0.5417*	*0.6111*
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Yager	A	*0.5778*	*0.5*	*0.4336*
	B	*0.6333*	*0.6222*	*0.5353*
	C	*0.8571*	*0.6986*	*84*
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chen	A	*0.4315*	*0.375*	*0.52*
	B	*0.5625*	*0.425*	*0.57*
	C	*0.625*	*0.55*	*0.625*
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Baldwin and Guild	A	*0.27*	*0.27*	*0.40*
	B	*0.27*	*0.37*	*0.42*
	C	*0.37*	*0.45*	*0.42*
Results		$A \sim B \prec C$	$A \prec B \prec C$	$A \prec B \sim C$
Chu and Tsao	A	*0.2847*	*0.25*	*0.24402*
	B	*0.32478*	*0.31526*	*0.26243*
	C	*0.350*	*0.27475*	*0.2619*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Cheng Distance	A	*0.7577*	*0.7071*	*0.7106*
	B	*0.8149*	*0.8037*	*0.7256*
	C	*0.8602*	*0.7458*	*0.7241*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$

Wang et al. Centroid	A	*0.2568*	*0.1778*	*0.1967*
	B	*0.2111*	*0.2765*	*0.1778*
	C	*0.2333*	*0.1889*	*0.1667*
Results		$B \prec C \prec A$	$A \prec C \prec B$	$C \prec B \prec A$
Wang Distance	A	*0.7289*	*0.6009*	*0.6284*
	B	*0.7157*	*0.7646*	*0.6289*
	C	*0.7753*	*0.6574*	*0.6009*
Results		$B \prec A \prec C$	$A \prec C \prec B$	$C \prec A \prec B$
Yao and Wu	A	*0.575*	*0.5*	*0.475*
	B	*0.65*	*0.625*	*0.525*
	C	*0.7*	*0.55*	*0.525*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Deng et al. area method	A	*0.3169*	*0.2369*	*0.2523*
	B	*0.3240*	*0.3503*	*0.2495*
	C	*0.3240*	*0.2549*	*0.2473*
Results		$A \prec B \sim C$	$A \prec C \prec B$	$C \prec B \prec A$
Cheng CV uniform distribution	A	*0.0328*	*0.0133*	*0.0693*
	B	*0.0246*	*0.0304*	*0.0385*
	C	*0.0095*	*0.0275*	*0.0433*
Results		$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \prec A$
Cheng CV proportional distribution	A	*0.026*	*0.008*	*0.0471*
	B	*0.0146*	*0.0234*	*0.0236*
	C	*0.0057*	*0.0173*	*0.0255*
Results		$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \sim A$

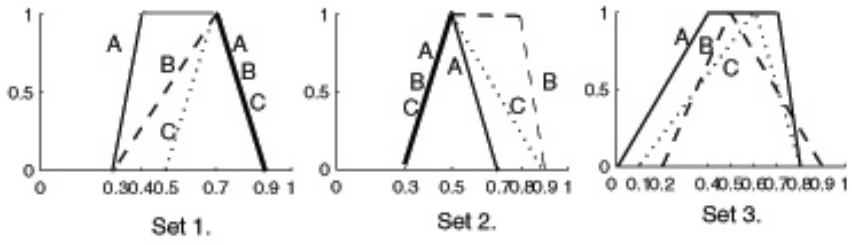

Fig. 3. Fuzzy numbers A,B,C in sets 1, 2, 3.

Authors	*Fuzzy number*	*set 1*	*set 2*	*set 3*
Goetschel and Voxman	A	*0.5667*	*0.5*	*0.5*
	B	*0.6667*	*0.6333*	*0.5167*
	C	*0.7*	*0.5333*	*0.55*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Abbasbandy and Asady sign distance	A	*1.15*	*1*	*0.95*
	B	*1.3*	*1.25*	*1.05*
	C	*1.4*	*1.1*	*1.05*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Asady and Zendehnam	A	*0.575*	*0.5*	*0.475*
	B	*0.65*	*0.625*	*0.525*
	C	*0.7*	*0.55*	*0.525*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Saeidifar	A	*0.5667*	*0.5*	*0.5*
	B	*0.6667*	*0.6333*	*0.5167*
$f(\gamma) = (2\gamma, 2\gamma)$	C	*0.7*	*0.5333*	*0.55*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$

Abbasbandy and Hajjari	A	*0.5558*	*0.5*	*0.5250*
	B	*0.6334*	*0.6416*	*0.5084*
	C	*0.7*	*0.5166*	*0.5750*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$
Extended method	A	*0.5604*	*0.5042*	*0.5333*
	B	*0.6917*	*0.6458*	*0.5146*
$f(\gamma) = (3\gamma^2, 2\gamma)$	C	*0.7042*	*0.5208*	*0.5854*
Results		$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$

We surmise the ranking results that is agreement with the extended method as the following Table based on the percentage.

Table 3.

Sets	*Set 1*	*Set 2*	*Set 3*
Extended method result	$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$
Agreement percentage	*67*	*78*	*11*

Note that in set 3, the results of ranking fuzzy numbers are very different.

Example 3.4 *Let* $A = (0.1, 0.3, 0.3, 0.5), B = (0.2, 0.3, 0.3, 0.4)$, *and* $C = (1, 1, 1, 1)$, *and* $f = (2\gamma, 3\gamma^2)$. *Then,*$Mag^f(A) = 0.2958, Mag^f(A) = 0.2979, Mag^f(A) = 1$. *Therefore the ranking order is* $A \prec B \prec C$.

Example 3.5 *Let* $A = (0.1, 0.3, 0.3, 0.5), B = (-0.5, -0.3, -0.3, -0.1)$, *and* $f = (2\gamma, 2\gamma)$. *Then,* $Mag^f(A) = 0.3, Mag^f(B) = -0.3$. *Therefore,* $A \succ B$
The above examples results are the same as the method result of Phani Bushan Rao et al. [18].

Example 3.6 *[18] Consider four fuzzy numbers* $A_1 = (0.1, 0.2, 0.3), A_2 = (0.2, 0.5, 0.8), A_3 = (0.3, 0.4, 0.9)$ *and* $A_4 = (0.6, 0.7, 0.8)$ *which were ranked earlier by Yager [28] , Fortemps and Roubens [11], Liou and Wang [16], and Chen and Lu [3] as shown in Table 4. It can be seen from Table 4 that none of the methods discriminates fuzzy numbers. Yager [28] and Fortemps and Roubens [11] methods failed to discriminate the fuzzy num-*

bers A2 and A3, whereas the methods of Liou and Wang [16] and Chen and Lu [3] cannot discriminate the fuzzy numbers A_2, A_3 and A_1, A_4.
By Extended method, and weighting functions $f_1 = (2\gamma, 2\gamma)$, and $f_2 = (2\gamma, 3\gamma^2)$, we get
$Mag^{f_1}(A) = 0.2$, $Mag^{f_1}(B) = 0.5$, $Mag^{f_1}(C) = 0.4333$, $Mag^{f_1}(D) = 0.7$
$Mag^{f_2}(A) = 0.1979$, $Mag^{f_2}(B) = 0.4937$, $Mag^{f_2}(C) = 0.4229$, $Mag^{f_2}(D) = 0.6979$.
Therefore the ranking order is $A_4 \succ A_2 \succ A_3 \succ A_1$. Observe that the our method result is the same as method of Phani Bushan Rao1 et al. [18].

Table 4: Comparison of various ranking methods.

Method		A_1	A_2	A_3	A_4	*Ranking order*
Ranking order Yager		*0.20*	*0.50*	*0.50*	*0.70*	$A_4 \succ A_2 \sim A_3 \succ A_1$
Fortemps and Roubens		*0.20*	*0.50*	*0.50*	*0.70*	$A_4 \succ A_2 \sim A_3 \succ A_1$
Liou andWang	$\alpha = 1$	*0.25*	*0.65*	*0.65*	*0.75*	$A_4 \succ A_2 \sim A_3 \succ A_1$
	$\alpha = 0.5$	*0.20*	*0.50*	*0.50*	*0.70*	$A_4 \succ A_2 \sim A_3 \succ A_1$
	$\alpha = 0$	*0.15*	*0.35*	*0.35*	*0.65*	$A_4 \succ A_2 \sim A_3 \succ A_1$
Chen and Lu [37]	$\beta = 1$	*-0.20*	*0.00*	*0.00*	*-0.20*	$A_2 \sim A_3 \succ A_1 \sim A_4$
	$\beta = 0.5$	*-0.20*	*0.00*	*0.00*	*-0.20*	$A_2 \sim A_3 \succ A_1 \sim A_4$
	$\beta = 0$	*-0.20*	*0.00*	*0.00*	*-0.20*	$A_2 \sim A_3 \succ A_1 \sim A_4$
Phani Bushan Rao1 et al.		*0.4591*	*0.6320*	*0.6146*	*0.8129*	$A_4 \succ A_2 \succ A_3 \succ A_1$
Extended method	$f_1 = (2\gamma, 2\gamma)$	*0.2*	*0.5*	*0.4333*	*0.7*	$A_4 \succ A_2 \succ A_3 \succ A_1$
Extended method	$f_2 = (2\gamma, 3\gamma^2)$	*0.1979*	*0.4937*	*0.4229*	*0.6979*	$A_4 \succ A_2 \succ A_3 \succ A_1$

4 Conclusion

In this paper, we develops a ranking method of trapezoidal fuzzy numbers to all fuzzy numbers which uses from a weighting function and a defuzzification for ranking fuzzy numbers. The properties of defuzzification ($Mag^f(.)$) are given by theorems and propositions. The flexibility is one of the most important properties of extended ranking method, because decision makers can select the different weighting functions as $f = (\underline{f}, \overline{f})$ such that functions $\underline{f}, \overline{f} : [0, 1] \to R$ are weighting functions for the lower and upper $\gamma-cuts$ sets of a fuzzy number, respectively. This

means that the functions $\underline{f}(\gamma)$ and $\overline{f}(\gamma)$ can be treated as the subjective weights indicating neutral, optimistic, or pessimistic preferences of the decision maker. Therefore, our method is more general and interesting for ranking fuzzy numbers. Also, the maximum entropy of weighting function $(f(\gamma))$ is discussed in [15]. It can be used to choose a suitable weighting function.

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