An Approximate Solution for the Scattering of High-Frequency Plane Electromagnetic Waves from a Perfectly Conducting Strip

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ABSTRACT:
An analytical method has been developed for the scattering of high-frequency plane electromagnetic waves from a perfectly conducting strip. The solution is much simpler compared to the other methods and gives quite accurate results for ka>>1. Using Green's Theorem, the scattering field has been expressed by an integral of the current induced on the strip. With the integral expression of Hankel function, a Fourier transform of the induced current and thus, an integral equation in spectral domain has been derived. Using some required transformation on the induced surface current, the obtained spectral equation has been reduced to its simplest form and then an approximate solution could be derived for the reduced spectral equation for ka>>1. Using this approximate solution the field related quantities such as radiation pattern and radar cross section can be obtained easily; but the induced current and current related quantities requires the numerical solution of the algebraic set of equations obtained by expressing the current in the form of an infinite series which satisfies the boundary conditions on the surface of the conducting strip.

KEYWORDS: Strip, Impedance, Scattering, Perfect Conductor.

1. INTRODUCTION

The solution of canonical problems such as half-plane, cylinder or sphere are important in the sense of diffraction theory and strip is one of the most important canonical structure. Due to its geometry, it's frequently used to investigate the multiple diffraction phenomenon. Furthermore, especially in remote sensing, a large number of practical problems can be simulated by conducting strip. On the other hand, diffraction by a slit in a infinite conducting plane can be reduced to a perfectly conducting strip problem by using the duality principle. Therefore, due to its conformity to many practical problems, strips have been extensively investigated by many authors by using different analytical and numerical methods [2-10].

As known, the electrical size of body limits the tractability of numerical methods while the geometrical complexity of the object restricts the applicability of the analytical methods. Therefore, hybrid methods are frequently used for the asymptotic solution of the problem at high frequencies.

Essentially, the hybrid methods involve the combined usage of Geometrical Theory of diffraction (GTD) and Moment Method (MM) (Field based

analysis) [11-13] or Physical Theory of Diffraction (PTD) and MM (Current based analysis) [14].

An alternative method was proposed by Veliev [15]. In this analytical-numerical method, the integral equation derived in spectral domain is reduced to a set of linear equations in terms of unknown Fourier coefficient of the induced current density function. After determining the unknown coefficient solving the set of linear equations, the surface current density and scattered field and then, current and field related quantities can be expressed. Scattering from an impedance strip was solved by using this method by us [1]. The reduction of integral equation in spectral domain into a set of linear equations is the essence of this method and includes analytical solutions of integrals. There are some mathematical difficulties in this stage. In this study, when only field expression and radar cross section are required at high frequencies, an approximate solution was obtained by overcoming these mathematical difficulties. Besides conducting strip, this approach can be used for resistive and impedance strips as well as for the systems of formed by any kind of strips.

2. FORMULATION OF THE PROBLEM

An infinitely long perfectly conducting strip of width 2a is placed in coordinate system as show in Figure 1. Since the strip is uniform along the z-axis, the problem can be reduced to a two dimensional problem. The time dependence of the fields are assumed exp(-jωt) and suppressed throughout the analysis.
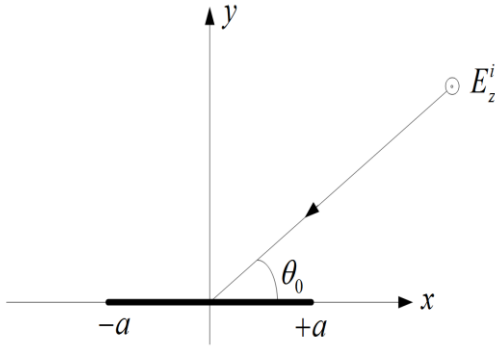

Fig. 1. The geometry of the problem.

The incident field is given as a linearly polarized plane wave as

$$E_z^i(x, y) = e^{-jk(x\alpha_0 + y\sqrt{1-\alpha_0^2})}. \tag{1}$$

Here, $\alpha_0 = \cos\theta_0$ and k is the wave number. Total field will be expressed as the sum of incident and scattered fields:

$$E_z(x, y) = E_z^i(x, y) + E_z^s(x, y). \tag{2}$$

By using Green's theorem, the scattered field can be expressed in terms of the currents induced on the surface of the strip as

$$E_z^s(x, y) = \frac{1}{4}\int_{-a}^{+a}\left\{kZ_0I_e(x') + jI_m(x')\frac{\partial}{\partial y}\right\}$$

$$H_0^{(1)}\left(k\sqrt{(x-x')^2 + y^2}\right)dx'. \tag{3}$$

Here $I_e(x')$ and $I_m(x')$ are the electric and magnetic currents respectively induced on the strip and $H_0^{(1)}(.)$ is Hankel function. Using boundary conditions the currents are expressed as

$$I_e(x) = E_z(x, +0) - E_z(x, -0) \quad , \tag{4}$$

$$I_m(x) = H_x(x, +0) - H_x(x, -0) \tag{5}$$

and considering the boundary conditions for a perfectly conducting strip, it is easily calculated that the magnetic current induced on the strip is zero. Induction of both currents is possible for an impedance strip only. So Eq. (3) is reduced to

$$E_z(x, y) = e^{-jk(x\alpha_0 + y\sqrt{1-\alpha_0^2})} + \frac{1}{4}\int_{-a}^{+a}kZ_0I_e(x').$$

$$H_0^{(1)}\left(k\sqrt{(x-x')^2 + y^2}\right)dx'. \tag{6}$$

By using the following Maxwell's equation

$$H_x = \frac{1}{jw\mu}\frac{\partial E_z}{\partial y} \tag{7}$$

in Eq. (5), one can obtain that

$$I_e(x) = \frac{1}{jw\mu}\left\{\frac{\partial E_z(x, +0)}{\partial y} - \frac{\partial E_z(x, -0)}{\partial y}\right\} \tag{8}$$

and

$$kZ_0I_e(x) = -j\left\{\frac{\partial E_z(x, +0)}{\partial y} - \frac{\partial E_z(x, -0)}{\partial y}\right\}. \tag{9}$$

Defining a new function, $f_e(x)$ as the difference of the terms in parentheses

$$kZ_0I_e(x) = -jf_e(x) \tag{10}$$

is found. In terms of this new function Eq. (6) becomes

$$E_z(x, y) = e^{-jk(x\alpha_0 + y\sqrt{1-\alpha_0^2})} - \frac{j}{4}\int_{-a}^{+a}f_e(x').$$

$$H_0^{(1)}\left(k\sqrt{(x-x')^2 + y^2}\right)dx'. \tag{11}$$

On the surface, the total field must be zero: using y=0 at Eq. (11) the integral equation for the function $f_e(x)$ is derived as follows:

$$-4je^{-jkx\alpha_0} = \int_{-a}^{+a}f_e(x')H_0^{(1)}(|x-x'|)dx'. \tag{12}$$

If one use the integral expression of Hankel function given below

$$H_0^{(1)}\left(k\sqrt{(x-x')^2 + y^2}\right) =$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}e^{jk\{(x-x')\alpha + \sqrt{1-\alpha}|y|\}}\frac{d\alpha}{\sqrt{1-\alpha^2}} \tag{13}$$

for y=0 in Eq. (12)

$$-4je^{-jkx\alpha_0} = \int_{-a}^{+a}f(x')\left\{\frac{1}{\pi}\int_{-\infty}^{+\infty}\left\{e^{jk(x-x')\alpha}\frac{d\alpha}{\sqrt{1-\alpha^2}}\right\}dx'\right\} \tag{14}$$

is obtained. Changing the order of the integrals

$$-4je^{-jkx\alpha_0} = \frac{1}{\pi}\int_{-\infty}^{+\infty}\left\{\int_{-a}^{+a}f(x')e^{-jkx'\alpha}dx'\right\}\frac{e^{jkx\alpha}}{\sqrt{1-\alpha^2}}d\alpha \tag{15}$$

is found. Considering that

$$F(\alpha) = \int_{-a}^{+a} f(x')e^{-jkx'\alpha}dx', \tag{16}$$

Eq. (16) can be reduced to the following form:

$$-4j\pi e^{-jkx\alpha_0} = \int_{-\infty}^{+\infty} F(\alpha)\frac{e^{jkx\alpha}}{\sqrt{1-\alpha^2}}d\alpha. \tag{17}$$

This equation is the integral equation of $F(\alpha)$ in spectral domain. In order to normalize the boundaries of the integral in Eq. (16), let

$$x = a\eta \text{ and } \varepsilon = ka. \tag{18}$$

By using these transformations in Eq. (16)

$$F(\alpha) = \int_{-1}^{+1} af(a\eta')e^{-j\varepsilon\alpha\eta'}d\eta' \tag{19}$$

and assuming

$$\overline{f}(\eta') = af(a\eta') \tag{20}$$

the following expression is derived

$$F(\alpha) = \int_{-1}^{+1} \overline{f}(\eta')e^{-j\varepsilon\alpha\eta'}d\eta'. \tag{21}$$

If Eq. (21) is used in Eq. (17)

$$-4j\pi e^{-j\varepsilon\eta\alpha_0} = \int_{-\infty}^{+\infty} F(\alpha)e^{j\varepsilon\eta\alpha}\frac{d\alpha}{\sqrt{1-\alpha^2}} \tag{22}$$

is obtained. When both sides of this equation are multiplied by $e^{-j\varepsilon\beta\eta}$ and by taking the integrals in the range of -1 to +1

$$-4j\pi\int_{-1}^{+1} e^{-j\varepsilon\eta(\beta+\alpha_0)}d\eta = \int_{-\infty}^{+\infty} F(\alpha)\left\{\int_{-1}^{+1} e^{j\varepsilon\eta(\alpha-\beta)}d\eta'\right\}d\alpha \tag{23}$$

is found. The expression on the left side of this equation will be

$$\int_{-1}^{+1} e^{-j\varepsilon\eta(\beta+\alpha_0)}d\eta = \frac{2\sin\varepsilon(\alpha_0+\beta)}{\varepsilon(\alpha_0+\beta)}, \tag{24}$$

and the integral in the parentheses on the right side will be

$$\int_{-1}^{+1} e^{-j\varepsilon\eta(\alpha-\beta)}d\eta = \frac{2\sin\varepsilon(\alpha-\beta)}{\varepsilon(\alpha-\beta)}. \tag{25}$$

By using equations (24) and (25) in Eq. (23)

$$-4j\pi\frac{\sin\varepsilon(\alpha_0+\beta)}{\alpha_0+\beta} = \int_{-\infty}^{+\infty}\frac{F(\alpha)}{\sqrt{1-\alpha^2}}\frac{\sin\varepsilon(\alpha-\beta)}{\alpha-\beta}d\alpha \tag{26}$$

is obtained. Although the solution of this integral equation is possible analytically, it is quite difficult at the same time. However for $\varepsilon = ka \gg 1$ we have that

$$\pi\delta(\alpha-\beta) = \frac{\sin\varepsilon(\alpha-\beta)}{\alpha-\beta} \tag{27}$$

and

$$\int_{-\infty}^{+\infty} \delta(x-a)f(x)dx = f(a). \tag{28}$$

So using Eq. (27) in Eq. (26)

$$-4j\pi\frac{\sin\varepsilon(\alpha_0+\beta)}{\alpha_0+\beta} = \pi\int_{-\infty}^{+\infty}\frac{F(\alpha)}{\sqrt{1-\alpha^2}}\delta(\alpha-\beta)d\alpha \tag{29}$$

And again using Eq. (28)

$$F(\beta) = -4j\sqrt{1-\beta^2}\frac{\sin\varepsilon(\alpha_0+\beta)}{\alpha_0+\beta} \tag{30}$$

is derived. Here θ is the observation angle, $\alpha_0 = \cos\theta_0$ and $\beta = \cos\theta$: so, substituting to Eq. (30) the following expression is derived:

$$F(\cos\theta) = -4j\sin\theta\frac{\sin\varepsilon(\cos\theta_0+\cos\theta)}{\cos\theta_0+\cos\theta}. \tag{31}$$

2.1. Asymptotic Expression of the Scattered Field

One can get the expression of the scattered field from Eq. (11) as follows

$$E_z^s(x,y) = -\frac{j}{4}\int_{-a}^{+a} f_e(x')H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx'. \tag{32}$$

Since the asymptotic expression of Hankel function is given as

$$H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right) = \sqrt{\frac{2}{j\pi kr}}e^{jkr\left(1-\frac{x'}{r}\cos\theta\right)}. \tag{33}$$

So using Eq. (33) in Eq. (32)

$$E_z^s(x,y) = -\frac{j}{4}\sqrt{\frac{2}{j\pi kr}}e^{jkr}\int_{-a}^{+a} f_e(x')e^{jkx'\cos\theta}dx' \tag{34}$$

or with Eq. (20)

$$E_z^s(x,y) = -\frac{j}{4}\sqrt{\frac{2}{j\pi kr}}e^{jkr}\int_{-1}^{+1}\overline{f}(\eta')e^{-j\varepsilon\eta'\cos\theta}d\eta' \tag{35}$$

is found. With Eq. (21), the scattered field can be obtained as

$$E_z^s(x,y) = -\frac{j}{4}\sqrt{\frac{2}{j\pi kr}}e^{jkr}F(\cos\theta) \tag{36}$$

or

$$E_z^s(x,y) = \sqrt{\frac{2}{\pi kr}}e^{j\left(kr-\frac{\pi}{4}\right)}\sin\theta\frac{\sin\varepsilon(\cos\theta_0+\cos\theta)}{\cos\theta_0+\cos\theta}. \tag{37}$$

Expressing the scattered field in a cylindrical wave form as

$$E_z^s(x,y) = A(r)P(\theta,\theta_0) \tag{38}$$

where

$$P(\theta,\theta_0) = \sin\theta \frac{\sin\varepsilon(\cos\theta_0 + \cos\theta)}{\cos\theta_0 + \cos\theta} \qquad (39)$$

is the radiation pattern of the scattered field while

$$A(r) = \sqrt{\frac{2}{\pi k r}} e^{j\left(kr - \frac{\pi}{4}\right)} \qquad (40)$$

is the amplitude function.

3. NUMERICAL RESULTS

Bistatic radar cross section for a TM-wave is given as [16]

$$\sigma(\varphi,\varphi_i) = \frac{kZ_0^2}{4} \left| \int_0^L J(x) e^{jkx\cos\varphi} dx \right|^2 . \qquad (41)$$

When the expression of the transformed current is used in Eq. (41)

$$\frac{\sigma(\theta,\theta_0)}{\lambda} = \frac{2}{\pi} \left| \int_{-1}^{+1} \bar{f}(\eta') e^{-j\varepsilon\eta'\cos\theta} d\eta' \right|^2 \qquad (42)$$

is obtained. Bistatic radar cross sections were given for strips of widths of 4.5λ ($2a=4.5\lambda$) and 10λ ($2a=10\lambda$) in Figure 2 and Figure 3 respectively.

(a)

(b)
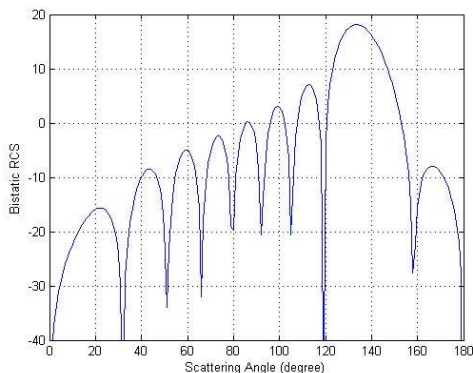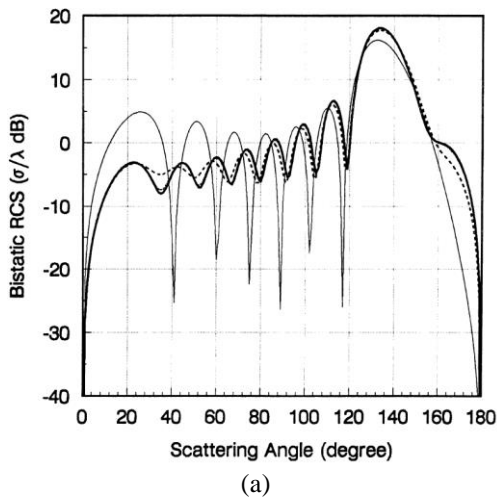
Fig. 2. Bistataic radar cross sections obtained by (a) Su [16] and (b) us respectively for 2a=4.5λ.
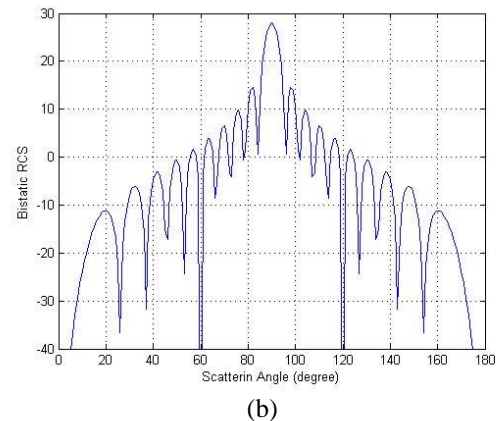
(a)

(b)

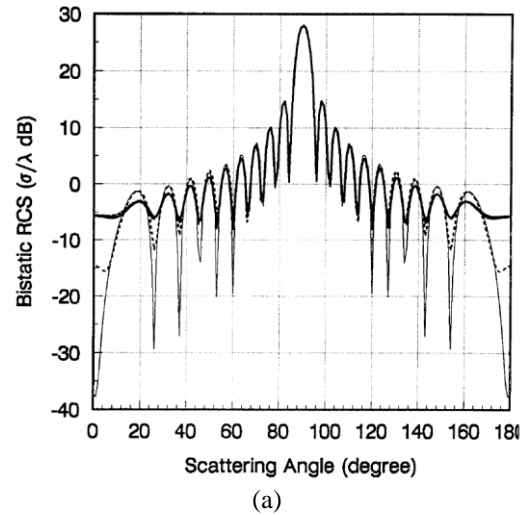**Fig. 3.** Bistataic radar cross sections obtained by (a) Su [16] and (b) us respectively for 2a=16λ.

For the strip of width of 4.5λ, there is difference about 10dB between our and Su's results. For the strip width of 4,5λ, this difference is reduced. In other words, the approach gives more accurate results with increasing frequency. However, scattered patterns overlap with the solutions of Su for each strip.

4. CONCLUSION

Scattering from a perfectly conducting strip was solved eliminating the mathematical complexity by an approximate method. Although this method was applied on perfectly conducting strip, it can be developed on impedance and resistive strips and strips which have different surface impedances.

While the frequency ($\varepsilon = ka$) increases, the accuracy of the method increases also. However, more importantly, form of the scattering pattern can be obtained accurately for any frequency. This means that

the form of the scattering pattern can be obtained easily for any strip at any frequency if it's the only required quantity. It is thought that this property of the approximation is valuable.

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