Formulation of the Diffraction Problem of Almost Grazing Incident Plane Wave by an Anisotropic Impedance Wedge

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ABSTRACT
In this paper, formulation of the problem of plane wave diffraction by a wedge with anisotropic impedances is given for the case of almost grazing incidence. All steps of problem are given in detailed. Wedge is a canonic structure and diffraction from wedge may be used in modelling scattering from a variety of complex structures. In this study, by using the Maxwell's equations the field components can be expressed in terms of z-components. By applying appropriate boundary conditions, a coupled system of equations is obtained in terms of field component and derivatives of field components with respect to φ and r. By using similarity transform to the coupled system of equations, the coupling is reduced to the simplest form in which Malyuzhinets theorem can be applied. The solution of field components is sought in the form of Sommerfeld integrals. The Malyuzhinets theorem is applied to the Sommerfeld integrals. By using Sommerfeld integrals the problem is reduced to a system of coupled functional equations. Solution of homogeneous functional equations is given in terms of χ_Φ functions. For a small parameter of the problem ($\sin\theta_0 << 1$ where θ_0 is the angle between z-axis and incident wave) the perturbation procedure is used to reduce the coupled functional equations to a system of linear equations with this small parameter being at the integral terms of equations. As a result the closed form solution is given for functional equations. The obtained analytic expression for the spectral functions is substituted to the Sommerfeld integrals, which are evaluated by means of steepest descent technique.Then, the analytical expressions for the diffraction coefficient for both magnetic and electric field components are derived. Considering these different geometries and small skewness angle, it is concluded that this approach enlarge the class of solvable diffraction problem in a small range. Additionally, the results are valuable for the comparison purposes for the other approximate methods.

KEYWORDS: Impedance Wedge, Functional Equations, Sommerfeld Integrals, Maliuzhinets Theorem, Perturbation Procedure.

1. INTRODUCTION

As known, the solution of the problem of the scattering waves from complex shaped objects needs some well-defined geometrical structure such as cylinder, strip, half-plane, sphere, ect. These structures are named as canonical structures. Wedge is also an important canonic structure for the electromagnetic scattering problems. Since the solutions for the wedge with parameters in a wide range may be used for the simulations of scattering from complex structures. The studies about the wedge diffraction problem date back to early 1950's. There have been great deals of theoretical studies by using different techniques.

The problem in the present work is started with to expression of the field components in terms of z-components by using the Maxwell's equations. These field components should satisfy the Helmholtz equation. Applying appropriate boundary conditions yields a differential equations system. For an arbitrary incidence case, the differential equations system is a coupled system. This coupled system may be transformed to uncoupled one for some special cases. In this study, similarity transform is used to reduce the coupling to its simplest form. The solution of field components is sought in the form of Sommerfeld integrals. With the aim of getting to know the Malyuzhinets functional equations the Malyuzhinets theorem is applied to the Sommerfeld integrals. The solution of homogeneous functional equations is investigated. For a small parameter of the problem ($\sin\theta_0 << 1$ where θ_0 is the angle between z-axis and incident wave) , the perturbation procedure is used to reduce the coupled functional equations to a system of linear equations with this small parameter being at the integral terms of equations. The obtained analytic expression for the spectral functions is substituted to the Sommerfeld integrals and Sommerfeld integrals are evaluated by means of steepest descent technique. The numerical results are obtained for different parameters of the problem.

There are great deals of investigation about the problem of wedge diffraction.

Malyuzhinets (Malyuzhinets, 1950) solved the generalized problem with impedance boundary conditions on the wedge faces in his D.Sc. Dissertion. This study also described by Malyuzhinets (1955a, 1955b, 1958a, 1958b), separately. The brief solution resumed by Malyuzhinets (1958c). His solution was reduced in the form of a Sommerfeld integral with an integrand involving a new special function $\Psi(z)$.

Williams (Williams, 1959) solved the problem of diffraction of an E-polarized plane wave by an imperfectly conducting wedge. The problem of the original boundary value is reduced to an ordinary difference equation. The wedge's conductivity is large but it is not infinite. The first investigation of the effect of the conductivity is that E-polarized plane wave on the wedge is proportional to its normal derivative. This case is valid for H-polarized plane wave. But results of the solution are very complicated.

Senior (Senior, 1959) gave a solution of the diffraction by an imperfectly conducting wedge. He used the Laplace transform, too. He took into account the case of finite conductivity.

Malyuzhinets (Malyuzhinets, 1960) gave a short review of his method.

The problem of the solution of a nonstationary problem of diffraction by an impedance wedge in tabulated functions was studied by Sakharova and Filippov (Sakharova and Filippov, 1967). Filippov (Filippov, 1967) investigated the solution of a nonstationary problem of diffraction of a plane wave by an impedance wedge.

Most of papers on this subject have examined the case of normal incidence. The problem's the case of oblique incidence has been investigated by Bucci and Franceschetti (Bucci and Franceschetti, 1975). But they have been investigating the case of oblique incidence, the wedge aperture has been assumed to be zero.

Bucci and Franceschetti (Bucci and Franceschetti, 1976) studied the problem of the electromagnetic scattering by a half plane with two face impedances. The solution for the problem of electromagnetic scattering by a half plane with two different face impedances is presented for both normal and oblique incidence. Illustrative examples are discussed and a chart relative to the existence of surface wave contributions is presented.

Vaccaro (Vaccaro, 1981) studied on electromagnetic diffraction from a right-angled wedge with soft conditions at one face. The diffraction of EM-plane wave from a wedge which has $\pi/2$-aperture is studied in the case of oblique incidence with respect to axis. Vaccaro's studying deals with the case of oblique incidence. To this end, we make use of the generalized reflection method, pioneered by Maliuzhinets, and extend to EM-waves by Vaccaro.

This method is efficient for both oblique and normal incidence because of the different wedge aperture. At the same time there can be different impedances on the two faces (Z^+ or Z^-). Hence, the solution of the problem becomes easier. The boundary conditions are transformed into functional difference equations, which are solved in a closed form. The utter solution for the field is given under the form of a Sommerfeld integral of simple trigonometric functions.

Mohsen (Mohsen, 1982) studied on the diffraction of an arbitrary wave by a wedge. He gave generalizations to some of the previous investigations.

Kim, Ra and Shin (Kim et. al., 1983), took into account the calculation of edge diffraction by a right-angled dielectric wedge. Then they had extended to a dielectric wedge of general angle in the study.

Senior and Volakis (Senior and Volakis, 1986) dealt with the problem of electromagnetic wave at oblique incidence on a right-angled imperfectly conducting wedge. One of wedge face is perfect conductor, while another is imperfect conductor. An exact integral expression for the total field is derived. The need is critical that if one of wedge's faces is imperfect conductor. The solution is obtained by a generalization of Maliuzhinets' technique. A uniform solution was derived in accordance with the UAT (Uniform Asymptotic Theory). If a plane wave is incident on the perfect conductor face of the wedge, computed data for the total field was found to be almost independent according to another face.

Tiberio, Pelosi, Manara and Pathak (Tiberio et. al., 1989) studied for the solution of High-Frequency Scattering from a wedge with impedance faces illuminated by a Line Source. A plane wave scatters from edges in nonperfectly conducting surfaces. Hence, surface impedance boundary conditions are important to be may provide a useful model is a important canonical problem in the geometrical theory of diffraction. An exact solution of this problem when a plane wave is right-angled on wedge's edge was given by Maliuzhinets. Because of this solution was found useful, this solution has been wanted to extend on some special cases. First of all, without "High-Frequency", Scattering from a wedge with impedance faces illuminated by a line source has examined in Part-1 of this solution. The main aim of this solution is the evaluation of the field scattered from a two-dimensional impedance wedge. This solution deals with two important aspects. First one is that of deriving uniform asymptotic solution for the diffracted field. Second one is that of examining the effect of wavefront curvature.

Because of two important aspects, this solution has been divided into two parts. While both source and observation points are located at finite distance from the wedge's edge, the two-dimensional problem of the diffraction has been investigated. Both the plane wave and far-field response of the wedge have been used to derive an exact integral representation for the total field.

Rawlins (Rawlins, 1990) produced a solution about the boundary-value problem depended on the physical problem of diffraction of an E- or H-polarized plane wave incident on a imperfectly conducting right-angled wedge.

Liu and Ciric (Liu and Ciric, 1993) improved formulas for the diffraction by a wedge. These formulas are about new analytical expressions for the diffraction integral in the case of a perfectly conducting wedge of an arbitrary angle illuminated by a plane wave or by a line source field.

The Malyuzhinets technique is reviewed by Osipov and Norris (Osipov and Norris, 1999). They focused around the basic problem of determining the wave field scattered from the edge of a wedge of exterior angle 2ø with arbitrary impedance conditions on either face. They begin by establishing a direct relationship between the Sommerfeld integral representation and the Laplace transform. This provides fresh insight into Maliuzhinets' inferences about functions representable via the Sommerfeld integral and, simultaneously, allows us to prove both the inversion formula for the Sommerfeld integral and the crucial nullification theorem.

Osipov and Norris (Osipov and Norris, 1999) tried to find a solution for the basic problem of determining the far-field scattered from the edge of a wedge of exterior angle 2ø with arbitrary impedance conditions on both faces of the wedge.

Lyalinov and Zhu (Lyalinov and Zhu, 1999) studied the problem of the diffraction of a skewly incident plane wave by an anisotropic impedance wedge. The Sommerfeld-Malyuzhinets' technique and the special function $\chi_ø$, which is originally introduced in the study of wave diffraction by a wedge located in a gyroelectric medium, have been used to find the exact solution for diffraction of a skewly incident and arbitrarily polarized plane wave by wedges with an arbitrary opening angle.

The problem of the diffraction of plane waves by a two-impedance wedge in cold plasma is studied by İkiz and Karaömerlioğlu (İkiz and Karaömerlioğlu, 2004). A two-impedance wedge in cold plasma may practically be used in modeling the electromagnetic scattering from a variety of large and complex objects.

As known many investigations have been done about the scattering by wedge. But while investigations were done, some cases had been taken into account. For example; certain opening angles of the wedge, certain incidence angles and some specific values of wedge surface impedance. The aim of study is abolish the limited cases. It is believed that solution at the end of this study should be useful for comparison purposes for more general analytic solutions to be found and for checking the other numerical methods proposed for investigating wave diffraction by anisotropic impedance wedges.

2. FORMULATION OF THE PROBLEM

The problem under consideration is a wedge with a wedge opening angle 2Φ, where the edge coincides with the z-axis. The direction of propagation of incidence wave is specified by the angles θ_0 *and* ϕ_0 as shown in figure1.

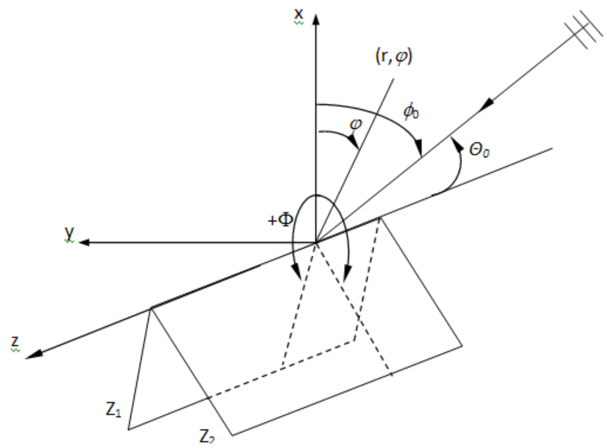


Fig. 1. The Geometry of the Problem

Φ is the angle between wedge faces and x-axis. The skewness angle, θ_0 is between the incident wave and z-axis. "(r, φ)" represents observation point.

The z-component of the incident wave is given by,

$$(Z_0 H_z{}^i, E_z{}^i)^T = \vec{V}_0 e^{(-ik_0' r\cos(\varphi-\varphi_0)+ik_0'' z)}. \quad (1)$$

Where $k_0' = k_0 \sin\theta_0$, $k_0'' = k_0 \cos\theta_0$, $Z_0 = (\mu_0/\varepsilon_0)^{1/2}$ and θ_0 is the skewness angle.

The vector $\vec{V}_0(r,\varphi)$ is defined as,

$$\vec{V}_0(r,\varphi) = (V_{10}, V_{20})^T. \quad (2)$$

Due to the invariance of the wedge geometry and the tensor impedance with respect to z, the z-component of the total field behave as,

$$(Z_0 H_z, E_z) = \vec{V}(r,\varphi) e^{ik_0'' z} \quad (3)$$

where the vector $\vec{V}(r,\varphi)$ is given as,

$$\vec{V}_0(r,\varphi)=(V_1,V_2)^T e^{-ik_0' r\cos(\varphi-\varphi_0)}. \tag{4}$$

Outside of the wedge $\vec{V}(r,\varphi)$ must satisfy the scalar Helmholtz equation

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi^2}+(k_0')^2\right]\vec{V}(r,\varphi)=0 \tag{5}$$

The following anisotropic surface impedance

$$\bar{\bar{\eta}}=(\eta_{11}\hat{a}_r+\eta_{12}\hat{a}_z)\hat{a}_r+(\eta_{21}\hat{a}_r+\eta_{22}\hat{a}_z)\hat{a}_z \tag{6}$$

and the anisotropic impedance boundary condition

$$\hat{a}_n\times\vec{E}=\bar{\bar{\eta}}.\hat{a}_n\times(\hat{a}_n\times\vec{H}) \tag{7}$$

Will be used to derive the coupled differential equations in the next section.

3. APPLICATION OF BOUNDARY CONDITIONS

The electric and magnetic field vectors can be expressed in cylindrical coordinate systems as follows:

$$\vec{E}=E_r\hat{a}_r+E_\varphi\hat{a}_\varphi+E_z\hat{a}_z \tag{8}$$

$$\vec{H}=H_r\hat{a}_r+H_\varphi\hat{a}_\varphi+H_z\hat{a}_z. \tag{9}$$

Since at $\varphi=-\Phi$

$$E_r=\bar{\eta}_{12}H_r+\bar{\eta}_{22}H_z \tag{10}$$

$$E_z=-\bar{\eta}_{11}H_r-\bar{\eta}_{21}H_z. \tag{11}$$

From the second equation(11);

$$H_r=-\frac{1}{\bar{\eta}_{11}}E_z-\frac{\bar{\eta}_{21}}{\bar{\eta}_{11}}H_z \tag{12}$$

By substituting (12) into (10),

$$E_r=-\frac{\bar{\eta}_{12}}{\bar{\eta}_{11}}E_z+\left(\bar{\eta}_{22}-\frac{\bar{\eta}_{12}\bar{\eta}_{21}}{\bar{\eta}_{11}}\right)H_z. \tag{13}$$

The r-components of the fields can be derived in terms of z and derivative of the z-components of the fields by using Maxwell equations.

$$E_r=\frac{i}{k_0\sin^2\theta_0}\left(\frac{1}{r}\frac{\partial(Z_0H_z)}{\partial\varphi}+\cos\theta_0\frac{\partial E_z}{\partial r}\right). \tag{14}$$

and

$$H_r=\frac{i}{k_0Z_0\sin^2\theta_0}\left(-\frac{1}{r}\frac{\partial E_z}{\partial\varphi}+\cos\theta_0\frac{\partial}{\partial r}(Z_0H_z)\right). \tag{15}$$

Using (14, 15) in (12, 13).

$$\frac{i}{k_0\sin^2\theta_0}\left(\frac{1}{r}\frac{\partial(Z_0H_z)}{\partial\varphi}+\cos\theta_0\frac{\partial E_z}{\partial r}\right)= -\frac{\bar{\eta}_{22}}{\bar{\eta}_{11}}E_z+\left(\bar{\eta}_{22}-\frac{\bar{\eta}_{12}\bar{\eta}_{21}}{\bar{\eta}_{11}}\right)H_z \tag{16}$$

and

$$\frac{i}{k_0Z_0\sin^2\theta_0}\left(-\frac{1}{r}\frac{\partial E_z}{\partial\varphi}+\cos\theta_0\frac{\partial}{\partial r}(Z_0H_z)\right)= -\frac{1}{\bar{\eta}_{11}}E_z-\frac{\bar{\eta}_{22}}{\bar{\eta}_{11}}H_z. \tag{17}$$

are obtained.

By rearranging (16-17);

$$\frac{i}{rk_0}\frac{\partial}{\partial\varphi}(Z_0H_z)=\sin^2\theta_0\left((\bar{\eta}_{22}-\frac{\bar{\eta}_{12}\bar{\eta}_{21}}{\bar{\eta}_{11}})H_z-\frac{\bar{\eta}_{12}}{\bar{\eta}_{11}}E_z\right)+ \frac{i}{k_0r}\cos\theta_0\left(-\frac{\partial E_z}{\partial r}\right) \tag{18}$$

and

$$\frac{i}{rk_0}\frac{\partial E_z}{\partial\varphi}=\sin^2\theta_0\left(Z_0\frac{\bar{\eta}_{21}}{\bar{\eta}_{11}}H_z+Z_0\frac{1}{\bar{\eta}_{11}}E_z\right)+ \frac{i}{k_0r}\cos\theta_0\frac{\partial}{\partial r}(Z_0H_z) \tag{19}$$

are obtained. Using $k_o^{'} = k_o.\sin\theta_o$ equations (18) and (19) may be combined by the matrix form.

$$\frac{i}{rk_0^{'}}\frac{\partial}{\partial\varphi}\begin{pmatrix} Z_0H_z \\ E_z \end{pmatrix}$$

$$= \sin\theta_0 \begin{pmatrix} (\eta_{22}^{-} - \dfrac{\eta_{12}^{-}\eta_{21}^{-}}{\eta_{11}^{-}})/Z_0 & -\dfrac{\eta_{12}^{-}}{\eta_{11}^{-}} \\ \dfrac{\eta_{21}^{-}}{\eta_{11}^{-}} & \dfrac{Z_0}{\eta_{11}^{-}} \end{pmatrix} . \begin{pmatrix} Z_0H_z \\ E_z \end{pmatrix} +$$

$$\frac{i}{rk_0^{'}}\cos\theta_0\frac{\partial}{\partial r}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} Z_0H_z \\ E_z \end{pmatrix} \tag{20}$$

4. SIMILARITY TRANSFORMATION

This equation

$$\frac{i}{rk_0^{'}}\frac{\partial \vec{V}(r,\pm\phi)}{\partial\varphi} = \pm\sin\theta_0 \overline{\overline{A}}^{\pm}\vec{V}(r,\pm\phi) + \frac{i}{rk_0^{'}}\cos\theta_0\overline{\overline{B}}\frac{\partial}{\partial r}\vec{V}(r,\pm\phi) \tag{21}$$

reduce to

$$\frac{i}{rk_0^{'}}\frac{\partial}{\partial\varphi}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\Bigg|_{\varphi=\pm\phi} = \pm\sin\theta_0 P^{-1}\overline{\overline{A}}^{\pm}P\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\Bigg|_{\varphi=\pm\phi} + \frac{i}{rk_0^{'}}\cos\theta_0 P^{-1}\overline{\overline{B}}P\frac{\partial}{\partial r}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\Bigg|_{\varphi=\pm\phi}. \tag{22}$$

By using similarity transform to the coupled system of equations, the coupling is reduced to the simplest form in which Maluizhinets theorem can be applied.

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = P^{-1}V. \tag{23}$$

It is obvious that u also satisfies Helmholtz equation and the solution in terms of Sommerfeld integral is,

$$u_j(r,\varphi) = \frac{1}{2\pi i}\int_{\gamma} e^{-irk_0^{'}\cos\alpha} f_j(\alpha+\varphi)d\alpha \tag{24}$$

5. MALIUZHINETS THEOREM

If

$$\frac{1}{2\pi i}\int_{\gamma} f(\alpha)e^{-ik_0^{'}r\cos\alpha d\alpha} = 0 \tag{25}$$

Let $f(\alpha)$ be regular function inside the loop γ+ and γ- everywhere, besides the possible exception at infinity, and satisfies the estimate

$$f(\alpha) = 0(\exp(n+1-a)|\mathrm{Im}\,\alpha|), \quad |\mathrm{Im}\,\alpha| \to \infty \tag{26}$$

where $0 < \alpha < 1$ and $\mathrm{n} \geq 0$ is an integer. Then

$$f(\alpha) = f_e(\alpha) + \sin\alpha\sum_{v=1}^{n} C_v\cos^{v-1}\alpha \tag{27}$$

where $f_e(\alpha)$ is an arbitrary even function, and C_v are arbitrary constants. As a corollary, $f(\alpha)$ should satisfy the functional equations of

$$f(\alpha) - f(-\alpha) = \sin\alpha\sum_{v=1}^{n} C_v\cos^{v-1}\alpha. \tag{28}$$

the resultant functional equations are;

$$\begin{aligned}&(\sin(\alpha-\theta)\pm\sin\sigma_1^{\pm})f_1(\alpha\pm\phi) - \\ &(-\sin(\alpha+\theta)\pm\sin\sigma_1^{\pm})f_1(-\alpha\pm\phi) \\ &= \pm q_{12}^{\pm}(f_2(\alpha\pm\phi) - f_2(-\alpha\pm\phi)).\end{aligned} \tag{29}$$

$$\begin{aligned}&(\sin(\alpha+\theta)\pm\sin\sigma_2^{\pm})f_2(\alpha\pm\phi) - \\ &(-\sin(\alpha-\theta)\pm\sin\sigma_2^{\pm})f_2(-\alpha\pm\phi) \\ &= \pm q_{21}^{\pm}(f_1(\alpha\pm\phi) - f_1(-\alpha\pm\phi)).\end{aligned} \tag{30}$$

6. CLOSED FORM SOLUTION

Let us assume that, the unknown functions f_1 and f_2 may be represented in terms of two new unknown functions ξ_1 and ξ_2 , such as

$$f_1(\alpha) = f_{10}(\alpha)\xi_1(\alpha)\sigma_{\varphi_0}(\alpha) \tag{31}$$

$$f_2(\alpha) = f_{20}(\alpha)\xi_2(\alpha)\sigma_{\varphi_0}(\alpha) \tag{32}$$

where

$$\sigma_{\varphi_0}(\alpha) = \frac{\mu\cos(\mu\varphi_0)}{\sin\mu\alpha - \sin\mu\varphi_0}, \quad \mu = \frac{\pi}{2\phi} \tag{33}$$

which satisfies that,

$$\sigma_{\varphi_0}(\alpha \pm \phi) = \sigma_{\varphi_0}(-\alpha \pm \phi) \tag{34}$$

and, $f_{10}(\alpha)$ and $f_{20}(\alpha)$ are the homogeneous solutions of equations (29) and (30) which is given by;

$$f_{j0} = \frac{\chi_\phi(\alpha + \phi + \pi - (-1)^j \theta - \sigma_j^+)}{\chi_\phi(\alpha + \phi - \pi + (-1)^j \theta + \sigma_j^+)} \times \frac{\chi_\phi(\alpha + \phi - (-1)^j \theta + \sigma_j^+)}{\chi_\phi(\alpha + \phi + (-1)^j \theta - \sigma_j^+)}$$

$$\times \frac{\chi_\phi(\alpha - \phi + \pi + (-1)^j \theta - \sigma_j^-)}{\chi_\phi(\alpha - \phi - \pi - (-1)^j \theta + \sigma_j^-)} \times \frac{\chi_\phi(\alpha - \phi + (-1)^j \theta + \sigma_j^-)}{\chi_\phi(\alpha - \phi - (-1)^j \theta - \sigma_j^-)}. \tag{35}$$

By using S integral theory $\xi_1(\alpha)$ and $\xi_2(\alpha)$ are determined as,

$$\xi_1(\alpha) = \frac{1}{f_{10}(\varphi_0)} - \frac{iq_{12}^+}{8\phi} \int_{iR} \frac{f_{20}(\tau+\phi)\xi_2(\tau+\phi) - f_{20}(-\tau+\phi)\xi_2(-\tau+\phi)}{(\sin(\tau-\theta) + \sin\sigma_1^+) f_{10}(\tau+\phi)} \bullet \{\sigma_1(\tau,\alpha) - \sigma_1(\tau,\varphi_0)\} d\tau$$

$$- \frac{iq_{12}^-}{8\phi} \int_{iR} \frac{f_{20}(\tau-\phi)\xi_2(\tau-\phi) - f_{20}(-\tau-\phi)\xi_2(-\tau-\phi)}{(\sin(\tau-\theta) + \sin\sigma_1^-) f_{10}(\tau-\phi)} \{\sigma_2(\tau,\alpha) - \sigma_2(\tau,\varphi_0)\} d\tau,$$

$$|\mathrm{Re}\,\alpha| < \phi \tag{36}$$

and,

$$\xi_2(\alpha) = \frac{1}{f_{20}(\varphi_0)} - \frac{iq_{21}^+}{8\phi} \int_{iR} \frac{f_{10}(\tau+\phi)\xi_1(\tau+\phi) - f_{10}(-\tau+\phi)\xi_1(-\tau+\phi)}{(\sin(\tau+\theta) + \sin\sigma_2^+) f_{20}(\tau+\phi)} \bullet \{\sigma_2(\tau,\alpha) - \sigma_2(\tau,\varphi_0)\} d\tau$$

$$- \frac{iq_{21}^-}{8\phi} \int_{iR} \frac{f_{10}(\tau-\phi)\xi_1(\tau-\phi) - f_{10}(-\tau-\phi)\xi_1(-\tau-\phi)}{(\sin(\tau+\theta) + \sin\sigma_2^-) f_{20}(\tau-\phi)} \{\sigma_2(\tau,\alpha) - \sigma_2(\tau,\varphi_0)\} d\tau,$$

$$|\mathrm{Re}\,\alpha| < \phi \tag{37}$$

where

$$\sigma_j(\alpha, z) = \frac{\sin \mu z}{\cos \mu z + (-1)^j \sin \mu z}. \tag{38}$$

For small θ_0 $(\sin\theta_0 \to 0)$ the right hand side of (36) and (37) approaches to zero and therefore perturbation theory can be used.

7. NUMERICAL RESULTS

In figure 2 the variation of diffraction coefficient D_1 with respect to the observation angle for different opening angles and skewness angles are given. As shown in this figures, D_1 decrease dramatically with increasing skewness angle, while there is no such a dependence between D_2 and skewness angle as shown in figure 3.

In figures 4 and 5 the variation of D_1 and D_2 with respect to ϕ_o are given. While ϕ_o increases, both of D_1 and D_2 also increase.

In figure 6 the variations of D_1 and D_2 with respect to the surface impedances are given. While values of surface impedances increase both of D_1 and D_2 increase also.

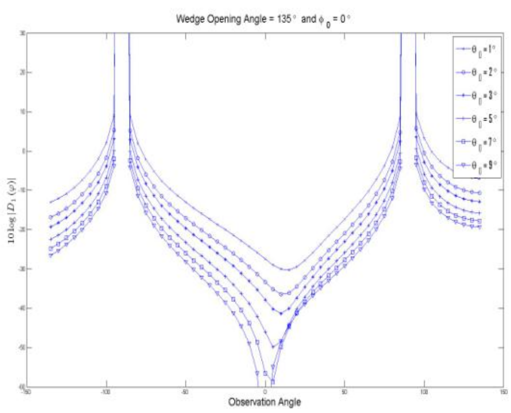


Fig. 2: Diffraction coefficient $10\log_{10}|D_1(\varphi)|$ versus observation angle with Φ=135°, Ø0=0°, θ_0 =1°, 2°, 3°, 5°, 7°, 9° $a_{12}^+ = 1.0, a_{12}^- = 1.0, a_{21}^+ = 1.0, a_{21}^- = 1.0$

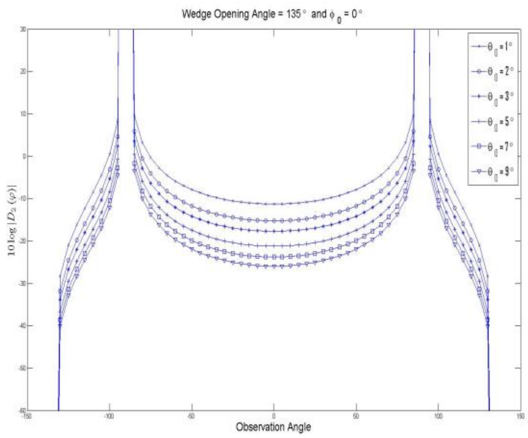


Fig. 3: Diffraction coefficient $10\log_{10}\left|D_2(\varphi)\right|$ versus observation angle with Φ=135°, Ø$_0$=0°, θ_0 =1°, 2°, 3°, 5°, 7°, 9° $a_{12}^{+}=1.0, a_{12}^{-}=1.0, a_{21}^{+}=1.0, a_{21}^{-}=1.0$

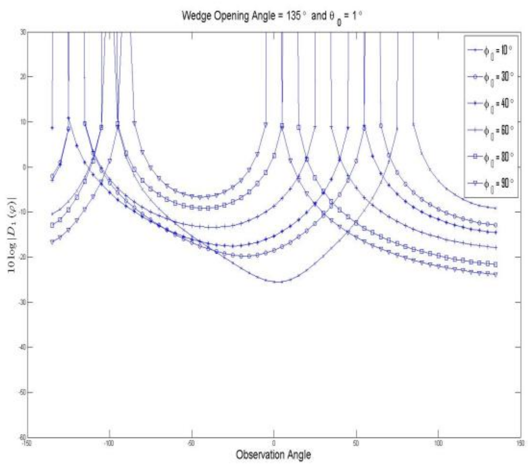


Fig. 4: Diffraction coefficient $10\log_{10}\left|D_1(\varphi)\right|$ versus observation angle with Φ=135°, θ_0 =1°, Ø$_0$=10°, 30°, 40°, 60°, 80°, 90°

$$a_{12}^{+}=1.0, a_{12}^{-}=1.0, a_{21}^{+}=1.0, a_{21}^{-}=1.0$$

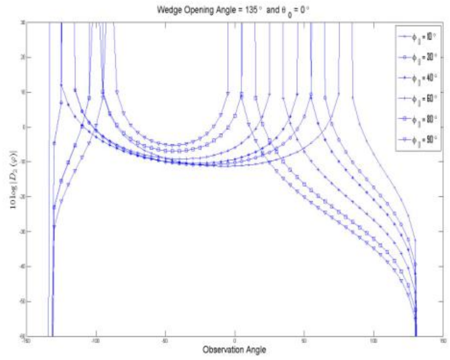
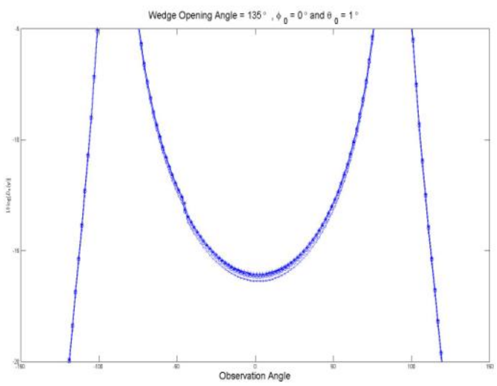


Fig. 5: Diffraction coefficient $10\log_{10}\left|D_2(\varphi)\right|$ versus observation angle with Φ=135°, θ_0 =0°, Ø$_0$=10°, 30°, 40°, 60°, 80°, 90°

$$a_{12}^{+}=1.0, a_{12}^{-}=1.0, a_{21}^{+}=1.0, a_{21}^{-}=1.0$$



x	o	$*$	$+$
$a_{12}^{+}=0.5+0.2i$	$a_{12}^{+}=0.5+0.2i$	$a_{12}^{+}=5.0+0.2i$	$a_{12}^{+}=0.5+0.2i$
$a_{12}^{-}=2.4+3.2i$	$a_{12}^{-}=2.4+3.2i$	$a_{12}^{-}=2.4+3.2i$	$a_{12}^{-}=2.4+3.2i$
$a_{21}^{+}=2.0+4.0i$	$a_{21}^{+}=4.0+6.0i$	$a_{21}^{+}=6.0+8.0i$	$a_{21}^{+}=8.0+10.0i$
$a_{21}^{-}=2.0+4.0i$	$a_{21}^{-}=4.0+6.0i$	$a_{21}^{-}=6.0+8.0i$	$a_{21}^{-}=8.0+10.0i$

Fig. 6: Diffraction coefficient $10\log_{10}\left|D_2(\varphi)\right|$ versus observation angle with Φ=135°, Ø$_0$=0°, θ_0 =1°

8. CONCLUSION

In this paper, formulation of the problem of plane wave diffraction by a wedge with anisotropic impedances is given for the case of almost grazing incidence. Even though there are numerous studies about the effects of impedance wedge on the propagation of electromagnetic waves, for some specific wedge opening angle, surface impedance and incidence angle, the diffraction from an arbitrary wedge for almost grazing incidence case is being investigated for the first time in this paper. For a small parameter of the problem, the perturbation procedure enables us to reduce the coupled functional equations to a set of linear equations.

One of the advantages of this study, using these values of opening angles, the results for the related geometries can be obtained easily. Considering these different geometries and small skewness angle, we conclude that this approach enlarge the class of solvable diffraction problem in a small range. Additionally, we hope that, the results are valuable for the comparison purposes for the other approximate methods.

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