# Performance Improvement of MIMO-OFDM Block Codes by Achieving a Suboptimum Permutation Distance

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#### **ABSTRACT:**

This paper propose a method in order to achieve a suboptimum permutation distance for every multi-input multioutput orthogonal frequency-division multiplexing (MIMO-OFDM) block code in the absence of perfect channel state information at the transmitter. As a matter of fact, our method is based on artificial delay power profiles (ADPPs) to obtain this permutation distance. Furthermore, we show that the performance of every MIMO-OFDM block code can be improved by applying our proposed suboptimum permutation distance in comparison with fixed, random and maximum permutation distance. Also, simulation results show that the performance of recent MIMO-OFDM block codes can be improved more than 3 dB by considering the proposed permutation.

**KEYWORDS:** Fading channels; frequency-selective channels; MIMO-OFDM systems; delay power profiles (DPPs).

## 1. INTRODUCTION

In high rate communication, wireless channels confront with different unwanted problems. One of the most challenging problems is channel fading which damages the performance of communication system. This is due to the fact that fading can negatively influence the phase and the amplitude of the transmitted signals. It is well-known that one of the best solutions in order to combat the fading effect is diversity (diversity is mainly transmitting copy of the transmitted signals more than once). Since all copies rarely face a deep fade, the performance of the transmission is improved.

Multiple-input multiple-output (MIMO) systems were employed for combating the fading effect by exploiting the spatial diversity in the frequency-flat fading channels. Space time coding were proposed for exploiting the spatial diversity by acquiring the space time codes (STCs) while the channels have the flat fading [1-5].

STCs are unusable in frequency selective channels while inter-symbol-interference (ISI) happens [6]. Orthogonal frequency-division multiplexing (OFDM) has been proposed to encounter the ISI effect of the frequency-selective channels. As we need to use the benefits of both OFDM and MIMO systems, MIMO-OFDM systems were introduced. Later on, space frequency codes (SFCs) and Space Time Frequency codes (STFCs) were proposed in MIMO-OFDM systems for exploiting spatial-frequency diversities and spatial-frequency and temporal diversity respectively. Moreover, design criteria of SFCs have been presented in [7-10].

Achieving full-diversity for a SFC or STFC is one of the basic goals in order to improve the performance of MIMO-OFDM codes. In addition to full-diversity, permutation is proposed by [14, 10] for the purpose of obtaining much better performance of the MIMO-OFDM block codes. For example, the authors in [12] apply fixed permutation distance on their code while Mr. W. Su *et al* consider a random one. Also, in [14] the distance for permuting is assumed as farther distance as possible.

In this paper, we propose a method to obtain the suboptimum permutation distance for *every* block code in MIMO-OFDM systems when the channel delay power profiles (DPPs) are unknown. Our method is based on maximizing coding advantage of the MIMO-OFDM block code in order to achieve the suboptimum permutation. Whereas the coding advantage depends on the channel DPPs, we have to select an estimated channel model for the goal of using its' DPPs. Therefore, we utilize the artificial DPPs (ADPPs) channel model, which is introduced in [13]. The simulation results shown that our proposed method outperforms the existence MIMO-OFDM block codes; which apply fixed [12], random [10] and with

Vol. 2, No. 4, December 2013

maximum [14] permutation distance, in the situation of unknown DPPs.

The rest of paper is organized as follows. In the next section, we describe the system model of MIMO-OFDM space-time-frequency codes. In section 3, we express the structure of our proposed method in detail. Finally, we present simulation results and conclusion through sections 4 and 5, respectively.

**Notations:** The capital boldface letters and boldface letters indicate matrices and vectors, respectively. We present superscripts  $(\cdot)^T, (\cdot)^H$ , and  $(\cdot)^*$  as transpose, Hermitian and complex conjugation, respectively.  $E\{\cdot\}$ ,  $\circ$  and  $\otimes$  denote expectation, Hadamard product and tensor product, respectively. Also, notation diag $(a_0, a_1, \ldots, a_N)$  stands for a diagonal  $N \times N$  matrix which diagonal entries are  $a_0, a_1, \ldots, a_N$ .

#### 2. SYSTEM MODEL

In this section, we assume a STF-coded MIMO-OFDM system with  $M_t$  transmit antennas and  $M_r$  receive antennas and  $N_c$  subcarriers within K successive OFDM blocks (time slots), whereas we have SF-coded MIMO-OFDM system for K = 1. Channel impulse response between *i*-th transmit antenna and *j*-th receive antenna, during the *k*-th OFDM block considered as:

$$h_{i,j}^{k}(\tau) = \sum_{l=0}^{L-1} \alpha_{i,j}^{k}(l) \delta(\tau - \tau_{l}), \ k = 1, 2, \dots, K$$
(1)

where  $\delta(\cdot)$  denotes the impulse function,  $\tau_l$ 's and  $\alpha_{i,j}^k(l)$ 's indicate time delays and the complex amplitude of *l*-th path from transmit antenna*i*to receive antenna *j*, respectively, where *L* is the number of paths from every transmit antenna to the other antennas. Here,  $\alpha_{i,j}^k(l)$ 's are zero-mean complex Gaussian random variables with variances  $\vartheta_l^2$ . For the purpose of having normalized power, it is assumed that  $\sum_{l=0}^{L-1} \vartheta_l^2 = 1$ .

Consider a  $KN_c \times M_t$  space-time-frequency code matrix as follows:

$$\boldsymbol{C} = [\boldsymbol{C}_1^T \quad \boldsymbol{C}_2^T \quad \cdots \quad \boldsymbol{C}_K^T]^T, \tag{2}$$

Where

 $C_{\nu}$ 

$$= \begin{bmatrix} c_1^k(0) & c_2^k(0) & \dots & c_{M_t}^k(0) \\ c_1^k(1) & c_2^k(1) & \dots & c_{M_t}^k(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1^k(N_c - 1) & c_2^k(N_c - 1) & \dots & c_{M_t}^k(N_c - 1) \end{bmatrix}.$$
 (3)

 $c_i^k(n)$  for  $1 \le i \le M_t$  and  $1 \le k \le K$ , refers to data symbols which are considered to be transmitted from the *i*-th transmit antenna on the*n*-th subcarrier of *k*-th OFDM block. At the transmitter, an  $N_c$ -point inverse fast Fourier transform is applied over each column of  $C_k$ . Then, after adding cyclic prefix, the *i*-th transmit antenna transmit the *i*-th column of  $C_k$ . At the *k*-th OFDM block of *j*-th receive antenna, after match filtering and removing the cyclic prefix,  $N_c$ -point FFT is applied. Therefore, at *n*-th frequency subcarrier we have:

$$r_{j}^{k}(n) = \sqrt{\frac{\rho}{M_{t}}} \sum_{i=1}^{M_{t}} c_{i}^{k}(n) H_{i,j}^{k}(n) + \mathcal{N}_{j}^{k}(n), \qquad (4)$$

For  $n = 0, 1, ..., N_c - 1$ ,

Where  $H_{i,j}^k(n)$  is the channel frequency response at the *n*-th subcarrier of the *k*-th OFDM block from transmit antenna *i* to the receive antenna *j*:

$$\mathcal{F}\{h_{i,j}^{k}(\tau)\} = H_{i,j}^{k}(f) \mid_{f = n\Delta f} \triangleq H_{i,j}^{k}(n) =$$
$$= \sum_{l=0}^{L-1} \alpha_{i,j}^{k}(l) e^{-j2\pi n\Delta f \tau_{l}}.$$
(5)

The symbol  $\mathcal{F}$  represents the Fourier transform.  $\Delta f = 1/T_s = BW/N$  where  $T_s$  and BW are OFDM symbol period and total bandwidth, respectively.  $\mathcal{N}_j^k(n)$  indicates the zero-mean additive white complex Gaussian noise with unit variance of the *n*-th subcarrier, at the receive antenna *j* and during *k*-th OFDM block. Also, in Equation (4) the expression  $\sqrt{\frac{\rho}{M_t}}$ guarantees that at each receive antenna  $\rho$  is the average signal-to-noise ratio (SNR).

#### 3. Our proposed method

In this section, we introduce a method to access suboptimum permutation distance for every MIMO-OFDM block code in case of unknown channel at the transmitter. We achieve this goal by maximizing the coding advantage of MIMO-OFDM block code.

First of all, we should describe the generalized coding advantage expression on all block codes in MIMO-OFDM systems. As discussed in [10, 11], the minimum rank of  $\Delta \circ \mathbf{R} \in \mathbb{C}^{KN_c \times KN_c}$  over all distinct codewords  $\mathbf{C}$  and  $\hat{\mathbf{C}}$  of the MIMO-OFDM code describes the maximum achievable diversity of a MIMO-OFDM code, where  $\Delta = (\mathbf{C} - \hat{\mathbf{C}})(\mathbf{C} - \hat{\mathbf{C}})^H$  and K indicates number of time slots which is 1 for SFC in a specific manner.

Now, the coding advantage  $(C_A)$  is defined as the determinant of a sub-matrix of  $\Delta \circ \mathbf{R}$ , which reaches the minimum rank for  $\Delta \circ \mathbf{R}$ , with generalized size of  $LM_tK$  [10, 11]. We present this sub-matrix as  $\hat{\Delta} \circ \hat{\mathbf{R}}$ . Thus, the coding advantage  $(C_A)$  is equal to:

$$C_A = \det\left(\widehat{\Delta} \circ \widehat{\boldsymbol{R}}\right),\tag{6}$$

Where

$$\boldsymbol{R} = E\{H_{i,j}H_{i,j}^H\} = \boldsymbol{R}_K \otimes \boldsymbol{R}_F,\tag{7}$$

and  $\mathbf{R} \in \mathbb{C}^{KN_c \times KN_c}$ . Also,  $\mathbf{R}_K \in \mathbb{C}^{K \times K}$  and  $\mathbf{R}_F \in \mathbb{C}^{N_c \times N_c}$  are the time and frequency correlation matrix, respectively [10], [11]. These correlation matrices also

are expressed as follows:

$$E\{a_{i,j}a_{i,j}^{H}\} = R_{K} \otimes \Lambda,$$
(8)  
for = diag{ $\vartheta_{0}^{2}, \vartheta_{1}^{2}, ..., \vartheta_{L-1}^{2}$ },  
 $a_{i,j} =$   
{ $\alpha_{i,j}^{1}(0), \alpha_{i,j}^{1}(1), ..., \alpha_{i,j}^{1}(L-1), ..., \alpha_{i,j}^{K}(0), \alpha_{i,j}^{K}(1), ..., \alpha_{i,j}^{K}(L-1)$ }).  
 $R_{F} = E\{H_{i,j}^{k}H_{i,j}^{k}\} =$   

$$\begin{bmatrix} 1 & \sum_{l=0}^{L-1} \vartheta_{l}^{2}w^{-\tau_{l}} & \cdots & \sum_{l=0}^{L-1} \vartheta_{l}^{2}w^{-(N_{c}-1)\tau_{l}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{l=0}^{L-1} \vartheta_{l}^{2}w^{(N_{c}-1)\tau_{l}} & \sum_{l=0}^{L-1} \vartheta_{l}^{2}w^{(N_{c}-2)\tau_{l}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{l=0}^{L-1} \vartheta_{l}^{2}w^{(N_{c}-1)\tau_{l}} & \sum_{l=0}^{L-1} \vartheta_{l}^{2}w^{(N_{c}-2)\tau_{l}} & \cdots & 1 \end{bmatrix}$$
, (9)

where  $w = e^{-j2\pi\Delta f}$ . Also, It is notable that we do not have  $\mathbf{R}_K$  in Equation (7) for case of using SFCs *i.e.* in this specific situation we have  $\mathbf{R} = \mathbf{R}_F$ .

As we consider in Equations (8) and (9), the expression of both  $\mathbf{R}_F$  and  $\mathbf{R}_K$  depend on the channel parameters (DPPs). Now, the problem of maximizing coding advantage is that the channel DPPs are unknown at the transmitter. For the solution, we consider the DPPs of an estimated channel (ADPPs) which is introduced in [13] and is approximately close to typical channels. So, the ADPPs in [13] are achieved as follows:

For artificial delay profile, *L* delays times as  $\{\tau_{A_l}\}_{l=0}^{L-1}$  are considered.

$$\tau_{A_l} = \begin{cases} 0 & , \quad l = 0 \\ \frac{\tau_{max}}{d^{L-l}}, \quad l = 1, 2, \dots, L-1 \end{cases}$$
(10)

Where d = 3 for  $2 \le L \le 8$ , d = 1.5 for  $L \ge 9$  and  $\tau_{max}$  is the length of cyclic prefix (CP). Also, corresponding to each delay time  $\tau_{A_l}$ , which is defined in Equation (10), the artificial power profile  $\vartheta_{A_l}^2$  is assumed to be:

$$\vartheta_{A_l}^2 = \frac{e^{-0.26\,\tau_{A_l}}}{\sum_{k=0}^{L-1} e^{-0.26\,\tau_{A_k}}}, \ l = 0, 1, \dots, L-1 \tag{11}$$

Therefore, we employ ADPPs parameters in expression of  $\mathbf{R}_K$  and  $\mathbf{R}_F$ . Now, the permutation distance is only unknown parameter in Equation (6). So, by maximizing the coding advantage we can achieve suboptimum permutation distance which is much closer to the optimum one in comparison with values in [15], [12], [10], [14] which are selected as fixed, random and the maximum permutation distance.

At last, it is worth to state that we can apply this method to every block codes in MIMO-OFDM systems when the channel is unknown at the transmitter. It means that, if we employ the ADPPs in order to maximize the coding advantage, we can achieve suboptimum permutation distance which performs superior than the other methods in [12], [10], [14], [15]. Also, it is obvious that since our method just applies the new permutation distance to a MIMO-OFDM code, thus the diversity of MIMO-OFDM code and the receiver complexity remain constant. But the code performance improves effectively.

## 4. SIMULATION RESULTS

In the simulations, we consider a MIMO-OFDM coded system with one receive antenna, BW = 1 MHz and CP equal to 20µs. Also, the type of detector which is assumed in this simulation is maximum likelihood (ML). Simulation results aim to demonstrate the improvement of the full-diversity SFCs in [12] by maximizing the coding advantage when there is no channel state information at the transmitter. Instead of unknown channel parameters, we exploit ADPPs at the coding advantage expression.

Fig. 1 demonstrates simulation results for both the main and the improved version of the block circular delay diversity (BCDD) SFC when there are 4 transmit antennas and  $N_c = 2048$ . Also, the equal power two ray channel of {0, 5} µs is applied to the simulations of Fig.1. The value of permutation distance ( $\gamma_{dpi}$ ) is obtained as 256 from ADPPs while  $\gamma_{dpi}$  is assumed to be 16 in [12]. As Fig. 1 illustrates, our proposed method outperforms the BCDD SFCs' performance effectively. For example, at a BER=10-3, it achieves more than 3 dB gains over the BCDD SFCs.



Fig. 1. BER performance,  $N_c = 2048$ , equal power channel 5µs, $M_t = 4$ , BPSK modulation, 1 bit/s/Hz

For Fig. 2 and 3 we use the equal power two ray channel of {0, 5} µs and {0, 5} µs respectively. Also, it is considered two transmit antennas and  $N_c = 1024$  for both figures. Since the channel is unknown for us, we achieve  $\gamma_{dpi} = 76$  for both channels. Although, in [12], the permutation distance ( $\gamma_{dpi}$ ) is assumed to be 8 for channel {0, 5} µs and 4 for channel {0, 15} µs, the channel is still unknown. Both figures demonstrate that our proposed permutation distance outperforms the

situation of using fixed one. For example, at a BER=10-4 of Fig. 2, it performs more than 5 dB gains over the BCDD SFCs. And for Fig. 3, the performance improves about 2 dB.



Fig. 2. BER performance,  $N_c = 1024$ , equal power channel 5µs, $M_t = 2$ , BPSK modulation, 1 bit/s/Hz



Fig. 3. BER performance,  $N_c = 1024$ , equal power channel 15µs, $M_t = 2$ , BPSK modulation, 1 bit/s/Hz

Finally, it should be stated that our proposed method can be implemented to the recent SF and STF codes to improve their performances.

## 5. CONCLUSION

In this paper, we proposed a method for improving the performance of block codes in MIMO-OFDM systems when the perfect channel state information is not available at the transmitter. Our method is based on achieving the suboptimum permutation distance by considering an artificial delay power profiles. Furthermore, it is stated that not only the proposed method does not affect receiver complexity and diversity of MIMO-OFDM block codes, but also it improves the code performance effectively.

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# Vol. 2, No. 4, December 2013

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