

Analysis of Available Methods for Maximum Unambiguous Range Demystification in High Pulse Repetition Frequency Radars

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Received: May 2017

Revised: May 2017

Accepted: May 2017

ABSTRACT:

Today's, Pulse Doppler Radars are used in most of industries needed radar, but it is necessary to note that despite these radars have some benefits, they also face several problems including range and velocity ambiguity. Obtaining range and velocity because is base of use of pulse Doppler radars, therefore these radars should use new methods in their technologies to use them to eliminate range and velocity ambiguity. Using high pulse repetition frequency radars will create velocity ambiguity. Introducing the methods mentioned in this paper, we will seek to eliminate ambiguity in range. despite, there are many methods to eliminate ambiguity, we will engage to assess Particle Filter and Chinese Remainder model in this study and finally, we will explain method of using Vernier on the timing gates in the different distances.

KEYWORDS: Unambiguity, Particle filter, Chinese Remainder Theorem, Vernier frequency, High pulse repetition frequency.

1. INTRODUCTION

The main application of pulse Doppler radar to detect moving targets and find the range and speed as well. pulse Doppler radar are divided into three categories; Radar systems employ low, medium, and high PRF schemes. Low PRF waveforms can provide accurate, long, unambiguous range measurements, but exert severe Doppler ambiguities. Medium PRF waveforms must resolve both range and Doppler ambiguities; however, they provide adequate average transmitted power as compared to low PRFs. High PRF waveforms can provide superior average transmitted power and excellent clutter rejection capabilities.[1]

Alternatively, high PRF waveforms are extremely ambiguous in range. When radar systems transmitted by Low pulse repetition frequency to detect and get the exact range is convenient but since the Nyquist sampling rate when the repetition frequency is low, we do not observe it. Therefore, in determining the speed targets will be ambiguous.

There are different methods to unambiguity the pulse Doppler radar, which is used in similar ways, but the principles work in all manner of mathematical formulation of these methods is different.

The present methods to range ambiguity resolving mainly are signal processing methods and data processing methods Data processing methods mainly are Chinese Remainder Theorem, permutation and combination method, multiple hypothesis tracking (MHT) and so on.[2]

The Chinese Remainder Theorem is simple and of low computation complexity, but it requires at least three HPRFs and has the limitation that the numbers of range cells corresponding to HPRFs must be coprime; the permutation and combination method and MHT method are of high computation complexity; the hybrid filter and IMM methods perform well in range ambiguity resolving, but they do not consider the clutter.

Particle filter based method for target tracking with the HPRF radar in clutter. The method makes full use of the particle filter (PF) that each particle represents a possible target state, updates target state with the ambiguous measurement directly, and thus avoids the problem that each possible measurement must be assigned a filter which may increase the computation complexity remarkably.[3]

This paper proposed a Vernier frequency method for range unambiguity with the HPRF radar. The

proposed method makes full use of veriner frequency. Proposed method can solve range ambiguity and target tracking in dense clutter simultaneously.

2. CHINESE REMAINDER THEOREM

In the method we assume K batches with different PRF is transmitted. The K PRFs create a set shown by R.

In order to detect a target and consequently estimate its unambiguous range, the target must be detected at least in l batches from the K transmitted batches and the ambiguous ranges in the detected batches should be estimated correctly. All l dimensional subsets from the set R is denoted by R_i $i = 1, \dots, L$, where L is the total number Of l dimensional subsets from the set R and equal to:

$$L = \binom{k}{l} = \frac{k!}{l!(k-l)!} \tag{1}$$

The aim in signal design is to have a radar which can detect targets with ranges up to R_{max} . The radar carrier frequency is f_c , and v_{max} is the maximum target's velocity. The minimum PRF of the radar is PRF_{min} and equal to:

$$PRF_{min} = fd_{max} = \frac{2 \times v_{max} \times f_c}{c} = \frac{2 \times v_{max}}{\lambda} \tag{2}$$

The aim of the signal design is to have a radar that satisfies the limitation in (2) and has the ability to resolve ambiguity in the ranges up to R_{max} using the closed form robust CRT. PRIs in several batches are written in the following form:

$$\frac{1}{PRF_i} = PRI_i = (Mr_s M_i) \frac{2}{c}; \quad i = 1, \dots, k \tag{3}$$

Where r_s is the range cell intervals in meter, c is the speed of wave propagation in the free space, and M_i is the multiplier corresponding to the i th batch. All M is must not have the common divisor so that the only divisor of the several batches are $\frac{2Mr_s}{c}$. [4]

The following in equalities must be satisfied in order to have a HPRF radar with maximum target's range equal to R_{max} :

$$\begin{aligned} PRI_i &\leq \frac{1}{PRF_{min}}; \quad i = 1, \dots, k \\ M \times r_s \times \prod_{i \in \mathcal{R}_l} M_i &\geq R_{max}; \quad i = 1, \dots, k \end{aligned} \tag{4}$$

So, the signal design is limited to finding at least K quantities for M_i , the common multiplier M, and an appropriate value for r_s . The rs is related to the sampling rate in the base band. In radar applications

where carrier frequency is large, the Doppler frequencies are large and consequently PRIs are small in HPRF radars. The more the sampling rate is, the more D of in choosing the M is are. On the other hand, power of the received echoes must be so large that the ambiguous range can be estimated preciously. The PRF selection problem is written in the following form:

$$\begin{aligned} \arg \max_{M_i} & \sum_{j=1}^L \prod_{i \in \mathcal{R}_j} M_i \\ \text{subject to } & M_i \leq \frac{\lambda f_s}{2Mv_{max}}; \quad i = 1, \dots, k \end{aligned} \tag{5}$$

$$\prod_{i \in \mathcal{R}_j} M_i \geq \frac{R_{max}}{Mr_s}; \quad i = 1, \dots, k$$

$$r_s = \frac{c}{2f_s}$$

where f_s is the sampling rate in baseband. Problem (4) can be solved by using numerical methods. [5]

Finding optimum quantities for r_s and M_i , $i = 1, \dots, k$, PRIs are obtained by (5).

3. PARTICLE FILTER METHOD

3.1. HPRF Radar Range Ambiguity

Assume that R_{max} is the maximum range of interest, and $\{F_{r,i} | i = 1, 2, \dots, L\}$ is the set of HPRFs used for range ambiguity resolving. Without loss of generality, the i th HPRF, $F_{r,i}$ is taken to formulate the problem of HPRF radar range ambiguity. The maximum unambiguous range $R_{u,i}$ corresponding to $F_{r,i}$ is given by

$$R_{u,i} = \frac{C}{2F_{r,i}} \tag{6}$$

where C is the speed of light. As illustrated in Fig. 1, let $r_{i,k}$ ($i = 1, 2, \dots, L$) denote the ambiguous range measurement at time k. Then, all possible ranges are generated by

$$r_{i,k}^j = R_{u,i} \times (j-1) + r_{i,k}, \quad i = 1, 2, \dots, L; j = 1, \dots, P_i \tag{7}$$

with

$$P_i = \text{floor} \left(\frac{R_{max}}{R_{u,i}} \right), \quad i = 1, 2, \dots, L \tag{8}$$

Denoting the maximum unambiguous number. The function Floor(x) means to get the nearest integer less

than or equal to x . The value $j \in \{1, 2, \dots, P_i\}$ is defined as the pulse interval number (PIN) corresponding to HPRF $F_{r,i}$ such that $r_{i,k}^j$ reflects the true range of target at time k .

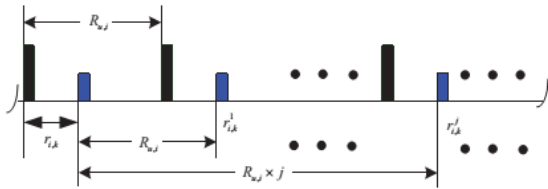


Fig.1. All possible Ranges corresponding to an ambiguous range.

Fig. 1 demonstrates that the true range of target must be one of ranges represented by (7), however, it is impossible to tell directly which one is true. Therefore, it is necessary to provide an integrated approach to the joint estimates of the target state and PIN [6].

3.2. System setup

In this section, the system model is formulated in polar coordinates and has ambiguity in the range measurements. It is assumed that a 2-D radar located at the origin of the coordinate system and responsible for detection of a single target with constant velocity. The dynamic model and measurement model are described as follows.

3.3. Dynamic Model

The target state $x_k = [r_k \ v_k \ PIN_k]^T$ contains target radial range r_k , radial velocity v_k and pulse interval number PIN_k where $[.]^T$ represents the transpose of a matrix $[.]$. The state propagation from time k to $k+1$ is given by

$$x_{k+1} = F_k x_k + G_K v_k \tag{9}$$

with

$$F_k = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

and

$$F_k = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

respectively denoting the transition matrix and the distribution matrix of process noise, where T is the sampling interval, and v_k is a zero-mean white process noise with covariance

$$Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \tag{12}$$

3.4. Measurement Model

The measurement set is given by $Z_k = \{z_{k,1}, \dots, z_{k,n_k}\}$ is the number of n_k , where $z_{k,i}$ measurements at time k , and $z(k) = [r_{app}(k) \ f_D(k)]^T$ contains the apparent radial range and Doppler measurement. The ambiguous measurement equation

at time k is given by

$$z_{k,m} = H_k x_{k,i} + W_k, \ m = 1, \dots, n_k \tag{13}$$

Where

$$H_k = \begin{bmatrix} 1 & 0 & -R_{u,i} \\ 0 & -\frac{2}{\lambda} & 0 \end{bmatrix} \tag{14}$$

is the measurement transitional matrix, λ is radar wavelength, $R_{u,i}$ is given by (6) and W_k is zero-mean white Gaussian noise process of known covariance R_k defined by

$$R_k = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_d^2 \end{bmatrix} \tag{15}$$

where σ_r^2 and σ_d^2 are the range and Doppler measurement variance respectively.

4. VERNIER FREQUENCY METHOD

4.1. Using Vernier on the Timing gates

The target range tracking systems operates based on range and then produce actual value of range for tracked objectives, which should be calculated to launch and control missile on the scope of target.[7]

Moreover, the target range tracking system operates the processing automatically to receive information from external target and if it fails to operate and process this tracking, the automatic conduct of system

tracking is performed by using the range scan on the target.

To perform tracking operation, the radar suggests three type of mode to us which include external target allocation mode, local conduct mode and automatic tracking by the target-returned signal mode.

External target allocation mode and local conduct mode take priority over automatic tracking mode. The external radar-allocated target range data which stored in the computer of system and then placed into the integrator of tracking system is known the external target allocation mode.

Until the target allocation and determination data don't perform operation of conduct with acceptable accuracy, reformation on the target range determination system also will be performed by an operator. Reformation will be done by a manual rotation or hand wheel.[8]

Conduction must be done by observed image on the range- velocity monitor, when reformation is done by an operator.

In case of accuracy of conduct on the target, the range tracking system should act in such a way that the target is settled between the Range Mark domain and on the screen.

The local conduct mode engages to control the amount of range received from target allocation stage and performs operation of conduct of range by a handwheel which can transfer range domain.

The range will be transferred regularly, until we change hand wheel. Amount of range is delivered to semi-automatic tracking range device that when this system integrates from range coordinate, sends it to automatic tracking range system.

This amount of range is delivered to the tracking system to implement the local conduct mode and the speed of hand wheel rotation is main factor in the local conduct mode. Therefore, the measure of range variations on the screen corresponds with the speed of variation and the handwheel rotation which be entered to the tracking system.

The range sensor that is called manual wheel or hand wheel, can perform the range fine or course reformation for tracking system.

In tracking mode, the target range error signal effects on the range determination system, which controls

and regulates the range- gate middle delay by ϕ_B , ϕ_H , Σ chanals.

4.2. Basic principles of using vernier

In first of this section, we will examine the basic principles and process of using Vernier with an example and will verify finally our findings with a numerical example.

In the Vernier method, the pulse repetition frequency is selected in such a way that has smaller pulse repetition periods compared with total time of signal emission and the target reception. Accordingly, determination of range domain of target will arise ambiguity in range. [9]

The ambiguity in range creates several problems for the target automatic tracking to choose a range without ambiguity. Indeed, as soon as target moves, the pulses which returned by that are coincided and corresponded with those signals observed in the periods corresponding to the ambiguous range and gated pulses.

During emission, the receiver will be out of reach in order to prevent the returned signal.

The signals reflected from target do not receive at this time and the target tracking systems can't act properly. In radar, advisable activities have been predicted to prevent this problem that its explanation exceeds the topic of this section. To determine the amount of correct target range, the Vernier frequencies method is implemented in radar. General principles of this method will be explained as follows.

In this method, the pulses sent by radar, act periodically on two repetition frequencies. First is main frequencies and another is selected among group is corresponded with modes that is multiples of main frequency for having correct number of periodicity in T_{unique} period.

$$T_{\text{unique}} = n_1 PRI_1 = n_2 PRI_2 \quad (16)$$

T_{unique} is collective period in which the ambiguity in range will not to be existed.

In principles of tracking Vernier, we also will evaluate the average and course size Vernier method. We will explain that the number of n_1 and n_2 periods, it is better to differ in unit size with each other in the large size Vernier method that this causes to create the maximum amount of T_{unique} . This matter will be verified by using of transactions as follows.

In this case, when the returned signals caused by main frequency are received with t_1 delay, the pulses caused through range gate are settled on the target-returned signals with t_1 delay and will be corresponded with returned signal too.

To explain the Vernier frequencies method which is to divide degrees by part, the timing diagram of emitted pulses, the target-returned signals and gates of tracking will be examined and n values will be selected to enhance understanding of this topic.

Figure 2 will describe this point by ($n_1=5$, $n_2=4$) values.

The a and b Figure 2 displays the received packets of pulses with repetition courses deals

$$T_1 = 1/f_1, T_2 = 1/f_2 (T_1 = T_u/n_1, T_2 = T_u/n_2) \quad (17)$$

In this figure, part c and part d will engage to represent the target-returned signals packages with t_1 delay in comparison to emitted pulses for both repetition frequencies.

Similarly, part e and part f engage to the range gate packages which is settled on the target-returned signals with t_1 delay in comparison to the emitted pulses and these parts lead to gate the signals returned by target.

Also, h and g diagrams of this figure focus on packages of the same range gates with t_1+T_1 delay, that is, they advance the returned range gate of the T_1 size and is placed exactly on the gated signal. Then, the gate order will be shifted of the T_1 size, which caused by second signal, that is mean, gating of both is done from same point and they are overlapped in the especial multiple; that point will be the unambiguity range of our target.

Hence, it can be inferred that we should fix error and difference rate between the target-returned signals through delaying the range gate.

If one error resulted from deviation between the actual range of target and the range is gained by gating, take places during a frequency period, transferring to other frequencies will cause to mismatch between gates and the target-returned signals the Δ dimension [10]

Δ is the time difference between the range gates adjusted in the repetition period of T_1 and the target-returned signals in the repetition period of T_2 .

$$\Delta = \frac{T_u}{n_2} - \frac{T_u}{n_1} = \frac{n_1 T_u - n_2 T_u}{n_1 * n_2} = \frac{T_u}{n_1 * n_2} \quad (18)$$

That is, in equation (18) in Δ direct dependency with the T_u as well.

In cases where the error is the size of K period T_1 , relation (19) will be established.

$$\Delta = K * \frac{T_u}{n_1 * n_2} \quad (19)$$

Therefore, determination of unambiguous range of target will be done when $\Delta = 0$. In this case, two returned signals will be overlapped in one space and that point will be position of unambiguous range.

To increase the sensitivity of the Vernier frequencies method and the mobility of the target marks on the different repetition frequencies, in addition to large scale, an important scale also is employed by using major and minor frequency to make a difference between n values of more than unity size which is equal to a greater integer and $(n_1-n_3) = N$.

In this case, the existence of an error in the single range determination will enable us to decrease the greatest amount of unambiguous range of the N size.

So, other measures are used to determine total of uniform range.[11]

To facilitate the process, the measure which is corresponded to the middle single range (T'_0) is considered as average measure and the measure is used as the course measure which employed to determine all of uniform ranges.

We have to establish the following conditions, if we tend to use the Vernier frequencies method:

$$\frac{T'_o}{T_o} = 0, m \quad \frac{T'_o}{T} = N, \frac{m}{2} \quad (20)$$

Where T'_0 is period of new unambiguous range and T_0 indicates total unambiguous range and N is also an integer.

In this equation, m indicates the tenth decimal which obtained after dividing and equals $m < 1$.

In the total of covered domain, if amount of unambiguous range be greater than amount of the

average single range in the $\frac{1}{m}$ quantity, then the

adopted frequency period will be satisfaction based on the following conditions:

Interval of new unambiguous range T'_0 should include integer (N) of T periods and the remaining

period must be equal $\frac{m}{2}$.

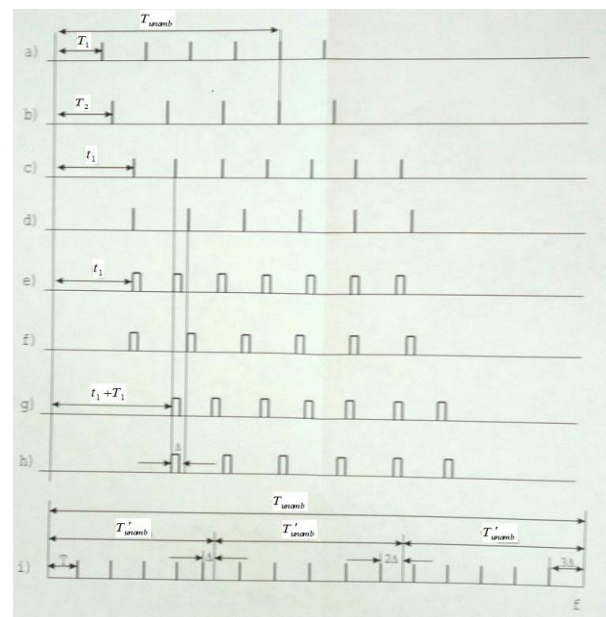


Fig. 2. Representation of the Vernier diagram on the target signal

To descript and explain this, we will evaluate figure 2:

In figure 2, to illuminate the subject, we suggest that amount of unambiguous range is greater than the new unambiguous range three times (0, m= 0.33) which has been determined based on average measure.

In this condition, T₀ internal includes a numerical value of the adopted frequency periods and the remaining time of that period is (0, m/2= 0.16= Δ).

By this diagram, it can be concluded that the mismatch between gate and the returned signal is great and a great error will be happened in determination of range based on average measure.

If an error is happened in a distance, then the mismatch between gate and signal is 0.16 of period (m/2= 0.16) or (0.16* T) and maximum error will be (3×0.16≈0.5) due to repetition of three of T₀' intervals in T₀ interval.

Accordingly, similar to fine and average Vernier methods it is necessary to create a mismatch (Δ=0) to determine the general unambiguous range. In this case, the requested gate delay is regulated in the stairs of multiples of the main frequency period.

The process of the target unambiguous range determination is operated in the controlled chanal and it can be used for modes of conduct of systems and the target tracking.

In the Vernier method, we will study based on the average and coarse measures. In the coarse measure, the difference between n values equals one but in the average measure, this difference is two [12].

When the difference between n values is more than unit, the unambiguous range is decreased according the difference of n values and the error rate will be greater.

We apply the average method to increase accuracy in the operation of decline in the error of gate and the returned signal.

Here, we will examine the Vernier method for small periods because this method should be analyzed in the system tracking discussion and in this section, we will represent only one example of how the system operates [13].

As mentioned in the previous section, we will have two periods as follows:

$$\begin{aligned}
 T_1 &= 10 \\
 T_2 &= 12 \\
 T_{unique} &= n_1PRI_1 = n_2PRI_2 \quad (21) \\
 n_1 &= 6, n_2 = 5 \\
 T_1 &= \frac{T_u}{n_1} = \frac{T_u}{6} \\
 T_2 &= \frac{T_u}{n_2} = \frac{T_u}{5} \\
 T_{unique} &= (6)(10) = (5)(12) = 60
 \end{aligned}$$

Since the difference between n₁ and n₂ is of the unity size, so we have used the course Vernier which will have the greatest unambiguous range in 60.

Equation (22) will show that if we have one error period, the difference between the range gates regulated on the target in the repetition period T₁ and the target-returned signals in the repetition period T₂ will be 2.

$$\begin{aligned}
 \Delta &= \frac{T_u}{n_2} - \frac{T_u}{n_1} = \frac{n_1T_u - n_2T_u}{n_1 * n_2} = \frac{T_u}{n_1 * n_2} \\
 \Delta &= \frac{60}{5} - \frac{60}{6} = \frac{6(60) - 5(60)}{(5)(6)} = \frac{60}{30} = 2 \quad (22)
 \end{aligned}$$

But if we have the error in the several periods, the difference between the range gates regulated in the repetition period T₁ and the target-returned signals in the repetition period T₂ will be equation (23):

$$\Delta' = K * \frac{T_u}{n_1 * n_2} \Rightarrow \Delta' = K \Delta \Rightarrow \Delta' = k * \frac{60}{30} \Rightarrow \Delta' = 2k \quad (23)$$

We will use one example to finely illustrate this matter.

According to figure (2), we send two minor and major pulses and receive the signals returned by part a in the part c and the pulses returned by part b in the part d.

In this case, the returned signals are received with t₁ delay, which resulted from the minor and major frequencies, therefore the pulses that created by the range gate with t₁ delay will be settled on the target-returned signals and corresponded to the returned signals[14].

This will be happened in the part e and part f. In the part g and part h, we engage to the pulse delay of the range gate of the t₁+T₁ size, when the difference between n values is equal unit because when we shift the pulse e of the T₁ size, that is means we shift the range gate of the T₁ size and this range gate is settled

exactly on the signal. Then, we advance the gate order that resulted from second signal of the T_1 size and this means that the gating of both is done from same point and they are overlapped on the especial and common multiple and this point will be our unambiguous range of target.

This means that we must fix the error which resulted from mismatch of the target signal in the T_1 frequency period with the range gate caused from the T_2 frequency period.

for $T_1 = 10$ 0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150
 for $T_2 = 12$ 0 12 24 36 48 60 72 84 96 108 120 130 ...
 $\Delta = 12 - 10 = 2 \Rightarrow K = 1 \Rightarrow$ first period

Figure 3 error in a period between gate and signal As seen in the figure (3), when the range cells are received in the screen in the receiver of tracker, which resulted from the T_1 and T_2 as above, in the cell of first period, the error rate is equal 2 in the first period, therefore because they differ from each other in a period, if the range gate resulted from T_2 is shifted of the T_1 size, in this case, both will begin to gate from one point and will overlap in one especial multiple. Here, we observe that both of target will overlap in the cell of 70. So, the unambiguous range will be obtained.

5 CONCLUSION

In this study, we tried to provide different methods to eliminate ambiguity in range for the high pulse repetition frequency radars. Although, there are enormous variety of these methods, each of these have their advantages and disadvantages and will effect on the accuracy of measurement. But our proposed method which is the Vernier method has enabled to improve accuracy using the error enlargement in the different states as well as to eliminate correctly the ambiguity in range.

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