Phase Noise Calculation in CMOS Single-Ended Ring Oscillators

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ABSTRACT:
All oscillators are periodically time varying systems, so to accurate phase noise calculation and simulation, time varying model should be considered. Phase noise is an important characteristic of oscillator design and defined as the spectral density of the oscillator spectrum at an offset from the center frequency of the oscillator relative to the power of the oscillator. Linear time invariant (LTI) and linear time variant (LTV) model's for calculating phase noise in ring oscillator is discussed. In this paper a new technique based on LTV model for impulse sensitivity function (ISF) calculation and thus phase noise estimation in CMOS single-ended ring oscillator is presented. This method is simpler than other methods and ISF can be simulated and calculated easily. Good results between theory and simulation is observed.

KEYWORDS: CMOS single-ended ring oscillator, Impulse sensitivity function, ISF, Phase noise.

1. INTRODUCTION

Ring oscillators are an essential building block in many digital and communication systems. They are used as voltage-controlled oscillators (VCO's) in applications such as clock recovery circuits for serial data communications, phase-locked-loops (PLL) for clock and data recovery, disk-drive read channels, on-chip clock distribution, and integrated frequency synthesizers.

With the growth of ring oscillator applications and stringent performance requirements, the issue of exact phase noise calculation in oscillators has become an important consideration for oscillators design. Phase noise is closely related to the performance of oscillators. Naturally the phase noise of a whole complex system can be measured, but it can be closely approximated by adding the phase noises of different oscillators together.

Phase noise is the undesired and uncontrolled fluctuations of the phase of signal. While phase noise is defining the doubt in the frequency domain, jitter is used to define the doubt in time domain. The factors that contribute of the phase noise of an oscillator can be classified into two categories. The first is the random factors that create random variations of timing of the signal edges. Major part of this noise originates from thermal noise and flicker noise of active and passive devices that constitute the circuits. The second is the systematic factors that generally be avoided by a careful design of the system. Systematic variations are mostly because of interfering signals from other parts of the integrated system [1]. Hajimiri and Lee [2] have proposed a time varying model based on the impulse sensitivity function to predict phase noise. This technique provides insight into the oscillators design. In this article a simple method for ISF calculation and simulation for CMOS single-ended oscillators is proposed.

The organization of this paper is as follows. In Section 2, the phase noise is described. In section 3, the phase noise models for ring oscillators are studied. The proposed theory is described in Section 4. The simulations results are provided in Section 5 and finally the conclusion is given.

2. PHASE NOISE

Phase noise is usually characterized in the frequency domain. In oscillators, there are fluctuations in amplitude and phase because of internal and external noise. The amplitude fluctuations are remarkably attenuated by the amplitude limiting mechanism which is present in any practical stable oscillator and is very strong in ring oscillators [3]. Therefore, focus on phase variation will be considered.

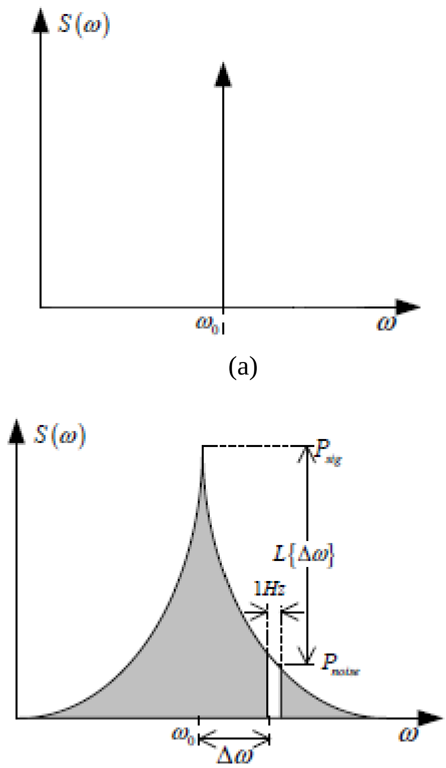
The spectrum of an ideal oscillator with no random fluctuations is like a pair of impulses at $\pm\omega_0$, Fig. 1(a). In a practical case, the output is more generally given by:

$$V_{out}(t) = A.f\left[\omega_0 t + \phi(t)\right] \tag{1}$$

Where $\phi(t)$ is random phase which is function of time and f is a periodic function with period $2\pi/\omega_0$. As a consequence of the fluctuation represented by $\phi(t)$, the spectrum of practical oscillator has sidebands close to the frequency of oscillation, Fig. 1(b). To quantify phase noise, consider a unit bandwidth at an offset $\Delta\omega$ with respect to ω_0, calculate the noise power in this bandwidth, and divide the result by the signal power, Fig. 1(b).

$$L(\Delta\omega) = 10\log\left[\frac{P_{noise(\omega_0+\Delta\omega,1Hz)}}{P_{sig}}\right],\left(\frac{dB_c}{Hz}\right) \tag{2}$$

Where P_{noise} is the noise power and P_{sig} is the signal power.



(a)

(b)

Fig.1. The definition of phase noise,
(a) ideal oscillator, (b) practical oscillator.

3. PHASE NOISE MODELS FOR RING OSCILLATORS

Several techniques have been proposed for estimation of phase noise in oscillators. The first models are presented based on LTI analysis. These models are not qualified to model the phase noise of ring oscillators which are nonlinearity and time variance. Recent models consider time variance for more accurate phase noise analysis in ring oscillators.

3.1. LTI MODELS

A. Leeson

In 1966, Leeson presented a phase noise model [4] based on an LTI assumption for tuned tank oscillators. Phase noise value, $L(\Delta\omega)$, which is based on Leeson modified model is given by:

$$L(\Delta\omega) = 10\log\left\{\frac{2FkT}{P_s}.\left[1+\left(\frac{\omega_0}{2Q_L\Delta\omega}\right)^2\right] .\left(1+\frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)\right\} \tag{3}$$

Where F is an empirical parameter (often called the “device excess noise number”), k is Boltzmann’s constant, T is the absolute temperature, P_s is the average power dissipated in the resistive part of the tank circuit, ω_0 is the oscillation frequency, Q_L is the effective quality factor of the tank with all the loadings in place, $\Delta\omega$ is the offset from the carrier and $\Delta\omega_{1/f^3}$ is the frequency of the corner between the $1/f^3$ and $1/f^2$ regions as shown in the sideband spectrum of Fig. 2. Since Leeson’s model is based on LTI model, it is not suitable for cyclostationary noise sources. Also this model has an empirical parameter.

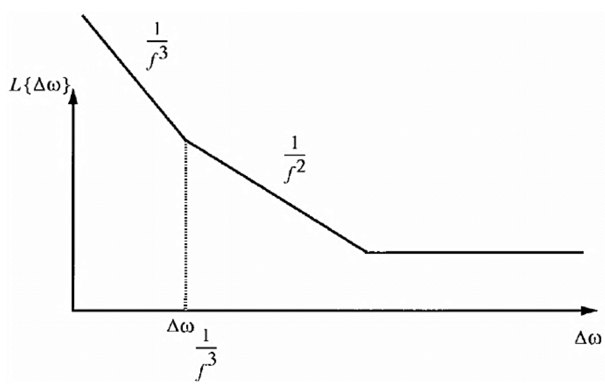


Fig. 2.Phase noise power spectral density regions of an oscillator versus offset from carrier [2].

B. Razavi

B. Razavi proposed a phase noise model for inductor less VCOs and is well suited for CMOS ring oscillators. In [5], [6] he proposed a new definition for *Q* factor, which makes Leeson's model applicable to inductor less oscillators. If an oscillator is modeled as in Fig. 3and open loop transfer function is assumed as $H(j\omega)=A(\omega)e^{j\phi(\omega)}$, an open-loop *Q* factor is defined as follows:

$$Q=\frac{\omega_0}{2}\sqrt{\left(\frac{dA}{d\omega}\right)^2+\left(\frac{d\phi}{d\omega}\right)^2} \tag{4}$$

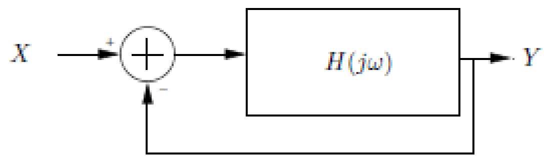


Fig. 3.Linear oscillatory system [5].

Consequently, the phase noise for an N-stage ring oscillator is given by [7]:

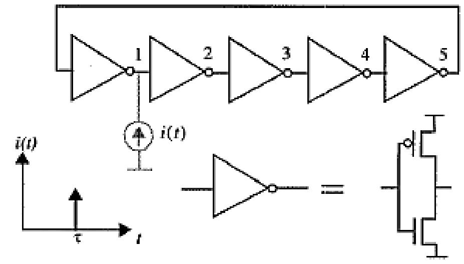
$$L(\Delta\omega)=\frac{2FkTN}{P_s}\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2 \tag{5}$$

The *Q* factor for a 3-stage ring oscillator is 1.3 and the *Q* factor for a 4-stage ring oscillator is 1.4. However, *Q* is not the only factor that determines the phase noise. A 4-stage ring oscillator has more noise sources than a 3-stage ring oscillator because of more delay stages. It also has to dissipate more power than a 3-stage ring oscillator with the same load capacitance [7].

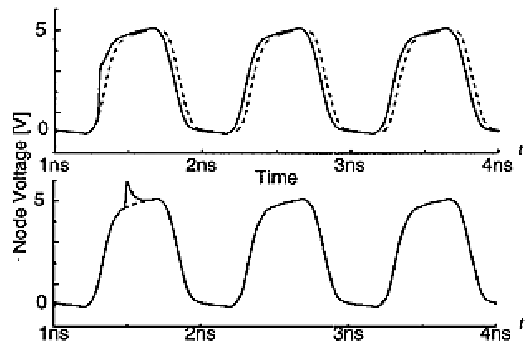
3.2. LTV MODEL

Hajimiri and Lee

The model of Leeson and Razavi suppose the oscillator circuit as a linear system; therefore, they are not precise. Hajimiri and Lee develop a general theory for phase noise calculation [2].The advantage in this model is it does not depend on any oscillator topology, and presents a normalized metric, the Impulse Sensitivity Function, with which one can compare relative performance between different oscillators. For example, consider the single-ended oscillator with a single current source on one of the nodes shown in Fig. 4(a). Suppose that the current source consists of an impulse of current with area Δq occurring at time $t=\tau$. If a current impulse injected to the circuit, the amplitude and phase of the oscillator will be affected similar to Fig. 4(b) and (c).



(a)



(b)

Fig. 4.(a) Five-stage ring oscillator (b) Current impulse injected at the peak changes the amplitude and has no effect on the phase, (c) Current impulse injected at zero crossing changes the phase and has minimal effect on the amplitude [3].

The instantaneous voltage change is given by [3], [8]:

$$\Delta V=\frac{\Delta q}{C_{node}} \tag{6}$$

Where C_{node} is the effective capacitance on that node at the time of charge injection. Phase shift is proportional to the voltage change ΔV, and hence to the injected charge Δq. Therefore $\Delta\phi$ can be written as [3]:

$$\Delta\phi=\Gamma(\omega_0 t)\frac{\Delta q}{q_{\max}}=\Gamma(\omega_0 t)\frac{\Delta V}{V_{swing}} \tag{7}$$

Where $q_{max}=C_{node}V_{swing}$ and V_{swing} is the voltage swing across the capacitor. The function $\Gamma(\omega_0 t)$, is the time-varying "proportionality factor". It is called the impulse sensitivity function, since it determines the sensitivity of the oscillator to an impulsive input. It is a dimensionless, frequency- and amplitude-independent periodic function that describes how much phase shift results from applying a unit impulse at any time [2].

Suppose the unit impulse response of the system as the amount of phase shift per unit current impulse, as [9]:

$$h_\phi(t,\tau)=\frac{\Gamma(\omega_0 t)}{q_{\max}}u(t-\tau) \tag{8}$$

Using equation (8) and the superposition integral, excess phase $\phi(t)$ by the following equation can be calculated:

$$\begin{aligned}\phi(t)&=\int_{-\infty}^{+\infty}h_\phi(t,\tau)i(\tau)d\tau=...\\ &=\frac{1}{q_{\max}}\int_{-\infty}^{t}\Gamma(\omega_0 t)i(\tau)d\tau\end{aligned} \tag{9}$$

The ISF is a periodic function, so it can be expanded into a Fourier series:

$$\Gamma(\omega_0 t)=\frac{c_0}{2}+\sum_{n=1}^{\infty}c_n Cos(n\omega_0\tau+\theta_n) \tag{10}$$

Where the coefficients are real valued, and θ_n is the phase of the n_{th} order harmonic, that can be overlooked. We can consider what happens when a sinusoidal current $i(\tau)=I_m Cos\left[\left(m\omega_0+\Delta\omega\right)t\right]$ for $m=0,1,2,...$ inject into anode. Suppose that $\Delta\omega \ll \omega_0$, therefore $\phi(t)$ is given by [9]:

$$\begin{aligned}\phi(t)&=\frac{c_0}{2q_{\max}}\int_{-\infty}^{t}I_m Cos\left[\left(m\omega_0+\Delta\omega\right)t\right]d\tau+...\\ &+\sum_{n=1}^{\infty}\frac{c_n}{q_{\max}}\int_{-\infty}^{t}I_m Cos\left[\left(m\omega_0+\Delta\omega\right)t\right]Cos(n\omega_0\tau)d\tau\end{aligned} \tag{11}$$

The only component of the integral which is preserved is for *n*=*m*. Therefore:

$$\begin{aligned}\phi(t)&=\frac{c_0 I_m Sin\left[(m\omega_0+\Delta\omega)t\right]}{2q_{\max}(m\omega_0+\Delta\omega)}+...\\ &+\frac{c_m I_m Sin\left[(2m\omega_0+\Delta\omega)t\right]}{2q_{\max}(2m\omega_0+\Delta\omega)}+\frac{c_m I_m Sin(\Delta\omega)t}{2q_{\max}(\Delta\omega)}\end{aligned} \tag{12}$$

Note that first and second terms are the negligible if $m\neq 0$, and if m = 0, the second and third terms are zero since the lowest Fourier coefficient c_n is c_1. Final value for $\emptyset(t)$ can be presented as [2], [9]:

$$\phi(t)\approx\frac{c_m I_m Sin(\Delta\omega)t}{2q_{\max}(\Delta\omega)} \tag{13}$$

Mathematically, the phase noise at a Δf offset from ω_0 arising from a white noise source of square magnitude $\overline{i_n^2}/\Delta f$ is equal to:

$$L\{\Delta\omega\}=10\log\left(\frac{\overline{i_n^2}/\Delta f}{8q_{\max}^2\Delta\omega^2}.\sum_{n=0}^{\infty}C_n^{\ 2}\right) \tag{14}$$

Note that I_m represents the peak amplitude, hence $I_n^2/2=\overline{i_n^2}/\Delta f$, for $\Delta f=1\,Hz$. Using Parseval's theorem we have:

$$\sum_{n=0}^{\infty}C_n^{\ 2}=\frac{1}{\pi}\int_0^{2\pi}\left|\Gamma(x)\right|^2 dx=2\Gamma_{rms}^2 \tag{15}$$

Where Γ_{rms} is the root mean square (rms) value of Γ(x) .Hence phase noise for a current noise is:

$$L\{\Delta\omega\}=10\log\left(\frac{\overline{i_n^2}/\Delta f\ .\Gamma_{rms}^2}{4q_{\max}^2\Delta\omega^2}\right) \tag{16}$$

- **Calculation of the ISF for Ring Oscillators**

To calculate phase noise using (16), needs to know the rms value of the ISF. To estimate Γ_{rms}, suppose that the ISF is triangular in shape and that its rising and falling edges are symmetric [9] as shown in Fig. 5.

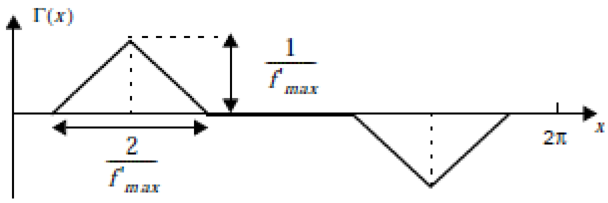


Fig. 5.Approximate ISF for ring oscillators [9].

The ISF has a maximum of $1/f'_{\max}$, where $f'_{\max}$ is the maximum slope of the normalized waveform f in (1). Therefore Γ_{rms} is given by:

$$\begin{aligned}\Gamma_{rms}^2&=\frac{1}{2\pi}\int_0^{2\pi}\Gamma^2(x)dx=...\\ &=\frac{4}{2\pi}\int_0^{1/f'_{\max}}x^2dx=\frac{3}{2\pi}\left(\frac{1}{f'_{\max}}\right)\end{aligned} \tag{17}$$

Stage delay is proportional to the rise time:

$$t_D=\frac{\eta}{f'_{\max}} \tag{18}$$

Where t_D is the stage delay normalized to the period and η is a proportionality constant, which is typically close to unity. The period is *2N*times longer than a single stage delay:

$$2\pi=2Nt_D=\frac{2N\eta}{f'_{\max}} \tag{19}$$

Using (17) and (19), the following approximate expression for Γ_{rms} is obtained:

$$\Gamma_{rms} = \sqrt{\frac{2\pi^2}{3\eta^3}} \cdot \frac{1}{N^{1.5}} \quad (20)$$

$1/N^{1.5}$ term in above equation is independent of the value of η. For single-ended ring oscillator $\eta = 0.75$ and for differential ring oscillators $\eta = 0.9$. Fig. 6 shows the shape of the ISF for a group of single-ended CMOS ring oscillators.

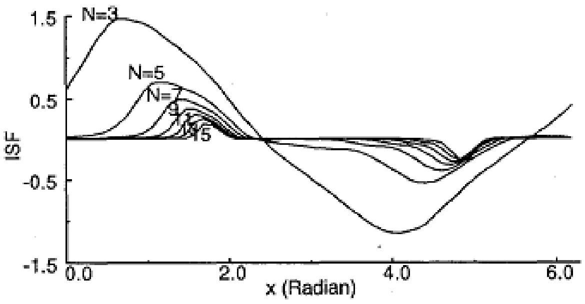


Fig. 6.ISF for single-ended CMOS ring oscillators of the same frequency with different number of stages [8].

4. PROPOSED METHOD

According to equation (1), ISF for CMOS single-ended ring oscillators can be obtained using the following equation approximately:

$$ISF = \ldots$$

$$= \begin{cases} \dfrac{1}{A^2\omega_0} \cdot \dfrac{dV_{out}(t)}{dt}, & N = 3 \\ \dfrac{1}{\beta} \cdot \displaystyle\sum_{i=1}^{i=N-2} \left(\dfrac{1}{A^2\omega_0} \cdot \dfrac{dV_{out}(t)}{dt} \right)^i, & N \ge 5 \end{cases} \quad (21)$$

Where A is the maximum voltage amplitude, ω_0 is the oscillation frequency and β is ISF correction factor which is given by:

$$\beta = \left(0.6 + \frac{N}{10} \right)^{N-5} \quad (22)$$

In order to investigate the accuracy of the proposed method, single-ended ring oscillator is simulated using ADS software in 0.18 µm CMOS process, the circuit is shown in Fig. 7. Γ_{rms} is calculated, and is shown in Table 1. The frequency of oscillation is kept constant (through adjustment of channel length), while the number of stages is varied from 3 to 9 (in odd numbers).According to the Table 1, the proposed method has high accuracy.

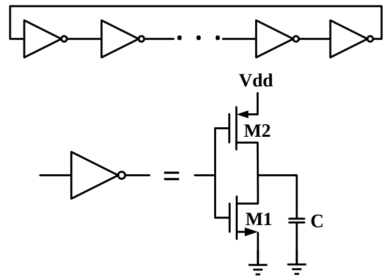


Fig. 7.N-stage single-ended ring oscillator.

Table 1.RMS Values of the ISF.

Number of stages	3	5	7	9
Proposed method	0.752	0.340	0.203	0.124
Hajimiri method	0.760	0.353	0.213	0.146
Proposed method in [10]	0.752	0.283	0.185	0.102

5. SIMULATION RESULTS

For compares the prediction and simulation of the ISF and phase noise:

1- Consider 5-stage single ended 384.6 MHz ring oscillator (Fig. 7) that simulated using ADS software in 0.18 µm CMOS process. According to circuit data gate oxide thickness $t_{ox} = 4.08nm$ and threshold voltages $V_{t,NMOS} = 0.475\ V$ and $|V_{t,PMOS}| = 0.45\ V$. All five inverters are similar with $(W/L)_{M1} = 3.2\,\mu m/0.8\mu m$ and $(W/L)_{M2} = 6.4\,\mu m/0.8\mu m$ .The $C_{ox} \approx 8.4 fF/m^2$, $V_{dd} = 1.8\ V$ and the total capacitance on each node $C_{tot} = 0.5\ pF$, therefore, $q_{max} = 0.93\ pC$ .Also $\left(\overline{i_n^2}/\Delta f\right) = (8/3).kT\,\mu C_{ox}\,(W/L)\left[(V_{dd}/2) - V_t\right]$ (sum of the current noise powers due to NMOS and PMOS devices)thus $\left(\overline{i_n^2}/\Delta f\right) = 6.2 * 10^{-24} A^2/Hz$ .Using proposed method $\Gamma_{rms}^{tot} = 0.34$ while using Hajimiri method $\Gamma_{rms} = 0.353$ (using proposed method in [10] ISF is 0.297). With inserting the values in equation (16), $L\{\Delta f\} = 10\log\left(0.0052/\Delta f^2\right)$ .The simulation results are shown in Fig. 8.In the all oscillators, ISF has its maximum value near the zero crossings of the oscillation, and a zero value at maxima of the oscillation waveform [11].

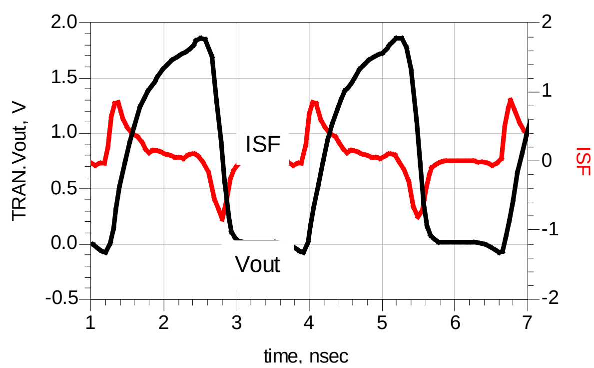


Fig. 8.Output waveform and ISF waveform.

2- Consider 5-stage single ended 53.9 MHz ring oscillator that simulated using ADS software, Fig. 9.According to circuit data gate oxide thickness $t_{ox} = 9.5nm$ and threshold voltages $V_{t,NMOS} = 0.665\ V$ and $|V_{t,PMOS}| = 0.9\ V$. All five inverters are similar with $(W/L)_{N1} = 8\ \mu m/1\mu m$, $(W/L)_{N2} = 4\ \mu m/1.5\mu m$ and $(W/L)_P = 8\ \mu m/1\mu m$. Note that $C_{ox} \approx 13.11 fF/m^2$,$V_{dd} = 2\ V$ and the total capacitance on each node $C_{tot} \approx 0.1\ pF$,therefore, $q_{max} = 0.2\ pC$.

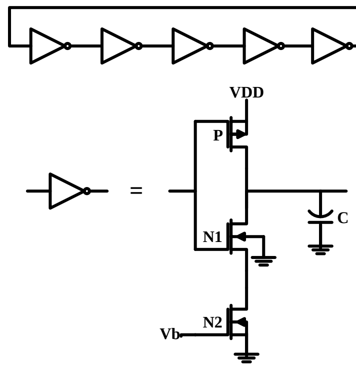


Fig. 9.5-stage single ended 53.9 MHz ring oscillator.

Using proposed method $\Gamma_{rms} = 0.358$ while using Hajimiri method $\Gamma_{rms} = 0.353$ (using proposed method in [10] ISF is 0.281). Also $\left(\overline{i_n^2}/\Delta f\right)_{tot} = 17.36*10^{-24}\ A^2/Hz$ thus $L\{\Delta f\} = 10\log\left(0.352/\Delta f^2\right)$.The simulation result is shown in Fig. 10. Phase noise is calculated and simulated and is shown in Table 2 and Fig. 11.

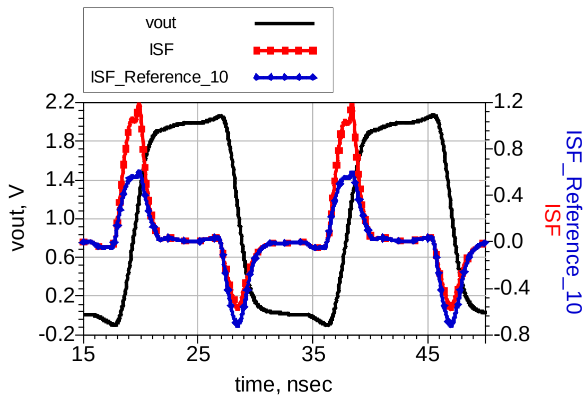


Fig. 10.5-Output waveform and ISF waveform.

Table 2. Phase Noise Values at 1 MHz Frequency.

	Proposed	**Hajimiri**	**simulation**	**[10]**
Phase noise @ 1 MHz, dBc	-124.53	-124.657	-125.1	-126.63

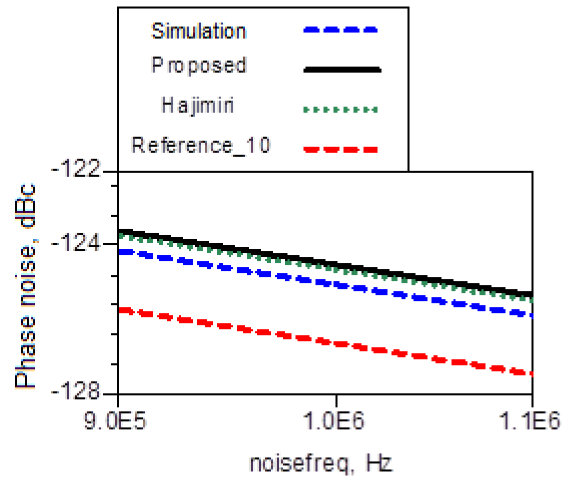


Fig. 11.Phase noise.

3-Consider 7-stage single ended 2.87 GHz ring oscillator, (Fig. 7). Using proposed method $\Gamma_{rms} = 0.22$ while using Hajimiri method $\Gamma_{rms} = 0.213$ (using proposed method in [10] ISF is 0.184). The output waveform and ISF waveform are shown in Fig. 12.

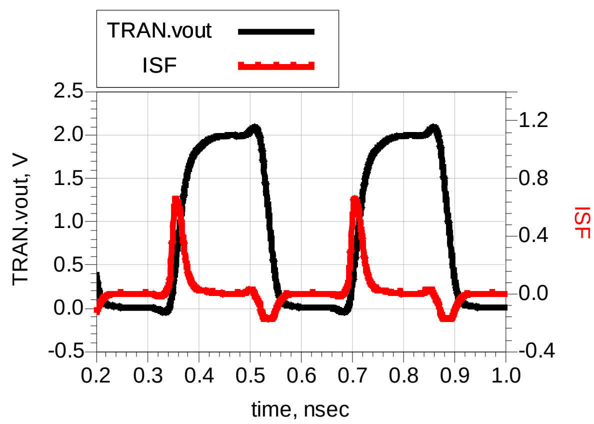


Fig. 12.Output waveform and ISF waveform.

Other ring oscillators in 0.18 μm CMOS process are simulated and in Table 3 results are presented.

Table 3. Other Simulation.

		Proposed	**Hajimiri**	**[10]**
9-stage 2.23 GHz	**ISF (RMS)**	0.132	0.146	0.108
	Phase noise @ 1 MHz, dBc	-151.05	-150.18	-152.8
11-stage 1.83 GHz	**ISF (RMS)**	0.0903	0.1082	0.0723
	Phase noise @ 1 MHz, dBc	-154.35	-152.78	-156.3

6. CONCLUSION

The Hajimiri phase noise analysis employs a linear time variant model for the oscillator. It gives physical insight into how device noise contributes to the overall phase noise. A new method based on Hajimiri method for analysis ISF and phase noise of single-ended ring oscillators was presented. According to simulation results, proposed method in this paper is almost exact and for simulating and calculating ISF is easier and faster than other methods. Note that proposed method, derived from a Hajimiri method and this method is suitable for simulating the ISF.

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