Comprehensive Comparison and Evaluation of Interpolation Methods for Channel Estimation without Channel Statistical Information in OFDM Systems

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ABSTRACT:
Orthogonal frequency division multiplexing (OFDM) is a multicarrier transmission technology in wireless environment that use a large number of orthogonal subcarriers to transmit information. In order to be able to demodulate OFDM signals accurately, channel estimation is needed for received signals. It is important to select a suitable interpolation method for estimation block. The goal of this paper is compering the methods of interpolation in OFDM system without using channel statistics information. Therefore, in this paper, we used pilots for obtaining the information of channel, and by the method of estimation without use of channel statistics information, the channel primary frequency response estimated in pilot's frequencies. At the end, the whole channel frequency response estimated of pilot frequency response by the different interpolation methods. In this paper, different interpolation methods such as Linear, Second order, Spline, Cubic Hermite and Low pass interpolation are compared.

KEYWORDS: OFDM- estimation- pilot- interpolation

1. INTRODUCTION

By deployment of information transfer rate and bandwith limitation, we seek methods to optimize the use of bandwith. Orthogonal frequency division multiplexing (OFDM) is a multicarrier transmission technology in wireless environment that use a large number of orthogonal subcarriers to transmit information. In this system, the information with the high bit rate send on several subcarriers with low bit rate in parallel. Orthogonally of subcarrier helping to bandwith limitation and the other hand preventing of information interference. At this system, the channel is a kind of wireless channel and assumed as frequency selective channel that often modeled on the base of Rayleigh fading with additive white Gaussian noise (AWGN) [1]. The channel is named frequency selective if the channel coherence bandwith was smaller than bandwith of transmitted signal, when the carrier information signal sending on the channel. In this case, the channel cause the distortion in transmitted signal [2].

The signal in output of channel has a large distortion than the input signal caused by fading and additive noise parameters in channels, finally the appeared error is extended. So is required to design equalizer in the receiver on the base of the certain characteristic of channel to reduce these effects. As for that the characteristic of channel is time variable, so required that the channel characteristic estimate alternately and corresponding to the given estimation, the output signal equalized to reduce the engender distortion [3].

Channel estimation of OFDM system divided in two ways:

1-blind channel estimation2-pilot based channel estimation

Blind channel estimation without knowing the information of the transmitted data, explores the statistical information of channel and certain properties of transmitted signal. It is focusing on the correlation between the data sent and received. Although it yields higher spectral and power efficiencies by using blind channel estimation but it needs more data to analyze.it just effective at this condition. Hence it is suitable for slow varying channel and excellent for applications where bandwidth is limited. This is clearly a disadvantage in the case of mobile wireless because the time-varying channel would preclude accumulation of a large amount of data [1-4- 5].

2. PILOT BASED CHANNEL ESTIMATION

Pilots are a kind of educational data and In order to obtain the channel information inserted into some fixed positions of signals sent by transmitter. Pilot symbols is known by transmitter and receiver. The pilot symbol sent by transmitter makes spectral efficiency and power utilization lower with the trade-off of quick response to the channel variation. For inserting the pilot in OFDM symbols, two major methods were proposed: Block-type and Comb-type [1].

2.1. Block-type pilot

If the channel has a slow variation that it means the coherence time was larger than the length of transmitted symbol, we can alternate channel estimation in certain interval and helping of this estimation for data estimation in next time interval symbols. At this method for channel estimation, pilots or educational data send at all of subcarriers and the receiver can obtain frequency or time information of channel by them. You can see block pilot in Fig. 1 [3].

2.2. Comb-type pilot

If the channel has a fast variation that it means the coherence time was smaller than the length of transmitted symbol, the channel estimation must doing in each transmitted symbol interval. In this way for channel estimation, some pilots with certain distance along with data send in each OFDM symbol and on the base of these transmitted pilots, the channel frequency information calculated in certain points of frequency at the receiver, and with using current interpolation methods the other channel frequency points can be calculated. You can observe comb pilot in Fig.2 [3].

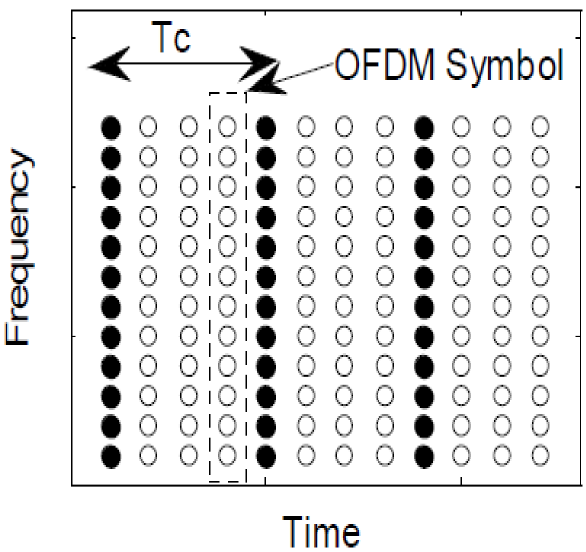

Fig. 1. Block-type pilot

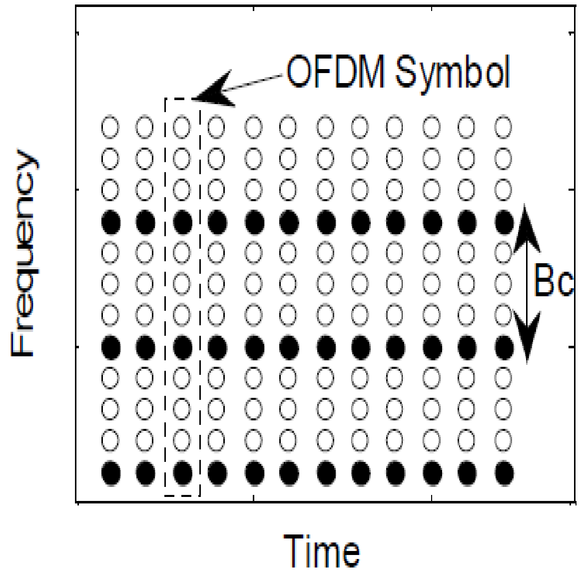

Fig. 2. Comb-type pilot

3. OFDM SYSTEM IN CHANNEL ESTIMATION BASED ON PILOT

OFDM system block diagram in channel estimation based on pilot showed in Fig. 3. First binary information are mapped according to modulation. Then serial to parallel conversion takes place. After that, pilot subcarriers are inserted along with data subcarrier. Finally, the samples of $X(k)$ are sending to the next block, as you see at (1):

$$X(k) = X(m N_f + L) = \begin{cases} x_p(m) & l = 0 \\ Data & l = 1,2,\ldots N_f - 1 \end{cases} \quad (1)$$

$x_p(m)$ is the value of m^{th} subcarrier pilot and N_f is the frequency interval of inserted pilot.

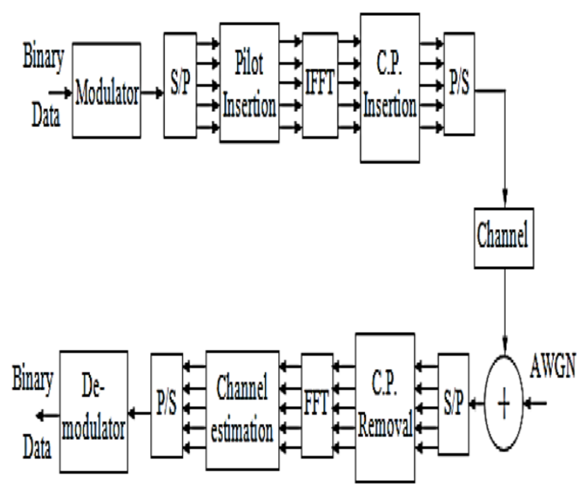

Fig. 3. OFDM system block diagram

Then IFFT is applied on $X(k)$ samples that transform frequency domain samples $X(k)$ into time domain $x(n)$. Use of guard interval or insert CP is the next level at each OFDM symbol that this prevent of inter symbol interference (ISI). The operation is that copy the N_g number of last each OFDM symbol at the first of it, as you see at (2):

$$x_g(n) = \begin{cases} x(N_{FFT}+n) & n = -N_G, -N_G+1, \ldots, -1 \\ x(n) & n = 0,1,\ldots N_{FFT}-1 \end{cases} \quad (2)$$

The noticeable point is that the length of CP must be larger than maximum channel delay that can limited the ISI and ICI. Then serial to parallel conversion takes place, signal is send and received signal in the receiver is $y_g(n)$, as you see at (3):

$$y_g(n) = x_g(n) \otimes h(n) + w(n) \quad (3)$$

$w(n)$ represents additive white Gaussian noise and $h(n)$ represents Impulse response of frequency selective multi-path channel. After serial to parallel conversion, removal of guard interval takes place from $y_g(n)$. Then reach FFT block de-multiplex the multicarrier signals and frequency domain samples $Y(k)$ are received. Then received pilot subcarrier $Y_p(k)$ achieved from $Y(k)$. The next level is getting primary estimation of channel or $H_p(k)$.

As respect of, that the paper's goal is channel estimation without use of channel statistics information; and as often the signal has large variance in each part of the path in OFDM system, we use channel estimation methods based on comb pilot (without use of channel statistics information) for achieving $H_p(k)$. So we can obtain whole channel estimation or $\widehat{H}(k)$ by using of interpolation methods.

Transmitted data samples or $X(k)$, calculated as (4). At the final, map inverse is done to getting binary data at receiver's output [6]-[7].

$$X(k) = \frac{Y(k)}{\hat{H}(k)} \quad (4)$$

Estimated performance based on comb pilot could be improved with more pilots. Because increasing the number of subchannels for the pilot is equivalent to reduce the interval between the channels, so that the correlation between two adjacent pilots increases, and estimated values of subchannel characteristic [1].

3.1. LS estimation

This is the simplest estimator used for channel estimation. In this method without notice to noise, ICI and knowledge of channel statistics information, channel estimation is done. LS based, channel transfer function $H_{p,ls}$ can be shown as (5):

$$H_{P,LS} = X_P^{-1} Y_P = \begin{bmatrix} \frac{Y_P(1)}{X_P(1)} & \frac{Y_P(2)}{X_P(2)} & \cdots & \frac{Y_P(N_P)}{X_P(N_P)} \end{bmatrix}^T \quad (5)$$

Where X_p and Y_p are input and output values of pilot subcarrier. LS has low computational complexity as compared to other channel estimators, but its main drawback is high MSE value [6].

3.2. DFT estimation

At this method that the goal is optimized the given result of LS method, we use of maximum channel delay and eliminate the external noise of the range of channel. The operation is dividing the received impulse response from LS method into the noise and Impulse response sub area, as show in (6). Then noise area eliminated and finally the level of noise effect on estimation factors is reduced.

$$h_{LS}(n) = \begin{cases} h(n) + w(n) & 0 \le n \le L-1 \\ w(n) & otherwise \end{cases} \quad (6)$$

In (6), L is maximum length of channel delay and the result of estimation can be better by the eliminate the external noise of the range of channel that shown in (7) [3].

$$h_{DFT}(n) = \begin{cases} h(n) + w(n) & 0 \le n \le L-1 \\ 0 & otherwise \end{cases} \quad (7)$$

4. INTERPOLATION

After that pilot subchannel frequency response estimated by no statistical methods such as DFT, LS and …, used the interpolation in frequency domain to obtain final channel estimation. Different method of interpolation cause different degree of accuracy and with more accuracy, the complexity is increase.

4.1. Nearest-neighbor interpolation

This method is a simplest interpolation method in one or more dimensions. The Nearest neighbor algorithm selects the value of the nearest point and does not consider the values of neighboring points at all. This method is not suitable for simple issues, but it is fast and simple way in more dimensions that issues have more complexity [8].

4.2. Linear interpolation

One of the simple methods of interpolation is Linear interpolation. Generally, use two data points in Linear interpolation which the Linear interpolant is the straight line between two points. This paper use two adjacent pilots for channel estimation by Linear interpolation as seen at (8) [1].

$$\hat{H}(k)=\hat{H}(mN_f+L)=(\hat{H}_P(m+1)-\hat{H}_P(m))\frac{1}{N_f}+\hat{H}_p(m) \quad (8)$$

$$m=0,1,\ldots,N_p-1$$

Linear interpolation is for one spatial dimension. For two spatial dimensions, Bilinear interpolation used that is an extension of Linear interpolation for interpolating functions of two variables on a 2D grid, And The key idea is to perform Linear interpolation first in one direction, and then again in the other direction. Although each step is linear but the Bilinear interpolation as a whole is not linear and quadratic. For three spatial dimensions, Trilinear interpolation is an extension of Linear interpolation on a 3D grid. In practice, a Trilinear interpolation is identical to three successive Linear interpolation [9].

4.3. Second Order Interpolation

Second order interpolation is better than Linear interpolation. channel estimation with Second order interpolation is calculated by used linear combination of three adjacent pilots and for this reason has better operation and more complexity than Linear interpolation because of it's increasing of order. This interpolation defining as shown at (9):

$$\hat{H}(k)=\hat{H}(mN_f+L)=c_1\hat{H}_P(m-1)+c_0\hat{H}_P(m)+\hat{H}_p(m)+c_{-1}\hat{H}_p(m+1) \quad (9)$$

$$m=0,1,\ldots,N_p-1$$

Second order interpolation factors defining at (10) [1].

$$c_1=\frac{\alpha(\alpha-1)}{2},c_0=-(\alpha-1)(\alpha+1),c_{-1}=\frac{\alpha(\alpha+1)}{2},\alpha=\frac{1}{N_f} \quad (10)$$

4.4. Spline interpolation

$S(x)$ is an n^{th} order Spline interpolation function in the [a, b] interval with nodal points such as x_0, x_1, …, x_n as this interval divided to n subinterval [x_i, x_{i-1}] and $a=x_0 \prec x_1 \prec \ldots \prec x_{n-1} \prec x_n = b$ and for each subinterval, $S(x)$ is equal to $f_i(x)$ and other point is that the $S(x)$ and its derivatives are continues to $(n-1)^{th}$ order in [a, b] interval.

The most commonly used Splines are Cubic Spline that has third order. As you seen, this interpolation equal to (11), that x_i, x_{i-1} are two adjacent pilots. M_i, M_{i-1} are function's second derivatives on this two pilots.

$$S(x)=(\chi_i-x)\left[\frac{(\chi_i-x)^2-h_i^2}{6h_i}\right]M_{i-1}+(x-\chi_{i-1})\left[\frac{(x-\chi_{i-1})^2-h_i^2}{6h_i}\right]M_i+\frac{1}{h_i}(\chi_i-x)f_{i-1}+\frac{1}{h_i}(x-\chi_{i-1})f_i \quad (11)$$

$$h_i=\chi_i-\chi_{i-1}$$

$$i=1,\ldots,n$$

The Spline curve has monotone figure for the reason that the curve made of third order polynomial curve's derivatives inter of both input points [10]- [11].

4.5. Cubic Hermite Spline interpolation

A Cubic Hermite Spline is a Spline where each piece is a third-degree polynomial specified in Hermite form. Cubic Hermite Splines are typically used for interpolation of numeric data specified at given argument values $x_1, x_2, \ldots, x_n$, to obtain a smooth continuous function. The data should consist of the desired function value and derivative at each x_k. The Hermite formula is applied to each interval [x_k, x_{k+1}] separately. The resulting spline will be continuous and will have continuous first derivative. This interpolation define as seen at (12):

$$P(x)=h_{00}(t)p_0+h_{10}(t)m_0+h_{01}(t)p_1+h_{11}(t)m_1$$

$$t=\frac{x-\chi_k}{\chi_{k+1}-\chi_k} \quad (12)$$

Where (h) s are Hermite basis functions. These can be written in different ways, each way revealing different properties. Some samples of Hermite basis function's calculating methods shown at Table 1.

p_0, p_1 are the function values and m_0, m_1 are first order derivatives in pilot points x_k, x_{k+1}; that can achieved by several methods such as finite difference, Cardinal Spline and Catmull-Rom Spline [12].

Bernstein polynomials of order 3 in Table 1 are shown at (13).

$$B_k(t)=\binom{3}{k}\cdot t^k\cdot(1-t)^{3-k} \quad (13)$$

4.6. Low pass interpolation

Low pass interpolation add L zeros to the sequence of main data and cause to increase sampling rate; then use FIR low pass filter with π/L Cut-off frequency, to the main data passed not variable and interpolation is done. Low pass interpolation does minimization of the mean-square error between the interpolated points and their ideal values [13].

Table 1. Hermite basis functions

	expanded	factorized	Bernstein
$h_{00}(t)$	$2t^3-3t^2+1$	$(1+2t)(1-t)^2$	$B_0(t)+B_1(t)$
$h_{10}(t)$	t^3-2t^2+t	$t(1-t)^2$	$\frac{1}{3}\cdot B_1(t)$
$h_{01}(t)$	$-2t^3+3t^2$	$t^2(3-2t)$	$B_3(t)+B_2(t)$
$h_{11}(t)$	t^3-t^2	$t^2(t-1)$	$-\frac{1}{2}\cdot B_2(t)$

5. CONCLUSION

In this paper we analyzed the estimated channel interpolation methods than not used channel statistic information for channel estimation.

Different method of interpolation cause different degree of accuracy and with more accuracy, the complexity is increase.

According to the result of simulation, as shown in Fig.4, the Second order interpolation has better operation and less bit error rate than Linear interpolation. The reason is that the Second order interpolation used linear combination of three adjacent pilots, but Linear interpolation used linear combination of two adjacent pilots and this multiplication of order cause to increase accuracy and complexity. The Cubic Spline interpolation even has better operation and less bit error rate than Second order interpolation but more complex, for that it is higher at orders, neither use second order derivation [1].

According to the result of simulation, as shown in Fig.5, Cubic Hermite Spline interpolation and Cubic Spline interpolation have same operation; but the first case has less calculating complexity and time than the second. The reason of this difference is that Cubic Spline calculated second order derivation but Cubic Hermite spline just calculated first order derivation [14].

According to the result of simulation, as shown in Fig. 6, the Low pass interpolation has better operation than Spline interpolation. Low pass interpolation has less error than other interpolation methods and minimize MSE between interpolated points and its ideal values [13].

Therefore, in choosing the methods of interpolation, it is important to notice priority of paragons such as complexity or simplicity, more or less accuracy and calculating time, to select best method.

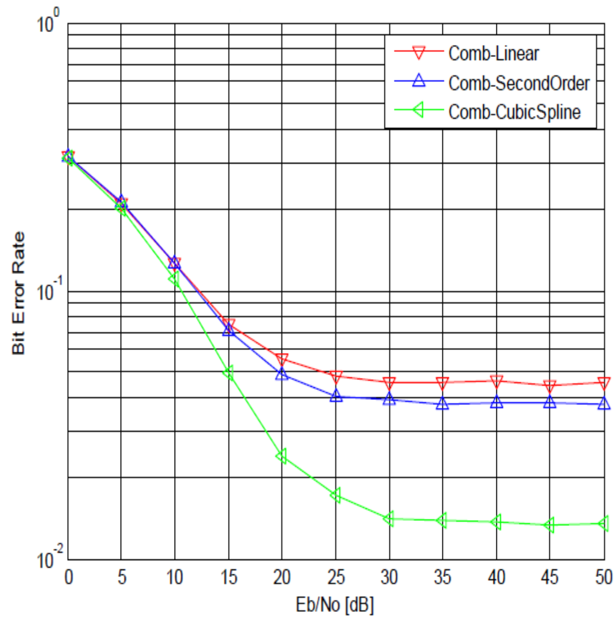

Fig.4. Comparison of 3 Interpolation methods

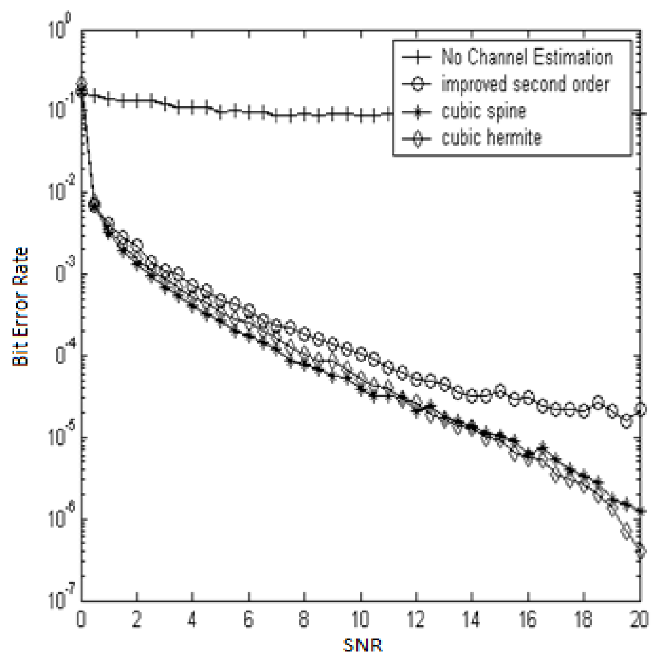

Fig.5. Comparison of 3 Interpolation methods

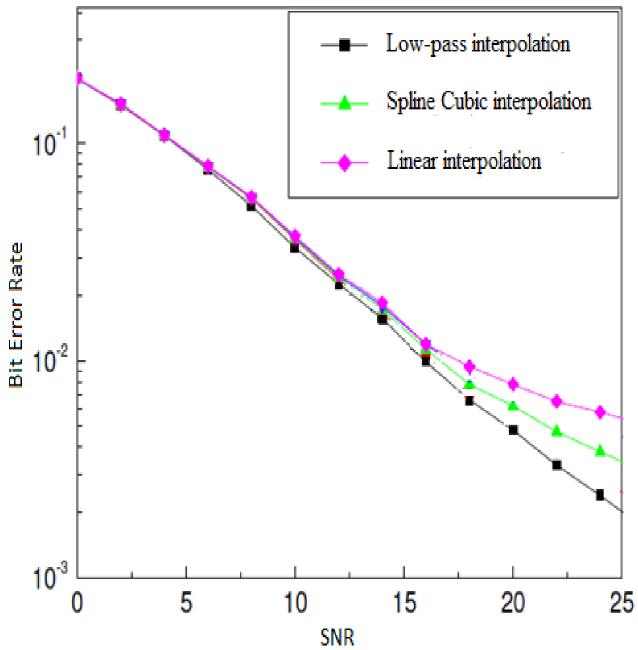

Fig.6. Comparison of 3 Interpolation methods

REFERENCES

[1] Fei Wang, **"A Thesis entitled Pilot-Based Channel Estimation in OFDM System,"** *University of Toledo*, pp. 1-95, May 2011.

[2] John G. Prokis and Masoud salehi, **"Digital communication"**, 5th edition 2007, *published by McGraw-Hill*.

[3] A .asadi and B. mozafari **"channel estimation by Wavelet transform in OFDM system"**, 1392.

[4] R. Hajizadeh, K. Mohamedpor and M.R. Tarihi, **"Channel Estimation in OFDM System Based on the Linear Interpolation, FFT and Decision Feedback"**, *18th Telecommunications forum Telfor* 2010, pp. 484-488, November 23-25, 2010.

[5] Rohit Negi and John Cioffi, **"Pilot Tone Selection for Channel Estimation in A Mobile OFDM System"**, *IEEE Transactions on Consumer Electronics*, Vol. 44, No. 3, pp. 1122-1128, AUGUST 1998.

[6] A. M. Khan, Varun Jeoti and M. A. Zakariya, **"Improved Pilot–based LS and MMSE Channel Estimation using DFT for DVB-T OFDM Systems"**, 2013 *IEEE Symposium on Wireless Technology and Appliction (ISWTA)*, pp. 120-124,September 22-25, 2013.

[7] Hala M. Mahmoud, Allam S. Mousa and Rashid Saleem, **"Channel Estimation Based in Comb-Type Pilots Arrangement for OFDM System over Time Varying Channel"**, *Journal Of Networks*, Vol. 5, No. 7, pp. 766-772, July 2010.

[8] https://en.wikipedia.org/wiki/Nearest-neighbor_Interpolation.

[9] https://en.wikipedia.org/wiki/Linear_Interpolation.

[10] dif-antegral pdf, Available at http://rozup.ir/up/stcomputer/studownload/dif-antegral.pdf.

[11] https://en.wikipedia.org/wiki/Spline_(mathematics).

[12] https://en.wikipedia.org/wiki/Cubic_Hermite_spline.

[13] A.Z.M. Touhidul Islam and Indraneel Misra, **"Performance of Wireless OFDM System with LS-Interpolation-Based Channel Estimation in Multi-Path Fading Channel"**, *International Journal on Computational Sciences & Applications (IJCSA)*, Vo2, No.5, pp. 1-10, October 2012.

[14] Dun CAO, Hongwei DU and Ming FU, **"Cubic Hermite Interpolation-based Channel Estimator for MIMO-OFDM"**, *Journal of Computational Information Systems 6:14 (2010) 4699-4704*, pp. 4699-4704, Available at http://www.Jofcis.com.