Synchronization new chaotic system using optimal nonlinear controller

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ABSTRACT:

Chaos is one of the most important phenomenons based on complex nonlinear dynamics. In this paper, we investigate chaos synchronization problems of a new 3D chaotic system. Based on the Lyapunov stability theory, a nonlinear control approach is presented for the synchronization between two three- dimensional chaotic systems. This system is a new three-dimensional autonomous chaotic system which is different from the other existing attractors and has larger Lyapunov exponent than the Lorenz system. In order to optimize this controller, Genetic algorithm is applied. Genetic algorithm detects the best optimized values for gain of this controller. Fitness function is also considered as Total Square Error between master and slave systems. Numerical simulations show the effectiveness of the presented controller.

KEYWORDS: New Chaotic System, Nonlinear Controller, Synchronization, Genetic Algorithm, Fitness Function.

1. INTRODUCTION

Chaos, as a very interesting and complex nonlinear phenomenon, occurs widely in both natural and man-made systems. Chaos synchronization, an important topic in non-linear science, has been developed and studied extensively in the last few years. Nowadays, many different techniques and methods have been proposed to achieve chaos synchronization. In [1], based on stability criterion of linear system and Lyapunov stability theory respectively, the chaos synchronization problems for energy resource demand–supply system are discussed using two different control methods. [2] Presented the synchronization of chaos by designing united controller. Chaos synchronization of a new 3D chaotic T system was investigated via three different methods in [3], active control and back stepping design methods are applied when system parameters are known, and adaptive control method was employed when system parameters are unknown or uncertain. Based on Lyapunov stability theory and Ruth–Hurwitz criteria, [4] have been proposed a generalized active control for synchronizing non-identical parametrically excited oscillators in master–slave configuration. In [5], a novel parameter identification and synchronization method has been proposed for the hyper chaotic systems with unknown parameters via the Lyapunov stability theory. In [6], the direct design method based on tree diagonal structure has been used to synchronize chaotic systems. [7] Focused on the synchronization of a class of master–slave chaotic systems with uncertainty and disturbance. Back stepping design was proposed for adaptive synchronization of a class of chaotic systems with unknown bounded uncertainties in [8]. [9] Investigated the modified projective synchronization (MPS) of a new hyper chaotic system. The different nonlinear feedback controllers were designed by an active control method for synchronization of two hyper chaotic systems with the same or different structures. [10] Demonstrated that hyper chaos synchronization between two identical hyper chaotic Chen systems and between two identical hyper chaotic Lu systems using active control was achieved. In [11], based on the stability theory of fractional-order differential equations and Lyapunov equations for fractional-order systems, two criteria are provided to ensure the inverse projective synchronization between fractional-order hyper chaotic Lorenz system and fractional-order hyper chaotic Chen system. Based on the nonlinear control theory, the anti-synchronization between two different hyper chaotic systems was investigated in [12].

The paper is organized as follows: Section 2 describes new chaotic system. Section3. Presents nonlinear control method. Section4. Nonlinear controller is designed to synchronize new chaotic system. Section5. Genetic algorithm is used to optimize the controller. Section6. Presented the numerical Simulation results. Section7. Provides the conclusion.

New 3D Autonomous Chaotic System

Recently, Huibin Lu and Xia Xiao [13] derived a new 3D chaotic system that takes the following form.

$$\dot{x} = a(y - x) + yz$$
$$\dot{y} = -bxz + cx \qquad\qquad (1)$$
$$\dot{z} = dxy - ez$$

Where $a = 20, b = 5, c = 40, d = 4, e = 3$ are constant parameters of system (1) and x, y, z are state variables with the initial conditions $[2,3,6]^T$. Dynamical behavior including is displayed in Figure 1 and 2.
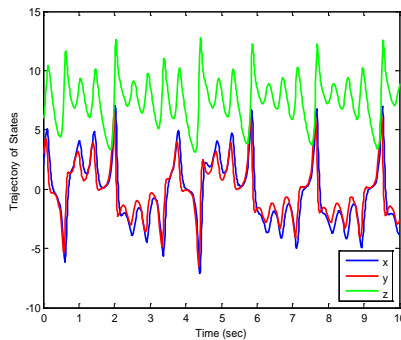
Figure1 and 2.

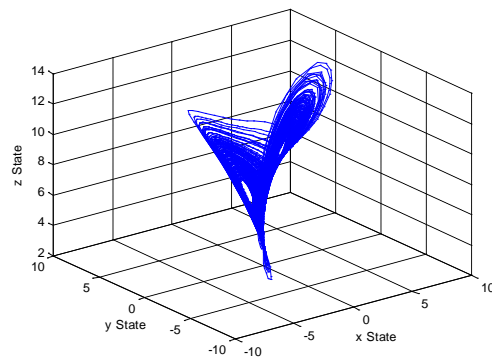Fig .1: Phase portrait of system (1).

Fig.2: States trajectory variation of system (1).

Design of controller via nonlinear control method

In this section we will propose a nonlinear control scheme to investigate the nonlinear synchronization between two identical chaotic systems. Consider the following system described by

$$\dot{x} = Ax + Bf(x) \qquad\qquad (2)$$

Where $x \in R^n$ is the state vector, $A \in R^{n \times n}, B \in R^{n \times n}$ are metrics and vectors of system parameters, and $f: R^n \rightarrow R^n$ is a nonlinear function. Equation (2) is considered as a drive system. By introducing an

additive control $u \in R^n$, then the controlled response system is given by

$$\dot{y} = Ay + Bg(y) + u \qquad\qquad (3)$$

Where $y \in R^n$ denotes the state vector of the response system and $g: R^n \rightarrow R^n$ is a nonlinear Function the synchronization problem is to design a controller u which synchronizes the states of both the drive and response systems. We subtracting equation (2) from equation (3) and get

$$\dot{e} = Ae + B\big(g(y) - f(x)\big) + u \qquad\qquad (4)$$

Where $e = y - x$, the aim of the synchronization is to make $\lim_{t \to \infty} \|e(t)\| = 0$. Then let Lyapunov error function be $V(e) = \frac{1}{2} e^T e$, where $V(e)$ a positive definite function is. Assuming that the parameters of the drive and response systems are known and the states of both systems are measurable, we may achieve the synchronization by selecting the controller u to make the first derivative of $V(e), i. e, \dot{V}(e) < 0$. Then the states of response and drive systems are synchronized asymptotically globally [12].

Synchronization of the New Chaotic System

In order to achieve the behavior of the synchronization between two new chaotic systems by using the proposed meted, suppose the drive system takes the following form

$$\dot{x}_1 = a(y_1 - x_1) + y_1 z_1$$
$$\dot{y}_1 = -bx_1 z_1 + cx_1 \qquad\qquad (5)$$
$$\dot{z}_1 = dx_1 y_1 - ez_1$$

and the response system is given as follows

$$\dot{x}_2 = a(y_2 - x_2) + y_2 z_2 + u_1$$
$$\dot{y}_2 = -bx_2 z_2 + cx_2 + u_2 \qquad\qquad (6)$$
$$\dot{z}_2 = dx_2 y_2 - ez_2 + u_3$$

Where $u_1(t), u_2(t)$ and $u_3(t)$ are control functions to be determined for achieving synchronization between the two systems (5) and (6). Define state errors between systems (5) and (6) as follows

$$e_x = x_2 - x_1$$
$$e_y = y_2 - y_1 \qquad\qquad (7)$$
$$e_z = z_2 - z_1$$

We obtain the following error dynamical system by subtracting the drive system (5) from the response system (6).

$$\dot{e}_x = a(e_y - e_x) + y_2 z_2 - y_1 z_1 + u_1$$
$$\dot{e}_y = ce_x - b(x_2 z_2 - x_1 z_1) + u_2 \qquad (8)$$
$$\dot{e}_z = -ee_z + d(x_2 y_2 - x_1 y_1) + u_3$$

In order to determine the controller, let

$$u_1 = u_{11} + u_{12}, \; u_{12} = y_1 z_1 - y_2 z_2$$
$$u_2 = u_{21} + u_{22}, \; u_{22} = b(x_2 z_2 - x_1 z_1) \quad (9)$$
$$u_3 = u_{31} + u_{32}, \; u_{32} = -d(x_2 y_2 - x_1 y_1)$$

Then we rewrite (8) in the following form:

$$\dot{e}_x = a(e_y - e_x) + u_{11}$$
$$\dot{e}_y = ce_x + u_{21} \qquad (10)$$
$$\dot{e}_z = -ee_z + u_{31}$$

By taking a Lyapunov function for equation (10) into consideration

$$V(e) = \frac{1}{2} e^T e \qquad (11)$$

We get the first derivative of $V(e)$:

$$\dot{V} = e_x(a(e_y - e_x) + u_{11}) + e_y(ce_x + u_{21}) + e_z(-ee_z + u_{31}) \qquad (12)$$

Therefore, if we choose u as follows:

$$u_{11} = -k_1 e_x - ce_y$$
$$u_{21} = -k_2 e_y - ae_x \qquad (13)$$
$$u_{31} = -k_3 e_z$$

Then

$$\dot{V} = -(a + k_1)e_x^2 - k_2 e_y^2 - (e + k_3)e_z^2 \quad (14)$$

Where $\dot{V}(e) < 0$ is satisfied. Since $\dot{V}(e)$ is a negative-definite function, the error states $\lim_{t \to \infty} \|e(t)\| = 0$. Therefore, this choice will lead the error states e_x, e_y, e_z to converge to zero as time t tends to infinity and hence the synchronization of two new chaotic systems is achieved.

Genetic Algorithm

The genetic algorithms are used to search the optimal parameter k in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response [14-16]. Table (1) shows the genetic algorithm parameters. The fitness function used is.

$$f(e_x, e_y, e_z) = \frac{1}{3}\sqrt{\int \left(e_x^2(t) + e_y^2(t) + e_z^2(t)\right) dt} \qquad (15)$$

by the training, we obtain these optimal parameters.

$$k_1 = 21, k_2 = 6.5, k_3 = 25.5 \qquad (16)$$

Numerical Simulation

State trajectory between master and slave systems for states x, y, z are indicated in Figure 3,4,5 respectively. Also error trajectory between two systems and control signals u_1, u_2, u_3 are respectively displayed in Figure 6 and 7.
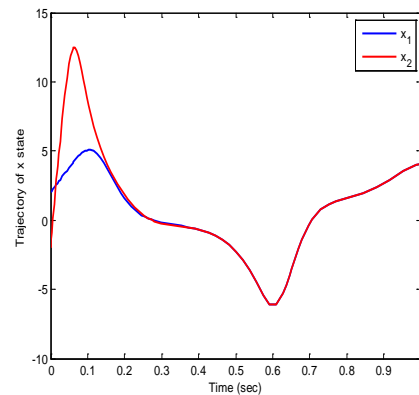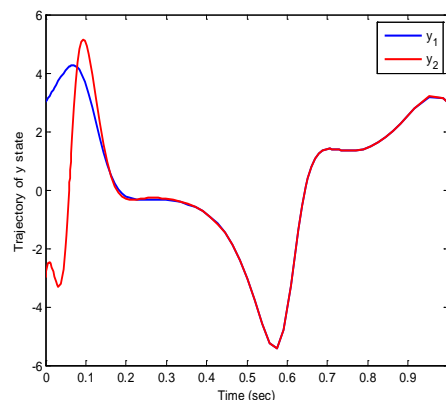

Fig.3: Time response of the state x.

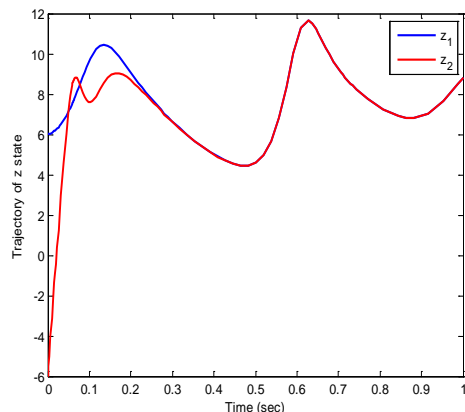Fig.4: Time response of the state y.
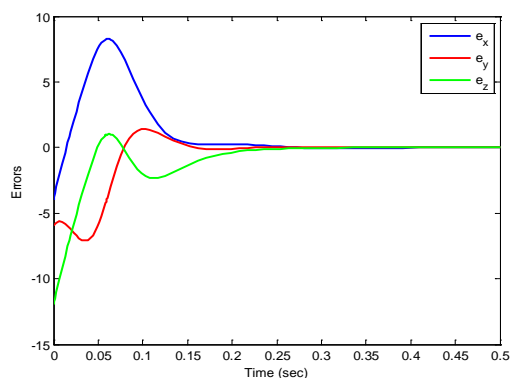
fig.5: Time response of the state z.

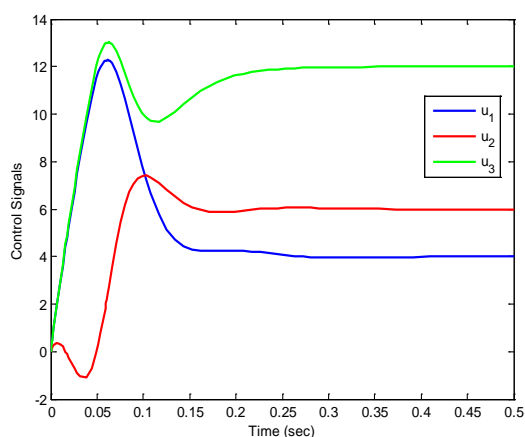fig.6: Error trajectory between master and slave system

Fig.7. Control signals for synchronization of the master and slave systems.

Conclusion

In this paper, we investigate chaos synchronization problems of a new 3D chaotic system. Synchronization method has been proposed for the new three-dimensional autonomous chaotic system which is different from the other existing attractors and has

larger Lyapunov exponent than the Lorenz system via the Lyapunov stability theory. In order to find the best values for gain of the controller, Genetic algorithm was utilized. Genetic algorithm minimizes Fitness function. Also this function is considered based on Total Square Error. Numerical stimulations are used to verify the effectiveness of the proposed control methods.

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