Adaptive Beamforming Using Projection Method in Large-Scale Antenna Arrays

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ABSTRACT:
Due to geometrical errors, complexity, and interference, the steering vector in large-scale antenna arrays needs to be accurately calculated. Beamforming methods based on adaptive processing can effectively overcome complexities and limitations of large-scale arrays. Here, we present the projection-based parallel linearly-constrained minimum variance (PBPLCMV) algorithm which calculates the steering vector of the desired signal through projecting the assumed steering vector onto the signal subspace. Then, the optimum weights of the adaptive beamformer are calculated using the parallel linear constrained minimum variance (PLCMV) algorithm. The simulation results show that for low input SNRs, the proposed method has better performance and output SINRs up to 18 dB more than the output SINRs of the parallel robust recursive linearly constrained minimum variance (PRRLCMV).
KEYWORDS: Adaptive beamforming, Large-Scale array, PLCMV, Projection method.

1. INTRODUCTION

In many array systems, the adaptive beamforming is used for receiving the desired signal from a particular direction, and for cancelling the interference signals. The performance of beamforming algorithms improves by increasing the number of the array elements; however, the complicacy of the calculations and the rate of data transmission are also increased [1]. Moreover, the array gets more sensitive to imperfections like mismatch of direction of arrival (DOA) and the array geometry error [2].

To overcome the abovementioned weaknesses, many methods have been suggested so far which can be classified into different groups. One type of the methods are based on partially adaptive processing [2-3]; in these methods, a portion of the dimensions of the adaptive array is used for adaptation, which reduces the computational complicacy, and increases the convergence speed; however, the performance of the beamforming is reduced due to the lowered degrees of freedom. Another type of the methods is based on processing of sub-arrays, in which an adaptive array is divided into many sub-arrays, and each sub-array independently adjusts its adaptive weights. For the performance of these methods to be similar to the main beamformer, the number of the elements of the smallest sub-array should be more than the number of the signal sources, which then makes it difficult to reduce the complicacy of the calculations. This condition is not necessary for the efficient recursive least square (ERLS) algorithm [4] which indeed, unlike traditional subarray- based methods, the adaptive weights of each sub-array is adjusted with the use of the data received in that sub-array and the average results of other sub-arrays. In this algorithm, an appropriate decomposition of the weight vector enhances the performance of the beamforming. Nonetheless, this algorithm will suffer from more complicate calculations than the traditional recursive least square if there is a high number of sub-arrays.

One of the methods based on the sub-array processing is the parallel linearly constrained minimum variance (PLCMV) algorithm put forward in 2005 on the basis of the linearly constrained minimum variance (LCMV) method [5]. Although the number of the degrees of freedom of the PLCMV algorithm is the same as that of the ERLS method, the later suffers from more expensive calculations. In the LCMV method, the optimum weight vector is obtained through minimizing the output power under a linear constraint determined with the previous knowledge of the steering vector of the desired signal [6]. However, in practice, the actual steering vector is not known, and thus, the PLCMV uses an assumed steering vector instead. This assumption can be invalid due to some imperfections in the array, and consequently, lead to errors in the steering vector, which finally translates into low performance of the PLCMV algorithm [5].

The parallel robust recursive linearly constrained minimum variance (PRRLCMV), proposed in 2009 [7], models imperfections of an array as generalized phase errors in the steering vector, and employs a gradient-based method [8] to search for the actual steering vector; thereafter, the optimum weights in each sub-array are obtained with the use of the LCMV algorithm. The PRRLCMV algorithm fails to function effectively in low signal-to-noise ratios (SNRs).

A recently suggested new projection-based method estimates the actual steering vector through projecting the assumed steering vector onto the signal space [9]; this method does not require any estimation of the number of the sources, and enjoys better performance than the traditional projection-based methods, even in low SNR ratios.

In this paper, we propose a new method called projection-based parallel linearly constrained minimum variance (PBPLCMV) which extracts the optimum weight vector through employing the PLCMV algorithm and estimating the actual steering vector with the use of the projection method presented in Ref. [9]. Our simulations results show that, in comparison with the PRRLCMV algorithm, the present method has better performance, and gives a more accurate estimation of the actual steering vector for low signal-to-noise ratios. The rest of the paper is organized as follows; the signal modeling is presented in section 2, followed by the steps of the PLCMV algorithm in section 3; our proposed algorithm is presented in section 4; thereafter, section 5 gives the simulations results obtained using MATLAB software, and finally, the paper ends with the conclusion in section 6.

2. SIGNAL MODELING

Assume that M narrow-band signals collide with a uniform linear array (ULA) having N elements spaced half a wavelength from each other. The $N\times 1$-dimensional vector of the received signal at time t can be written as:

$$x(t)=As(t)+n(t) \tag{1}$$

where $A=\left[a(\theta_1),...,a(\theta_M)\right]$ is the response matrix of the $N\times M$ array; $n(t)$ is the $N\times 1$-dimensional additive white Gaussian noise; $s(t)$ is the $M\times 1$-dimensional vector of the received signal; $a(\theta_i)$ is the array steering vector of input signal $s_i(t)$ with the angle θ_i :

$$a(\theta_i)=\left[1,e^{j\pi\sin\theta_i},...,e^{j\pi(N-1)\sin\theta_i}\right]^T \tag{2}$$

where $(.)^T$ is the transpose operator.

The correlation matrix of the array is obtained as follows:

$$R_{xx}=E\left[xx^H\right]=AR_{ss}A^H+\sigma_n^2 I \tag{3}$$

where $(.)^H$ is the conjugate transpose operator; E is the expectation; R_{ss} is the source correlation matrix, and I is the identity matrix.

The output of the array is obtained as the product of the conjugate transpose of the weight vector of the array and the received signal vector:

$$y(t)=w^H x(t) \tag{4}$$

3. THE PLCMV ALGORITHM

In the PLCMV algorithm, the $N\times 1$-dimensional vector of the received signal $x(k)$ is divided into M subsections [5]:

$$x(k)=\left[x_1^T(k),x_2^T(k),...,x_M^T(k)\right]^T \tag{5}$$

where $x_i(k)$ is an $N_i\times 1$ vector for N_i , $i=1,2,...,M$ elements of the arrays in the *i*th subsection so that $\sum_{i=1}^{M}N_i=N$.

The assumed steering and weight vectors are also divided using Eqs. (6) and (7).

$$a(\theta_p)=\left[a_1^T(\theta_p),a_2^T(\theta_p),...,a_M^T(\theta_p)\right]^T \tag{6}$$

$$w(k)=\left[w_1^T(k),w_2^T(k),...,w_M^T(k)\right]^T \tag{7}$$

Through dividing the vectors, the PLCMV algorithm is summarized in relations (8) to (14).

$$w_i(k+1)=w_i(k)+S_{DQ}(k)a_i(\theta_p)-\mu_y(k)x_i(k) \tag{8}$$

$$S_{DQ}(k)=S\left[1-D(k)+\mu_y(k)Q(k)\right] \tag{9}$$

$$S=\left(\sum_{i=1}^{M}S_i\right)^{-1}=\left(\sum_{i=1}^{M}a_i^H(\theta_p)a_i(\theta_p)\right)^{-1} \tag{10}$$

$$Q(k)=\sum_{i=1}^{M}Q_i(k)=\sum_{i=1}^{M}a_i^H(\theta_p)x_i(k) \tag{11}$$

$$y(k)=\sum_{i=1}^{M}y_i(k)=\sum_{i=1}^{M}w_i^H(k)x_i(k) \tag{12}$$

$$D(k)=\sum_{i=1}^{M}D_i(k)=\sum_{i=1}^{M}a_i^H(\theta_p)w_i(k) \tag{13}$$

$$\mu_y(k)=\mu y^H(k) \tag{14}$$

Where k is the repetition index, and μ is the size of the step, being a positive small number.

4. PROPOSED ALGORITHM

The aim of our proposed algorithm is to estimate the steering vector in large-scale arrays, while still enjoying advantages of other methods used in small arrays for achieving optimum accuracy and performance. Here, the same method of the algorithm presented in Ref. [9] is used to estimate the steering vector of the desired signal; thereafter, the optimum weights of the adaptive beam-forming are calculated with the use of the PLCMV algorithm. We now first explain how to estimate the steering vector and then the proposed algorithm itself.

The decomposition of the eigenvalues of the sample correlation matrix, which is an approximation of the array correlation matrix, is done with the use of the following relation:

$$\hat{R}_{xx} = \sum_{i=1}^{N} \lambda_i \upsilon_i \upsilon_i^H \qquad (15)$$

where $\lambda_i, i = 1, 2, ..., N$ and υ_i are respectively the eigenvalues and eigenvectors of the sample correlation matrix.

The eigenvectors corresponding to the large projections of the steering vector of the desired signal on υ_i are used for forming matrix P which spans the subspace of the desired signal. Since in practice the mismatch between the assumed and actual steering vectors is not high, large projections of the assumed steering vector on υ_i vectors can be used for building matrix P ; these projections are determined using the following relation:

$$p(i) = \left|\upsilon_i^H a(\theta_p)\right|^2, i = 1, 2, ..., N \qquad (16)$$

Then, N projections are sorted in descending order as follows:

$$p(N) \geq p(N-1) \geq ... \geq p(1) \qquad (17)$$

The N eigenvectors corresponding to the eigenvalues of the sample correlation matrix are also sorted as $[\upsilon_N, \upsilon_{N-1}, ..., \upsilon_1]$. Thereafter, the smallest n satisfying inequality (18) is determined.

$$(p(N) + p(N-1) + ... + p(n)) \big/ N \succ \rho \qquad (18)$$

Constant $0 \prec \rho \prec 1$ is used for selecting large projections of $a(\theta_p)$ on υ_i. Then, n projections are considered as large projections, the corresponding eigenvectors of which build matrix P according to the following relation:

$$P = [\upsilon_n \quad \upsilon_{n+1} \quad ... \quad \upsilon_N] \qquad (19)$$

Finally, through projecting $a(\theta_p)$ on the signal subspace spanned by P, the steering vector is estimated as follows:

$$\hat{a}(\theta_d) = PP^H a(\theta_p) \qquad (20)$$

Despite the traditional projection methods, no estimation of the number of sources is needed in the above-discussed method [9]. The assumed steering vector in the PLCMV algorithm is replaced with the estimated steering vector of the desired signal; the beam-forming weight vector is also updated using the PLACMV algorithm.

Therefore, the proposed algorithm for large-scale arrays is summarized in the following four steps: I) the estimation of the steering vector of the desired signal through projecting the assumed steering vector on the subspace of the signal, using Eq. (20); II) dividing the vector of the received signal, the steering and the weight vectors into M subsections according to Eqs. (5), (6), and (7); III) updating the weight vector in each subsection using Eq. (8), and IV) forming the optimum weight vector from the weight vectors of the subsections using Eq. (7).

5. SIMULATION RESULTS

The array used in our simulations, for the sake of making comparison with other methods [6-7], was made of 12 elements, each of which in one subsection. The angle of the desired signal was 0°, while the assumed steering angle was 3°. There were two interfering signals with the angles of 10° and -20°, the interference-to-noise ratio of which was $INR = 10dB$. The step size parameters of μ and μ' were respectively 0.002 and 0.007, and ρ was assumed to be 0.7. The number of the repetition steps in our simulations were 2500. The results of each step of the experiment were the average of the results of 50 runs of the algorithm.

In the first simulation, the performance of the proposed algorithm in beam-patterning has been compared with the PRRLCMV algorithm under the condition that there is an error in the steering direction. Figs. (1) and (2) show the beam-patterns formed by the two algorithms for the SNR values of 0 and -10 dB. When SNR is zero, both of the algorithms can well track the desired signal; however, the PRRLCMV algorithm fails to track the desired signal when the SNR is -10 dB, while our proposed algorithm tracks the desired signal at the position of 0 degree, and neutralizes the interference sources at the positions of 10 and -20 degrees, through forming the main lobe and appropriate nulls respectively at the locations of the main signal and interference sources.

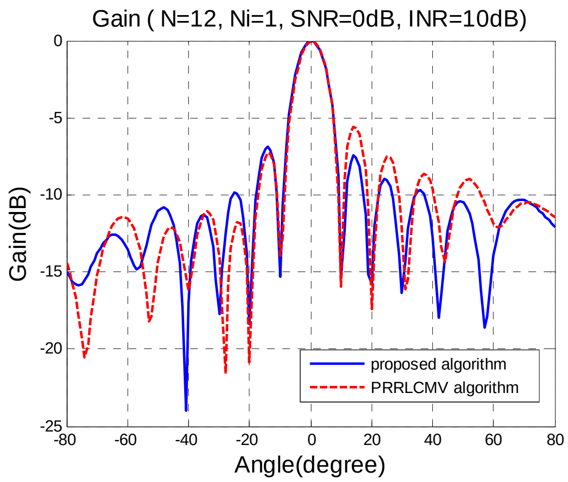

Fig. 1. Comparison of the beampatterns among the proposed algorithm and PRRLCMV algorithm in SNR=0dB

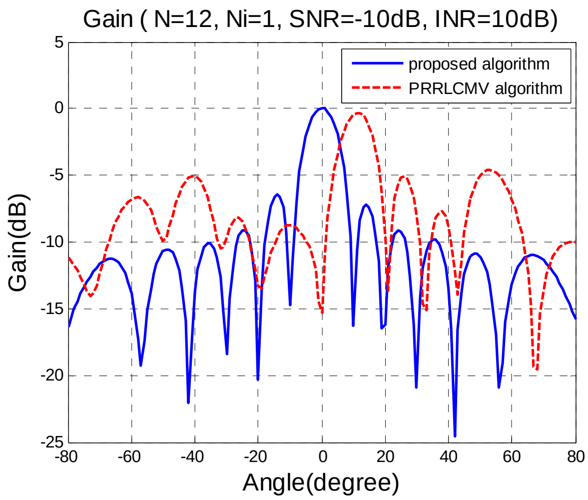

Fig. 2. Comparison of the beampatterns among the proposed algorithm and PRRLCMV algorithm in SNR=-10dB

The criterion for making comparison in the second simulation was the output signal-to-interference-plus-noise ratio (SINR). We have compared the standard linearly-constrained minimum variance (SLCMV) algorithm with the actual steering vector [6], the PRRLCMV algorithm with erroneous steering vector, and the present algorithm with erroneous steering vector. The variation of the output SINR versus the input SNR values has been shown in Fig. (3). It is inferred that, as compared with the PRRLCMV algorithm, the present algorithm has higher output SINR values for SNR values less than -5 dB; its output SINR values are only 1-2 dB less than those of the standard algorithm. Therefore, the proposed algorithm can remove errors in the steering vector estimation in beam-forming for very small SNRs, i.e. less than -5 dB.

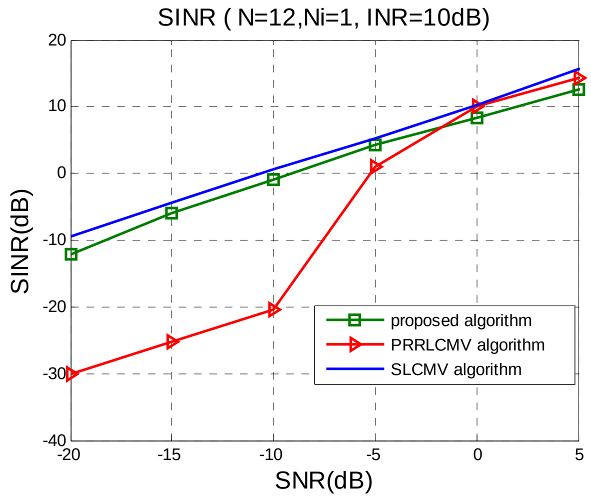

Fig. 3. The output SINR versus SNR

6. CONCLUSION

The PBPLCMV algorithm has been put forward in this paper. In this algorithm, the actual steering vector is estimated through projecting the assumed steering vector on the signal subspace, and then, the assumed steering vector in the PLCMV algorithm is replaced with the estimated steering vector. Our simulations results show that the proposed algorithm has better performance than the PRRLCMV algorithm for SNRs less than -5 dB. Moreover, the output SINRs of the present algorithm for input SNRs less than -5 dB is up to 18 dB higher than the output SINRs of the PRRLCMV algorithm.

REFERENCES

[1] R. A. Monzingo and T.W. Miller, **“Introduction to Adaptive Arrays, ”** New Yourk, Wiley, 1980.

[2] D. J. Chapman, **“Partial Adaptivity for the Large Array,”** *IEEE Trans. On Antennas and Propagation*, Vol. 24, pp. 685-696, Sep. 1976.

[3] D. R. Morgan, **“Partially Adaptive Array Techniques,”** *IEEE Trans. On Antennas and Propagation*, Vol. 26, pp. 823-833, Nov. 1978.

[4] S. J. YU, J. H. Lee, **“Adaptive Array Beamforming Based on an Efficient Technique,”** *IEEE Trans. On Antennas and Propagation*, Vol. 44, pp. 1094-1101, Aug. 1996.

[5] W. Wang, W. X. Sheng, B. Y Qi, **“Subarray Adaptive Array Beamforming Algorithm based on LCMV,”** *Asia Pacific Microwave Conference Proceedings*, Vol. 3, pp. 1964-1966, Dec. 2005.

[6] O. L. Frost, **“An Algorithm for Linearly Constrained Adaptive Array Processing,”** *Proceedings of The IEEE*, Vol. 60, No. 8,pp. 926-935, Aug. 1972.

[7] F. Huang, W. Sheng, X. Ma and W. Wang, **“Robust Adaptive Beamforming for Large-Scale Arrays, ”** *Elsevier Signal Processing*, pp. 165-172, June 2009.

[8] Z. L. Yu, Q. Zou, M. H. Er, **“A New Approach to Robust Beamforming against Generalized Phase Errors,”** *Proceedings of the Sixth Circuits and System*

Symposium on Emerging Technologies: Frontiers of Mobile and Wireless Communications, Vol. 2, pp. 775-778, June 2004.

[9] F. Huang, W. Shemg and X. Ma, **"Modified Projection Approach for Robust Adaptive Array Beamforming,"** *Elsevier Signal Processing*, pp. 1758-1763, June 2012.