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Sediment Transport in Unsteady Flow Conditions

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ABSTRACT

Sediment transport under unsteady flow condition is studied experimentally. In the first step, sediment transport under different steady flow conditions was measured and an empirical equation was derived for its calculation. In the next step, two continuous and three stepwise hydrographs were generated in the flume, and their sediment transport rate was measured. The continuous hydrographs were then approximated by different number of steps. Sediment transport for the hydrographs was then calculated by assuming a steady state condition in each step employing the empirical equation derived in the first step of the study. Results showed that in continuous as well as the stepped hydrographs the difference between the calculated and measured sediment rates is less than 10%. This shows that in the range of the tested hydrographs which conforms to many rivers in Iran, approximating the hydrograph with steady state steps for sediment transport calculations leads to acceptable results. In the next step, the flow and sediment transport in the flume under the tested hydrographs were simulated by using HEC-RAS software. Various sediment transport equations were used and calculated results were then compared with experimental measurements. Results showed that by increasing the number of steps in stepped hydrograph in HEC-RAS, calculated sediment transport rates by each equation tend to constant values.

KEYWORDS

Unsteady flow, Flood hydrograph, Sediment Transport, Bed load.

1. Introduction

Sediment transport in steady, uniform flow conditions has been studied by many researchers and many empirical equations are available for its calculation such as Shields (1936), Einstein (1950), Parker (1990), Yen and Lee (1995), Yang (1984), Yang (1996), and Wilcock (2001). The problem is more complicated when non-uniform and unsteady flow is taken into account. On the other hand, though the most intensive transport processes in rivers occur during the passage of a flood wave (Huygens et al. 2000, Rowinski and Czernuszenko, 1998), the initial theories and formula of sediment dynamics were all established based on steady and uniform flows. This may be attributed to the complexity of experimental works in unsteady flows with relatively fast variation of parameters such as discharge and flow depth. It should also be mentioned that, sediment transport studies under unsteady condition can be used for a better

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understanding of scouring under unsteady flow condition around hydraulic structures.

Previous experiments conducted by several researchers have provided a deeper view to sediment transport under unsteady flow condition. The pioneering investigation was done by Griffiths and Sutherland (1977). Results of their experiments with hydrograph durations longer than 30 min showed that there is no deference between the rate of sediment transport in flood hydrograph and equivalent stepwise hydrograph with steady flow condition in each step. However, Suzka (1988) showed that the time to the peak flow in a hydrograph is an important parameter on the rate of sediment transport. In addition, he proposed two unsteadiness parameters based on hydrograph characteristics including time to peak, and the difference between flow depths in peak and the base flow. He found that the sediment ratio was always larger than the calculated ratio derived from equivalent stepwise hydrograph if the unsteadiness parameters are large. However, if the duration of the hydrograph is long enough or the variation of flow depth is small, unsteadiness parameters will be very small. Yen and Lee (1992) studied experimentally the bed topography, total amount of sediment discharges and transverse sediment sorting in an alluvial channel bend under unsteady flow conditions with non-uniform sediments. They found that the scour depth, deposition height, transverse sorting and total sediment discharge increase with increasing unsteadiness of hydrographs.

Reid et al. (1996) rated different predictive bed load sediment transport equations against a unique set of field data collected by automatic slot samplers installed on a desert river during flash floods. Their analysis showed that the bed load fluxes measured in desert flash floods are close to the value predicted by using the Meyer-Peter and Muller (1948) and Parker (1990) equations with hydrograph peak discharge. Plate (1994) and Wang (1994) observed a lag time between the occurrence of peak discharge and that of the peak sediment transport rate. Lee et al. (2004) presented a method to estimate sediment transport based on equivalent stepwise hydrograph. They also found that the bed load yield during hydrograph rising time is smaller than recession time and the value of the ratio was approximately between 0.5 and 0.75. Esmaeili et al. (2007) preformed an experimental study and showed that the ratio of the mean discharge of sediment transport for unsteady conditions to the corresponding value for equivalent stepwise hydrograph was 1.41.

Despite all the previous studies there are still many relevant aspects in unsteady sediment transport that require a further clarification. The objective of the present study is to investigate the sediment transport rate under some flood hydrographs and to present a model for its calculation. In this study, flood hydrographs similar to large rivers in Iran such as Karun and Bakhtiyari rivers are considered.

2. Experimental setup

Experiments were conducted in a horizontal flume 14 m long, 0.75 m wide, and 0.6 m deep with glass sidewalls at the hydraulic laboratory of Amirkabir University of Technology. Except the first 2 m which was rigid, the channel bed was filled with about 20 *cm* thick, uniform sand with median size of 0.00075 *m* and density of 2630 (Kg/m^3). The geometric standard deviation of sediment grading was 1.2. In order to reduce the flow disturbance and turbulence, a honeycomb was used at the upstream section of the channel. Measured velocity profiles when the flume bed was fixed showed that the flow was fully developed after 5 m from the flume intake.

Figure 1 shows the general view of the laboratory flume and its different components. Water was circulated in the channel by a centrifugal pump with a maximum capacity of 120 (*Lit/s*). A magnetic flow meter was installed in the supply conduit to measure the discharge passing through the channel with 0.5 *lit/s* accuracy. The flow rate in the flume was controlled and preset by a speed control unit attached to the pump system. The desired hydrograph was defined in the computer as the target and the speed control unit commanded the pump system to modify its speed based on magnetic flow meter response to approach the target.

All of the experiments were conducted in live bed condition ($u^*/u^*c>1$; where u^* is bed shear velocity and u*c is critical shear velocity for the bed material). The range of flow intensity (u^*/u^*c) was between 1 and 1.68 for both steady and unsteady flow conditions. Time of experiments was set up in a way that the bed level did not substantially change during the experiment since there was no sediment injection from the upstream. Observations showed that the bed level decreased unacceptably in the time of experiment when the flow intensity was more than 1.7. The flow regime in all experiments was subcritical with maximum Froude number of 0.3. Shear velocity (u^*) in the channel was determined by calculating water surface profile and slope of the energy line. The water depth at upstream and downstream of the channel was measured by a point gage with an accuracy of ±0.1 mm. To calculate water surface profile, first Chezy coefficient was calculated for the flow based on the bed roughness (Van Rijn, 1993). Then the calculated water surface elevation upstream of the flume was compared with measurement results and if they were different, Chezy coefficient was slightly modified. After determination of the water surface, shear velocity in the middle of the channel was calculated. In fact, because of the very mild slope of the water surface (in the order of 0.0001) shear velocity variation along the flume was only 2 %.

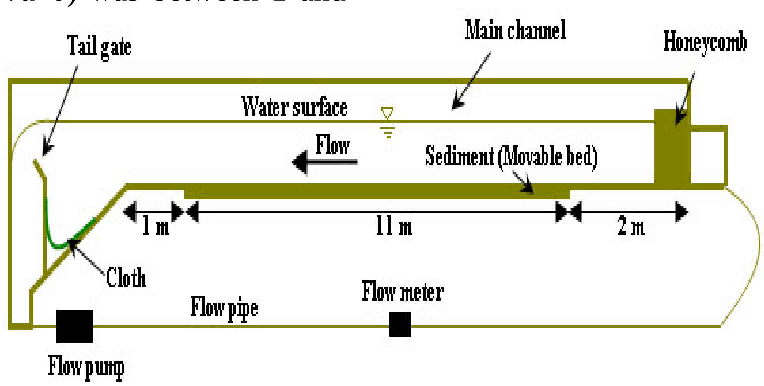

Fig. 1. Side view of the laboratory flume and its different components

Sediment at the end of the flume in each experiment was trapped and collected by a piece of fabric. The collected sediment was then dried and weighed. According to the time of experiments and the weight of the transported sediment, the sediment rate was calculated.

Experiments at different flow intensities were first carried out in steady state condition. The objective of this part of experiments was to derive an empirical equation for sediment transport rate in the flow and sediment conditions of the experimental flume. By employing the derived equation, errors in using the existing empirical equations were avoided. Table 1 shows the discharge and total time of experiments in steady flow condition. As it is shown in this table, discharge was changed from 55 *lit/s* to 87 *lit/s* which was equivalent to flow intensity of 1 to 1.68. Time of the experiment was the longest possible time at which the bed level change was negligible. Downstream flow depth in all experiments was 0.2 *m*.

Modeled hydrographs in the experiments were based on natural condition in large rivers in Iran such as Karun and Bakhtyari Rivers. Data was collected from large river in Iran showed that the pick discharge changed from 500 to 7000 (m^3/s) for a return period of 2 to 200 years. The pick discharge of 2500 (m^3/s) was therefore selected by the model scale of 1/60 for all the experiments.

Table 1. Experiments of sediment transport in steady flow condition

Number	**Flow discharge (m3/s)**	**Flow intensity** (u_*/u_{*c})	**Experiments duration (min)**
1	0.055	1	60
2	0.061	1.12	35
3	0.07	1.28	40
4	0.075	1.37	16
5	0.082	1.53	6
6	0.087	1.68	5

Table 2. Characteristics of different tested hydrographs

Hydrograph Number	**Number of steps**	**Peak discharge (m^3/s)**	**Rising duration (min)**	**Volume of water under hydrograph curve**	**Description**
1	----	0.087	20	127.3	Continuous hydrograph including rising and falling limb
2	----	0.087	52	173	Continuous hydrograph with rising limb only
3	6	0.087	18	63	Stepwise hydrograph model for hydrograph No.1 with equal steps but with only the rising limb
4	5	0.087	41	164	Stepwise hydrograph model for hydrograph No.2 with equal steps
5	17	0.087	50	168	Stepwise hydrograph model for hydrograph No.2 with more steps

Table 2 shows the characteristics of different hydrographs tested in the present study. During passage of a particular hydrograph, changes of flow depth in the channel for different flow discharges were negligible.

According to the table, hydrograph (1) is continuous and has both rising and recession limbs. Hydrograph (2) is also continuous but has only a rising limb. Other hydrographs in Table 2 (Hydrographs 3 to 5) are stepwise hydrographs that are models of the continuous hydrographs (1) and (2). In contrast to a continuous hydrograph, the stepwise hydrograph can be easily modeled and generated in a laboratory flume; therefore, the difference between approximating a continuous hydrograph and an equivalent stepwise hydrograph is interesting. On the other hand, it was intended to know the accuracy of the empirical sediment transport equations in a continuous hydrograph approximated by steps and a real stepped hydrograph. Figure 2 shows the shape of the different hydrographs in the present study.

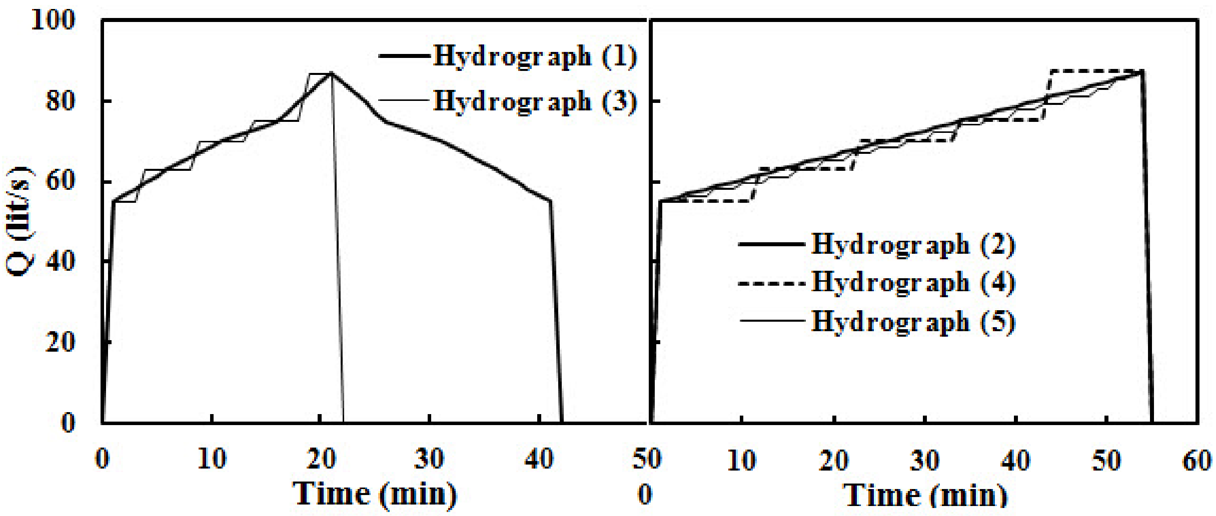

Fig. 2. Hydrographs of the present study

Table 3. Result of the steady flow experiments

Number	Flow intensity (u_*/u_{*c})	Flow discharge Q (m^3/s)	sediment discharge $Q_s \times 10^{-7} (m^3/s)$	$(Q_s/Q)\times 10^3$	Flow intensity (u_*/u_{*c})
1	1.00	0.055	3.51	0.0070	1.00
2	1.12	0.061	9.25	0.0162	1.12
3	1.28	0.070	30.47	0.0484	1.28
4	1.37	0.075	36.36	0.0542	1.37
5	1.53	0.082	53.61	0.0744	1.53
6	1.68	0.087	79.44	0.1032	1.68

3. Experimental Results

3.1. Sediment transport in steady flow condition

In steady flow, estimation of sediment transport by using different empirical equations is not the same, since each equation is developed in an especial condition of flow and sediment (Van Rijn, 1993). To avoid such errors, experiments were carried out with different flow intensities to derive a sediment transport equation in the flow and sediment conditions of the existing flume.

Table 3 shows the experimental results of sediment transport in the steady flow condition. As can be seen, by increasing flow discharge or flow intensity, the sediment to flow discharge ratio was increased. In Fig. 3, the sediment to flow discharge ratio (Column 5 in Table 3) is plotted versus flow intensity (Column 2 in Table 3). This figure shows that as the flow intensity approaches unity, the amount of transported sediment tends to zero as expected. Finally, by fitting a linear curve to the collected data, the following linear equation was developed with correlation factor of R^2 = 0.98.

$$\frac{Q_s}{Q} = 0.00014 \times \left(\frac{u_*}{u_{*c}} - 1 \right) \qquad (1)$$

Where Q_s is the transported sediment discharge and Q is the flow discharge in a steady state flow. It should be noted that Eq. (1) is valid for flow intensity range of 1 to 1.7.

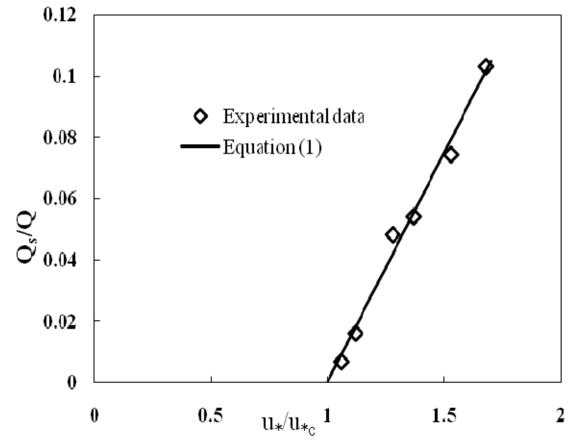

Fig. 3. Variation of sediment discharge to flow discharge ratio with the flow intensity

3.2. Variation of sediment discharge to flow discharge ratio with the flow intensity

3.2.1. Sediment Transport in a Continuous Hydrograph

Continuous Hydrograph (1) in Table 2 was generated in the flume and the transported sediment under it was measured and shown in Table 4. To calculate sediment transport by using this hydrograph on the other hand, it was simulated with different number of steps. Each step was then considered as a steady state flow and its sediment transport was calculated using Eq. (1). It is obvious that by increasing the number of steps the shape of the hydrograph is better simulated. For example, the volume of water under hydrograph (1) with 20 steps is only about 0.5% different from the continuous hydrograph (1) (Fig. 4). Figure 4 shows the result of this analysis. In Figure 4-a the effect of the number of steps is illustrated. As can be seen, the calculated sediment transport approaches a constant value by increasing the number of steps. Results showed that the amount of the calculated sediment with 20 steps was about 9.5% different (less) from the measured value (Table 4). The same process was also carried out for Hydrograph (2). Results of the analysis are presented in Fig. 5. This figure illustrates that for the number of steps more than 30, the amount of calculated sediment transport is similar. The volume of water under stepwise hydrograph with 30 steps is almost equal to the volume of water under continuous Hydrograph (2). In this case, the amount of calculated sediment transport for stepwise hydrograph with 30 steps was only about 4.5% different (less) from the measured value (Table 4).

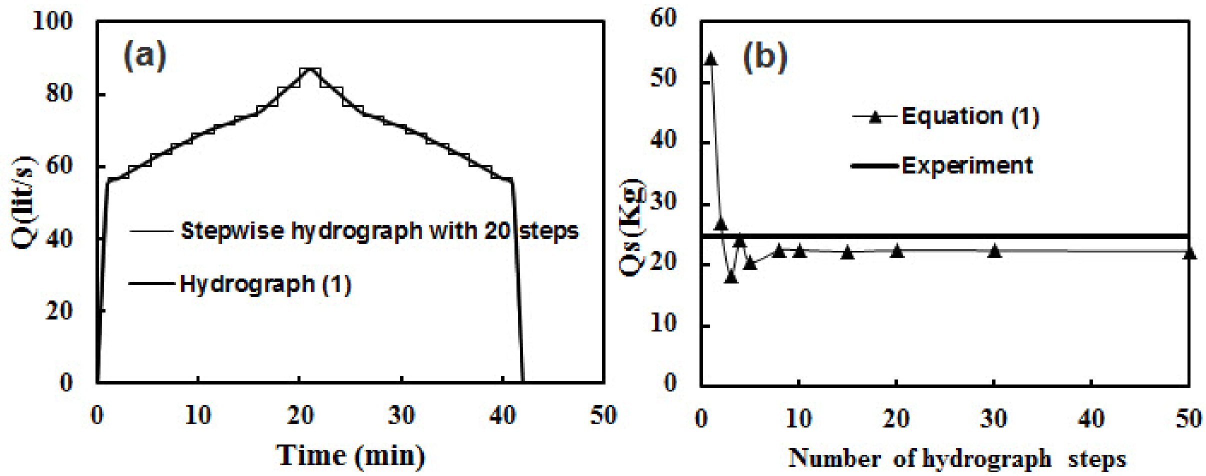

Fig. 4 Modeling of Hydrograph (1): (a) Stepwise hydrograph with 20 steps, (b) Variation of transported sediment due stepwise hydrograph with different steps

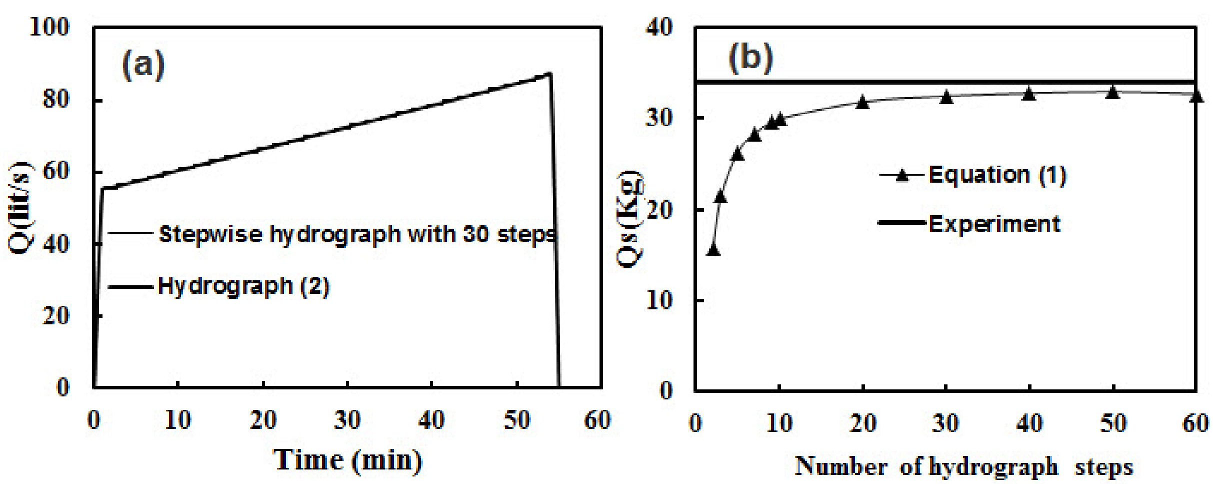

Fig. 5 Modeling of Hydrograph (2): (a) Stepwise hydrograph with 30 steps, (b) Variation of transported sediment due to stepwise hydrograph with different steps

Based on these results, it can be concluded that sediment transport for a hydrograph can be calculated by simulating the hydrograph by steps and assuming a steady flow at each step. In such an analysis the number of steps should be increased so that the calculated sediment would be independent of the number of steps and that the volume of the flow under the hydrograph should be the same as that under the simulated stepped one. To reach a comprehensive conclusion however, more experiments with different shapes of hydrographs are necessary.

3.2.2. Sediment Transport in a Stepped Hydrograph

The accuracy of sediment transport calculation in a stepped hydrograph in comparison to a real continuous hydrograph was then tested. In these tests stepped hydrographs of those continuous ones in the previous section were generated in the flume and their sediment transport was measured (Table 1). Sediment transport of each hydrograph was then calculated by assuming each step as a steady flow and using Eq. (1).

Hydrograph (3) in Table 1 is the stepped model of the raising limb of Hydrograph (1) with five equal steps with duration of 5 min at each step (Fig. 2). As shown in Table 4, measured sediment weight of this hydrograph was only 7.2% different from half of the measured value of the transported sediment by Hydrograph (1). Therefore, generation of a stepped hydrograph instead of a continuous one does not change considerably the amount of the transported sediment. In addition, calculation of sediment weight for stepped hydrograph (3) by using Eq. (1) was only about 0.5% more than that measured in the laboratory. This value is less than the difference of sediment calculations with continuous Hydrographs (1) and (2).

Hydrograph (4) (Table 2) was a stepped Hydrograph form of continuous Hydrograph (2) with five equal steps with duration time of 10 min at each step and the volume of water was about 6% less. The weight of transported sediment for this hydrograph was 16% less than Hydrograph (2) (Table 4). On the other hand, the calculated sediment transport for Hydrograph (4) by using Eq. (1) was 3.8% different form the measured value. At the end, Hydrograph (5) in Table 1 which is a model of continuous Hydrograph (2) with 18 steps and duration time of 3 min for each step was tested. The volume of water under this hydrograph was only about 3% less than Hydrograph (2). After conducting the experiment, the amount of transported sediment by this hydrograph was measured as 2% less than transported sediment by continuous Hydrograph (2) (Table 4). Furthermore, by using Eq. (1) the total amount of calculated transported sediment for Hydrograph (5) was 4.1% more than the measured value in the laboratory.

Based on these results, it can be concluded that without losing the accuracy, sediment transport for a continuous hydrograph can be studied in a laboratory with simulating the hydrograph by steps.

3.2.3. Calculating sediment transport under unsteady flow condition by using the existing empirical equations

In order to examine the accuracy of popular numerical models on sediment transport during a hydrograph event, experimental flume with hydrograph (1) was modeled by using HECRAS software version 4.0.0. This version of software can perform one-dimensional unsteady moving bed computations. The quasi-unsteady flow assumption in this software approximates a continuous hydrograph with a series of discrete steady flow profiles. Each discrete steady flow profile is continued for a particular flow duration which is then divided into shorter blocks of time for sediment transport computations.

In HEC-RAS, sediment transport is computed with one of the seven sediment transport functions which are explained in Yang (1984), Mayer and Muller (1948), Laursen, and Emmett (1958), Engelund and Hansen (1967), Toffaleti (1968), Ackers and White (1973). Sediment transport results are strongly dependent on the transport function which is selected. Careful comparison of the conditions at which an equation is derived with the flow condition in each problem helps the user to select the most convenient equation.

In the present study, laboratory flume geometry was modeled in the software with 11 similar equal distance cross sections as is shown in Fig. 6. Calibration of the flume with steady experimental data showed that the Manning coefficient in each section must be 0.013. This value is in the range of roughness of sand bed streams with plane bed which is similar to the present experiments' condition (Simons and Associates, 1985). In addition, by using Log law relations, the Manning coefficient is calculated as 0.0121 which is close to the calibration value. Hydraulic condition of Hydrograph (1) was set in the software in form of a stepwise hydrograph as an upstream boundary condition and the transported sediment during this hydrograph was studied. The flood hydrograph was simulated with different equal steps from 2 to 30 steps. Downstream boundary condition was set to 0.2 m in every time step similar to the experiment. In addition, Sediment fall velocity was calculated based on Van Rijn (1993) and the input sediment load was set to zero.

Table 5. Total transported sediment in the hydrograph with different steps and different empirical equations (N)

No. of Steps	MPM (1948)	Laursen (1958)	Engelund and Hansen (1967)	Toffaleti (1968)	Ackers and White (1973)	Yang (1984)	Wilcock (2001)
2	20.21	297.24	120.66	84.56	75.93	66.90	1.16
6	45.81	390.44	154.02	114.78	124.59	94.08	1.50
10	55.13	444.39	172.66	126.55	148.13	107.91	1.69
14	52.09	420.85	164.81	120.66	138.32	102.02	1.60
20	54.64	437.53	169.71	125.57	146.17	106.93	1.67
30	54.45	433.60	167.75	122.63	145.19	105.95	1.65

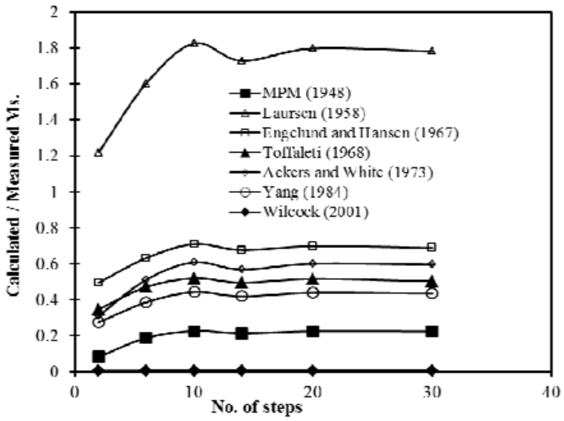

Fig. 6. Variations of the calculated to measured cumulative transported sediment (Ms.) ratio using different methods with number of steps in stepwise hydrograph

Table 5 shows the total transported sediment in the hydrograph with different steps and different empirical equations. As shown in this table, when the hydrograph is simulated by more than 14 steps the calculated sediment transport becomes independent of the number of steps. The difference of volume of water under the continuous (Hydrograph 1) and stepwise hydrographs with 14 steps (Modeled in HEC-RAS) is less than 2%. This trend is similar for all sediment transport equations. In addition, the calculated sediment transport by various equations to the laboratory measured values ratio for different stepwise hydrographs is shown in Fig. 6. As shown in this figure, among all the considered sediment transport equations, result of Engelund and Hansen (1967) sediment is close to the experimental value with a calculated to measured sediment ratio of 0.78. However, the result of the equations presented by Wilcock (2001) significantly under-predict the measured value.

3. Conclusion

In the present work, sediment transport under unsteady flow condition was studied experimentally. Flood hydrographs similar to large rivers in Iran rivers were considered. In the first step, experiments on sediment transport under steady flow condition were carried out to develop a sediment transport formula which conforms better to the present experimental condition. Secondly, the accuracy of using the steady state sediment transport formula for an unsteady hydrograph flow was studied. Two continuous hydrographs and three stepped hydrographs were generated in the flume, and their total transported sediment was measured. The continuous hydrographs were then approximated by different number of steps.

Sediment transport for the hydrographs was then calculated by assuming a steady state condition at each step employing the empirical equation derived in the first step of the studies. Results showed that sediment transport of continuous hydrographs can be calculated with simulating them with steps and assuming a steady flow at each step, providing that the number of steps is large enough so that the results are independent of the number of steps and that the volume of the flow under the stepped hydrograph is close to the continuous one. The difference between the calculated and measured sediment for continuous as well as the stepped hydrographs is less than 10%. On the other hand, calculation of sediment transport for an actual stepped hydrograph leads to more accurate results. Moreover, results of experiments showed that, instead of continuous hydrographs, equivalent stepped hydrographs can be generated in the laboratory for sediment transport studies with acceptable accuracy.

Finally, the flow and sediment transport in the flume under one of the continuous tested hydrographs was simulated by HEC-RAS software. The hydrograph was modeled in the program by different number of steps. In addition, various sediment transport equations were used and results were then compared with experimental data. It was concluded that, by increasing the number of steps in stepped hydrograph, calculated sediment transport by each equation tend to constant values. In addition, the best result was 0.78 times the experimental measured sediment by Engelund and Hansen (1967) equation.

References

Ackers, P. and White, W. R. (1973), “Sediment transport: New Approach and Analysis,” Journal of hydraulic Division, ASCE, 99(11), 2040-2060.

Einstein, H. A. (1950), “The Bed Load Function for Sediment Transportation in Open Channel Flows,” Technical Bulletin 1026, USDA, Soil Conservation Service, Washington D.C., 71 pp.

Engelund, F., Hansen, E., (1967). A monograph on sediment transport in alluvial streams.TekniskVorlag, Copenhagen, Denmark.

Esmaili, K.shafaiBajestan, M.,kashefipour, M. (2007), “A model for predicting the bed load in ephemeral rivers” Journal of water and soil, Vol. 22(2), 240-250 (In Persian).

Griffiths G. A., Sutherland A. J. (1977), “Bedload transport by translation waves,” Journal of Hydraulic Diversion, 103(11): 1279–1291.

Huygens, M., Verhoeven, R. & De Sutter, R. (2000) Integrated river management of a small Flemish river catchment. In: The Role of Erosion and Sediment Transport in Nutrient and Contaminant Transfer (ed. by M. Stone) (Proc. Waterloo Conf., July 2000), 191-198. IAHS Publ. no. 263.

Laursen, and Emmett M., (1958), “Total sediment load of streams,” Journal of Hydraulic Division, ASCE, 84(HY1), PP. 1530-1 to 1530-2.

Mayer-Peter, E., and Muller, R. (1948), “Formulas for bedload transport.” Proc., Int. Assoc. of Hydraulic Struct. Res., 39-64.

Parker, G. (1990), “Surface-based bed load transport relationship for gravel rivers,” 28(4), pp 417-436.

Plate, E. J. (1994).“The need to consider non-stationary sediment transport.” Int. J. Sediment Res., 9, 117–123.

Reid I., Powell D. M., Laronne, J. B.,(1996).“Prediction of bed-load transport by desert flash floods.”, Journal of Hydraulic Engineering, American Society of Civil Engineers 122(3), 170-173.

Rowinski, P. M. and Czernuszenko, W. (1998) Experimental study of river turbulence under unsteady conditions. ActaGeophys.Polonica XLVI, 461-480.

Shields, A. (1936), “Anwendung der echnlichkeit smechanik und der Turbulenz-forschung auf die Gescheibebewung”, MittelungenPreuss. Versuchs.WasserbauSchiffbau, 26, 98-109.

Simons, Li & Associates, (1985), “Design manual for engineering analysis of fluvial systems”, Arizona Department of Water Resources, Tucson, Arizona.

Suszka , L. (1988), “Sediment transport at steady and unsteady flow- a laboratory study", Doctoral dissertation, No. 704, Ecole Poly technique Federale de Lausanne, Lausanne, Switzerland.

Toffaleti, F. B. (1968), Technical Report No. 5.,"A procedure for computation of total River sand discharge and detailed distribution, Bed to surface", Committee on channel stabilization U. S. Army Corps of Engineers, November, 1968.

Van Rijn, L. C., (1993). "Principles of sediment transport in rivers, estuaries and coastal seas", Aqua publication, The Netherlands.

Wang, Z. (1994). "An experimental study of motion and deposition of gold particles in a mountain river." J. Hydraul. Res., 32(5), 643–648.

Wilcock, P. R. (2001), "Toward a Practical Method for Estimating Sediment-Transport Rates in Gravel Bed Rivers," Earth Surface Processes and Landforms, 26, PP. 1395-1408.

Yang, C. T. (1996). Sediment transport: Theory and practice, McGraw-Hill, New York.

Yang. C. T. (1984). "Unit stream Power equation for gravel" Journal of Hydraulic Division, ASCE, 110(12), PP. 1783-1797.

Yen, C. L. and Lee, K. T. (1995). "Bed topography and sediment sorting in channel bend with unsteady flow." J. Hydrol. Eng., ASCE. 125(8) P. 591-599.

Yen, C. L., Chang, S. Y., and Lee, H. Y. (1992)."Aggradation degradation process in alluvial channels." J. Hydraul. Eng., 118(12), 1651–1669.