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Research article

Free axial vibration of cracked nanotubes incorporating scale effects using doublet mechanics

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Abstract

This paper investigates the free axial vibration of cracked nanotubes with considering scale parameter under various boundary conditions. The cracked nanotube is modeled by dividing it into two segments connected by a linear spring. The stiffness of the spring is dependent to the crack severity and obtained using fracture mechanics principles. Governing equations and corresponding boundary conditions are derived with the aid of doublet mechanics (DM). The natural frequencies are obtained analytically with solving characteristics equation and the influence of the crack severity, the boundary conditions, the tube chirality, and the dimensions of nanotube on the free axial vibration of cracked nanotubes is studied in detail. It was shown that the frequency decreases with increase of the crack severity and scale parameter. This reduction is more apparent when the boundaries of the beam are changed from free end to clamped one. In addition, when the crack location is near the support, a larger decrease in the frequency can be observed. To validate accuracy and efficiency of the present method, the results obtained herein are compared with the available results in the literatures and good agreement is observed.

Keywords: Axial vibration, Cracked nanotube, Doublet mechanics, Natural frequency, Crack severity.

1. Introduction

The importance of the beam and its engineering applications is obvious, and it undergoes different kinds of loading. It is well known that presence of crack in a beam creates discontinuities varies its dynamic behavior and may cause the failure [1]. Cracks are classified base on the geometry and its orientation, cracks parallel to beam axis are known as longitudinal cracks, cracks that close and open when subjected to alternative stresses are known as breathing crack, crack which are perpendicular to the axis of shaft are known as transverse crack, the cracks on surface which is not visible known as sub-surface crack on the surface are known as surface crack. Cracked structures are susceptible to failure depending on the vibration mode. Failure is due to the resonance formed by the superposition of frequency of periodic force acting on structure and the natural frequency of the structure. Crack severity is defined by division of crack length to crack depth [2]. Natural frequency is the frequency at which a system or structure vibrates when subjected to an initial excitation in the absence of any driving or damping force [3]. Then, to determine natural frequency, free undamped vibration must be considered. For any cracked structure, the study of resonance is more important because it affects the structure in different ways [4]. When the frequency of applied load becomes equal to associated natural frequency, the structure vibrates theoretically at infinite amplitude leading to failure [5]. To avoid structural failure due to periodic load, it is important to determine resonant frequency. As material failure could lead to disastrous results, structures have regular costly and time consuming inspections. During the last decades, damage detection methods using vibration analysis have attracted growing interest because of their simplicity for implementation [6]. It is known that presence of the crack reduces its natural frequency and deviates its mode shape. It should be pointed out that the frequency reduction in cracked beam is not due to removal of mass rather the reduction of mass would increase natural frequency [7]. Indeed, a crack in a structure leads to a reduction in the stiffness and an increase in the damping of the structure. As frequencies are measured more easily than mode shapes, and on the other hand, mode shapes also affected by experimental errors, the investigation of the natural frequency is more significant. Therefore, it is possible to predict the crack depth and crack location by measuring changes in the vibration parameters.

In recent years, the study of the beamlike vibration in nanoscale devices has been of significant interest to researchers due to their use in NEMS [8]. Nowadays, it is still a challenge to study the mechanics of nanomaterials by means of experimental tests due to the difficulties exists in the nanoscale [9]. Furthermore, structures at the nanoscale are known to exhibit a size-dependent behavior. Therefore, the theoretical methods such as atomistic simulations and classical continuum mechanics (CCM) theories are often used to analyze the dynamic responses of cracked nanostructures [10- 12]. It is known that atomistic simulation methods are extremely costly and time-consuming task [13]. On the other hand, CCM theories are assumed to be scale independence ignoring the scale effect [14]. Couple stress theory and shear deformation maybe used to investigate vibration of nanostructure. Alimoradzadeh et al. studied free and forced vibration of a clamped microbeam based on the third order shear deformation and modified couple stress theories. The size dependent dynamic equilibrium equations along with boundary conditions derived using the variational approach. They found that dimensionless frequencies are strongly dependent on the scale parameter and power index [15].

To improve CCM, DM elasticity theory has been also used in the linear and nonlinear vibration analysis of carbon nanotubes [16, 17]. DM is a micro-mechanical theory based on a discrete material model wherein solids may be represented as arrays of points or nodes at finite distances. A pair of such nodes is referred to as a doublet, and the nodal spacing distances introduce length scales into the microstructural theory. Each node in the array is allowed to have translation and rotation where small translational and rotational displacements are expanded in Taylor series about the nodal point. The order at which the series is truncated defines the degree of approximation employed. The lowest order case using only a single term in the series does not contain any length scales, while using the terms beyond the first produce a multi length scale theory.

Due to different causes, cracks are often found in the nanostructures. For example, thermally-induced crack in the fabrication process of nanomaterials such as ZnO nanorods and nanowires may be created during heating [18]. The presence of the cracks in the nanodevices affects the safety and reliability in applications. However, few published papers investigated the aspect of mechanical analysis of cracked nanostructures [19]. Another field that recently attracted growing interest for the researchers is considering the scale effects on vibration of cracked nanobeams. Recently, nonlocal beam model has been adopted for the flexural [20, 21] and torsional [22, 23] vibration analysis of cracked nanostructures. Buckling behavior of imperfect axially compressed cylinder with an axial crack studied by many researcher [24, 25]. However, fewer researches have been so far conducted on the vibration behavior of cracked CNTs using DM theory. Although, there are several studies focusing on the axial responses of these kinds of nanostructures, none of them has incorporated the scale effect, explicitly. Gheshlaghi and Hasheminejad studied the axial vibration of CNTs using a modified couple stress theory [26]. Using a nonlocal elasticity model, the effects of crack on free axial vibration of nanorods and the effects of elastic medium on axial statics and dynamics of nanotubes were investigated. Zhang et al reported an order-of-magnitude reduction in the fatigue crack propagation rate for an epoxy system with the addition of five percent of carbon nanotube additives using fractography analysis and fracture mechanics modeling [27]. Rane et al. developed a method based on measurement of natural frequencies for detection of the location and size of a crack in a cantilever beam [28]. Hsu et al. studied the longitudinal frequency of a cracked nanobeam. They obtained the frequency equation of the nanobeam with different boundary conditions based on the nonlocal elasticity theory [29]. Singh introduced transcendental eigenvalue problems in axially vibrating rods to estimate the damage parameters in the continuous structure from natural frequencies [30]. Yali and Cercevik studied the axial vibration of cracked carbon nanotubes with arbitrary boundary conditions using the nonlocal elasticity theory. The crack severity and the supports were modeled by an axial spring representing the discontinuity in the axial displacement [31]. Loghmani and Hairi Yazdi studied free vibration of Euler-Bernoulli nanobeam with multiple cracks using Eringen's nonlocal elasticity theory based on wave approach [32]. Ebrahimi and Mahmudi proposed a finite element (FE) model to study the thermal transverse vibrations of cracked nanobeams resting on a double-parameter nonlocal elastic foundation using Hamilton’s principal [33]. Dilena and Morassi studied the identification of a single open crack in a vibrating beam, either under axial or bending vibration, based on measurements of damage-induced shifts in natural frequencies and antiresonant frequencies [34].

Although there are many papers like mentioned above that investigate the cracked nanotubes but none of them obtained the frequency explicitly dependent to chiral effect. Indeed, they enter the scale parameter in to equations of motion indirectly by being implicitly contained in the macrotensors of elasticity. In other words, unlike the elastic macrotensors in DM, the elastic macrotensors in the nonlocal theory are unknown functions of the underlying microstructural parameters. Indeed, and the parameters of the microstructure are not included in the mathematical model directly.

As far as known, however, there has been no investigation on the longitudinal vibration of a nanostructure with cracks explicitly incorporates scale effect in details. The lack prompted the authors to model the free axial vibration of cracked nanotubes based on DM theory and to investigate the scale effects on axial vibration frequency. In this paper, the axial vibration of a cracked nanobeam with different boundary conditions is studied using DM theory. The effects of the crack parameter, crack location, and scale parameter on the vibration frequency of the cracked nanotube are studied. The main purpose of the present work is to propose a comprehensive analytical model to study the free axial vibration of cracked CNTs. To this end, the governing equations of cracked nanotubes incorporating scale effects are derived using DM principle.

2- Free axial vibration analysis of SWCNT using DM

In this section, the governing equation of motion of a nanotube in presence of the scale parameter is derived by DM principle. To this end, a nanotube is considered with the circular cross-section of constant area A with the radius R and the length L, modulus of elasticity E, density ρ, scale parameter η as shown in Fig. 1.

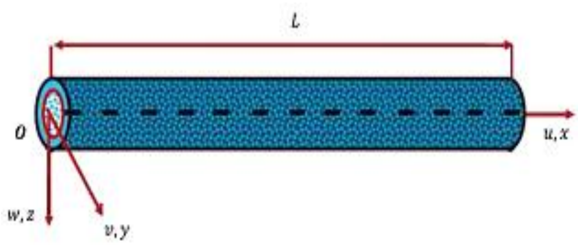

Fig. 1 A SWCNT [3].

The governing equation for axial vibration of SWCNTs considering scale effect is given by [3]

$$E\frac{\partial^2 u}{\partial x^2}+\kappa E\eta^2\frac{\partial^4 u}{\partial x^4}=\rho\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

wherein u is the axial displacement of the beam and κ is chiral dependent and given by

$$\kappa = \begin{cases} \frac{1}{12} & \text{for Zigzag nanotubes} \\ \frac{1}{16} & \text{for Armchair nanotubes} \end{cases} \quad (2)$$

Now, the mode shape function of the CNT in axial vibration mode is determined. To this end, the time response of the nanotubes is considered to be the following equation

$$\frac{\partial^2 u}{\partial t^2} = -\omega_n^2 u \quad (3)$$

Substituting (3) into (1) and solving the resulting equation, gives the following relation for displacement

$$u = A sin\left(\frac{n\pi}{L}x\right) + B cos\left(\frac{n\pi}{L}x\right) \quad (4)$$

The boundary conditions are as follow:
Clamped-clamped boundary condition

$$u(x,t) = 0 \quad at\ x = 0, L \quad (5)$$

Clamped-free boundary condition

$$u(x,t) = 0\ at\ x = 0\ and\ \frac{\partial u(x,t)}{\partial x} = 0\ at\ x = L \quad (6)$$

Free-free boundary condition

$$\frac{\partial u(x,t)}{\partial x} = 0 \quad at\ x = 0, L \quad (7)$$

However, in real supporting situations, idealized boundary conditions (clamped-clamped and clamped-free) never occur and therefore there exists axial or rotational restraint or both [5].
Solving Eq. (4) with considering related boundary condition yields the following mode shapes and natural frequencies for un-cracked nanobeams.

For clamped-clamped boundary conditions

$$u(x) = \sum_{n=1}^{\infty} C_n\, sin\left(\frac{n\pi}{L}x\right) \quad (8)$$

$$\omega_n^2 = \frac{E}{\rho}\left(\frac{n\pi}{L}\right)^2\left[1 - \eta^2\kappa\left(\frac{n\pi}{L}\right)^2\right] \quad (9)$$

For free-free boundary conditions

$$u(x) = \sum_{n=1}^{\infty} C_n\, cos\left(\frac{n\pi}{L}x\right) \quad (10)$$

$$\omega_n^2 = \frac{E}{\rho}\left(\frac{n\pi}{L}\right)^2\left[1 - \eta^2\kappa\left(\frac{n\pi}{L}\right)^2\right] \quad (11)$$

For clamped-free boundary conditions

$$u(x) = \sum_{n=1}^{\infty} C_n\left[1 - cos\left(\frac{n\pi}{L}x\right)\right] \quad (12)$$

$$\omega_n^2 = \frac{E}{\rho}\left(\frac{(2n+1)\pi}{2L}\right)^2\left[1 - \eta^2\kappa\left(\frac{(2n+1)\pi}{2L}\right)^2\right] \quad (13)$$

The axial force is also obtained by the following equation [8]

$$F = EA\left[\frac{\partial u}{\partial x} + \kappa\eta^2\frac{\partial^3 u}{\partial x^3}\right] \quad (14)$$

3- Crack modeling

A schematic diagram of a cracked nanobeam is depicted in Fig. 2. The nanobeam with previous specifications has a crack at location C located at a distance L_C from the left end. The crack is modeled by a linear elastic axial spring representing the discontinuity in the axial displacement. Crack severity or crack parameter shown with K defined by $K = \frac{EA}{kL}$. In the present model, the effect of the crack is taken into account following the methodology proposed in [29]. To this end, the CNT is divided into two intact nanobeam pieces which are connected by an axial spring located at the cracked section to consider the additional strain energy due to the presence of the crack. It is obvious that in the crack location, the axial displacement has discontinuity. However the axial force is presumed to be continuous. Natural frequencies for cracked nanobeams for different crack positions, crack severities, mode numbers, and dimensions of nanobeam on the free axial vibration of nanotubes are studied. In this work, the nanobeam with a single and double edge crack for a longitudinal vibration is explored based on the DM theory.

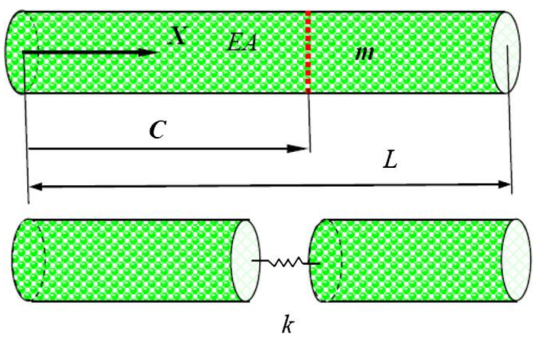

Fig. 2 A nanobeam with crack

It should be pointed out that presence of crack causes a complex geometrical property which is difficult to study. Analyzing the results in the presence of the crack, the equation of motion for two intact nanobeams given with (1) can be expressed as [3]

$$E\frac{\partial^2 u_1}{\partial x^2} + \kappa E\eta^2 \frac{\partial^4 u_1}{\partial x^4} = \rho \frac{\partial^2 u_1}{\partial t^2} \qquad (15)$$

$$E\frac{\partial^2 u_2}{\partial x^2} + \kappa E\eta^2 \frac{\partial^4 u_2}{\partial x^4} = \rho \frac{\partial^2 u_2}{\partial t^2} \qquad (16)$$

Similar to (4), the solution of (15) and (16) can be expressed as

$$u_1 = A_1 sin\left(\sqrt{\frac{\rho}{E}}\omega_n x\right) + B_1 cos\left(\sqrt{\frac{\rho}{E}}\omega_n x\right) \qquad (17)$$

$$u_2 = A_2 sin\left(\sqrt{\frac{\rho}{E}}\omega_n x\right) + B_2 cos\left(\sqrt{\frac{\rho}{E}}\omega_n x\right) \qquad (18)$$

Eqs. (17) and (18) are the free axial vibration solution of the segment one and two, respectively. (17) and (18) have four unknown coefficients must be determined. Obtaining the natural frequencies in axial mode, two more conditions than those given in (5)-(7) are needed. The conditions are compatibility conditions at the crack section given by

Jump in axial deflection,

$$u_1 - u_2 = CF_1 \qquad (19)$$

Continuity of the axial force,

$$F_1 = F_2 \qquad (20)$$

In (19), $C = \frac{1}{k}$ is the flexibility of the spring and obtained using fracture mechanics principles.

Applying the boundary and compatibility conditions, (17)- (20) yield a system of four homogeneous algebraic equations with A_1, B_1, A_2, and B_2 as unknowns.

For two ends clamped boundary conditions stated in (5), (17)- (20) can be written in matrix form as

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & sin(k_2) & cos(k_2) \\ cos(k_1) & -sin(k_1) & -cos(k_1) & sin(k_1) \\ u & w & -sin(k_1) & -cos(k_1) \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix} = 0 \qquad (21)$$

wherein $q = \sqrt{\frac{\rho}{E}}\omega_n, k_1 = qL_c, k_2 = qL, u = sin(k_1) - Ckcos(k_1) + Cq^3\kappa\eta^2 cos(k_1), w = cos(k_1) + Cksin(k_1) - Cq^3\kappa\eta^2 sin(k_1)$

For a nontrivial solution of A_1, B_1, A_2 and B_2, the determinant of the coefficients of the matrix must be set to zero for each boundary type. The process gives the explicit form of characteristic equation of the cracked nanobeam with fixed-fixed yields

$$-sin(k_2)[-1+psin(k_1)cos(k_1)]+pcos(k_2)cos^2(k_1)=0 \quad (22)$$

wherein $p=EACq(1-\kappa\eta^2q^2)$

For the other boundary conditions, similar calculations yield the following characteristic equations for the fixed- free and free-free cracked nanobeam,

For the fixed-free and free-free cracked nanobeam,

$$-cos(k_2)[-1+psin(k_1)cos(k_1)]-psin(k_2)cos^2(k_1)=0 \quad (23)$$

For the free-free cracked nanobeam,

$$-pcos(k_2)sin^2(k_1)-sin(k_2)[1+psin(k_1)cos(k_1)]=0 \quad (24)$$

One may introduce the frequency ratio as the frequency of nanobeam without considering crack to the frequency of the cracked nanobeam to obtain dimension less frequency as

$$F.R=\frac{Natrual\ Frequency\ without\ crack}{Natrual\ Frequency\ with\ crack} \quad (25)$$

The roots of the characteristic equations (22) – (24) are the axial frequencies for the cracked nanobeam incorporating the scale effects, explicitly. For example, for Zigzag, Armchair and other arbitrary chiral nanotubes, we obtained different frequency equations.

A MATLAB program is written to solve the characteristic equations. It should be pointed out here that, by setting $C=0$ in the above equations, it can be obtained the natural frequency, i.e. eigenvalue equations, of the corresponding uncracked nanobeams and $\eta=0$ the corresponding scale less nanobeam. Indeed, the computation of these vibration frequencies may also be used to detect the location and severity of cracks in a nanostructure.

4- Local flexibility of cracked nanostructure

Assume that a slender prismatic nanobeam with a circular cross section, having a non-propagating single edge crack (SEC) and double edge crack (DEC) (or flaw like crack) are shown in Fig. 3.

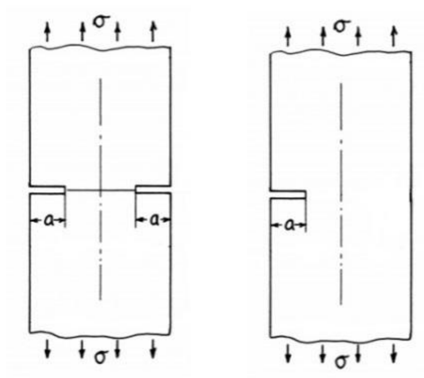

Fig. 3 SEC and DEC in axial loading of SWCNT

The cracked section is presented by a massless axial spring with flexibility C, where

$$F=k\Delta x \quad (26)$$

and

$$C = \frac{1}{k} = \frac{\Delta x}{F} \tag{27}$$

This quantity is a function of the crack severity and the axial stiffness (EA) of the cross section of the nanobeam, and can be written as suggested in [25] as below:

$$C = \frac{d}{EA} f(\xi) \tag{28}$$

wherein $f(\xi)$ is calculated by the following equations

for single edge crack

$$f(\xi) = 0.0007 + 0.3255\xi - 8.4253\xi^2 + 167.486\xi^3 - 831.418\xi^4 + 2268.89\xi^5 - 3154.06\xi^6 + 1852.87\xi^7 \tag{29}$$

For double edge crack

$$f(\xi) = -0.0445 + 11.77\xi - 393.721\xi^2 + 4813.5\xi^3 - 27314.8\xi^4 + 79935\xi^5 - 115803\xi^6 + 66031.8\xi^7 \tag{30}$$

where d is the diameter of the circular cross section and $f(\xi)$ is called the local flexibility function.

In this study an attempt is also made to calculate the stress intensity factor of the nanobeam explicitly incorporate scale effect using DM and principals of fracture mechanics. The predictive equation $f(\xi)$ for slender prismatic nanobeam with a SEC and DEC is proposed as [35]

$$K_I = \sigma\sqrt{\pi a} F(\frac{a}{d}) \tag{31}$$

wherein $F(\frac{a}{d})$ is a function of crack severity and crack type defined by the following equations for cracked nanobeam with circular cross section.

For single edge crack

$$F\left(\frac{a}{d}\right) = 1.122 - 0.231\left(\frac{a}{d}\right) + 10.550\left(\frac{a}{d}\right)^2 - 21.710\left(\frac{a}{d}\right)^3 + 30.382\left(\frac{a}{d}\right)^4 \tag{32}$$

For double edge crack

$$F\left(\frac{a}{d}\right) = \frac{1.122 - 0.561\left(\frac{2a}{d}\right) - 0.205\left(\frac{2a}{d}\right)^2 + 0.471\left(\frac{2a}{d}\right)^3 - 0.190\left(\frac{2a}{d}\right)^4}{\sqrt{1 - \frac{2a}{d}}} \tag{33}$$

In the above equations $\frac{a}{d}$ is introduced as normalized crack depth.

5- Results and discussion

In order to show the accuracy and capability of the proposed method, the results for the natural frequency of nanotubes with a single edge crack are presented and compered to the numerical in [30], experimental in [34] and nonlocal Eringens results in [29]. A comparative study for evaluation of the first five natural axial frequencies considering the crack ($\frac{L_C}{L} = 0.2002$) and scale effects and the results given by [34] is carried out in Table 1 for a beam with fixed-free boundary condition and $K = \frac{EA}{kL} = 0.1144$. It can be seen that Table 1 confirms the reliability of the present formulation and results.

From Table 1, it is obvious that the frequencies of nanobeam are approaching to the frequencies obtained from experimental and nonlocal simulations by choosing the corresponding nonlocal parameter.

Table 1: Natural frequencies of the clamped-free beam with a single crack for different methods.

Mode number	Numerical result [30]	Eringen theory [29]	Experimental result [34]	Present method
1	1.4278	1.4278	1.4451	1.4399
2	4.5579	4.5576	4.5585	4.5578
3	7.8540	7.8540	7.7590	7.7875
4	10.4471	10.4486	10.3564	10.3840
5	12.8476	12.8741	----	12.7989

Effect of Crack severity and tube chirality

Fig. 4 shows that effect of crack opening size on the frequency ratio of the cracked nanobeam for Zigzag and Armchair nanotubes for mode 3. The boundary condition is fixed-free and the crack is located as $\frac{L_c}{L} = 0.5$.From this figure, it can be seen that as the crack parameter increases, the frequency decreases. Moreover, this reduction is significant as the crack severity being smaller. As the crack severity is more increases, the two graphs approaches to single value. That's why the smaller crack severity is used for further modeling. It can also be concluded that in identical crack severity, the frequency of Armchair chiral is higher than the Zigzag one. This increase is more apparent in lower crack severities.

Effect of boundary condition and crack parameter

Effect of crack location for specified scale parameter has been represented in Fig. 5 for various crack severity for fixed-free boundary condition. The crack parameter is assumed to be $K = 0.11$ and the first mode of vibration is considered.

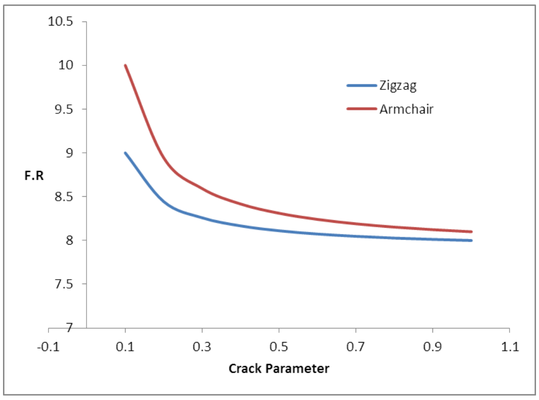

Fig. 4 Natural frequency versus crack severity

From this figure, it can be observed that frequency ratio varies for a certain crack parameter depending on the crack location. It can be seen that frequency ratio increases as the crack moves away from the fixed end. In other words, reduction in frequency is less for crack located near free end. It is also seen that as the crack severity increases, the frequency ratio decreases. This reduction is more apparent as the crack location moves to neighborhood of free end support.

This important note should be noticed carefully. As crack moves to fixed boundaries, the reduction is more apparent. To explain this effect sensibly, we can consider this example. Suppose a bar with free-free boundary conditions with a crack in the middle of the bar. As the crack moves from the middle to the end of the free boundaries, as it is expected, the effect of the crack is lessen. When crack reaches exactly to the end of the bar, it can be supposed that there is no crack in the bar. Therefore, we can conclude that as crack move to the free end boundaries, the effect of crack decreases.

Now, we can suppose a bar with fixed-fixed boundaries with a crack in the middle. In this case, as the crack moves toward the fixed end, the effect of the crack is more apparent. Especially when the crack reaches around the fixed ends of the bar, the frequency severely is affected by the crack. In this case the tube may even be separated from the support.

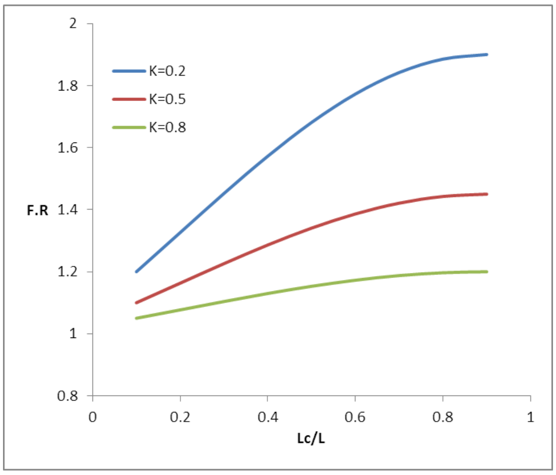

Fig. 5 Frequency ratio versus normalized crack position

Effect of scale parameter

To demonstrate the influence of the scale parameter on the free axial vibration of cracked nanobeams, variations of the frequency ratio versus the scale parameter is plotted for different boundary conditions in Fig. 6. From this figure, the following important notes can be achieved. Firstly, in addition to the crack severity, the scale parameter has a decreasing effect on the axial frequency. The decreasing effect of the crack is the result of the rigidity loss of the structure, and the more flexible the structure is, the smaller its frequency is. The decreasing effect of the scale parameter can be explained in this way that the scale has a negative modulus, and its negative amount is increased by the surface residual stress resulting in the decrease in the axial rigidity of nanobeam. Moreover, the scale parameter decreases the potential energy of the system, and as it is known, as the potential energy of the system decreases, its natural frequency decreases too.

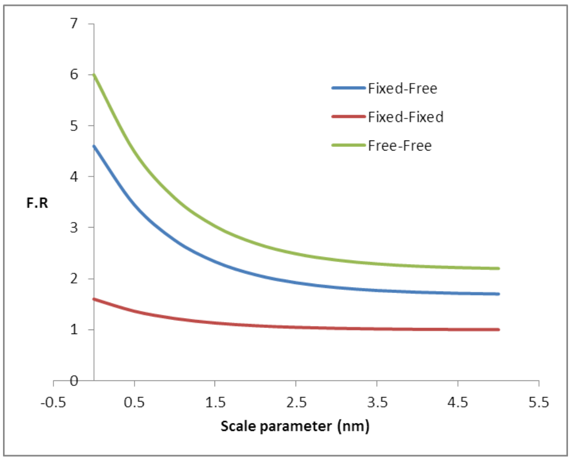

Fig. 6 Frequency ratio versus scale parameter for different boundary condition

Then, the scale effect is taken account in the analysis that makes the nanobeam stiffer. Therefore, a larger nonlocal parameter leads to a decrease of the crack effect on the frequency.

Effect of tube length

Next, the effects of the tube length on the axial frequency are noticed. For this purpose, in Fig. 7, the frequency ratio of nanobeams with respect to tube length for different mode numbers is depicted.

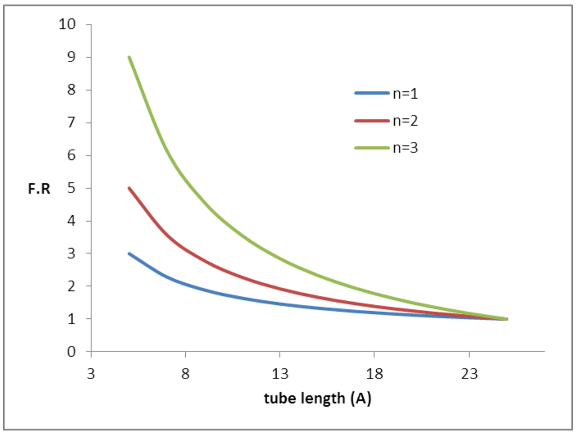

Fig. 7 Frequency ratio versus tube length (A) for different mode number

From this figure, it can be concluded that higher mode numbers reduce to higher frequencies. This increasing is more apparent in lower tube length. The frequency is decreases as the tube length increases. As the tube length increases more, the difference between mode number decreases. In other words, for nanobeams with enough large length, the effect of the mode number on frequency is negligible, and only the crack can have an apparent decreasing effect on lower length.

Effect of crack type

Effect of the scale parameter on the critical stress intensity factor (also known as fracture toughness) has been represented in Fig. 8 for various crack type for fixed-free boundary condition. It is assumed the tube length to be $L = 20A$ with crack length $a = 0.1\ nm$. The crack depth is assumed to be $\frac{a}{d} = 0.2$ and the first mode of vibration is considered. From this figure, it can be observed that fracture toughness decreases as the scale parameter increases. This reduction is more apparent in lower tube length.

As it is expected, it can be seen that the double edge cracked nanotube has lower fracture toughness than a single cracked nanotube. These discrepancies become more significant as the crack depth increases. In other words, the predicted local flexibility for a double edge cracked nanotube is more dominant when the crack depth increases.

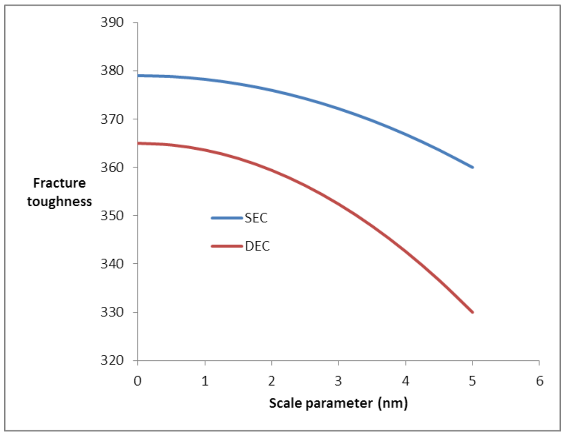

Fig. 8 Fracture toughness ($TPa\sqrt{nm}$) versus scale parameter for different crack type

6- Conclusion

In this study, the free axial vibration of cracked nanobeams in the presence of the scale effects is investigated using DM elasticity theory with different boundary conditions. The governing equation of motion is obtained with dividing the nanotube into two segment in crack location connected with a linear spring with flexibility determined from fracture mechanics principles. The numerical results reveal that both the crack severity and scale parameter have a decreasing effect on the natural frequency.

From the present study, the following notes are especially obtained:

1. By using the present method, the eigenvalue equation for a cracked nanorod with any kinds of boundary conditions can be conveniently

determined from a fourth order determinant.

2. The vibration frequency of nanorods is shown to be dependent on the crack severity, the end conditions and scale parameter.
3. Influence of a crack on the dynamic behavior of the nanorod is sensitive to its location and length not to nanotube material. Natural frequency reduces due to the presence of cracks. The amount of reduction depends on location and size of cracks. As crack moves to the fixed support, more reduction in frequency is observed. On the other hands, the effect of crack is more pronounced when the cracks are near to the fixed end than at free end.
4. As the scale parameter increases, the frequency decreases. The scale effects are more prominent for free end boundary condition. Also, for larger scale parameter, reduction in vibration frequency is not very sensible.
5. For a certain crack location, the natural frequencies of a cracked nanotube are inversely proportional to the crack severity. While for a certain crack severity, change in natural frequency is less as the crack position moves away from fixed end.
6. By increasing the length of the nanotube, the effect of the mode number decreases. For the nanotubes with enough large length, the effect of the mode number on the frequency of cracked nanotube can be neglected.
7. This study also shows that the effect of the crack with the presence of the scale parameter on the axial frequency decreases in comparison with the case that only the effect of the crack on the axial frequency is considered.
8. In the same conditions, Zigzag nanotubes has lower frequency that the Armchair one. This difference is more pronounced in lower cracked parameter.
9. For a certain crack parameter, change in natural frequency is less as the crack position moves away from fixed end.
10. The scale parameter has decreasing effect on fracture toughness. As expected nanotubes with DEC has lower fracture toughness than SEC.

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