Journal of Solid Mechanics Vol. 9, No. 4 (2017) pp. 849-864

Non-Linear Analysis of Asymmetrical Eccentrically Stiffened FGM Cylindrical Shells with Non-Linear Elastic Foundation

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Received 2 August 2017; accepted 5 October 2017

ABSTRACT

In this paper, semi-analytical method for asymmetrical eccentrically stiffened FGM cylindrical shells under external pressure and surrounded by a linear and non-linear elastic foundation is presented. The proposed linear model is based on two parameter elastic foundation Winkler and Pasternak. According to the von Karman nonlinear equations and the classical plate theory of shells, strain-displacement relations are obtained. The smeared stiffeners technique and Galerkin method, used for solving nonlinear problem. To finding the nonlinear dynamic response of fourth order Runge-Kutta method is used. The effect of parameters asymmetrical eccentrically stiffened on the nonlinear dynamic buckling response of FGM cylindrical shells have been investigated.

Keywords : FGM cylindrical shells; Non-linear dynamic analysis; Asymmetrical stiffened; Non-linear elastic foundation.

1 INTRODUCTION

THE eccentrically stiffened FGM cylindrical shells have more application in modern Engineering. Study on nonlinear behavior of these structures is important of the practical. Research on nonlinear stability of these structures has been of interest to scientists. By searching on nonlinear dynamic analysis can be said that study on analytical methods is very limited. Dung and Nam [1] studied the non-linear dynamic analysis of eccentrically stiffened FGM cylindrical shells under external pressure with an elastic foundation that material of stiffeners is metal. Darabi et al. [2] analyzed the linear and non-linear parametric resonance analysis of FGM cylindrical shell. Sofiyev and Schnack [3] continued to investigate the critical parameters for the cylindrical thin shell under a linear increase of dynamic torsional loading, and cyclic axial loading by using Galerkin and Ritz methods. The thermo-mechanical vibration analysis of FGM shell by flow was presented by Sheng and Wang [4]. Sofiyev [5] studied the non-linear dynamic buckling of FGM truncated conical shell. Thermal vibration of magnetostrictive FGM shell was presented by Hong [6]. Huang and Han [7] studied the nonlinear dynamic buckling of FGM cylindrical shells under axial load time dependent based on Budiansky-Roth criterion [8].

Dynamic stability of circular cylindrical shells with combined effect of compressive static and periodic axial loads was presented by Pellicano [9]. The Sanders-Koiter theory was applied to model the nonlinear dynamics of the

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system in the case of finite amplitude of vibration. Duc and Thang [10] continued to investigate the nonlinear dynamic response of the eccentrically stiffened FGM cylindrical shells by using the first-order shear deformation theory and the stress function with the full equations of motion.

A review of studies shows that very few studies on the analytical solution non-linear dynamic of asymmetrical eccentrically stiffened FGM cylindrical shells with linear and non-linear elastic foundation have been done. In this paper, the non-linear dynamic analysis of asymmetrical eccentrically stiffened FGM cylindrical shells with non-linear elastic foundation under uniform external pressure studied. The nonlinear equations is based on the classical theory and large deflection hypothesis using the smeared stiffeners technique and Galerkin method for nonlinear dynamic response of fourth order Runge-Kutta method is used.

2 FORMULATION

2.1 FGM power law properties

In this paper, FGM made of metal and ceramic. The volume-fraction to be given by a power law

$$\begin{aligned} &V_c = V_c(z) = \left(\frac{2z+h}{2h}\right)^k \\ &V_m = V_m(z) = 1 - V_c(z) \end{aligned} \tag{1}$$

which h is the thickness of shell, $k \geq 0$ is the volume-fraction index, z is the thickness coordinate, footnotes c and m show ceramic and metal, respectively.

Effective properties (Pr_{eff}) of FGM shell by linear combination law is as follows [11]

$$\mathrm{Pr}_{eff} = \mathrm{Pr}_{ou}(z)V_{ou}(z) + \mathrm{Pr}_{in}(z)V_{in}(z) \tag{2}$$

According to the mentioned law, The Young's modulus of the shell and stiffeners can be expressed in the following form

$$\begin{aligned} &E(z) = E_m V_m + E_c V_c = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k, \ -\frac{h}{2} \leq z \leq \frac{h}{2} \\ &E_s = E_c + (E_m - E_c)\left(\frac{2z-h}{2h_s}\right)^{k_2}, \ -\frac{h}{2} \leq z \leq \frac{h}{2} + h_s \\ &\rho(z) = \rho_m V_m + \rho_c V_c = \rho_m + (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^k, \ -\frac{h}{2} \leq z \leq \frac{h}{2} \\ &\rho_s = \rho_c + (\rho_m - \rho_c)\left(\frac{2z-h}{2h_s}\right)^{k_2}, \ -\frac{h}{2} \leq z \leq \frac{h}{2} + h_s \end{aligned} \tag{3}$$

which E_m, E_c and ρ_m, ρ_c are the Young's modulus and mass density of the metal and ceramic, respectively, E_s and ρ_s is the Young's modulus and mass density of stiffeners, respectively, $k_2 \geq 0$ is the volume-fraction index of stiffeners.

FGM cylindrical thin shell is assumed with length L, radius R, which is surrounded by an elastic foundation. Material properties of stiffeners are assumed FGM (Fig.1). Original coordinates, x, y, z are in the axial, circumferential, and inward radial directions respectively.

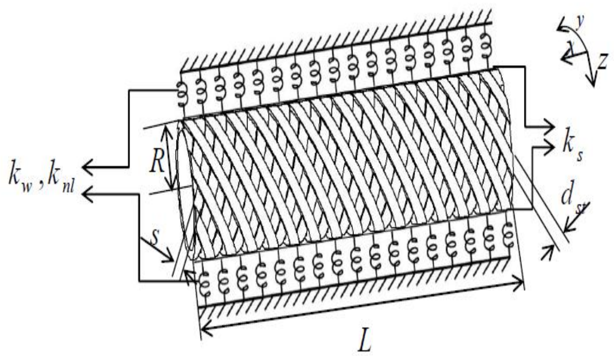


Fig.1
Configuration of an asymmetrical stiffened cylindrical shell surrounded with foundation.

The strains across the shell thickness at a distance z from the mid-surface are represented by

$$\varepsilon_x = \varepsilon_x^0 - z\,\chi_x\,,\ \varepsilon_y = \varepsilon_y^0 - z\,\chi_y\,,\ \gamma_{xy} = \gamma_{xy}^0 - 2z\,\chi_{xy} \tag{4}$$

where $\varepsilon_x^0, \varepsilon_y^0$ are normal strains, γ_{xy}^0 is the shear strain at the mid-surface, $\chi_x, \chi_y, \chi_{xy}$ are the change of curvatures and twist of shell.

2.2 Displacement-strain-stress relations

According to the von Karman nonlinear strain-displacement relations [12] the strain components at the mid-surface of cylindrical shells as:

$$\begin{aligned}
&\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\
&\varepsilon_y^0 = \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\
&\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y},\ \chi_x = \frac{\partial^2 w}{\partial x^2},\ \chi_y = \frac{\partial^2 w}{\partial y^2},\ \chi_{xy} = \frac{\partial^2 w}{\partial x\,\partial y}
\end{aligned} \tag{5}$$

where $u = u(x, y, t), v = v(x, y, t), w = w(x, y, t)$ are displacements along x, y, z axes, respectively.

According to Eq. (5), compatibility equation be expressed in the following form

$$\frac{\partial^2 \varepsilon_x^0}{\partial y^2} + \frac{\partial^2 \varepsilon_y^0}{\partial x^2} - \frac{\partial^2 \gamma_{xy}^0}{\partial x\,\partial y} = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{6}$$

The stress-strain relationship for FGM cylindrical shells can be written as follows

$$\begin{aligned}
&\sigma_x^{sh} = \frac{E(z)}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \\
&\sigma_y^{sh} = \frac{E(z)}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \\
&\tau_{xy}^{sh} = \frac{E(z)}{2(1+\nu)}\gamma_{xy}
\end{aligned} \tag{7}$$

The Poisson's ratio (ν) is assumed to be constant, $\sigma_x^{sh}, \sigma_y^{sh}$ normal stress in x, y coordinates, respectively, τ_{xy}^{sh} is shear stress on the un-stiffened shell. By rotation of the strain tensor, the stress-strain relations of the asymmetrical stiffeners are obtained. With transformation of strains from the xy -axis to the $1'2'$ -axis and $1''2''$ -axis (Fig. 2), Eqs. (8) and (9) can be made [13]

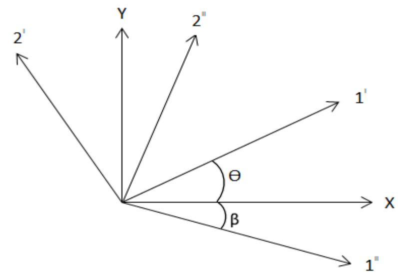


Fig.2
Rectangular coordinates rotation.

$$\begin{aligned}\varepsilon_1' &= \varepsilon_x \cos^2\theta + 2\gamma_{xy}\sin\theta\cos\theta + \varepsilon_y \sin^2\theta \\ \varepsilon_2' &= \varepsilon_x \sin^2\theta - 2\gamma_{xy}\sin\theta\cos\theta + \varepsilon_y \cos^2\theta\end{aligned} \tag{8}$$

$$\begin{aligned}\varepsilon_1'' &= \varepsilon_x \cos^2\beta - 2\gamma_{xy}\sin\beta\cos\beta + \varepsilon_y \sin^2\beta \\ \varepsilon_2'' &= \varepsilon_x \sin^2\beta + 2\gamma_{xy}\sin\beta\cos\beta + \varepsilon_y \cos^2\beta\end{aligned} \tag{9}$$

According to the uniaxial Hooke's law

$$\varepsilon_1' = \frac{P'}{hdE_s},\ \varepsilon_1'' = \frac{P''}{hdE_s} \tag{10}$$

where d is the width of stiffeners, p', p'' are stiffener loads in the $1'2'$ -axis and $1''2''$ -axis, respectively and θ, β are the angle of the stiffeners.

According to Fig. 3, the length of the stiffener grid

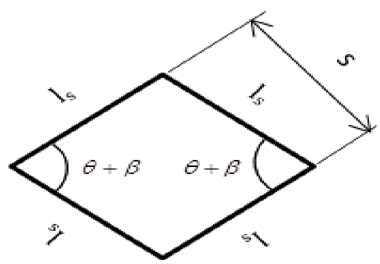


Fig.3
View a rhombic stiffener grid.

$$l_s = \frac{s}{\sin(\theta+\beta)} \tag{11}$$

where s is the stiffener spacing.

The stress-strain relations for FGM asymmetrical stiffeners are

$$
\begin{aligned}
\sigma_x^s &= \frac{P^{'}\cos\theta}{l_s h\left(\sin\theta+\sin\beta\right)}+\frac{P^{"}\cos\beta}{l_s h\left(\sin\theta+\sin\beta\right)} \\
&= \frac{P^{'}\cos\theta+P^{"}\cos\beta}{l_s h\left(\sin\theta+\sin\beta\right)}=\frac{h_s dE_s}{l_s h_s}\frac{\left(\varepsilon_1^{'}\cos\theta+\varepsilon_1^{"}\cos\beta\right)}{\left(\sin\theta+\sin\beta\right)} \\
&= \frac{h_s dE_s}{sh_s}\frac{\left(\varepsilon_1^{'}\cos\theta+\varepsilon_1^{"}\cos\beta\right)}{\left(\sin\theta+\sin\beta\right)}\sin\left(\theta+\beta\right) \\
&= Z_1\left[\varepsilon_x\left(\cos^3\theta+\cos^3\beta\right)+2\gamma_{xy}\left(\sin\theta\cos^2\theta-\sin\beta\cos^2\beta\right)+\varepsilon_y\left(\sin^2\theta\cos\theta+\sin^2\beta\cos\beta\right)\right]
\end{aligned}
\tag{12}
$$

where

$$
Z_1=\frac{h_s dE_s}{sh_s}\frac{\sin\left(\theta+\beta\right)}{\left(\sin\theta+\sin\beta\right)}
\tag{13}
$$

Similarly

$$
\begin{aligned}
\sigma_y^s &= \frac{h_s dE_s}{sh_s}\frac{\left(\varepsilon_1^{'}\sin\theta+\varepsilon_1^{"}\sin\beta\right)}{\left(\cos\theta+\cos\beta\right)}\sin\left(\theta+\beta\right) \\
&= Z_2\left[\varepsilon_x\left(\sin\theta\cos^2\theta+\sin\beta\cos^2\beta\right)+2\gamma_{xy}\left(\sin^2\theta\cos\theta-\sin^2\beta\cos\beta\right)+\varepsilon_y\left(\sin^3\theta+\sin^3\beta\right)\right] \\
\tau_{xy}^s &= \frac{h_s dE_s}{2sh_s}\left(\varepsilon_1^{'}-\varepsilon_1^{"}\right)\sin\left(\theta+\beta\right) \\
&= Z_3\left[\varepsilon_x\left(\cos^2\theta-\cos^2\beta\right)+2\gamma_{xy}\left(\sin\theta\cos\theta+\sin\beta\cos\beta\right)+\varepsilon_y\left(\sin^2\theta-\sin^2\beta\right)\right]
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
Z_2 &= \frac{h_s dE_s}{sh_s}\frac{\sin\left(\theta+\beta\right)}{\left(\cos\theta+\cos\beta\right)} \\
Z_3 &= \frac{h_s dE_s}{2sh_s}\sin\left(\theta+\beta\right)
\end{aligned}
\tag{15}
$$

σ_x^s, σ_y^s is the normal stress of stiffeners. τ_{xy}^s, h_s are shear stress and thickness of the stiffeners, respectively. θ, β are the asymmetrical angle of stiffeners. To consider the effect of stiffeners on the shell used the smeared stiffeners technique. By integrating the stress-strain equations and calculating the resultant forces and moments for stiffened FGM cylindrical shells will be [14, 15]

$$
\begin{aligned}
N_x &= B_{11}\varepsilon_x^0+B_{12}\varepsilon_y^0-B_{14}\chi_x-B_{15}\chi_y \\
N_y &= B_{21}\varepsilon_x^0+B_{22}\varepsilon_y^0-B_{24}\chi_x-B_{25}\chi_y \\
N_{xy} &= B_{33}\gamma_{xy}^0-2B_{36}\chi_{xy}
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
M_x &= B_{14}\varepsilon_x^0+B_{15}\varepsilon_y^0-B_{41}\chi_x-B_{42}\chi_y \\
M_y &= B_{24}\varepsilon_x^0+B_{25}\varepsilon_y^0-B_{51}\chi_x-B_{52}\chi_y \\
M_{xy} &= B_{36}\gamma_{xy}^0-2B_{63}\chi_{xy}
\end{aligned}
\tag{17}
$$

B_{ij} are components of the extensional, bending and coupling stiffeners of FGM cylindrical shells that can be found in Appendix A. N_x, N_y, N_{xy} are in-plane normal force and shearing force intensities, respectively. M_x, M_y, M_{xy} are bending moment and twisting moment intensities, respectively. Sort by Eq. (16) in terms of the strain as follows

$$\begin{aligned}
\varepsilon_x^0 &= A_{22}^* N_x - A_{12}^* N_y + B_{11}^* \chi_x + B_{12}^* \chi_y \\
\varepsilon_y^0 &= A_{11}^* N_y - A_{21}^* N_x + B_{21}^* \chi_x + B_{22}^* \chi_y \\
\gamma_{xy}^0 &= A_{33}^* + 2B_{36}^* \chi_{xy}
\end{aligned} \tag{18}$$

Substituting Eq. (18) in Eq. (17) can be written

$$\begin{aligned}
M_x &= B_{11}^{**} N_x + B_{21}^{**} N_y - D_{11}^* \chi_x - D_{12}^* \chi_y \\
M_y &= B_{12}^{**} N_x + B_{22}^{**} N_y - D_{21}^* \chi_x - D_{22}^* \chi_y \\
M_{xy} &= B_{36}^* N_{xy} - 2D_{36}^* \chi_{xy}
\end{aligned} \tag{19}$$

Non-linear equations thin circular cylindrical shell based on the classical shell theory follow as [2, 3 and 16]

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\
&\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
&\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} + \frac{1}{R} N_y + q_0 \\
&-k_w w + k_s \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + k_{nl} w^3 = \rho_1 \frac{\partial^2 w}{\partial t^2} + 2\rho_1 \varepsilon \frac{\partial w}{\partial t}
\end{aligned} \tag{20}$$

In Eq. (20) k_w is Winkler foundation modulus, k_s is shear stiffness layer based on Pasternak, k_{nl} is nonlinear foundation modulus, q_0 is external pressure, t is time (s), ε is damping coefficient and

$$\rho_1 = \int_{-h/2}^{h/2} \rho(z) dz + \int_{-h/2}^{h/2+h_s} \rho(z) \frac{d_s}{S_s} dz + \int_{-h/2}^{h/2+h_r} \rho(z) \frac{d_r}{S_r} dz = \left(\rho_m + \frac{\rho_c - \rho_m}{k+1} \right) h + \left(\rho_c + \frac{\rho_c - \rho_m}{k_2 + 1} \right) \frac{d_s h_s}{S_s} \tag{21}$$

According to the first two of Eq. (20), a stress function φ may be defined as:

$$N_x = \frac{\partial^2 \varphi}{\partial y^2},\ N_y = \frac{\partial^2 \varphi}{\partial x^2},\ N_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y} \tag{22}$$

Substituting Eq. (18) in to Eq. (6) and Eq. (19) in to the third of Eq. (20) and according to the Eq. (5) and (22)

$$\begin{aligned}
&A_{11}^* \frac{\partial^4 \varphi}{\partial x^4} + \left(A_{33}^* - A_{12}^* + A_{21}^* \right) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + A_{22}^* \frac{\partial^4 \varphi}{\partial y^4} + B_{21}^* \frac{\partial^4 w}{\partial x^4} + \left(B_{11}^* + B_{22}^* - 2B_{36}^* \right) \frac{\partial^4 w}{\partial x^2 \partial y^2} \\
&\qquad + B_{12}^* \frac{\partial^4 w}{\partial y^4} + \frac{1}{R} \frac{\partial^2 w}{\partial x^2} - \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] = 0
\end{aligned} \tag{23}$$

$$\rho_1 \frac{\partial^2 w}{\partial t^2} + 2\rho_1 \varepsilon \frac{\partial w}{\partial t} + D_{11}^* \frac{\partial^4 w}{\partial x^4} + \left(D_{12}^* + D_{21}^* + 4D_{36}^*\right) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}^* \frac{\partial^4 w}{\partial y^4} - B_{21}^{**} \frac{\partial^4 \varphi}{\partial x^4}$$
$$-\left(B_{11}^{**} + B_{22}^{**} - 2B_{36}^*\right) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} - B_{12}^{**} \frac{\partial^4 \varphi}{\partial y^4} - \frac{1}{R} \frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 \varphi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \tag{24}$$
$$-\frac{\partial^2 \varphi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - q_0 + k_w w - k_s \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - k_{nl} w^3 = 0$$

Eqs. (23) and (24) are a non-linear equation system in terms of two unknown parameter φ and w.

3 DYNAMIC GALERKIN APPROACH

Suppose the asymmetrical stiffened FGM cylindrical shell is simply supported. The deflection of cylindrical shells consider the three-term as follows [17,18]

$$w = f_0(t) + f_1(t) \sin\frac{m\pi x}{L} \sin\frac{ny}{R} + f_2(t) \sin^2\frac{m\pi x}{L} \tag{25}$$

In which $f_0(t), f_1(t)$ and $f_2(t)$ time dependent pre-buckling, linear and nonlinear unknown amplitude, respectively. Also $\sin\frac{m\pi x}{L}\sin\frac{ny}{R}, \sin^2\frac{m\pi x}{L}$ are linear and nonlinear buckling shape and *m, n* are the number of half wave and full wave in the axial and circumferential direction, respectively.

Substituting Eq. (25) in Eq. (23) and solving it, obtained the equation for the unknown function φ as follows

$$\varphi = \varphi_1 \cos\frac{2m\pi x}{L} + \varphi_2 \cos\frac{2ny}{R} - \varphi_3 \sin\frac{m\pi x}{L} \sin\frac{ny}{R} + \varphi_4 \sin\frac{3m\pi x}{L} \sin\frac{ny}{R} - \sigma_{0y} h \frac{x^2}{2} \tag{26}$$

σ_{0y} is the average circumferential stress and the coefficients φ_i as follows

$$\begin{aligned}
\varphi_1 &= \frac{n^2\lambda^2}{32A_{11}^* m^2\pi^2} f_1^2 - \frac{\left(4\lambda L - 16A_{24}^* m^2\pi^2\right)}{32A_{11}^* m^2\pi^2} f_2 \\
\varphi_2 &= \frac{m^2\pi^2}{32A_{22}^* n^2\lambda^2} f_1^2 \\
\varphi_3 &= \frac{B}{A} f_1 + \frac{m^2 n^2 \pi^2 \lambda^2}{A} f_1 f_2 \\
\varphi_4 &= \frac{m^2 n^2 \pi^2 \lambda^2}{G} f_1 f_2
\end{aligned} \tag{27}$$

In addition, cylindrical shell must be satisfy the circumferential close conditions as:

$$\int_0^{2\pi R}\int_0^L \frac{\partial v}{\partial y} \mathrm{d}x\,\mathrm{d}y = \int_0^{2\pi R}\int_0^L \left[\varepsilon_y^0 + \frac{w}{R} - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \right] \mathrm{d}x\,\mathrm{d}y = 0 \tag{28}$$

Substituting Eq. (25) and (26) in to Eq. (24) and by applying the Galerkin method in the ranges $0 \le x \le L$ and $0 \le y \le 2\pi R$ and doing mathematic, we have

$$\left(\frac{d^2 f_0}{dt^2}+2\varepsilon\frac{df_0}{dt}\right)+b_{11}f_0-b_{12}f_1^2-b_{13}f_1^2f_2+b_{14}f_2-a_{13}q_0+b_{15}k_wf_0$$
$$+b_{16}k_wf_2-b_{17}k_sf_2+b_{18}k_{nl}f_0^3+b_{19}k_{nl}f_2^3+b_{110}k_{nl}f_0f_1^2$$
$$+b_{111}k_{nl}f_0f_2^2+b_{112}k_{nl}f_0^2f_2+b_{113}k_{nl}f_1^2f_2=0 \quad (29a)$$

$$\left(\frac{d^2 f_1}{dt^2}+2\varepsilon\frac{df_1}{dt}\right)+a_{22}f_1+b_{21}f_1f_0+b_{22}f_1f_2+a_{25}f_1f_2^2+b_{23}f_1^3$$
$$-b_{24}k_wf_1f_0-b_{25}k_wf_1f_2+b_{26}k_sf_1f_2+a_{27}k_wf_1+a_{28}k_sf_1$$
$$+b_{27}k_{nl}f_1^3+b_{28}k_{nl}f_0^2f_1+b_{29}k_{nl}f_1f_2^2+b_{210}k_{nl}f_0f_1f_2$$
$$+b_{211}k_{nl}f_1f_0^3+b_{212}k_{nl}f_1f_2^3+b_{213}k_{nl}f_0f_1^3+b_{214}k_{nl}f_0f_1f_2^2$$
$$+b_{215}k_{nl}f_0^2f_1f_2+b_{216}k_{nl}f_1^3f_2=0 \quad (29b)$$

$$\left(\frac{d^2 f_2}{dt^2}+2\varepsilon\frac{df_2}{dt}\right)+a_{31}f_1^2+a_{32}f_1^2f_2+a_{33}f_2+a_{34}k_w\left(\frac{3}{4}f_2+f_0\right)$$
$$+b_{31}k_{nl}f_0^3+b_{32}k_{nl}f_2^3+b_{33}k_{nl}f_0f_1^2+b_{34}k_{nl}f_0f_2^2$$
$$+b_{35}k_{nl}f_0^2f_2+b_{36}k_{nl}f_1^2f_2+a_{35}k_sf_2=0 \quad (29c)$$

If $f=W_{\max}$, then according to Eq. (19), with replacing $x=iL/2m$ and $y=j\pi R/2n$ (i, j are odd integer numbers), the maximal deflection of the shells as follow

$$f=f_0+f_1+f_2 \quad (30)$$

3.1 Nonlinear vibration analysis

To study the nonlinear vibration of asymmetrical eccentrically stiffened FGM cylindrical shells under external pressure with the intensity $q_0=Q\sin\Omega t$, Eqs. (29a-c) is as follows

$$\left(\frac{d^2 f_0}{dt^2}+2\varepsilon\frac{df_0}{dt}\right)+b_{11}f_0-b_{12}f_1^2-b_{13}f_1^2f_2+b_{14}f_2-a_{13}Q\sin(\Omega t)$$
$$+b_{15}k_wf_0+b_{16}k_wf_2-b_{17}k_sf_2+b_{18}k_{nl}f_0^3+b_{19}k_{nl}f_2^3$$
$$+b_{110}k_{nl}f_0f_1^2+b_{111}k_{nl}f_0f_2^2+b_{112}k_{nl}f_0^2f_2+b_{113}k_{nl}f_1^2f_2=0 \quad (31a)$$

$$\left(\frac{d^2 f_1}{dt^2}+2\varepsilon\frac{df_1}{dt}\right)+a_{22}f_1+b_{21}f_1f_0+b_{22}f_1f_2+a_{25}f_1f_2^2+b_{23}f_1^3$$
$$-b_{24}k_wf_1f_0-b_{25}k_wf_1f_2+b_{26}k_sf_1f_2+a_{27}k_wf_1+a_{28}k_sf_1$$
$$+b_{27}k_{nl}f_1^3+b_{28}k_{nl}f_0^2f_1+b_{29}k_{nl}f_1f_2^2+b_{210}k_{nl}f_0f_1f_2$$
$$+b_{211}k_{nl}f_1f_0^3+b_{212}k_{nl}f_1f_2^3+b_{213}k_{nl}f_0f_1^3+b_{214}k_{nl}f_0f_1f_2^2$$
$$+b_{215}k_{nl}f_0^2f_1f_2+b_{216}k_{nl}f_1^3f_2=0 \quad (31b)$$

$$\left(\frac{d^2 f_2}{dt^2}+2\varepsilon\frac{df_2}{dt}\right)+a_{31}f_1^2+a_{32}f_1^2f_2+a_{33}f_2+a_{34}k_w\left(\frac{3}{4}f_2+f_0\right)$$
$$+b_{31}k_{nl}f_0^3+b_{32}k_{nl}f_2^3+b_{33}k_{nl}f_0f_1^2+b_{34}k_{nl}f_0f_2^2$$
$$+b_{35}k_{nl}f_0^2f_2+b_{36}k_{nl}f_1^2f_2+a_{35}k_sf_2=0 \quad (31c)$$

Q is amplitude of excitation and Ω is excitation frequency. By using Eqs. (31a-c), the fundamental frequencies of natural vibration and nonlinear response of system can be calculated. To finding the nonlinear dynamic response of fourth order Runge-Kutta method is used. Ignoring the uniform buckling shape and nonlinear buckling shape and assuming free vibration without the linear damping and regardless of the higher-order terms, we have

$$\frac{d^2 f_1}{dt^2} + \left(a_{22} + a_{27}k_w + a_{28}k_s\right)f_1 = 0 \tag{32}$$

The fundamental frequencies of natural vibration of asymmetrical eccentrically stiffened FGM cylindrical shells as follow

$$\omega_{mn} = \sqrt{a_{22} + a_{27}k_w + a_{28}k_s} \tag{33}$$

4 NUMERICAL RESULTS

In this section, the asymmetrical stiffened FGM cylindrical shells by an elastic foundation are considered with $R = 0.5m, L = 0.75m$. The combination of materials consists of Aluminum $E_m = 70GPa, \rho_m = 2702\,kg/m^3$ and Alumina $E_c = 380GPa, \rho_c = 3800\,kg/m^3$. The Poisson's ratio is chosen to be 0.3. The height of stiffeners is 0.01 *m* and width is 0.0025 *m*. Each of the stiffener system includes 25 stiffeners distributed regular.

In order to verify the formulation, in Fig. 4 natural frequencies of perfect stiffened isotropic cylindrical shells without elastic foundation compared with the results of the analysis Sewall and Naumann [19] and Sewall et al. [20]. In Table 1. the natural frequencies of isotropic cylindrical shells with elastic foundation to compared with the results given by Paliwal et al. [21] and Sofiyev et al. [22]. As can be seen, good agreement is obtained in this comparison. The calculated based on data of Table 2. is considered.

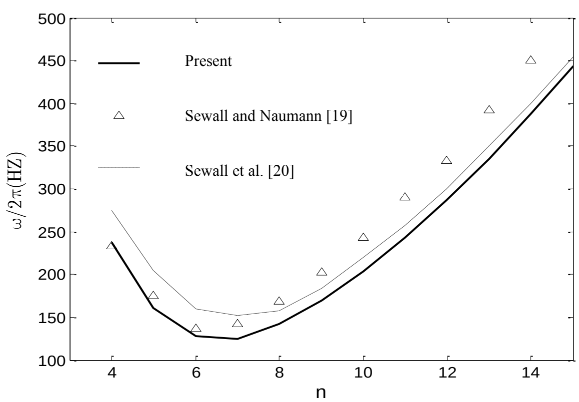


Fig.4
Comparison of natural frequency of isotropic stiffened shells.

Table 1
Comparison of natural frequency of un-stiffened shells with elastic foundation.

n	Present	Sofiyev et al. [22]	Errors (%)	Paliwal et al. [21]	Errors (%)
1	0.675	0.679	0.65	0.678	0.60
2	0.362	0.364	0.66	0.363	0.47
3	0.206	0.208	0.65	0.205	0.70
4	0.137	0.138	0.56	0.127	7.29

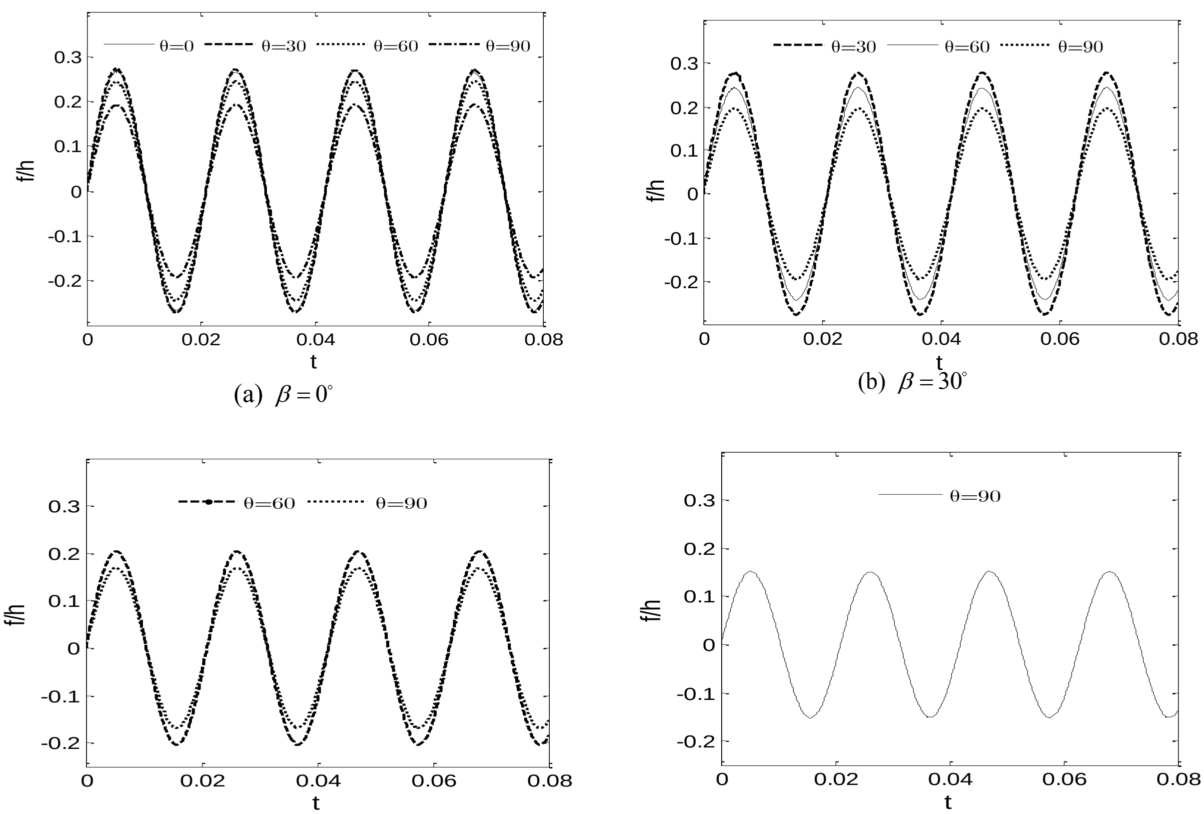
Table 2
Data required.

$k_s(N/m^3)$	$k_w(N/m)$	$k_{nl}(N/m^5)$	k	k_2	R/h	$h_s(m)$	$d_s(m)$	m
5×10^5	2.5×10^4	$\pm1\times10^{10}$	1	1	250	0.01	0.0025	1

Fig. 5 show effects of stiffeners with different angles on the non-linear responses of cylindrical shells with elastic foundation and $n=5$. In previous works, only stringer and ring stiffeners are investigated that amplitude of vibrations ring stiffeners is lower than stringer stiffeners and this subject in the present work is confirmed. According to Fig. 5 for the mid-states have been chosen the stiffeners with various angle that can be observed the effects of them on the non-linear responses of cylindrical shells. As can be seen of Fig. 6 minimum amplitude of vibrations related to the shells with ring stiffeners and maximum of the amplitude of vibrations when the angle of both series stiffeners together is 30^o.

Fig. 6 shows effects of stiffeners with different angles on the natural frequencies of cylindrical shells with elastic foundation. Fig. 6 also shows in the high modes, by increasing angles of stiffeners, natural frequencies of shell is increases. (So in the higher modes, maximum natural frequencies for the state to know that angle of stiffener is 90^o and minimum natural frequencies for the state to know that angle of stiffener is 0^o.)

Fig. 7 shows effects of stiffeners with different angles on the dynamic responses of cylindrical shells with elastic foundation for loading speed (5×10^6 N/m^2s). In this section, the height of stiffeners is 0.005 *m* and width is 0.002 *m* and $n=8$. Fig. 7 shows by increasing angles of stiffeners, critical dynamic buckling load (q) is increases. So maximum critical dynamic buckling load (q) and maximal amplitude response for the state to know that angle of stiffener is 90^o and minimum critical dynamic buckling load (q) for the state to know that angle of stiffener is 0^o.



(a) $\beta=0^\circ$ (b) $\beta=30^\circ$

(c) $\beta=60^\circ$ (d) $\beta=90^\circ$

Fig.5
Non-linear responses of cylindrical shells with various angle of stiffeners.

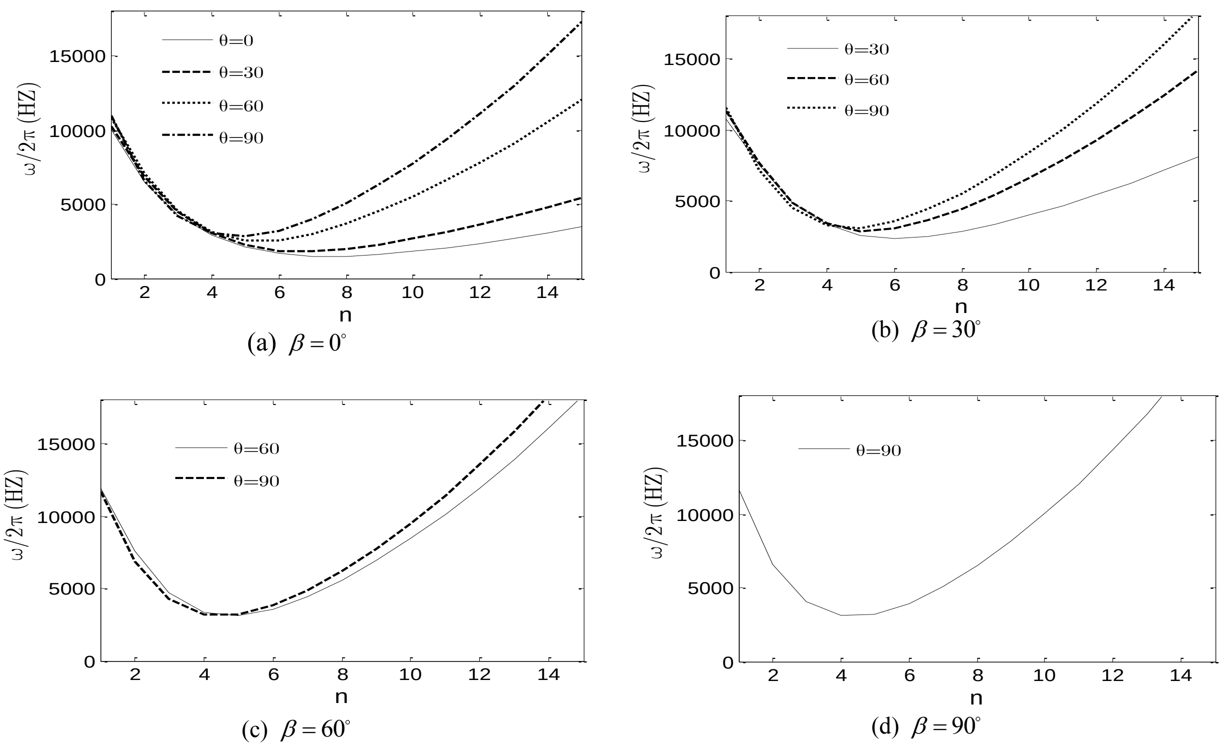


Fig.6
Natural frequencies of cylindrical shells with various angle of stiffeners.

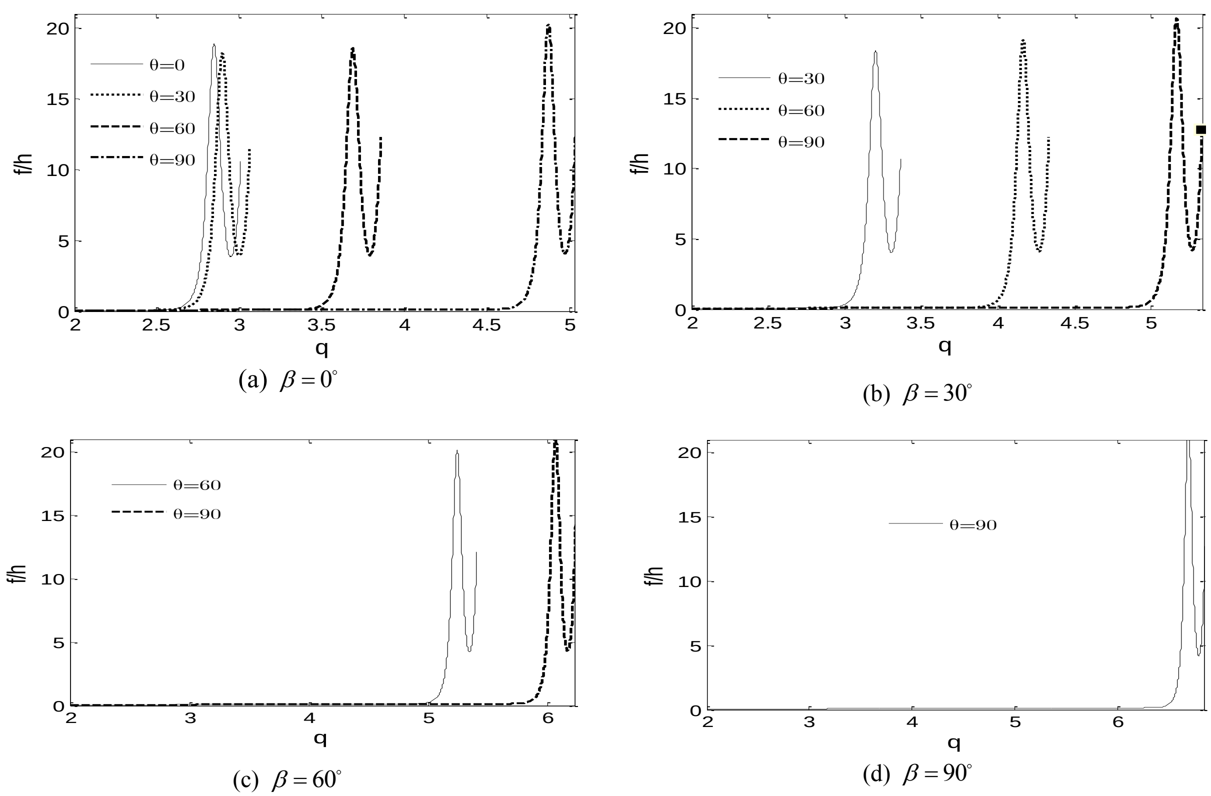
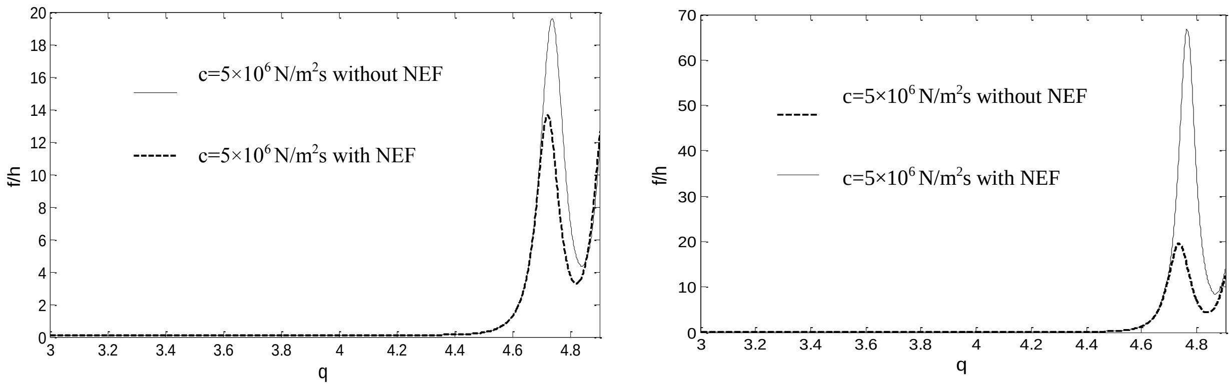


Fig.7
Dynamic responses of cylindrical shells with various angle of stiffeners.

Figs. 8 show effects non-linear elastic foundation on the dynamic responses of cylindrical shell with for two values of loading speed (5×10^6 N/m^2s) and $n = 8$. As can be observed the maximal amplitude response increases and reduces, when the coefficient of non-linear elastic foundation is positive and negative, respectively. Figs. 8 show the critical dynamic buckling load and maximal amplitude response increases when the loading speed increases.



(a) (b)

Fig.8
Effect of non-linear elastic foundation on the dynamic responses of cylindrical shells. a) Negative non-linear elastic foundation b) Positive non-linear elastic foundation.

5 CONCLUSIONS

The semi-analytical method for asymmetrical eccentrically stiffened FGM cylindrical shells with linear and non-linear elastic foundation under uniform external pressure is presented. The proposed linear model is based on Winkler and Pasternak elastic foundation parameters. According to the Von Karman nonlinear equations and the classical plate theory (CPT) of shells, strain displacement relations are obtained. The smeared stiffeners technique and Galerkin method, used for solving nonlinear problem. With considering three terms approximation for the deflection shape, the relation for frequency-amplitude of non-linear vibration is obtained and the non-linear dynamic responses are analyzed by using fourth order Runge-Kutta method.

Some conclusions are obtained from this study:

a) The amplitude of vibration for shell with asymmetric stiffeners, it be concluded that maximum amplitude, both of the stiffeners have angle of both series stiffeners together is 30^o.
b) In the high modes, the maximum natural frequencies for the state to know that angle of stiffener is 90^o and minimum natural frequencies for the state to know that angle of stiffener is 0^o.
c) By increasing angles of stiffeners, critical dynamic buckling load is increases.
d) Maximum critical dynamic buckling load and maximal amplitude response for the state to know that angle of stiffener is 90^o and minimum critical dynamic buckling load for the state to know that angle of stiffener is 0^o.
e) The maximal amplitude response increases and reduces, when the coefficient of non-linear elastic foundation is positive and negative, respectively
f) The critical dynamic buckling load and maximal amplitude response increases when the loading speed increases.

APPENDIX A

$$\begin{aligned}
&B_{11}=\frac{E_1}{1-\nu^2}+Z_1E_{1s}\left(\cos^3\theta+\cos^3\beta\right),\ B_{12}=\frac{E_1\nu}{1-\nu^2}+Z_1E_{1s}\left(\sin^2\theta\cos\theta+\sin^2\beta\cos\beta\right)\\
&B_{14}=\frac{E_2}{1-\nu^2}+Z_1E_{2s}\left(\cos^3\theta+\cos^3\beta\right),\ B_{15}=\frac{E_2\nu}{1-\nu^2}+Z_1E_{2s}\left(\sin^2\theta\cos\theta+\sin^2\beta\cos\beta\right)\\
&B_{21}=\frac{E_1\nu}{1-\nu^2}+Z_2E_{1s}\left(\sin\theta\cos^2\theta+\sin\beta\cos^2\beta\right),\ B_{22}=\frac{E_1}{1-\nu^2}+Z_2E_{1s}\left(\sin^3\theta+\sin^3\beta\right)\\
&B_{24}=\frac{E_2\nu}{1-\nu^2}+Z_2E_{2s}\left(\sin\theta\cos^2\theta+\sin\beta\cos^2\beta\right),\ B_{25}=\frac{E_2}{1-\nu^2}+Z_2E_{2s}\left(\sin^3\theta+\sin^3\beta\right)\\
&B_{33}=\frac{E_1}{2(1+\nu)}+2Z_3E_{1s}\left(\sin\theta\cos\theta+\sin\beta\cos\beta\right),\ B_{36}=\frac{E_2}{2(1+\nu)}+2Z_3E_{2s}\left(\sin\theta\cos\theta+\sin\beta\cos\beta\right)\\
&B_{41}=\frac{E_3}{1-\nu^2}+Z_1E_{3s}\left(\cos^3\theta+\cos^3\beta\right),\ B_{42}=\frac{E_3\nu}{1-\nu^2}+Z_1E_{3s}\left(\sin^2\theta\cos\theta+\sin^2\beta\cos\beta\right)\\
&B_{51}=\frac{E_3\nu}{1-\nu^2}+Z_2E_{3s}\left(\sin\theta\cos^2\theta+\sin\beta\cos^2\beta\right),\ B_{55}=\frac{E_3}{1-\nu^2}+Z_2E_{3s}\left(\sin^3\theta+\sin^3\beta\right)\\
&B_{63}=\frac{E_3}{2(1+\nu)}+2Z_3E_{3s}\left(\sin\theta\cos\theta+\sin\beta\cos\beta\right)
\end{aligned}\tag{A.1}$$

where

$$\begin{aligned}
&E_1=\int_{-h/2}^{h/2}E_{sh}(z)dz=\left(E_{ou}+\frac{E_{in}-E_{ou}}{k+1}\right)h\\
&E_2=\int_{-h/2}^{h/2}zE_{sh}(z)dz=\frac{(E_{in}-E_{ou})kh^2}{2(k+1)(k+2)}\\
&E_3=\int_{-h/2}^{h/2}z^2E_{sh}(z)dz=\left[\frac{E_{ou}}{12}+(E_{in}-E_{ou})\left(\frac{1}{k+3}-\frac{1}{k+2}+\frac{1}{4k+4}\right)\right]h^3
\end{aligned}\tag{A.2}$$

$$\begin{aligned}
&E_{1s}=\int_{-h/2}^{h/2+h_s}E_s(z)dz=\left(E_c+\frac{E_m-E_c}{k_2+1}\right)h_s\\
&E_{2s}=\int_{-h/2}^{h/2+h_s}zE_s(z)dz=\frac{E_c}{2}hh_s\left(\frac{h_s}{h}+1\right)+(E_m-E_c)hh_s\left(\frac{1}{k_2+2}\frac{h_s}{h}+\frac{1}{2k_2+2}\right)\\
&E_{3s}=\int_{-h/2}^{h/2+h_s}z^2E_s(z)dz=\frac{E_c}{3}h_s^3\left(\frac{3}{4}\frac{h^2}{h_s^2}+\frac{3}{2}\frac{h_s}{h}+1\right)+(E_m-E_c)h_s^3\left(\frac{1}{k_2+3}+\frac{1}{k_2+2}\frac{h}{h_s}+\frac{1}{4(k_2+1)}\frac{h^2}{h_s^2}\right)
\end{aligned}\tag{A.3}$$

APPENDIX B

$$\begin{aligned}
&\Delta=A_{11}A_{22}-A_{12}A_{21},\ A_{22}^*=\frac{A_{22}}{\Delta},\ A_{12}^*=\frac{A_{12}}{\Delta}\\
&A_{11}^*=\frac{A_{11}}{\Delta},\ A_{21}^*=\frac{A_{21}}{\Delta},\ A_{33}^*=\frac{1}{A_{33}},\ B_{36}^*=\frac{A_{36}}{A_{33}}\\
&B_{11}^*=A_{22}^*A_{14}-A_{12}^*A_{24},\ B_{12}^*=A_{22}^*A_{15}-A_{12}^*A_{25}\\
&B_{21}^*=A_{11}^*A_{24}-A_{21}^*A_{14},\ B_{22}^*=A_{11}^*A_{25}-A_{21}^*A_{15}
\end{aligned}\tag{B.1}$$

$$\begin{aligned}
&B_{11}^{**}=A_{22}^{*}A_{14}-A_{21}^{*}A_{15},\ B_{21}^{**}=A_{11}^{*}A_{15}-A_{12}^{*}A_{14}\\
&B_{12}^{**}=A_{22}^{*}A_{24}-A_{21}^{*}A_{25},\ B_{22}^{**}=A_{11}^{*}A_{25}-A_{12}^{*}A_{24}\\
&D_{11}^{*}=B_{11}^{*}A_{14}-B_{21}^{*}A_{15}-A_{41},\ D_{12}^{*}=B_{12}^{*}A_{14}-B_{22}^{*}A_{15}-A_{42}\\
&D_{21}^{*}=B_{11}^{*}A_{24}-B_{21}^{*}A_{25}-A_{51},\ D_{22}^{*}=B_{12}^{*}A_{24}-B_{22}^{*}A_{25}-A_{52}\\
&D_{36}^{*}=A_{36}A_{36}^{*}-A_{63}
\end{aligned} \tag{B.2}$$

$$\begin{aligned}
&A=A_{11}^{*}m^4\pi^4+\left(A_{33}^{*}-A_{12}^{*}-A_{21}^{*}\right)m^2n^2\pi^2\lambda^2+A_{22}^{*}n^4\lambda^4\\
&B=B_{21}^{*}m^4\pi^4+\left(B_{11}^{*}+B_{22}^{*}-2B_{36}^{*}\right)m^2n^2\pi^2\lambda^2+B_{12}^{*}n^4\lambda^4-\frac{L^2}{R}m^2n^2\\
&G=81A_{11}^{*}m^4\pi^4+9\left(A_{33}^{*}-A_{12}^{*}-A_{21}^{*}\right)m^2n^2\pi^2\lambda^2+A_{22}^{*}n^4\lambda^4\\
&\lambda=\frac{L}{R}
\end{aligned} \tag{B.3}$$

where

$$\begin{aligned}
&B_1=B_{21}^{**}m^4\pi^4+\left(B_{11}^{**}+B_{22}^{**}-2B_{36}^{*}\right)m^2n^2\pi^2\lambda^2+B_{12}^{**}n^4\lambda^4-\frac{L^2}{R}m^2n^2\\
&D=D_{11}^{*}m^4\pi^4+\left(D_{12}^{*}+D_{21}^{*}+4D_{36}^{*}\right)m^2n^2\pi^2\lambda^2+D_{22}^{*}n^4\lambda^4
\end{aligned} \tag{B.4}$$

$$a_{11}=\frac{1}{2A_{11}^{*}R^2\rho_1},\ a_{12}=\frac{n^2}{8A_{11}^{*}R^3\rho_1},\ a_{13}=\frac{1}{\rho_1},\ a_{14}=\frac{1}{2\rho_1} \tag{B.5}$$

$$\begin{aligned}
&a_{21}=\frac{Rn^2\lambda^2}{L^2},\ a_{22}=\frac{1}{L^4\rho_1}\left(D+\frac{B^2}{A}\right)\\
&a_{23}=\frac{1}{L^4\rho_1}\left[\frac{m^2n^2\pi^2\lambda^2}{A}+\frac{B}{A}m^2n^2\pi^2\lambda^2-\frac{n^2\lambda^2\left(\lambda L-4A_{24}^{*}m^2\pi^2\right)}{4A_{11}^{*}m^2\pi^2}\right]\\
&a_{24}=\frac{1}{L^4\rho_1}\left(\frac{m^4\pi^4}{16A_{22}^{*}}+\frac{n^4\lambda^4}{16A_{11}^{*}}\right)\\
&a_{25}=\frac{1}{L^4\rho_1}\left(\frac{m^2n^2\pi^2\lambda^2}{A}+\frac{m^2n^2\pi^2\lambda^2}{G}\right)\\
&a_{26}=\frac{Rn^2\lambda^2}{L^2\rho_1},\ a_{27}=\frac{1}{\rho_1},\ a_{28}=\frac{1}{L^2\rho_1}\left[\left(\lambda n\right)^2+\left(m\pi\right)^2\right]
\end{aligned} \tag{B.6}$$

$$\begin{aligned}
&a_{31}=\frac{1}{\rho_1}\left\{\left[4A_{24}^{*}\left(\frac{m\pi}{L}\right)^4-\frac{1}{R}\left(\frac{m\pi}{L}\right)^2\right]\frac{n^2\lambda^2}{4A_{11}^{*}m^2\pi^2}+2\frac{B}{A}\left(\frac{m\pi}{L}\right)^2\left(\frac{n}{R}\right)^2\right\}\\
&a_{32}=\frac{2}{\rho_1}m^2n^2\pi^2\lambda^2\left(\frac{m\pi}{L}\right)^2\left(\frac{n}{R}\right)^2\left(\frac{1}{A}-\frac{1}{G}\right)\\
&a_{33}=\frac{1}{\rho_1}\left\{16A_{44}^{*}\left(\frac{m\pi}{L}\right)^4-\left[4A_{24}^{*}\left(\frac{m\pi}{L}\right)^4-\frac{1}{R}\left(\frac{m\pi}{L}\right)^2\right]\frac{\left(\lambda L-4A_{24}^{*}m^2\pi^2\right)}{A_{11}^{*}m^2\pi^2}\right\}\\
&a_{34}=\frac{4}{\rho_1},\ a_{35}=\frac{4}{\rho_1}\left(\frac{m\pi}{L}\right)^2
\end{aligned} \tag{B.7}$$

$$b_{11}=2a_{11},\ b_{12}=\left(\frac{1}{2}a_{31}+a_{12}\right),\ b_{13}=\frac{1}{2}a_{32},\ b_{14}=\left(a_{11}-\frac{1}{2}a_{33}\right),\ b_{15}=\left(2a_{14}-\frac{1}{2}a_{34}\right)$$
$$b_{16}=\left(a_{14}-\frac{3}{8}a_{34}\right),\ b_{17}=\frac{1}{2}a_{35},\ b_{18}=a_{13}-\frac{a_{34}}{4},\ b_{19}=\frac{5}{16}a_{13}-\frac{35}{256}a_{34},\ b_{110}=\frac{3}{4}a_{13}-\frac{9}{32}a_{34} \tag{B.8}$$
$$b_{111}=\frac{9}{8}a_{13}-\frac{15}{32}a_{34},\ b_{112}=\frac{3}{2}a_{13}-\frac{9}{16}a_{34},\ b_{113}=\frac{9}{16}a_{13}-\frac{15}{64}a_{34}$$

$$b_{21}=-b_{11}a_{21},\ b_{22}=-b_{14}a_{21}-a_{33}\frac{a_{21}}{2}+a_{23},\ b_{23}=-b_{12}a_{21}-a_{31}\frac{a_{21}}{2}+a_{24},\ b_{24}=\left(a_{21}b_{15}+\frac{a_{21}}{2}a_{34}\right)$$
$$b_{25}=\left(a_{21}b_{16}+\frac{3}{8}a_{21}a_{34}\right),\ b_{26}=\left(a_{21}b_{17}-\frac{a_{21}}{2}a_{35}\right),\ b_{27}=\frac{9}{16}a_{27},\ b_{28}=3a_{27},\ b_{29}=\frac{15}{8}a_{27} \tag{B.9}$$
$$b_{210}=\frac{9}{2}a_{27},\ b_{31}=\frac{a_{34}}{2},\ b_{32}=\frac{35}{128}a_{34},\ b_{33}=\frac{9}{16}a_{34},\ b_{34}=\frac{15}{16}a_{34},\ b_{35}=\frac{9}{8}a_{34},\ b_{36}=\frac{15}{128}a_{34}$$

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