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# Buckling Study of Thin Tank Filled with Heterogeneous Liquid

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**ABSTRACT**

Buckling of imperfect thin shell tank which is subjected to uniform axial compression is analyzed. The effect of internal pressure on the stability of a shell tank filled with a homogeneous-heterogeneous liquid was considered. Investigation of the liquid nature effect on reduction of the shell buckling load is performed by using the finite elements method. Calculating results in terms of analytical formula give a good agreement with the numerical results given by Abaqus when using actual measurements. The obtained results show the influence of the physical characteristics of liquid especially in the case of heterogeneous liquid. The study of combination between compression load, lateral pressure and the mechanical properties of liquid filling the tank is recommended for dimensioning the shell tanks to avoid the buckling phenomenon. 

**Keywords :** Thin Shell tank; Buckling strenght; Homogeneous-heterogeneous liquid; Imperfection; Finit elements method.

## 1 INTRODUCTION

THIN shells are widely used in various fields of civil and mechanical engineering. In particular, they are used in aircraft construction, ship building, rocket construction, the nuclear, aerospace, and aeronautical industries, as well as the petroleum and petrochemical industries (pressure vessels, pipelines), etc. In addition, anisotropic, laminated composite shells are increasingly used in a variety of modern engineering fields. Several studies dealing with the effect of initial geometric imperfections on strength buckling of thin shell structures have been realized. [2] have studied experimental buckling of cylindrical shells subjected to general initial imperfections. They have shown that huge reduction of the buckling critical load could be obtained, as compared with the perfect shell. [8] and [11] studies the pre- and post-buckling compression behaviors of concentric multi-walled cylindrical shells filled with low-shear-modulus (or fluid like) materials, which are widely observed in biological composites. [12] have considered a parabolic localized imperfection and have obtained it by using an analytical approach large reduction of the buckling load for thin cylindrical axisymmetric shell under uniform axial compression. The axisymmetric linear bending theory of shells is treated for thin-walled orthotropic cylindrical shells under any smooth axial distribution of normal and shear pressures ,the equations are developed, solved and explored in [15]. A probabilistic approach to the determination of stability reliability of an imperfect supporting cylindrical shell under combined loading, which is based on the main postulates of Bolotin's approach is presented in [17].More recently [9,10] present the Analytical research on dynamic buckling of thin cylindrical shells with thickness variation under axial

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pressure. [13] studied thr buckling and its effect on the confined flow of a model capsule suspension.A review of recent researches on FGM cylindrical structures under coupled physical interactions [14], The main aim of this review is to collate the research performed in the area of coupled mechanics on FGM cylindrical structures during the last 10 years, thereby giving a broad perspective of the state of art in this field. [16] reviewed recent research on the strength, stability and vibration behaviour of liquid-containment shell structures, and traces the developments pertaining to the design of these facilities to withstand various loading and environmental effects such as liquid pressure. Manufacturing processes that are used to fabricate shell structures cannot eliminate initial geometric imperfections. One can reduce the overall weight of a structure. Also, it worthy of note that Fluid-Structure Interaction (FSI) occurs across many complex systems of engineering disciplines ranging from nuclear power plants and turbo machinery components, naval and aerospace structures, and dam reservoir systems to flow through blood vessels to name a few. The forces generated by violent fluid/structure contact can be very high; they are stochastic in nature and thus difficult to describe. They do, however, often constitute the design loading for the structure. The problem is a tightly-coupled elasto-dynamic problem in which the structure and the fluid form a single system. Solution of these problems is obviously complex and technically challenging.

This work presents the effect of internal pressure on the stability of a shell tank filled initially by a homogeneous and heterogeneous liquid and the effect of the nature of the liquid on the critical buckling pressure of thin imperfect elastic thin shell tank subjected to uniform axial compression .Our structure has a localized axisymmetric defect having triangular imperfection. The analysis of its effect on the critical buckling load is driven directly by calculating finite elements using the element of S8R Abaqus code [7].

This paper is organized as follows. Section 2 describes the fundamental equations of cylindrical shell with imperfection. Section 3 introduces the analytical formula for the reduction of buckling load that will allow us validation of the results obtained by our analysis study. Section 4 describe our modeling of thin shell tank having the local imperfection, this modeling is based on the finite elements method. Finally, Section 5 presents our results and discussions.

## 2 FUNDAMENTAL EQUATIONS OF CYLINDRICAL SHELL WITH IMPERFECTION

The model of Von Karman-Donnell introduced by [1] to study the buckling of circular cylindrical shells under torsion Describing moderately large deflection of the middle surface of shell. Another version of this model has been dated in 1950, introduced a small initial disturbance $\bar{w}$ on the structure to study the effect of imperfections was presented by [3] . The Von Karman-Donnell equations are given by:

$$\nabla^4 F - \frac{Et}{R} w_{,xx} + \frac{1}{2}\Lambda(w, w + 2\bar{w}) = 0 \tag{1}$$

$$D\nabla^4 w + \frac{t}{R} F_{,xx} - h\Lambda(F, w + \bar{w}) = 0 \tag{2}$$

where $w$ is the radial displacement of the shell counted positively outwards , $E$ is the Young's modulus , $\bar{w}$ is the geometric defect, $h$ is the thickness of the shell, $F$ the Airy function of stress ,$D$ is the flexural rigidity of the shell, $x$ is The axial coordinate, and $y$ is the coordinate orthoradial, finally $\nabla^4$ is the Laplacian operator.

$$\Lambda(X, Y) = X_{,xx} Y_{,yy} - 2X_{,xy} Y_{,xy} + X_{,yy} X_{,xx}$$

In the case of elastic axisymmetric shell with perfect geometry under axial compression, the critical stress is

$$\lambda_{cl} = \frac{h}{R\sqrt{3(1-\upsilon^2)}} \tag{3}$$

Parameters $h$, $\upsilon$ and $R$ designate, respectively shell thickness, Poisson's ratio and shell mean radius. Deviate from the Eq.(3) the critical pressure relation becomes:

$$P_{cl} = \frac{2\pi E h^2}{3\sqrt{6}(1-\upsilon^2)^{\frac{3}{4}} RL}\sqrt{\frac{h}{R}} \tag{4}$$

## 3 ANALYTICAL FORMULA FOR THE REDUCTION OF BUCKLING LOAD

In this section, the analytical formula is presented, that will allow us validation of the results obtained by our analysis. The aim of this work is to extend the previous procedure as in [5,6]. They developed an analytical formula for the critical buckling load based on the shell parameters and the defect parameters. This formula is well developed and detailed in [5]:

$$\frac{\lambda_{\max}}{\lambda_{cl}} = 1 - \frac{1}{\lambda_c^{1/3}}\left(\frac{9(C_{COR1}(1-\beta^2)^2(\delta_a{}^2+\delta_c{}^2)+C_{COR2}(1+\beta^2)^2\delta_b{}^2}{8(1+\beta^2)^2}\right)^{1/3} \tag{5}$$

with $C_{COR1}$ = 0.14 and $C_{CO2}$= 0.8 correction coefficients in the formula to increase its range of validity, $\beta$ is the modal aspect ratio (axial wavelength / circumferential) ), $\delta_A$ , $\delta_B$ and $\delta_C$ are coefficient which are connected to the Fourier transform of localized imperfection, n is the wave number in the circumferential direction $\lambda_{\max}$ is the value of the critical buckling load .

## 4 MODELING OF THIN SHEL TANK HAVING THE LOCAL DEFECT

This study based on shells in interaction with liquid under internal pressure. The stability analysis may be conducted by assuming a branching diagram according to the method known Euler. This consists in solving the nonlinear problem of balance during the pre-buckling stage, followed at each loading increment by solving a problem to the eigenvalues in order to detect any balance bifurcation. The analysis of the effect of a liquid existing inside the shell as an internal pressure is considered, the influence of the nature of the liquid such as the homogeneity and heterogeneity is modeled like a buckling strength of the thin cylindrical shells subjected to a lateral pressure load.

### *4.1 Element type*

Several shell elements are available in code Abaqus [7] these elements differ in the number of nodes per element and the number of degrees of freedom per node. Two types of elements are particularly advantageous in the case of buckling of the thin cylindrical shells. The first type is composed by linear geometry elements (S4R, S4R5) with four nodes and admitting respectively 6 and 5 five degrees of freedom per node, the second contains the quadratic geometry elements (S8R, S8R5) with eight nodes and admitting respectively 6 degrees of freedom and 5 degrees of freedom per node. [4] have used the analytical formula in the case of an axisymmetric shell flawless under uniform axial loading to calculate the critical stress, then they compared the results with those obtained by finite element in Abaqus using the S8R elements S4R and S4R5.The element that proved most accurate is S8R. The numerical simulations presented in this work are carried out using this element.

### *4.2 Analysis method*

The procedure Buckle of calculating finite element method is used. This procedure uses a technique of linear disturbances to solve the eigenvalues problem associated to the buckling of the shell. First we applied a load increment $Q^N$ . With $N$ the overall degree of freedom defined by the discretization of the shell.

The amplitude of the loading is not important. So the load will be proportionate with the load multipliers. Those multipliers will be calculated following the resolution of the problem associated to the eigenvalues:

$$\left(K_0^{NM} + \lambda_i K_\Delta^{NM}\right)V_i^M = 0 \tag{6}$$

where $K_0^{NM}$ is the initial rigidity matrix including the effect of a pre-loading $P_0^N$ ,$K_\Delta^{NM}$ is the matrix of the geometric rigidity or the matrix of the initial stresses due to the loading $Q^N$ ,$\lambda_i$ are the eigenvalues of the problem, Eq.(6) and $V_i^M$ are the buckling modes associated and N are the global degrees of freedom of the model and is the its buckling mode. The eigenvalues can be extracted using Abaqus by two methods. The first is Lanczos method, this method is the faster to calculate a large number of eigenmodes for a system admitting a large number of freedom degrees, the second is the iterations sub spaces method, this method can used it in the case of small number of eigenmodes.

## 5 NUMERICAL RESULTS AND DISCUSSIONS

### *5.1 Buckling load of thin shell tank with axisymmetric imperfection*

As shown in Fig. 1, a thin cylindrical shell tank filled with liquid under the lateral pressure.

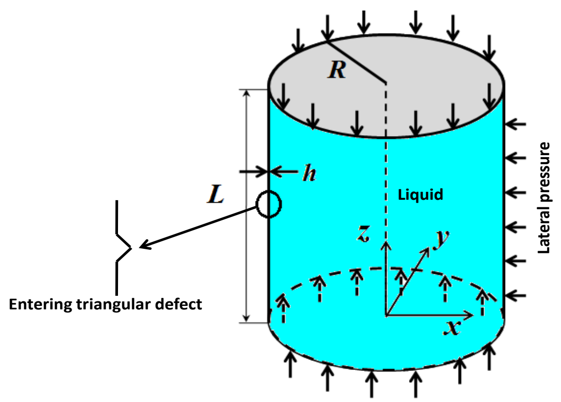



**Fig.1**
Geometry and coordinate of the axially compressed cylindrical shell.

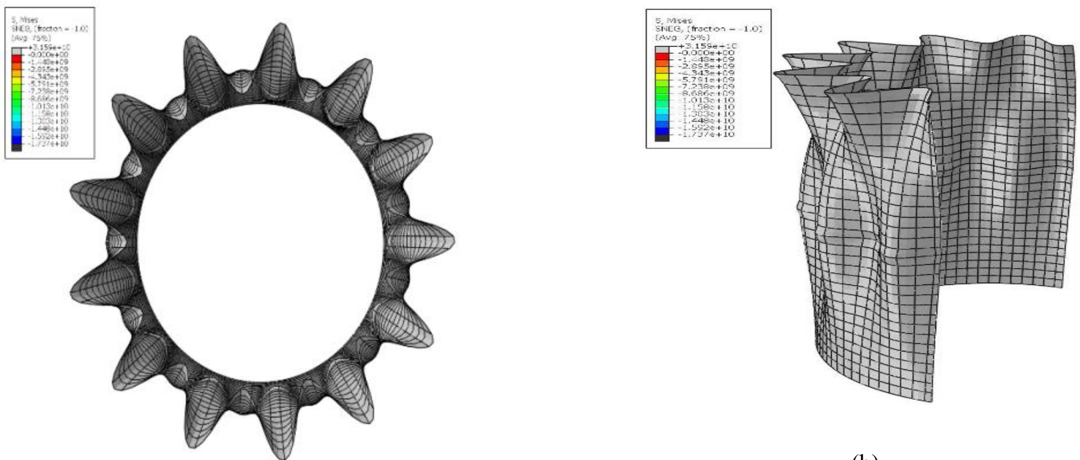
The mechanical characteristics of the elastic material constituting the shell tank are the Table1.

**Table1**
The mechanical characteristics and the considered geometry *L=5R*.

| Young's modulus (*MPa*) | Poisson's ratio | Radius (*m*) | Thickness (*mm*) | Length (*m*) | Critical stress (*Pa*) |
|---|---|---|---|---|---|
| 70000 | 0.3 | 0.135 | 0.09 | 0.27 | $2.8\text{x}10^7$ |

For cylindrical axisymmetric shells, the finite element mesh contains two variables that change the total number of elements in the model. The first variable is the number of divisions along the meridian axial direction of the shell. The second variable is the number of divisions along the shell circumference. Varying both these numbers affects the accuracy of the results. The elements throughout the convergence study that was conducted in this work have aspect ratios, which are chosen to be close to unity. Beginning with a coarse mesh, the deflections and stresses along a meridian as well as the lowest buckling load are computed and stored. For each subsequent refined mesh, considered as a next step in an iterative process, the obtained results are compared to those of the previous step. The refinement process is stopped when the results are found to be less than 1% different from those of the last step mesh. the use of twenty seven circumferential elements and a hundred elements in the axial direction are largely enough to reach convergence finite elements.

Fig. 2 show the buckling mode obtained in this case. The critical stress reduced is $\lambda_{\max} / \lambda_{cl} = 0.893$ . This is an asymmetric mode.



(a) (b)

**Fig.2**
First buckling mode using Abaqus code ( $\varepsilon / h = 1$ and $\lambda_{\max} / \lambda_{cl} = 0.893$ ) (a) axial view and (b) lateral view.

In Figs. 3 and 4, the results obtained are presented by the corrected analytical formula Eq. (5) And the numerical model developed in Abaqus. They show that there is very good agreement between these two results for two wavelength values $\zeta = 14.85\,mm$ and $\zeta = 4.32\,mm$. The analytical formula for the reduction of critical load allows estimating the critical load in our case and this remains valid for small values of defect amplitude considered. These values can reach the value of the shell thickness. Other parametric studies are needed to better assess the capacity of the analytical formula, to account for the effect of localized defects on the buckling strength.

The presented numerical model using the finite element method using the S8R element in ABAQUS is validated by the corrected analytical formula presented. And that, for to evaluate the critical buckling load of thin cylindrical shells subjected to a uniform loading in axial compressive .these shells are subject to localized imperfections geometric axisymmetric.

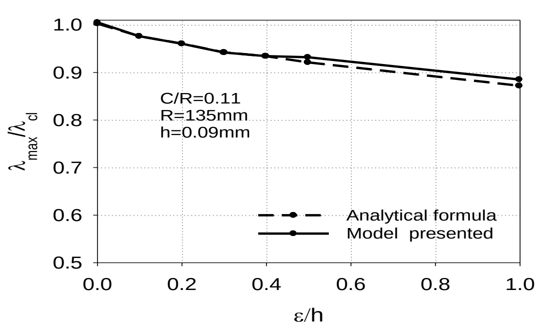



**Fig.3**
Effect of the shape defect: $\varepsilon$/h, $\zeta = 4.32\,mm, \beta = 2, n = 15$ **:**
Comparison between numerical model and analytical results for the reduction of the critical load.

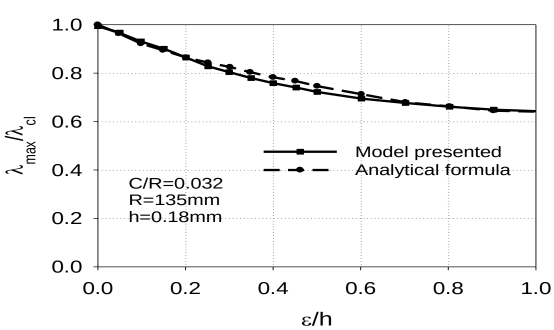



**Fig.4**
Evolution of the critical load according to the reduced amplitude $\varepsilon$/h, $\zeta = 4.32\,mm, \beta = 2, n = 15$ : Comparison between numerical model and analytical results for the reduction of the critical load.

*5.2 Instability of thin cylindrical shell tank filled with a liquid under lateral pressure*

Our study examines the behavior of cylindrical shells subject to internal pressure loading indicating the presence of a liquid and lateral pressure. The problem presented in this model is how to model the overall behavior buckling taking into account the nature of the liquids developed. So the liquid is considered like an internal pressure who admits the physical characteristic (Homogeneity and heterogeneity).If the liquid is homogeneous therefore each finite element S8R of the liquid exerts a uniform internal pressure on the interior surface of the tank as well as the sides. In the case of heterogeneous liquid is modeled by the periodic distribution axisymmetric variable pressure in the two directions (circumferential and axial).So we have a great opportunity to control the variation of the total density and to consider this parameter in the analysis of bifurcations mode. The model simplifies approximates to the model of Euler, so it is a non-linear calculation of bifurcation in the sense of Euler.

At least at the level of the analysis of the mechanical behavior of a shell-type tank, the critical reference load is calculated, and consequently the critical stress and critical pressure. The behavior of an imperfect shell filled with a homogeneous-heterogeneous liquid is presented, then we will complete this presentation by the case load that particularly concerns us, it is the lateral pressure. The type of defect like triangle directed inward is chosen. This defect has the dual advantage of being very easily modeled and to be perfectly representative of the defects that result from welding and that appear at the junction of two cells for assembling the shell. It also helps to account for the essential characteristics of defects, to check the axial wavelength, and amplitude.

The default will be located halfway up the cylinder generator to avoid interaction with the limit conditions of the shell. The limit conditions restraint are type fitting at both ends of the shell. The mechanical characteristics of the elastic material constituting the imperfect shell are as follows Table 2.

**Table2**
The mechanical characteristics and the considered geometry *L=2R*.

| Young's modulus(*MPa*) | Poisson's ratio | Radius (*m*) | Thickness (*mm*) | Length (*m*) | Critical stress (*Pa*) |
|---|---|---|---|---|---|
| 70000 | 0.3 | 0.135 | 0.09 | 0.675 | $2.8\text{x}10^7$ |

The use of twenty seven circumferential elements and a hundred elements in the axial direction are largely enough to reach convergence finite elements. Fig.5 shows the buckling mode obtained in the case of a tank filled with a homogeneous liquid. The critical stress reduced is $\frac{\lambda_{\max}}{\lambda_{cl}} = 1.59.$ This is an asymmetric mode according to the axial axis.

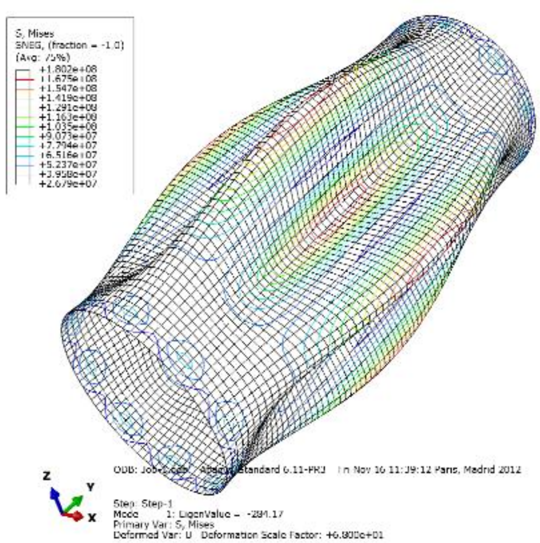



**Fig.5**
First mode shape of cylindrical shell tank using Abaqus code (Deformed shape without lateral pressure).

Fig. 6 shows the buckling mode obtained in the case of the same tank filled with a homogeneous liquid under lateral pressure. The critical stress reduced is $\frac{\lambda_{\max}}{\lambda_{cl}} = 1.91$. This is always an asymmetric mode. The lateral pressure increases the buckling strength of our tank shell.

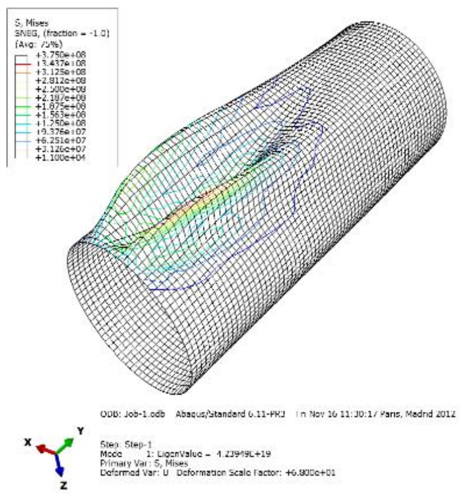


**Fig.6**
First mode shape of cylindrical shell tank using Abaqus code (Deformed shape with lateral pressure).

The cylindrical tank solicited by lateral pressure loses more than 53% of its load capacity compared with the tank without lateral pressure. This result shows that the lateral pressure contributes to making the structure instable instead. From the graphical representation of results, the critical stress seems to be a decreasing function of the lateral pressure Fig.7. The case of the lowest dimensioning which represents a reservoir of the stability domain is obtained by an equal lateral pressure from 30 MPa. The following results using the stress-strain diagram are presented in the Fig.8.

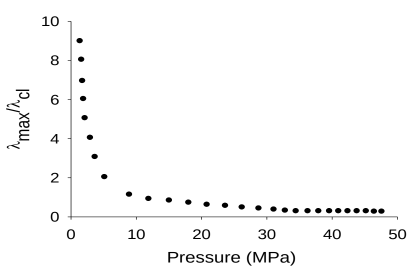



**Fig.7**
Effect of lateral pressure: Evolution of the critical load according to the lateral pressure amplitude.

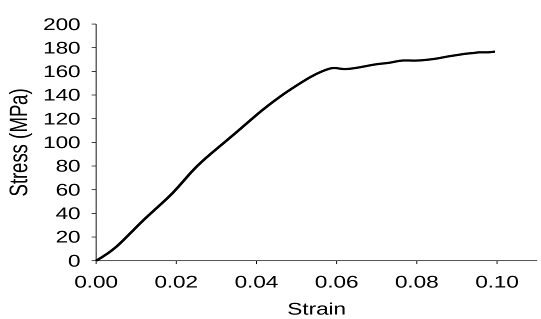



**Fig.8**
Stress-strain diagram (Case of a homogeneous liquid under lateral pressure).

The examination of the net critical stress (background effect) shows nonlinearity especially for high loads (plastic domain), then the curve is a straight line (elastic domain). The internal lateral pressurization induced a slight increase in the bearing capacity. The Increasing of the critical load depends on the setting of the pressurization threshold.

The Table 3., present a comparison of the influence of a homogeneous liquid and the influence of a heterogeneous liquid on the critical bifurcations stress. A case study is given from six numerical tests.

**Table 3**
Comparison between the influences of a homogeneous-heterogeneous liquid on the critical buckling pressure.

| tests | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Critical pressure ($x10^9$ Pa) case of the heterogeneous liquid | 15.6 | 7.6 | 3.89 | 2.59 | 1.94 | 1.56 |
| Critical pressure ($x10^9$ Pa) case of the homogeneous liquid | 15.9 | 7.83 | 4.03 | 2.95 | 2.21 | 1.67 |

It is noticed that the critical buckling pressure values found for a tank filled with a heterogeneous liquid are inferior to those found by a homogeneous liquid. The structure filled with a heterogeneous liquid loses more than 20% of its load capacity compared with the case of a homogeneous liquid. Pressure distribution and variable density according to both axial and circumferential directions is an influential factor on the mechanical behavior of the shell tank.

Finally, our study shows that the analysis of the general defects considering the coupling position between internal and lateral pressure has a large interest in the case of sizing of thin cylindrical shells filled with liquid, because it can lead to the worst case of instability.

## 6 CONCLUSIONS

The study shows a fall in the critical load and this is due to the influence of the characteristics of the defects and especially its amplitude. This change has a material interest in the case of sizing of thin cylindrical tank shells. The finite element method using the element S8R Abaqus was used to study the variation of the critical load in the buckling problem of thin cylindrical shells filled by liquids subject to a lateral pressure load when they have a localized imperfection. This model is validated by a theoretical approach using the analytical formula. The study results indicate a fall in the critical load and this is due to the influence of the physical characteristics of liquids (homogeneous or heterogeneous) and especially the case of heterogeneous liquid. The influence of lateral pressure is important because it degrades the buckling performance. This parameter has a large interest on the resistance to instability. It is recommended to integrate it during the final dimensioning of thin cylindrical shells. In the future work, the analytical model based on the Von Karman-Donnell equation will be developed, this theoretical approach will integrate the variable density of the liquid to study its effect on the critical buckling load of thin imperfect cylindrical shell tank.

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