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Three-Dimensional Finite Element Analysis of Stress Intensity Factors in a Spherical Pressure Vessel with Functionally Graded Coating

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ABSTRACT

This research pertains to the three-dimensional (3D) finite element analysis (FEA) of the stress intensity factors (SIFs) along the crack front in a spherical pressure vessel coated with functionally graded material (FGM). The vessel is subjected to internal pressure and thermal gradient. The exponential function is adopted for property of FGMs. SIFs are obtained for a wide variety of crack shapes and layer thickness. The reported results clearly show that the material gradation of coating and the crack configuration can significantly affect the variation of SIFs along the crack front. The results are given which are applicable for fatigue life assessment and fracture endurance of FGM coating spherical pressure vessel and can be used in design purposes.

Keywords : Spherical pressure vessel; Functionally graded coating; Stress intensity factor; 3D crack; Finite element analysis.

1 INTRODUCTION

SPHERICAL pressure vessels find extensive use as classical problems in engineering mechanics [1] and for various functions in the chemical, aeronautical, space, nuclear, and arm ament industries [2]. Since spherical pressure vessels are susceptible to cracking during manufacturing [3] or in-service conditions, though consideration of fracture mechanic criterion in their design process is essential for reliable applications. In spite of its practical interest, has been analyzed by only a few researchers so far. Hakimi et al. [4] considered pressurized cylindrical and spherical hollow vessels, containing axisymmetric or semi-elliptic cracks emerging on the internal or external surface. They evaluated the stress intensity factor in the linear elastic domain and the J integral in the elastoplastic range using the finite element method. Perl and Bernshtein studied the 3D stress intensity factors for a wide range of inner radial lunular and crescentic surface cracks in a typical [5] and in thin and thick[3] spherical pressure vessel. They [2] also considered three-dimensional stress intensity factors for ring cracks and arrays of coplanar cracks emanating from the inner surface of a spherical pressure vessel. Parallel to new industrial developments, the outstanding properties of functionally graded material have led to receive considerable attention by many researchers to investigate their usage capability in spherical pressure vessels. The problem of the elastoplastic deformation behavior of functionally graded spherical pressure vessels under internal pressure is investigated by Akis [1]. Elastic analysis of thick-walled spherical pressure vessels of functionally graded materials under different

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boundary conditions are reported in literature [6-7]. FGM structures under thermo-mechanical loading can lead to induce the surface cracks and affect the structural safety [8-9].

The present study concerns the 3D finite element investigation of the stress intensity factors through the semi elliptical surface crack in a pressurized spherical vessel coated with functionally graded material. The effects of material non-homogeneity of coating, the crack configuration, the geometry of the vessel and thermo-mechanical loading on the distribution of the prevailing mode-I SIF along the crack front is also studied. As depicted in Fig. 1, the crack configuration is described by two dimensionless parameters $\xi = \frac{a}{c}$ and $\eta = \frac{a}{t}$, the so-called aspect ratio and the relative depth of crack, respectively. As seen, the *x*-coordinate of any arbitrary point *P* along the crack front (X_P) is normalized with the *x*-coordinate of the corner point *G*, as follow; $\psi = \frac{X_P}{X_G}$.

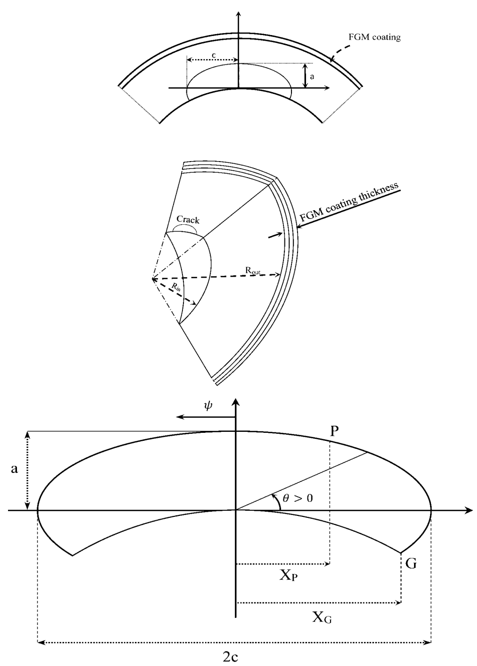


Fig.1
A functionally graded spherical pressure vessel containing a semi-elliptical surface crack.

2 STRESS INTENSITY FACTOR CALCULATIONS

2.1 Crack opening displacements technique for 3D crack extraction in FGM spherical vessel

After obtaining the displacement values around the crack front of the FGM spherical vessel with the help of finite element code, the Crack Opening Displacements (COD) technique can be utilized in order to calculate the prevailing mode-I stress intensity factors.

As shown by Eischen [10], the nature of the stress singularity for continuously non-homogenous, isotropic and linear elastic solid is precisely the same as the well-known form applicable to homogeneous materials, irrespective of the particular form of the Young's modulus variation. Thus, the stress intensity factors can be obtained from crack opening displacements (CODs) as [11]:

$$K_I = \frac{E_{tip}\sqrt{2\pi}}{8(1-\nu^2)} \lim_{\delta\to 0}\left(\frac{\Delta u(r)}{\sqrt{\delta}}\right) = \frac{E_{tip}\sqrt{2\pi}}{8(1-\nu^2)} \lim_{\delta\to 0}\left(\frac{u(\delta,\pi)-u(\delta,-\pi)}{\sqrt{\delta}}\right) \tag{1}$$

where E_{tip} is the elastic modulus at the crack front, δ which approaches zero, is a small distance from the crack-front to the considered node on the crack-surfaces as shown in Fig. 2.

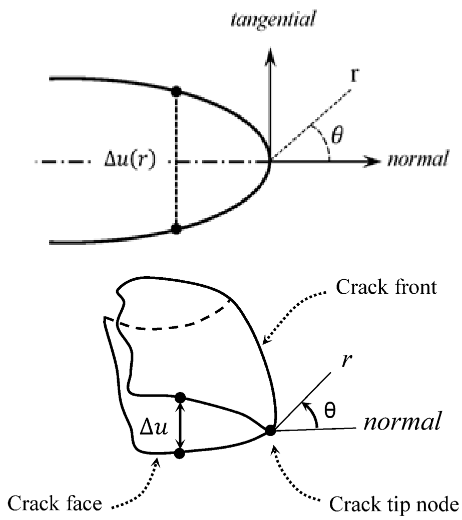


Fig.2
Deformed shape of the crack surface.

2.2 *Finite element modeling*

A spherical pressure vessel of the inside diameter of 20 *mm* with the wall thickness of 1 *mm* and the thickness of 0.1 *mm* for functionally graded coating is considered. An internal pressure, P_{in} , is applied at $r = r_i$, and the vessel is fixed in all directions in a small region far away enough from the crack position. In thermo-mechanical analysis we applied a nodal temperature as an external body force through the vessel thickness and assumed the gradation of the thermal expansion coefficient through the coating.

The problem has also been examined through the commercial software, ANSYS, which has been used as a finite element solver. The finite element modeling of spherical vessel with FGM coating is shown in Fig. 3(a). As shown in Fig 3(b). , the isoparametric brick elements are used everywhere except near the crack front and the singularity elements are applied around the crack front.

In this study, a finite element code using the ANSYS Parametric Design Language (APDL) is used to account for spatial variation in material property of FGM coating. Here at first a constant value of 0.3 for Poisson's ratio is assumed throughout the vessel and coating. Whereas recent works [12-14] deal on the influence of the Poisson's ratio on the values of SIFs in FGMs, therefore the effects of non-homogeneous Poisson's ratio through the coating thickness on the distribution of the SIF along the crack front is also studied. The material is assumed to be isotropic with exponentially varying elastic modulus (*E*), coefficient of thermal expansion ($CTE = \alpha$) and Poisson's ratio (υ) through the thickness as follow,

$$\begin{aligned} E(r) &= E_i e^{\phi(r-r_0)} \\ \alpha(r) &= \alpha_i e^{\gamma(r-r_0)} \\ v(r) &= V_i e^{\varsigma(r-r_0)} \end{aligned} \tag{2}$$

where r_0 is the outer radius of the vessel and ϕ, γ, ς are the constant of material, thermal and Poisson's ratio non-homogeneity which defined as:

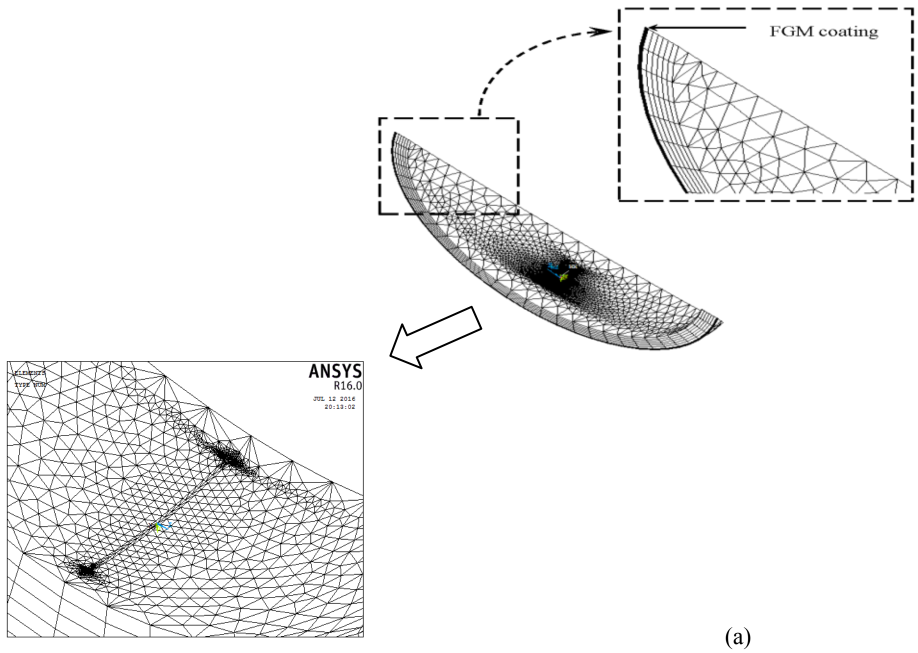
$$\begin{aligned} \phi &= \frac{1}{tc}\ln\left(\frac{E_{0c}}{E_{ic}}\right) \\ \gamma &= \frac{1}{tc}\ln\left(\frac{\alpha_{0c}}{\alpha_{ic}}\right) \\ \varsigma &= \frac{1}{tc}\ln\left(\frac{v_{0c}}{v_{ic}}\right) \end{aligned} \tag{3}$$

which *tc* denotes the thickness of the FGM coating, $E_{ic}, v_{ic}, \alpha_{ic}$ and $E_{0c}, v_{0c}, \alpha_{0c}$ are the values of elastic modulus, Poisson's ratio and coefficient of thermal expansions at the inner and outer radius of the FGM coating, respectively. In order to discuss the numerical results, the stress intensity factors are normalized with respect to K_O given by:

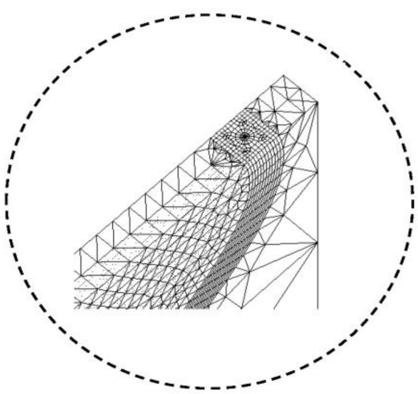
$$K_0 = \frac{2P_{in}(r_0)^2}{(r_0)^2 - (r_i)^2}\sqrt{\frac{\pi a}{Q}} \qquad \text{for plane stress} \tag{4}$$

where the shape factor for an elliptical crack, *Q*, is approximated by[15]

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{5}$$



(a)



(b)

Fig.3
Finite element model of a spherical pressure vessel containing inner crack: (a) the finite element model with FGM coating, (b) singular elements around the crack front.

2.3 Validation of the model

To the best of the literature survey, the problem of the stress intensity factor in cracked FGM spherical vessels subjected to the internal pressure is not reported. In order to justify the reliability of the method, the distribution of the SIF of the slender crescentic radial crack in a spherical pressure vessel with $\xi = 0.5, \eta = 0.2$ and the spherical vessel's radii ratio $\frac{R_0}{R_i} = 4.3$, are plotted in Fig. 4 and compared with those reported by Perl et al. [3] obtained by finite element implementation and using the standard-API 579-1, and show to be in good agreement for both cases.

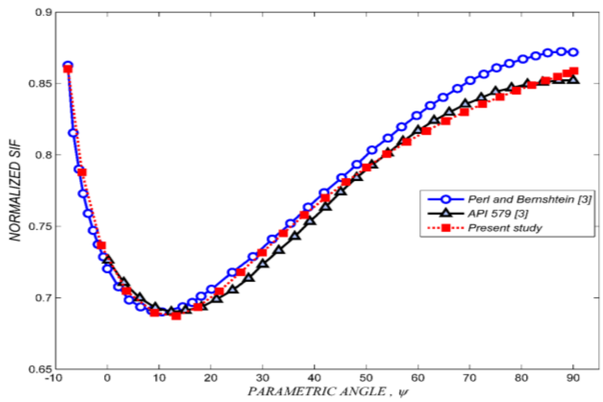


Fig.4
Distribution of the stress intensity factor along the front of a slender crescentic radial crack in a spherical pressure vessel with $\xi = 0.5, \eta = 0.2$ and $\frac{R_0}{R_i} = 4.3$.

3 RESULTS AND DISCUSSION

A semi-elliptical surface crack with constant aspect ratio ($\xi = 0.4$) and different values of crack depths in a spherical pressure vessel with FGM coating has been firstly analyzed. The resulting stress field is shown in Figs. 5(a-d). Each curve is plotted for a certain values of crack depth and aspect ratio and different values of material gradation ($RE = E_{0c}/E_{ic}$), *i.e.,* 0.1, 1, 5 and 20. Here the value of Poisson's ratio is set equal to $\nu = 0.3$.

For reason of symmetric distribution of the K_I along the crack front on both sides of the deepest point, the resulting figures are only plotted for positive values of the normalized coordinate system, ψ. It can be seen that K_I is considerably sensitive to the gradation of Young modulus and the overall trend of the SIF is affected by the values of material gradation of coating. As shown, higher the gradation of the coating, higher the values of the K_I along the crack front. In other words, increasing the material gradation of coating will increase the risk of crack propagation. The maximum deviation from the homogeneous coating is seen when $RE = 20$. This trend can be observed for any value of aspect ratios. An overall investigation through Figs. 5 (a-d) revealed that the deeper the crack depth, the higher the values of the SIFs along the crack front. It means that the risk of crack propagation for deeper cracks is higher than others. As seen, the points with zero parametric angle ($\theta = 0$) have the minimum value of the SIF.

The effect of relative crack depth on distribution of the stress intensity factor along the crack front with a constant value of aspect ratio ($\xi = 0.4$) in a FGM coated spherical pressure vessel with certain values of material gradation ($RE = 5, RCTE = 1$) is shown in Fig. 6. The graph demonstrates that the smaller crack depths produce the smaller stress intensity factors along the crack front. It is evident that for large values of the crack depths, the maximum SIF occurs at corners. Here it can be concluded that, in probable crack growth, corner points start to propagate sooner. Decreasing the crack depth causes to shift the location of critical stress intensity factor toward the deepest point for smaller crack depths. As a result, the location of the maximum value of the SIF along the crack front strongly depends on the crack profiles.

The effect of thermal expansion gradation ($RCTE = \alpha_{0c}/\alpha_{ic}$) of the vessel coating with $RE = 1$ on the distribution of the K_I along the crack front (with $\xi = \eta = 0.4$) is shown in Fig. 7. As expected, the stress intensity factor increases with the increasing the gradation of thermal expansion of the vessel coating. Another interesting point is that the stress intensity factor along the crack front is more affected by the graded thermal expansion than Young modulus. Also it can be seen that higher the graded CTEs, higher the difference between the maximum and minimum values of the SIFs.

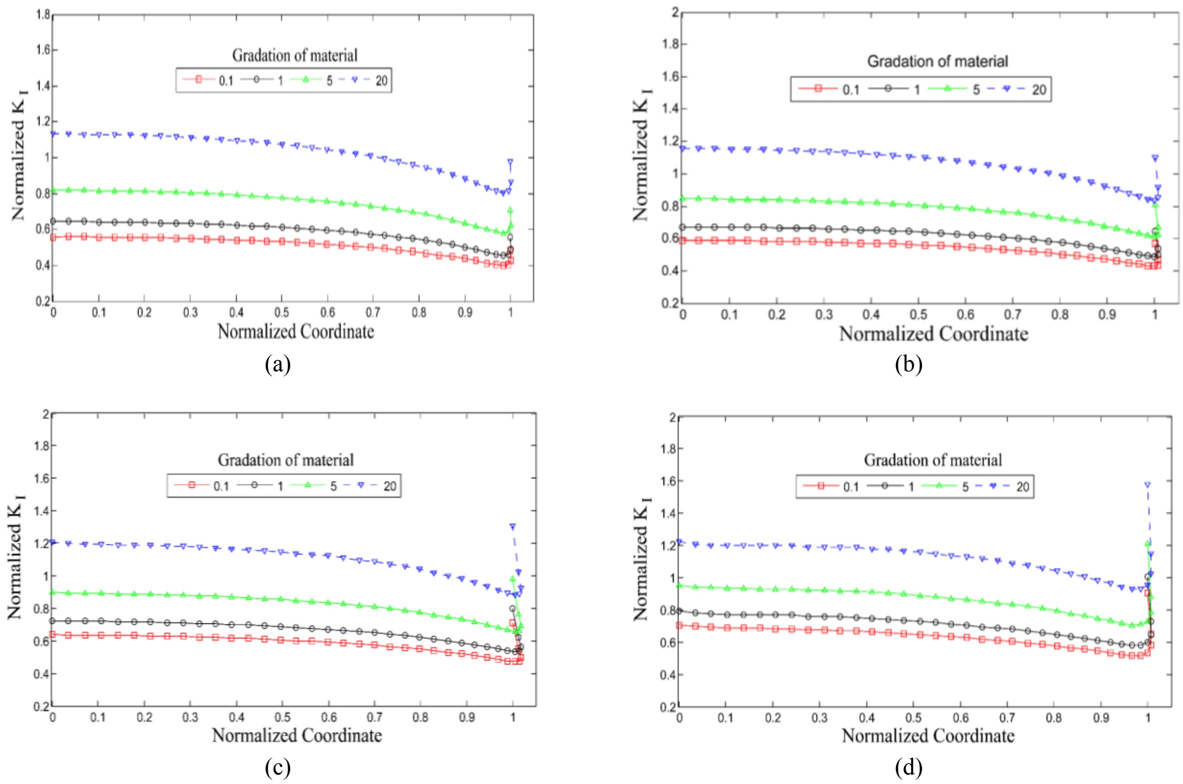


Fig.5
Distribution of the first mode stress intensity factor along the crack front in a spherical pressure vessel with FGM coating (RCTE=1) for constant $\xi = 0.4$ and different values of the material gradation with various crack depths (η); a) $\eta = 0.2$; b) $\eta = 0.4$; c) $\eta = 0.6$; d) $\eta = 0.8$.

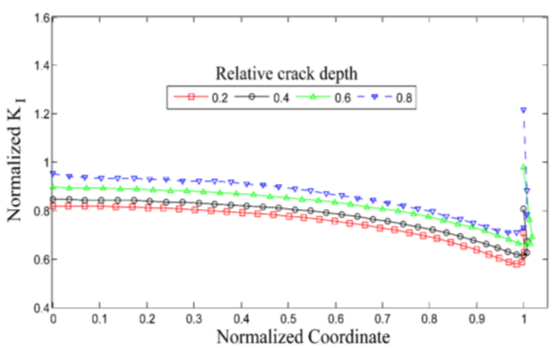


Fig.6
Variation of the K_I along the crack front in a FGM coated spherical pressure vessel (with RE=5, RCTE=1) for constant $\xi = 0.4$ and different values of relative crack depths (η), i.e., $\eta = 0.2, 0.4, 0.6$ and 0.8.

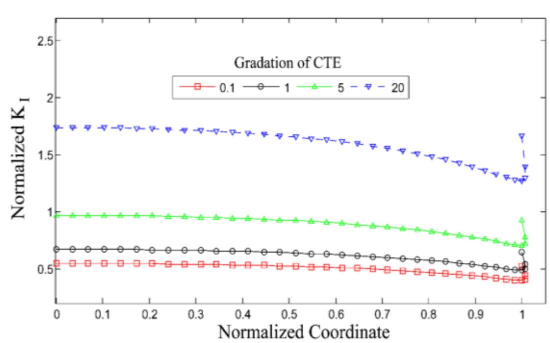


Fig.7
Distribution of the first mode stress intensity factor along the crack front in a spherical pressure vessel with exponentially varying thermal expansion coating for $\xi = \eta = 0.4$ and different values of RCTE $= \alpha_{0c} / \alpha_{ic}$.

The influence of the graded Poisson's ratio on the distribution of the K_I along the crack front is given in Fig. 8(a-c). As seen in Fig. 8(a), the modulus of elasticity is kept constant and the effect of graded Poisson's ratio has been studied. The results show that for points far from the crack cusps ($\psi \prec 1$), the SIF along the crack front decreases by decreasing the Poisson's ratio. However, the maximum deviation from homogeneous coating is lower than 3%. It is notable that the plotted lines near the crack cups ($\theta \prec 1$) overlap each other. The effect of graded Poisson's ratio by considering the different values of E_{0c}/E_{ic} , i.e., 5 and 20 is shown in Fig. 8(b)-8(c). It is evident that higher the Poisson's ratio or the RE, higher the K_I along the crack front. Here the difference in the SIFs is always below 10%. Variation of the K_I along the crack front for constant gradation of coating, $\eta = 0.2$ and different values of aspect ratios (ξ), i.e., $\xi = 0.2, 0.4, 0.6$ and 0.8 is depicted in Fig. 9. As shown, higher the aspect ratio, higher the SIFs near the crack tips. On the other hand, for points near the deepest point of the crack front an opposite trend is observed. Another interesting point is that as the aspect ratio increases, the distribution of the SIF for points near the midpoint of the crack front ($\psi \prec 0.5$) tends to straight line. In other word, for large values of aspect ratios ($\xi \geq 0.6$), the maximum stress intensity factors happen at corners. As the aspect ratio decreases ($\xi \prec 0.6$), the deepest point of the crack front experiences the maximum value of the stress intensity factor. As a result, narrower cracks tend to grow in depths sooner than in longitudinal. Influence of the FGM coating thickness on the value of K_I along the crack front for constant $\xi = \eta = 0.4$, RE=5 and RCTE=1 is shown in Fig. 10. As expected, the K_I increases with the increasing the thickness of FGM coating. Therefore, it can be concluded that the FGM coating thickness can significantly affect the fracture properties of spherical vessel.

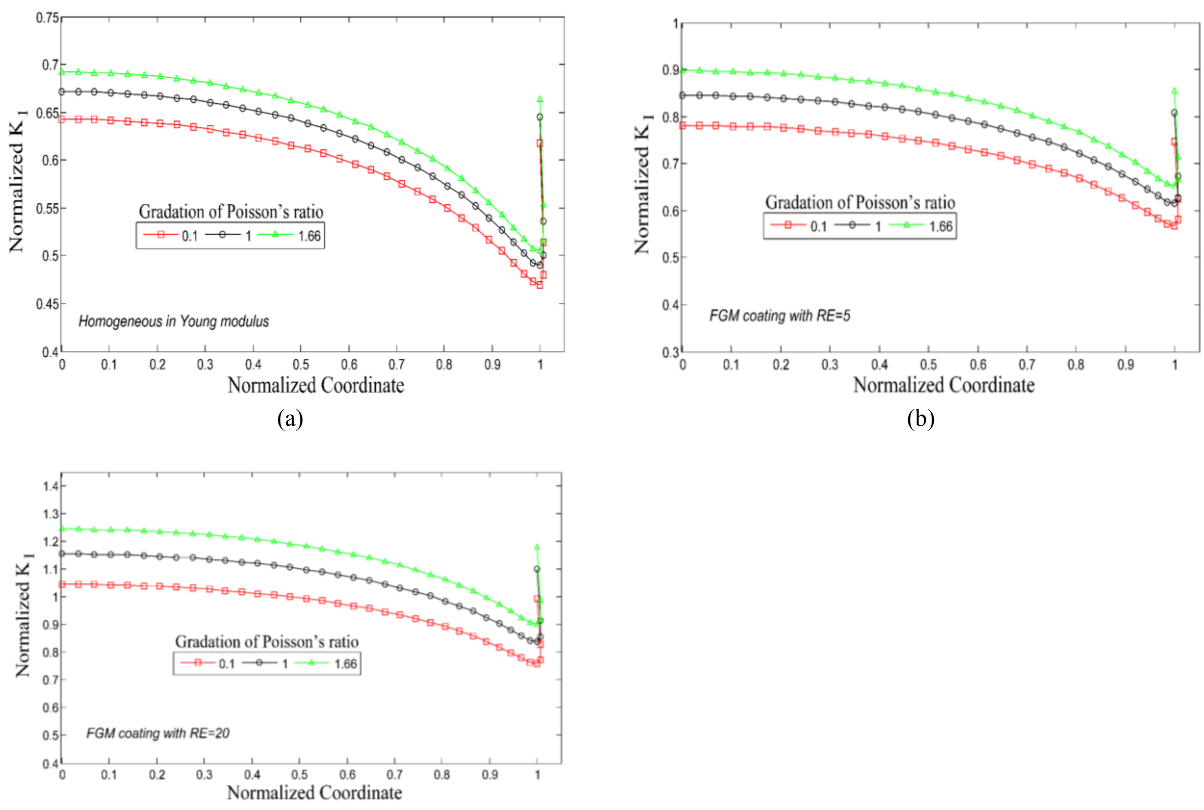


Fig.8
Influence of graded Poisson's ratio on the distribution of first mode SIF along the crack front in a spherical pressure vessel with FGM coating (RCTE=1) for different values of material gradation (RE), (a) homogeneous coating, (b) RE=5, (c) RE=20.

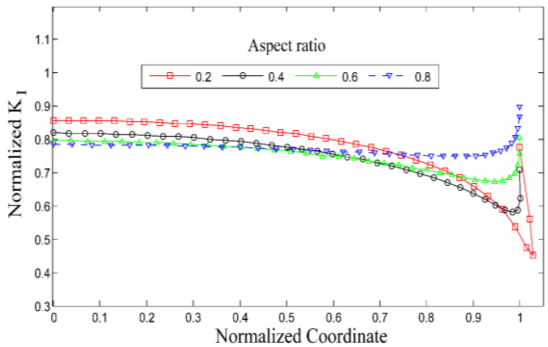


Fig.9
Variation of the K_I along the crack front in a FGM coated spherical pressure vessel (with RE=5, RCTE=1) for constant $\eta = 0.2$ and different values of aspect ratios (ξ), i.e., $\xi = 0.2, 0.4, 0.6$ and 0.8.

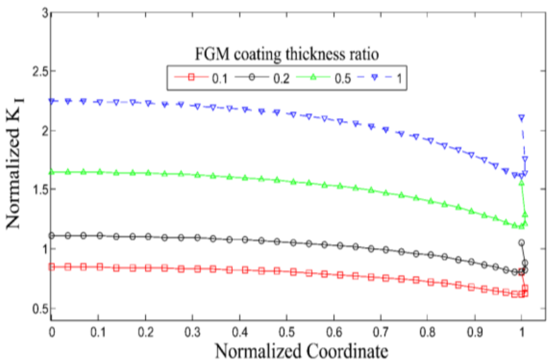


Fig.10
Influence of FGM coating thickness on the value of K_I along the crack front in spherical pressure vessel.

4 CONCLUSIONS

In this study, three-dimensional finite element analysis of the stress intensity factors along the crack front in a spherical pressure vessel coated with functionally graded material is considered. The vessel is subjected to the thermo-mechanical loading (internal pressure and thermal gradient).

The effect of graded Young modulus, coefficient of thermal expansion and Poisson's ratio on the distribution of the SIF along the crack front for different crack profiles is studied. The obtained results clearly show that the material gradation of coating, especially gradation of elasticity modulus and the crack configuration, i.e., aspect ratio and relative crack depth can strongly affect the variation of SIFs along the crack front. The numerical results provided herein can be used for fatigue life assessment of cracked FGM coating spherical pressure vessel.

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