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Dispersion of Love Wave in a Fiber-Reinforced Medium Lying Over a Heterogeneous Half-Space with Rectangular Irregularity

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ABSTRACT

This paper concerned with the dispersion of Love wave in a fiber-reinforced medium lying over a heterogeneous half-space. The heterogeneity is caused by the consideration of quadratic variation in density and directional rigidity of lower half-space. The irregularity has been considered in the form of rectangle at the interface of the fiber-reinforced layer and heterogeneous half-space. The dispersion equation of Love wave has been deduced for existing geometry of the problem under suitable boundary conditions using variable separation method. It has also been observed that for a homogeneous layer with rigidity lying over a regular homogeneous isotropic half-space, the velocity equation coincides with the classical results of Love wave. The effect of the medium characteristics on the dispersion of Love waves has been discussed and the results are displayed with graphs by means of MATLAB programming to clear the physical significance. The study of Love wave dispersion with irregular interface helps civil engineers in building construction, analysis of earthquake in mountain roots, continental margins, and so on. It is also beneficial for the study of seismic waves generated by artificial explosions.

Keywords : Fiber reinforcement; Rectangular irregularity; Heterogeneous half-space; Phase velocity.

1 INTRODUCTION

SURFACE wave characteristics are often studied to identify the properties and structure of the Earth's interior. Waves, propagating along surfaces or interfaces are very important to seismologist to find the reasons for damages resulting from earthquakes. In fact, the study of surface waves in heterogeneous and layered media has been of relevant interest to theoretical seismologists till recently. The study of generation and propagation of waves in the elastic solid has a long and recognized history in the field of seismology, geophysics and applied mathematics (Raoofian Naeeni and Eskandari-Ghadi [1, 2]). Love waves are transversally propagated surface waves, which we feel directly during the earthquake. The study of the propagation of Love waves through the crustal layer of the Earth gives us the concept about the inner shape of the Earth. The study of the mechanical behaviour of a self-

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reinforced material has extremely good significance in geo-mechanics. Many elastic fiber-reinforced composite materials are strongly anisotropic in behaviour. The characteristic property of a self-reinforced material is that its components act together as a single anisotropic unit as long as they remain in elastic condition, i.e. the two components are bound together so that there is no relative displacement between them. Self-reinforced materials are the collection of composite materials, where the polymer fibers are reinforced through relatively oriented polymer fibers, derived from the same fiber. Alumina is an example of self-reinforced material. Under certain temperatures and pressures, some fiber materials may also be modified to self-reinforced materials with the aid of reinforcing a matrix material of the same fiber. In real life, the fibers are probably carbon, nylon, or, conceivably, metallic whiskers. It has been watched that the propagation of elastic surface waves is laid low with the elastic properties of the medium through which they travel (Achenbach [3]).

The Earth's crust contains some hard and soft rocks or materials that may exhibit self-reinforcement property, and inhomogeneity is the trivial feature of the Earth. Particularly an excellent amount of literature about the fiber-reinforced material is available in the notable book via Spencer [4] and Richter [5]. Meissner [6] has demonstrated the existence of torsional surface waves in heterogeneous elastic half-space considering the quadratic and linear variation for shear modulus and density, respectively. Propagation, reflection, and transmission of magneto elastic shear waves in a self- reinforced medium have been analyzed by means of Chattopadhyay and Choudhury [7]. Abd-Alla et al. [8] have been talked about surface waves propagation in fibre-reinforced anisotropic elastic media subjected to the gravity field. Singh [9] thought about on wave propagation in thermally conducting linear fibre-reinforced composite materials and reflection of plane waves at the free surface of a fiber-reinforced elastic half-space was examined by way of Singh and Singh [10]. Kumar and Gupta [11] studied the deformation due to various sources in a fibre-reinforced anisotropic generalized thermoelastic medium. Chattopadhyay and Singh [12] discussed the propagation of magneto elastic shear waves in an irregular self-reinforced layer.

The present work is anxious about the dispersion of Love wave in a fiber-reinforced medium lying over a heterogeneous half-space with the irregular interface. It has been seen that the nearness of support of reinforcement, inhomogeneity, and irregularity in the dispersion equation approves the huge impact of these parameters on the dispersion of Love wave. Love wave at a layered medium bounded by irregular boundary surfaces has been studied by Singh [13]. Chattaraj et al. [14] concentrated on the dispersion of Love wave propagating in irregular anisotropic porous stratum under initial stress. Effects of irregularity and anisotropy on the propagation of shear waves were analyzed by Chattopadhyay et al. [15]. Gupta et al. [16] discussed the influence of irregularity and rigidity on the propagation of the torsional wave. Propagation of torsional surface waves under the effect of irregularity and initial stress was studied by Gupta et al. [17]. Propagation of torsional surface waves in heterogeneous half-space with the irregular free surface has been studied by Selim [18]. He has talked about the impact of irregularity and heterogeneity on the propagation of torsional surface waves. Gupta et al. [19] examined the effect of irregularity on the propagation of torsional surface waves in an initially stressed anisotropic poroelastic layer. Abd-Alla and Ahmed [20] pointed out the existence of Love waves in a non-homogeneous orthotropic elastic layer under initial stress overlying semi-infinite medium. Vaishnav et al. [21] pointed out the propagation of the torsional surface wave in an anisotropic layer sandwiched between heterogeneous half-space. Vishwakarma and Gupta [22] have been talked about the existence of torsional surface waves in an Earth's crustal layer lying over a sandy mantle. Wave propagation in an inhomogeneous cross-anisotropic medium is investigated by means of Wang et al. [23]. Longitudinal and shear waves in an elastic medium with void pores has been given by Dey and Gupta [24]. Influence of rigid boundary on the propagation of the torsional surface wave in an inhomogeneous layer has been studied by Gupta et al. [25].

In the present problem, the layer is taken as an elastic fiber-reinforced anisotropic medium, whose components are concrete and steel, collectively behave as a single unit as long as they continue to be in elastic condition, while the half-space is assumed to be non-homogeneous with rectangular irregularity, wherein directional rigidity, and density varies quadratically with respect to the depth z. Here, the thickness of the layer has been taken into consideration as H. The heterogeneity inside the half-space may be taken as $N = N_2(1+sz)^2$, $L = L_2(1+sz)^2$ (directional rigidity), $\rho = \rho_2(1+sz)^2$ (density). In which s is a constant and having dimension that is the inverse of the length. Here, N_2, L_2 and ρ_2 are the values of N, L and ρ at $z = 0$. For this reason, the present study with the reinforcement, irregularity, and the assumed variation in parameters associated with rigidity and density can be beneficial in predicting the character of the Love wave in inhomogeneous geo-media.

2 FORMULATION OF PROBLEM

We consider an elastic fiber-reinforced anisotropic layer of thickness *H*, whose components are concrete and steel, together behave as a single unit as long as they remain in elastic condition lying over heterogeneous half-space with rectangular irregularity, whereas the lower half-space is assumed to be anisotropic and non-homogeneous in nature, in which directional rigidities and density vary quadratically with space variable z , which is orthogonal to the *x*-axis i.e. direction of wave propagation. The rectangular irregular surface has been taken at the interface of the fiber-reinforced anisotropic layer and heterogeneous half-space with length $2a$ and depth *h*. The upper surface of the fiber-reinforced anisotropic layer is stress-free. The existing geometry of the problem is depicted in Fig. 1. The shape of the irregularity at the interface of the fiber-reinforced anisotropic layer and heterogeneous half-space is taken $z = \varepsilon f(x)$, where

$$f(x) = \begin{Bmatrix} 0; & |x| \succ a \\ 2a; & |x| \le a \end{Bmatrix}$$
$$\varepsilon = \frac{h}{2a} \ll 1 \tag{1}$$

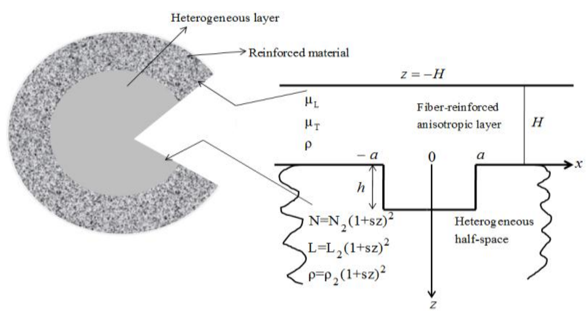

Fig.1
Geometry of the problem.

3 DYNAMIC OF THE LAYER

3.1 Dynamic of fiber-reinforced layer under pre-stress

The constitutive equations for a fiber-reinforced linearly anisotropic elastic medium with respect to preferred direction $\vec{a}$ is (Spencer [4])

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}\right) + 2\left(\mu_L - \mu_T\right) \times \left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta\left(a_k a_m e_{km} a_i a_j\right), \tag{2}$$

wherein τ_{ij} are the components of stress vector and δ_{ij} is a Kronecker delta. Now the components of the infinitesimal strain are

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{3}$$

and μ_T and μ_L are transverse and longitudinal elastic shear modules respectively, λ is elastic parameter, $\left(\mu_T - \mu_L\right)$ is also reinforced anisotropic elastic parameters with dimension of stress, and $u_i\,(i = 1,2,3)$ are the displacement vectors components. $a_i = (a_1, a_2, a_3)$ are the directional cosines of $\vec{a}$ with respect to the Cartesian coordinate system $\left(a_1^2 + a_2^2 + a_3^2 = 1\right)$ and u_i are displacement vector coponents. Also, α and β are specific stress components to take into account different layers for concrete part of the composite material.

Using the conventional Love wave conditions

$$u_1 \equiv 0, \quad u_2 \equiv u_2(x,z,t), \quad u_3 \equiv 0. \tag{4}$$

The equation of motion for Love wave in reinforced medium is

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{5}$$

Using Eqs. (2) and (4) in (5) and setting $a_2 = 0$ (taking transversely isotropic), we get

$$\begin{gathered} \tau_{12} = \mu_T \left[G \frac{\partial u_2}{\partial x} + Q \frac{\partial u_2}{\partial z} \right], \ \tau_{22} = 0, \ \tau_{23} = \mu_T \left[Q \frac{\partial u_2}{\partial x} + R \frac{\partial u_2}{\partial z} \right], \\ G \frac{\partial^2 u_2}{\partial x^2} + 2Q \frac{\partial^2 u_2}{\partial x \partial z} + R \frac{\partial^2 u_2}{\partial z^2} = \frac{\rho}{\mu_T} \frac{\partial^2 u_2}{\partial t^2}, \end{gathered} \tag{6}$$

where

$$G = 1 + \left(\frac{\mu_L}{\mu_T} \right) a_1^2, \quad Q = \left(\frac{\mu_L}{\mu_T} \right) a_1 a_3, \quad R = 1 + \left(\frac{\mu_L}{\mu_T} \right) a_3^2. \tag{7}$$

For the surface wave changing harmonically, we consider

$$u_2(x,z,t) = u_\theta(z) e^{ik(x-ct)}, \tag{8}$$

wherein $c = \left(\frac{\omega}{k} \right)$ is the velocity of simple harmonic waves of wavelength $\frac{2\pi}{k}$ traveling forward, ω is the angular frequency and k is the wave number. On substituting (8) into (6), we obtain

$$R \frac{\partial^2 u_\theta}{\partial z^2} + 2Qik \frac{\partial u_\theta}{\partial Z} + \left(\frac{\rho}{\mu_T} \omega^2 - k^2 G \right) u_\theta = 0. \tag{9}$$

Therefore, the solution of the aforementioned equation is

$$u_\theta(z) = A_1 e^{-ik\xi_1 z} + A_2 e^{ik\xi_2 z}, \tag{10}$$

where

$$\xi_1 = \frac{Q + \sqrt{Q^2 + R\left(\frac{c^2}{c_0^2} - G \right)}}{R}, \qquad \xi_2 = \frac{Q - \sqrt{Q^2 + R\left(\frac{c^2}{c_0^2} - G \right)}}{R},$$

where $c_0 = \sqrt{\frac{\mu_T}{\rho}}$ is the shear wave velocity, and $\frac{c}{c_0}$ is the phase velocity ratio. Therefore from Eq. (8), we have

$$u_2(x,z,t) = \left(A_1 e^{-ik\xi_1 z} + A_2 e^{ik\xi_2 z} \right) e^{ik(x-ct)}. \tag{11}$$

This is the displacement of the upper fiber-reinforced medium, in which A_1 and A_2 are arbitrary constants.

3.2 Dynamic of non-homogeneous anisotropic layer

Let v , u , and w be the displacement components in the *x*-, *y*- and *z*- directions, respectively. Starting from the general equation of motion and the using of the conventional Love waves conditions, i.e., $v = 0$, $w = 0$, and $u = u_{\Theta}(x, z, t)$, the only y component of the equation of motion in the absence of body force can be written as (Biot [26])

$$N\frac{\partial^2 u_{\Theta}}{\partial x^2}+\frac{\partial}{\partial z}\left(L\frac{\partial u_{\Theta}}{\partial z}\right)=\rho\frac{\partial^2 u_{\Theta}}{\partial t^2}. \tag{12}$$

For a wave propagating along the *x*- direction, we assume

$$u_{\Theta}(x, z, t)=U_{\Theta}(z)e^{ik(x-ct)}, \tag{13}$$

Using (13), (12) takes the form as:

$$\frac{d^2U_{\Theta}}{dz^2}+\frac{1}{L}\frac{dL}{dz}\frac{dU_{\Theta}}{dz}+\frac{k^2}{L}\left(c^2\rho-N\right)U_{\Theta}=0. \tag{14}$$

After putting, $U_{\Theta}=\frac{U_{\theta}^{'}}{\sqrt{L}}$ in (14), we obtain

$$\frac{d^2U_{\theta}^{'}}{dz^2}-\frac{1}{2L}\frac{d^2L}{dz^2}U_{\theta}^{'}+\frac{1}{4L^2}\left(\frac{dL}{dz}\right)^2U_{\theta}^{'}+\frac{k^2}{L}\left(c^2\rho-N\right)U_{\theta}^{'}=0. \tag{15}$$

The variations in rigidities and density are taken as:

$$\left.\begin{aligned} N &= N_2\left(1+sz\right)^2 \\ L &= L_2\left(1+sz\right)^2 \\ \rho &= \rho_2\left(1+sz\right)^2 \end{aligned}\right\}. \tag{16}$$

The usage of (16), (15) changes to

$$\frac{d^2U_{\theta}^{'}}{dz^2}+m_2^2U_{\theta}^{'}=0, \tag{17}$$

where,

$$m_2^2=\frac{N_2k^2}{L_2}\left(\frac{c^2}{c_1^2}-1\right), \tag{18}$$

where $c_1=\sqrt{\frac{N_2}{\rho_2}}$ is the velocity of the shear wave in the half-space. The solution of (17) may be assumed as:

$$U_{\theta}^{'} = A_3 e^{ikm_2 z} + A_4 e^{-ikm_2 z}. \tag{19}$$

Thus, the solution for the non-homogeneous, anisotropic lower semi-infinite medium will be of the form

$$u_{\Theta}(x,z,t) = U_{\Theta}(z) e^{ik(x-ct)} = \frac{U_{\theta}^{'}}{\sqrt{L}} e^{ik(x-ct)}.$$

As a result, the solution for the non-homogeneous, anisotropic lower semi-infinite medium will be of the form

$$u_{\Theta}(x,z,t) = \frac{A_4 e^{-ikm_2 z}}{\sqrt{L_2}\left(1+sz\right)} e^{ik(x-ct)}. \tag{20}$$

4 BOUNDARY CONDITIONS AND DISPERSION FREQUENCY

The following boundary conditions must be satisfied:

(i)At the free surface $z = -H$, boundary is stress free so that

$$\mu_L \frac{\partial u_2}{\partial z} = 0 \tag{21}$$

(ii) At the irregular interface $z = \varepsilon f(x)$, the continuity of the stress require that

$$\mu_L \frac{\partial u_2}{\partial z} = L_2 \frac{\partial u_{\Theta}}{\partial z} \tag{22}$$

(iii) Again at the irregular interface $z = \varepsilon f(x)$, the continuity of the displacement

$$u_2\left(z\right) = u_{\Theta}\left(z\right) \tag{23}$$

Using boundary condition **(i)** in Eq. (11) it reduces to

$$-A_1\left(Q - R\xi_1\right)e^{ik\left(Q-R\xi_1\right)H} + A_2\left(Q - R\xi_2\right)e^{ik\left(Q-R\xi_2\right)H} = 0. \tag{24}$$

Using boundary condition **(ii)** in Eqs. (11) and (20) it reduces to

$$-A_1\mu_L ik\left(Q - R\xi_1\right)e^{ik\left(Q-R\xi_1\right)\varepsilon f(x)} + A_2\mu_L ik\left(Q - R\xi_2\right)e^{-ik\left(Q-R\xi_2\right)\varepsilon f(x)} - A_4 S_2\sqrt{L_2} = 0. \tag{25}$$

Using boundary condition **(iii)** in Eqs. (11) and (20) it reduces to

$$A_1 e^{ik\left(Q-R\xi_1\right)\varepsilon f(x)} + A_2 e^{-ik\left(Q-R\xi_2\right)\varepsilon f(x)} - A_4\frac{S_1}{\sqrt{L_2}} = 0. \tag{26}$$

where,

$$S_1 = \left[\frac{e^{-ikm_2 z}}{\left(1+sz\right)}\right]_{z=\varepsilon f(x)} \tag{27}$$

$$S_2 = \frac{\partial}{\partial z}\left[\frac{e^{-ikm_2 z}}{(1+sz)}\right]_{z=\varepsilon f(x)} \tag{28}$$

Now eliminating A_1, A_2 and A_4 from Eqs. (24), (25) and (26) we have

$$\begin{vmatrix} -(Q-R\xi_1)e^{-ik(Q-R\xi_1)H} & (Q-R\xi_2)e^{ik(Q-R\xi_2)H} & 0 \\ -\mu_L ik(Q-R\xi_1)e^{ik(Q-R\xi_1)\varepsilon f(x)} & \mu_L ik(Q-R\xi_2)e^{-ik(Q-R\xi_2)\varepsilon f(x)} & -S_2\sqrt{L_2} \\ e^{ik(Q-R\xi_1)\varepsilon f(x)} & e^{-ik(Q-R\xi_2)\varepsilon f(x)} & -\frac{S_1}{\sqrt{L_2}} \end{vmatrix} = 0.$$

where

$$\xi_1 = \frac{Q+\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}}{R}, \quad \xi_2 = \frac{Q-\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}}{R},$$

Expanding the determinant, we have

$$\tan\left[\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}\,k\left(\varepsilon f(x)+H\right)\right] = \frac{L_2}{\mu_L}\frac{\sqrt{\frac{N_2}{L_2}\left(1-\frac{c^2}{c_1^2}\right)}-\frac{\frac{s}{k}}{\left(1+\frac{s}{k}\varepsilon f(x)k\right)}}{\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}}. \tag{29}$$

Therefore, Eq. (29) is the frequency equation for the propagation of Love waves, which gives the velocity of Love waves in an elastic fiber-reinforced anisotropic layer medium of finite thickness H overlying on an heterogeneous half-space with rectangular irregularity.

5 PARTICULAR CASES

5.1 Case I

If $\varepsilon \to 0$, $h \to 0$ i.e. in the absence of irregularity in the half-space, then Eq. (29) reduces to

$$\tan\left[kH\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}\right] = \frac{L_2}{\mu_L}\frac{\sqrt{\frac{N_2}{L_2}\left(1-\frac{c^2}{c_1^2}\right)}-\frac{s}{k}}{\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}}$$

which is the dispersion equation of Love-type wave when the interface of the layered half-space is regular.

5.2 Case II

When $s \to 0$, $\varepsilon \to 0$ and $h \to 0$ i.e. when the half space is homogeneous and free from irregularity, then (29) becomes

$$\tan\left[kH\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}\right]=\frac{L_2}{\mu_L}\frac{\sqrt{\frac{N_2}{L_2}\left(1-\frac{c^2}{c_1^2}\right)}}{\sqrt{Q^2+R\left(\frac{c^2}{c_0^2}-G\right)}}$$

which is the dispersion equation of Love wave in anisotropic fiber-reinforced layer in the absence of irregularity and heterogeneity.

5.3 Case III

When $\mu_L \to \mu_T \to \mu_1$, i.e. the upper layer is homogeneous with rigidity μ_1, which implies $G \to 1,\ Q \to 0$ and $R \to 1$ and the lower half-space is homogeneous, isotropic and regular that is ($s \to 0,\ L_2 \to N_2 \to \mu_2,\ \varepsilon \to 0,\ h \to 0$), then (29) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_0^2}-1}\right]=\frac{\mu_2}{\mu_1}\frac{\sqrt{\left(1-\frac{c^2}{c_1^2}\right)}}{\sqrt{\left(\frac{c^2}{c_0^2}-1\right)}}.$$

which is the classical dispersion relation of Love wave (as Love [27]) in a homogeneous layer over an homogeneous half-space.

6 NUMERICAL CALCULATIONS AND DISCUSSION

Numerical computations of Eq. (29) have been performed to demonstrate the effects of reinforcement, heterogeneity, and irregularity contains within the assumed medium on the dispersion of Love wave. For the computational reason, we represent a few numerical data for reinforced layer medium from Chattopdhyay and Chaudhury [28] and Gubbins [29] for the heterogeneous layer.

For reinforced medium

$$\mu_L = 7.07\times 10^9 N\ /\ m^2$$
$$\mu_T = 3.5\times 10^9 N\ /\ m^2$$
$$\rho_0 = 1600 Kg\ /\ m^3$$

For heterogeneous half-space

$$N_2 = 6.34\times 10^{10} N\ /\ m^2$$
$$L_2 = 3.99\times 10^{10} N\ /\ m^2$$
$$\rho_2 = 3364 Kg\ /\ m^3$$

Figs. 2-6 show the variation of dimensionless phase velocity c/c_0 against dimensionless wave number kH by using aforementioned numerical data.

Fig. 2 depicts the effect of heterogeneity parameter s/k on the dispersion of Love waves in the presence of reinforcement (i.e. $a_1^2=0.20,\ a_3^2=0.85$) in the layer and height of irregularity (i.e. $h/H=0.01$) in the interface of the fiber-reinforced anisotropic layer and heterogeneous half-space. The value of s/k for curve 1 , curve 2 , curve 3, curve 4, curve 5, and curve 6 has been taken as 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 respectively. It is found that as the heterogeneity parameter increases, the phase velocity of Love decreases at a particular wave number. This leads to the fact that heterogeneity of the medium is inversely proportional to the phase velocity of the Love wave.

In Fig. 3, curves have been drawn to shows the effect of heterogeneity parameter s/k associated with the directional rigidity and density of lower half-space in the presence of reinforcement (i.e. $a_1^2 = 0.20,\ a_3^2 = 0.85$) and absence of height of irregularity (i.e. $h/H = 0$). The value of s/k for curve 1, curve 2, curve 3, curve 4, curve 5, and curve 6 has been taken as 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 respectively. It is observed that the phase velocity of the Love wave decreases at a particular value of wave number for the heterogeneity of the half-space increases.

Fig. 4 has been plotted to depicts the effect of reinforcement parameter on the Love wave dispersion in the fiber-reinforced layer lying over heterogeneous half-space with rectangular irregularity. The values of a_1^2 and a_3^2 for curve 1 , curve 2, curve 3, curve 4, curve 5, and curve 6 have been taken as 0.10, 0.15, 0.25, 0.30 and 0.35 and 0.35, 0.30, 0.25, 0.20, 0.15 and 0.10 respectively. From this figure it is observed that the phase velocity of Love wave increases with increases of a_1 and decreases of a_3 .

In Fig. 5 , the curves show the effect of height of irregularity h/H on the phase velocity of Love wave in the heterogeneous half-space. The presence of irregularity at the interface of the fiber-reinforced layer and heterogeneous half-space has the significant impact on the dispersion of the Love wave. The value of height of the irregular space has been considered as 0.01, 0.02, 0.03, 0.04, 0.05 and 0.06 for curve 1, curve 2, curve 3, curve 4, curve 5, and curve 6 respectively. Under the taken values it is found that phase velocity of Love wave decreases as the height of irregularity increases, that is speed of Love wave is inversely proportional to the height of irregularity.

Fig. 6 manifests the variation of dimensionless phase velocity against non-dimensional wave number for different values of height of irregularity h/H , fixed value of heterogeneity parameter (i.e. $s/k = 0.2$) and the absence of the reinforcement (i.e. $a_1 = a_3 = 0$). The value of h/H for curve 1, curve 2, curve 3, curve 4, curve 5, and curve 6 has been taken as 0.030, 0.025, 0.020, 0.015, 0.010 and 0.005. Under the considered values it has been seen that speed of Love wave increases as the height of irregularity decreases thereby reflect the fact that phase velocity of Love wave is inversely proportional to the height of irregularity of the medium.

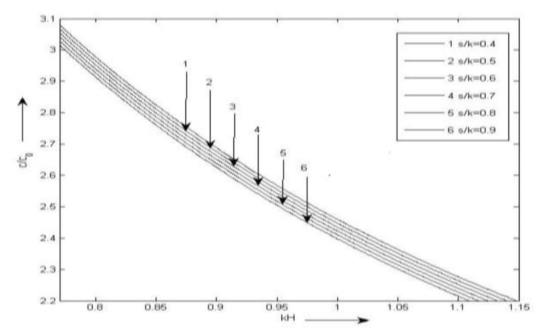

Fig.2
Variation of dimensionless phase velocity c/c_0 with non-dimensional wave number kH for different values of heterogeneity parameter s/k in the presence of reinforcement (i.e. $a_1^2 = 0.20,\ a_3^2 = 0.85$) and height of irregularity (i.e. $h/H = 0.01$).

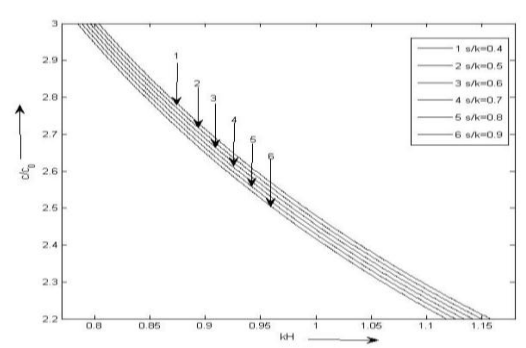

Fig.3
Comparison of dimensionless phase velocity c/c_0 against dimensionless wave number kH for various values of heterogeneity parameter s/k in the presence of reinforcement (i.e. $a_1^2 = 0.20,\ a_3^2 = 0.85$) and absence of height of irregularity (i.e. $h/H = 0$).

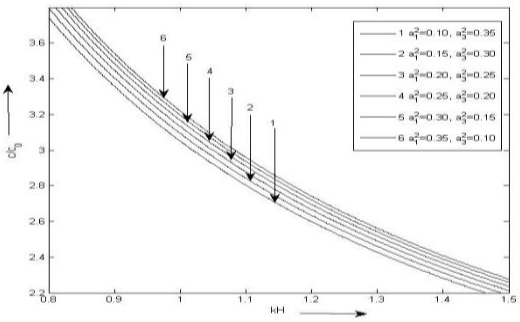

Fig.4
Variation of dimensionless phase velocity c/c_0 as a function of dimensionless wave number kH for different values of reinforcement a_1, a_3 when $h/H = 0.01$ and $s/k = 0.3$.

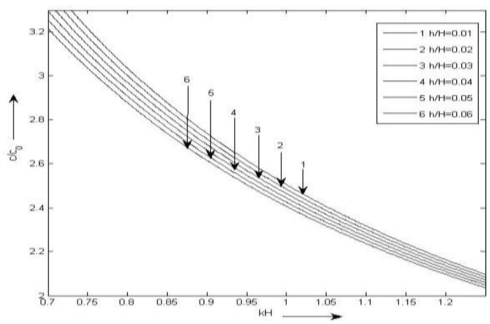

Fig.5
Comparison of dimensionless phase velocity c/c_0 with dimensionless wave number kH for various values of height of irregularity h/H in the presence of heterogeneity parameter (i.e. $s/k = 0.2$) and reinforcement (i.e. $a_1^2 = 0.20,\ a_3^2 = 0.85$).

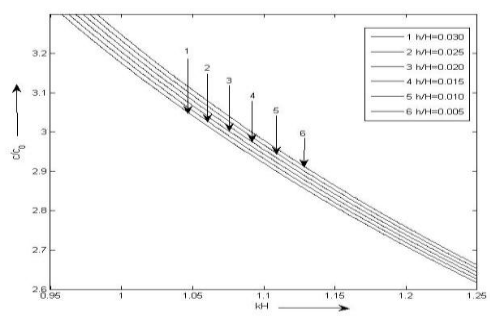

Fig.6

Variation of dimensionless phase velocity c/c_0 with respect to non-dimensional wave number kH for different values of height of irregularity h/H in the presence of heterogeneity parameter (i.e. $s/k = 0.2$) and the absence of reinforcement (i.e. $a_1 = a_3 = 0$).

7 CONCLUSIONS

In this paper, we have studied the dispersion of Love wave in a fiber-reinforced layer lying over a heterogeneous half-space with rectangular irregularity. The technique of separation of variables has been utilized to solve the dynamic equation of motion, for various media utilizing suitable boundary condition at the interface of the fiber-reinforced anisotropic layer and heterogeneous half-space with the irregular interface. The dispersion relation of Love wave has been obtained and corresponds with the classical dispersion relation of Love wave in particular cases. The effect of non-dimension phase velocity on the dispersion curve has been determined numerically and graphical representation has been given by means of MATLAB programming. So it can conclude that the phase velocity in the fiber-reinforced layer is affected by reinforcement, inhomogeneity, and irregularity. From the aforementioned figures, the outcome can be summarized as complying with:

Every one of the figures exhibits that phase velocity c/c_0 of the Love wave diminishes with the ascent of dimensionless wave number kH, which is properly nature of seismic wave, i.e., as depth raises, the rate of the surface wave decreases.

The phase velocity of Love wave diminishes as the heterogeneity parameter s/k increments in the presence of reinforcement parameters a_1, a_3 in the layer with the presence or absence of height h/H of irregularity.

The phase velocity of Love wave raises because of the reinforcement parameters a_1 increases and a_3 decreases in the presence of rectangular irregularity and heterogeneity within the half-space.

Within the presence of reinforcement parameters a_1, a_3 and heterogeneity parameter s/k, phase velocity diminishes with increments of height h/H of the irregularity.

The height h/H of the irregularity influenced the phase velocity of Love wave, and the phase velocity increases with decreases of the height of the irregularity in the presence of heterogeneity and absence of reinforcement parameters a_1, a_3.

At long last, when the upper layer is homogeneous and free from reinforcement and the lower half-space is homogeneous, isotropic and regular, the dispersion Eq. (29) reduces to the classical dispersion equation of Love wave (A.E.H. Love, [27]) and subsequently validating the solutions of the problem talked about in the present paper. The present study may be useful for geophysical applications involving the dispersion of Love waves in different layered media of Earth.

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