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Weight Optimum Design of Pressurized and Axially Loaded Stiffened Conical Shells to Prevent Stress and Buckling Failures

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ABSTRACT

An optimal design of internal pressurized stiffened conical shell is investigated using the genetic algorithm (GA) to minimize the structural weight and to prevent various types of stress and buckling failures. Axial compressive load is applied to the shell. Five stress and buckling failures as constraints are taken into account. Using the discrete elements method as well as the energy method, global buckling load and stress field in the stiffened shell are obtained. The stiffeners include rings and stringers. Seven design variables including shell thickness, number of rings and stringers, stiffeners width and height are considered. In addition, the upper and lower practical bounds are applied for the design variables. Finally, a graphical software package named as Optimal Sizer is developed to help the designers.

Keywords: Weight optimum design; Internal pressurized stiffened conical shell; Failure analysis; Genetic algorithm; Discrete elements method.

1 INTRODUCTION

BECAUSE of a strong interest in optimum design by industry, orthogonally stiffened conical shells are common structural elements in many engineering structures including wings and fuselages of airplanes, pressure vessels, submarine hulls, missiles and so on. Such structures are usually enforced by different loads as internal pressure and axial force. Hence, presentation of a new procedure for analysis and design of these structures are very important.

Most of previous studies on optimal design of stiffened shells have focused on stiffened cylindrical shells. Patel and Patel [1] made an attempt to obtain the weight optimum design software for the airplane fuselage type of stiffened cylindrical shell under pure bending load. They used the two numerical methods as penalty function technique and method of complex box. Simitses and Giri [2] presented a procedure including highly automated computer programs for the optimum weight design of fuselage type stiffened cylinders subjected to combined torsion and axial compression with and without lateral pressure. Simões et al. [3] considered a branch and bound strategy coupled with an entropy-based algorithm to solve the reliability-based optimum design of a welded stringer-stiffened steel cylindrical shell subject to axial compression and bending. Bushnell and Bushnell [4] expanded the PANDA2 computer program for minimum-weight design of stiffened composite panels to handle optimization of ring and stringer stiffened cylindrical panels and shells. Léné et al. [5] presented an optimization procedure which aims to choose the most efficient values of the geometric parameters defining the composite reinforcements and the

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most efficient position of the stiffeners on the surface of the cylinder. Simitsess and Sheinman [6] investigated on optimizing stiffened thin circular cylindrical shells under uniform axial compression against general instability in the presence of initial geometric imperfection.

A beneficial literature survey reveals that there are a few papers on the subject of optimum design of stiffened conical shells. Irisarri et al. [7] studied on a multiobjective optimization methodology for composite stiffened panels. Rao and Reddy [8] considered the design optimization of axially loaded, simply supported stiffened conical shells for minimum weight. Natural frequency, overall buckling strength and direct stress constraints are considered in the design problems.

Several structural optimization methods are available in the literature to quickly provide an optimal design for stiffened shells [9]. Due to probabilistic global optimization results, handling integer, discrete and continuous variables and ability to search complex and noisy space, genetic algorithms (GA) have gained much popularity in recent years. Ambur and Jaunky [10] presented a design strategy for optimal design of composite grid-stiffened structures with variable curvature subjected to global and local buckling constraints using a genetic algorithm. Luspa and Ruocco [11] performed the optimum topological design of simply supported composite stiffened panels based on structural weight minimization using genetic algorithms. Bagheri et al. [12] employed the genetic algorithm (GA) method to optimize ring stiffened cylindrical shells. The objective functions are the maximum fundamental frequency and minimum structural weight of the shell. Four constraints including the fundamental frequency, the structural weight, the axial buckling load, and the radial buckling load are considered. El Ansary et al. [13] developed an optimum design technique of stiffened liquid-filled steel conical tanks subjected to global and local buckling constraints using a numerical tool that couples a non-linear finite element model developed in-house and a genetic algorithm optimization technique. Maghsoudi Mehrabani et al. [14] performed vibration analysis of simply supported rotating cross-ply laminated stiffened cylindrical shell using an energy approach. Then, the optimal design of parameters due to shell and stiffeners is conducted by genetic algorithm (GA) method. Marín et al. [15] developed an optimization procedure for optimal design of a composite material stiffened panel with conventional stacking sequence using static analysis and hygrothermal effects. The procedure is based on a neural network system and a multi-objective optimization method as a genetic algorithm.

According to aforementioned literature and a good knowledge of authors, it is obvious that optimal design of pressurized stiffened conical shell under axial compression based on the genetic algorithm has not been investigated in published articles. The main difference in the present work with those developed by Rao and Reddy [8] is that genetic algorithm (GA) method is used in the present study and local buckling constraints is considered. In addition, all the previous studies used to only deal with stiffened conical shells while the effects of stiffeners in shells were evaluated by an averaging method. However, in order to accurately allow for the eccentric effects, stiffeners are treated as discrete element in present study. Moreover, analysis of the different buckling modes under combined loads (internal pressure and axial load) is one of the other contribution of this work.

This paper deals with the optimal design of internal pressurized stiffened conical shell subjected to axial compression. An optimization method named the genetic algorithm is employed to minimize the structural weight. In the optimization procedure, shell thickness, number of rings and stringers, stiffeners width and height are defined as design variables. The weight of structure is taken as the objective function. Skin local buckling, skin-stringer local buckling, crippling, global buckling, ultimate collapse stress and the upper and lower practical bounds of the design variables are set as constraint functions. Applying energy method, global buckling load and stress field in the shell are determined. Different design examples and parametric studies are considered to illustrate the stability and performance of proposed optimization procedure. Also, a user friend graphical software package is presented being very useful to engineers who are designers of shell structures, and even to engineering managers who need an overview of the subject.

2 GEOMETRICAL CONFIGURATION

The thin conical shell, as shown in Fig. 1, is considered, which is stiffened by circumferential and meridional stiffeners as rings and stringers, respectively. α is the cone angle, L is the length, a and b are mean radius of cone at the small and large end, respectively, and t is the shell thickness. Reference surface of the conical shell is taken to be at its middle surface of shell where an orthogonal coordinate system (x, θ, z) is fixed, and $r_x = r(x)$ is a radius at any coordinate point (x, θ, z). Displacement of the shell in the x, θ and z directions are denoted by u, v and w, respectively. Depth and width of the stiffeners are symbolized by $d_{s(or\ r)}$ and $b_{s(or\ r)}$, respectively and the ring intervals are denoted by *s*. Subscripts (*s*, *r*) indicate the stringer and ring stiffeners, respectively. Displacements from the middle surface of shell to any point located on the stiffeners are addressed by z .

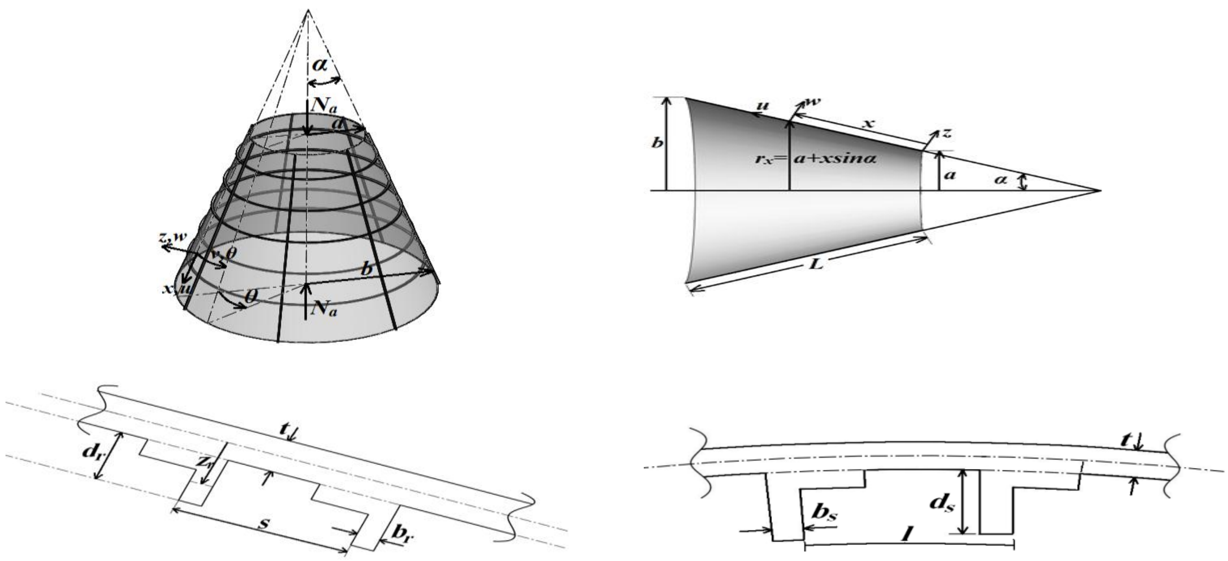

Fig.1
Geometry of orthogonal stiffened conical shell.

3 STRESS AND BUCKLING ANALYSES

In order to obtain optimum weight design of the stiffened conical shells, formulations of different stress and buckling failure modes of the axially loaded and pressurized shell should be extracted. Here, five failure modes as skin local buckling, skin-stringer local buckling, crippling, global buckling and ultimate collapse stress are considered.

3.1 Static and Global buckling analyses

The energy method and the discrete elements approach are employed to obtain the global buckling load and stress field of the pressurized stiffened conical shell under axial compression. For this purpose, strain energies of stiffeners and shell should be calculated, firstly. Then, the differential equations of equilibrium can be derived using Hamilton's principle. Finally, these equations are solved using the Fourier expansion method.

3.1.1 Strain energy of the shell

The strain energy of the conical shell is expressed as:

$$U_{\varepsilon} = \frac{1}{2}\int_0^L\int_0^{2\pi} \varepsilon^T \left[S\right] \varepsilon\, r_x \, d\theta dx \tag{1}$$

where $r_x(x) = a + x \sin\alpha$ and the strain vector $\{\varepsilon\}$ can be written as:

$$\varepsilon^T = \{e_x \quad e_\theta \quad e_{x\theta} \quad \kappa_x \quad \kappa_\theta \quad \kappa_{x\theta}\} \tag{2}$$

where symbols e_x, e_θ and $e_{x\theta}$ are middle surface strains and symbols κ_x , κ_θ and $\kappa_{x\theta}$ are middle surface curvatures.

Geometric relations of deformation for the reference surface of the conical shell can be written as [16]

$$
\begin{aligned}
&e_x=\frac{\partial u}{\partial x}, e_\theta=\frac{1}{r_x}\frac{\partial v}{\partial \theta}+\frac{u\sin\alpha+w\cos\alpha}{r_x}, e_{x\theta}=\frac{1}{r_x}\frac{\partial u}{\partial \theta}+\frac{\partial v}{\partial x}-\frac{v\sin\alpha}{r_x}\\
&\kappa_x=-\frac{\partial^2 w}{\partial x^2}, \kappa_\theta=-\frac{1}{r_x^{\,2}}\frac{\partial^2 w}{\partial \theta^2}+\frac{\cos\alpha}{r_x^{\,2}}\frac{\partial v}{\partial \theta}-\frac{\sin\alpha}{r_x}\frac{\partial w}{\partial x},\\
&\kappa_{x\theta}=2\left(-\frac{1}{r_x}\frac{\partial^2 w}{\partial x\,\partial \theta}+\frac{\sin\alpha}{r_x^{\,2}}\frac{\partial w}{\partial \theta}+\frac{\cos\alpha}{r_x}\frac{\partial v}{\partial x}-\frac{v\sin\alpha\cos\alpha}{r_x^{\,2}}\right)
\end{aligned}
\tag{3}
$$

It is assumed that the displacements are continuous functions of the thickness coordinate, which results in continuous transverse strains.

Meanwhile, stiffness matrix [S] for an isotropic shell is given by

$$
\left[S\right]=\begin{bmatrix}
A_{11} & A_{12} & 0 & 0 & 0 & 0\\
A_{21} & A_{22} & 0 & 0 & 0 & 0\\
0 & 0 & A_{66} & 0 & 0 & 0\\
0 & 0 & 0 & D_{11} & D_{12} & 0\\
0 & 0 & 0 & D_{21} & D_{22} & 0\\
0 & 0 & 0 & 0 & 0 & D_{66}
\end{bmatrix}
\tag{4}
$$

where $A=[A_{ij}]$ and $D=[D_{ij}]$, $(\mathrm{i},\mathrm{j}=1,2,6)$ are, respectively, the extensional and bending stiffness matrices, and these are defined as:

$$
\begin{aligned}
&A_{11}=A_{22}=\frac{Et}{\left(1-\upsilon^2\right)} \quad , \quad A_{12}=A_{21}=\frac{\upsilon Et}{\left(1-\upsilon^2\right)} \quad , \quad A_{66}=\frac{Et}{2\left(1+\upsilon\right)}\\
&D_{11}=D_{22}=\frac{Et^3}{12\left(1-\upsilon^2\right)} \quad , \quad D_{12}=D_{21}=\frac{\upsilon Et^3}{12\left(1-\upsilon^2\right)} \quad , \quad D_{66}=\frac{Et^3}{12\left(1+\upsilon\right)}
\end{aligned}
\tag{5}
$$

Besides, the work done on the shell due to axial compression and internal pressure is described as:

$$
U_{N_a}=\gamma N_a^{\,x}\int_0^{2\pi}\int_0^{L}\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 r_x\,dx\,d\theta+(1-\gamma)\int_0^{2\pi}\int_0^{L} N_a^{\,x}u\,r_x\,dx\,d\theta
\tag{6}
$$

$$
U_P=-\frac{P_i}{2}\tan\alpha\left(a+b\right)\int_0^{L}\int_0^{2\pi}\left[\left(\frac{\partial^2 w}{\partial \theta^2}+w\right)w\right]\left(1+\frac{x\sin\alpha}{b}\right)r_x\,d\theta dx
\tag{7}
$$

where $N_a^{\,x}$ and P_i are the axial load on the edge of shell in x direction and internal pressure, respectively. Also, $\gamma=0$ and 1 correspond to stress and buckling analyses, respectively.

3.1.2 Strain energy of the stiffeners

The stiffener-to-shell joints are the significant technology issues, either adhesively bonded or mechanically fastened to the shell or more recently fabricated without fasteners by co-curing the stringers and co-bonding the rings. In all of the abovementioned technologies, the shell and the stiffeners have same displacements. Therefore, the stiffeners (rings and stringers) are assumed to be an integral part of the shell. Meanwhile, when stiffeners of equal strength are closely and evenly spaced, the stiffened shell can be modeled as an equivalent orthotropic shell (smearing method). However, as the stiffener spacing increases or the wavelength of vibration becomes smaller than the stiffener spacing, determination of dynamic characteristics for the stiffened shell cannot be accurate anymore. If a more accurate mathematical model is needed, the stiffeners can be treated as discrete elements and therefore it is possible to use non-uniform eccentricity, unequally spaced and different materials for stiffeners. In order to maintain displacement compatibility between the stiffeners and the shell, a special transformation is used including coupling effects due to eccentricity of the stiffener. It should be also noted that the displacements vary through depth of the stiffeners. Therefore, displacement of a point at distance Z from the shell middle surface can be explained by shell displacement field [8]

$$u_{s(or\, r)} = u - z\frac{\partial w}{\partial x} \;,\; v_{s(or\, r)} = v - \frac{z}{r_x}\frac{\partial w}{\partial \theta} \;,\; w_{s(or\, r)} = w - \frac{z}{r_x}\frac{\partial v}{\partial \theta} \tag{8}$$

The strain of stringers in the meridional direction and the strain of rings in the circumferential direction are respectively defined as:

$$\varepsilon_{sx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{9}$$

$$\varepsilon_{r\theta} = \frac{1}{r_x}\left(\frac{\partial v}{\partial \theta} - \frac{z}{r_x}\frac{\partial^2 w}{\partial \theta^2} + u\sin\alpha - z\sin\alpha\frac{\partial w}{\partial x} + w\cos\alpha - \frac{z\cos\alpha}{r_x}\frac{\partial v}{\partial \theta}\right) \tag{10}$$

Using discrete stiffener theory, the strain energy for the stringers can be written as [17]

$$U_s = \frac{1}{2}\sum_{k=1}^{N_S} E_{sk}\int_0^L\int_{A_{sk}} \varepsilon_{sx}^2 dA_{sk}dx + \frac{1}{2}\sum_{k=1}^{N_S} G_{sk}J_{sk}\int_0^L\left[\frac{1}{r_{xk}^2}\frac{\partial^2 w_s}{\partial x\,\partial \theta} - \frac{2\sin\alpha}{r_{xk}^2}\frac{\partial w}{\partial \theta} - \frac{\cos\alpha}{r_{xk}}\frac{\partial v}{\partial x} + \frac{2\sin\alpha\cos\alpha}{r_{xk}^2}v\right]^2 dx \tag{11}$$

where $G_{sk}J_{sk}$, A_{sk} and N_s are torsional stiffness, cross sectional area of the k^{th} stringer and the number of stringers, respectively.

The strain energy of the rings can be written as [17]

$$U_r = \frac{1}{2}\sum_{k=1}^{N_r} E_{rk}\int_0^{2\pi}\int_{A_{rk}} \varepsilon_{r\theta}^2 dA_{rk}r_{xk}d\theta + \frac{1}{2}\sum_{k=1}^{N_r} G_{rk}J_{rk}\int_0^{2\pi}\left[\frac{1}{r_{xk}^2}\left(\frac{\partial^2 w_s}{\partial \theta\,\partial x}\right)^2\right]r_{xk}d\theta \tag{12}$$

where $G_{rk}J_{rk}$, A_{rk} and N_r are torsional stiffness, cross sectional area of the k^{th} ring and the number of rings, respectively.

3.1.3 Obtaining of global buckling load and Von Mises stress

The total energy of internal pressurized stiffened conical shell subjected to axial compression can be written as [17]

$$\Pi = U_\varepsilon + U_s + U_r - U_{N_a} - U_P \tag{13}$$

The simply supported boundary conditions are satisfied by the following expansions of the generalized displacement field

$$u(x,\theta) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A\cos\lambda x\cos(n\theta) \;,\; v(x,\theta) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B\sin\lambda x\sin(n\theta) \;,\; w(x,\theta) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C\sin\lambda x\cos(n\theta) \tag{14}$$

where $\lambda = m\pi/L$ and m, n denote longitudinal and circumferential wave numbers, respectively. Substituting Eqs. (1), (6), (7), (11), (12) and (14) into Eq. (13) and after differentiating the obtained equation with respect to A, B and C, the total energy of the shell can be written in matrix form as:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \gamma N_a^{x\,cr} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda^2 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} + (1-\gamma) N_a^x \begin{Bmatrix} c_{11} \\ 0 \\ 0 \end{Bmatrix} \tag{15}$$

where $a_{ij}\ (i, j = 1, 2, 3)$ and c_{11} are listed in Appendix A.

In stress analysis ($\gamma = 0$), for given compressive axial load per unit length N_a^x and internal pressure P, the stresses in x and θ directions can be obtained by substituting A, B and C coefficients obtained from Eq. (15) into Eq. (14) and then Eq. (3) as well as using the stress-strain relation of the shell in a compact form as:

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{16}$$

where $\{\sigma\}^T = \{\sigma_x, \sigma_\theta, \sigma_{x\theta}\}$ and $\{\varepsilon\}^T = \{\varepsilon_x, \varepsilon_\theta, \varepsilon_{x\theta}\}$ are the stress and the strain vectors of the shell at a distance z from the reference surface, and $[Q] = [Q_{ij}]\ (i, j = 1, 2, 6)$ is the stiffness matrix of the shell and it can be written as:

$$[Q] = \begin{bmatrix} \dfrac{E}{1-\upsilon^2} & \dfrac{\upsilon E}{1-\upsilon^2} & 0 \\ \dfrac{\upsilon E}{1-\upsilon^2} & \dfrac{E}{1-\upsilon^2} & 0 \\ 0 & 0 & G \end{bmatrix} \tag{17}$$

Hence, the maximum Von Mises stress might be occurred on the length of shell and its magnitude is given by

$$\sigma_{VM} = \left(\sigma_x^2 + \sigma_\theta^2 - \sigma_x \sigma_\theta + 3\sigma_{xy}^2\right)^{1/2} \tag{18}$$

For global buckling analysis ($\gamma = 1$) and for each mode (m, n), and rearrangement of Eq. (15) with respect to $N_a^{x\,cr}$, the eigenvalue problem is obtained as:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = N_a^{x\,cr} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda^2 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} \tag{19}$$

The solution of Eq. (19) gives the eigenvalues $N_{a,m,n}^x$ and the minimum of eigenvalues is the critical buckling load.

3.2 Local buckling modes

Different modes of local instability might occur in thin-walled stiffened shells. Particularly, three types of local instability can be more important in stiffened conical shells, namely as local shell panel buckling between stiffeners, local buckling of the stiffener-skin panel and crippling instability.

3.2.1 Local shell panel buckling between stiffeners

The critical buckling stress of the conical shell panel between stiffeners is given by [18]

$$\sigma_{cr,l}^{p}\left(x\right)=\frac{E}{\sqrt{3\left(1-\upsilon^{2}\right)}}\frac{t}{\rho\left(x\right)}\;\; for\;\; \xi\left(x\right)\geq 2.44/\sqrt{\cos\left(\alpha\right)} \tag{20}$$

$$\sigma_{cr,l}^{p}\left(x\right)=\frac{4\pi^{2}E}{12\left(1-\upsilon^{2}\right)}\left(\frac{t}{l\left(x\right)}\right)^{2}\;\; for\;\; \xi\left(x\right)< 2.44/\sqrt{\cos\left(\alpha\right)} \tag{21}$$

where $\rho\left(x\right)=R\left(x\right)/\cos\left(\alpha\right), \xi\left(x\right)=l\left(x\right)/\sqrt{R\left(x\right)t}$ and *l(x)* is distance between evenly spaced stringers. The larger value between Eqs. (20) and (21) is considered as the critical stress for panel local buckling.

3.2.2 Local buckling of the stringer-skin panel

Another local buckling mode is the local buckling of the stringer -skin panel. This failure mode is a column instability in which the cross-section of the stringer translates. The highest stress sustained by the skin-stiffener panel may be given by [18]

$$\sigma_{cr,l}^{sp}=\sigma_{0}-\frac{\sigma_{0}^{2}\left(\frac{s}{\kappa\sqrt{c}}\right)^{2}}{4\pi^{2}E}+\frac{PR}{2t''} \tag{22}$$

where $\sigma_{0}=0.3\sqrt{\sigma_{y}E}\,(b_{s}/d_{s})^{3/4}, s$ is distance between evenly spaced rings and κ is radius of gyration of stringer and is given by

$$\kappa=\sqrt{\frac{I_{ss}}{A_{ss}}} \tag{23}$$

3.2.3 Crippling instability

Crippling is a local instability failure mode involving local failure of only the stringers. The cross sections of the stringers are distorted in their own plane without any translation or rotation. Crippling generally occurs in stringers having wide thin flanges. In order to determine the crippling stresses, the local buckling stresses of the stringer webs and flanges must be determined, firstly. For each stringer element, the crippling stress is given by

$$\sigma_{cr,l}^{st}=\frac{\sum_{i=1}^{m}\sigma_{c_i}A_{i}}{\sum_{i=1}^{m}A_{i}} \tag{24}$$

In the present study, the L-shaped stiffeners are considered. Therefore, the crippling stress can be obtained by rewriting Eq. (24) as:

$$\sigma_{cr,l}^{st}=\frac{\sigma_{c_1}A_{1}+\sigma_{c_2}A_{2}}{A_{1}+A_{2}}\;;\;\sigma_{c_i}=C_{i}\sqrt{\sigma_{y}E}\,(\frac{b_{i}}{d_{i}})^{3/4} \tag{25}$$

where

- C_i =0.3 is constant coefficient and has been proposed by Ref. [18].
- σ_y is yield stress.
- E is the Young's modulus.

4 OPTIMIZATION PROCEDURE

A discrete structural optimization problem in standard mathematical forms can be formulated in the following form

$$\begin{aligned} &Minimize\ \ F(X) \qquad Subject\ \ to: \\ &\qquad g_j(X) \le 0 \qquad j = 1,2,3,...,m \ \ ; \ \ X = \{X_1, X_2, X_3, ..., X_n\} \\ &\qquad X_i^l \le X_i \le X_i^u \qquad i = 1,2,3,...,n \end{aligned} \tag{26}$$

where $F(X), g_j(X), X_i^l$ and X_i^u are the objective function, constraints functions, lower bound and upper bound of variable design, respectively.

4.1 General approach of the Genetic Algorithm

A weight optimization for the axial buckling loads of orthogonal stiffened conical shells with internal pressure is performed using GA. For each GA process, the design variables should be identified. The aim of this study is to minimize the weight of stiffened conical shell by altering the number of stiffeners, cross section of stiffener and the thickness of shell. Genetic algorithm is searching procedures based on the mechanics of natural genetics and natural selection. The GA starts from an initial set or first generation of randomly chosen design variables with a uniform probability distribution. In this paper, a simple GA with Reproduction, Crossover, and Mutation operators is used for the optimum weight of stiffened shell. The reproduction operator emphasizes the survival of the fittest in GA. Individual strings from a set of populations are selected on a proportionate basis for reproduction according to their fitness. Fitness is defined as a figure of merit, which is minimized. In an effective reproduction, individuals with higher fitness values have higher probability of being selected for mating and subsequent genetic action. In order to illustrate the configuration of a GA-optimization procedure, a comprehensive flowchart is presented in Fig. 2.

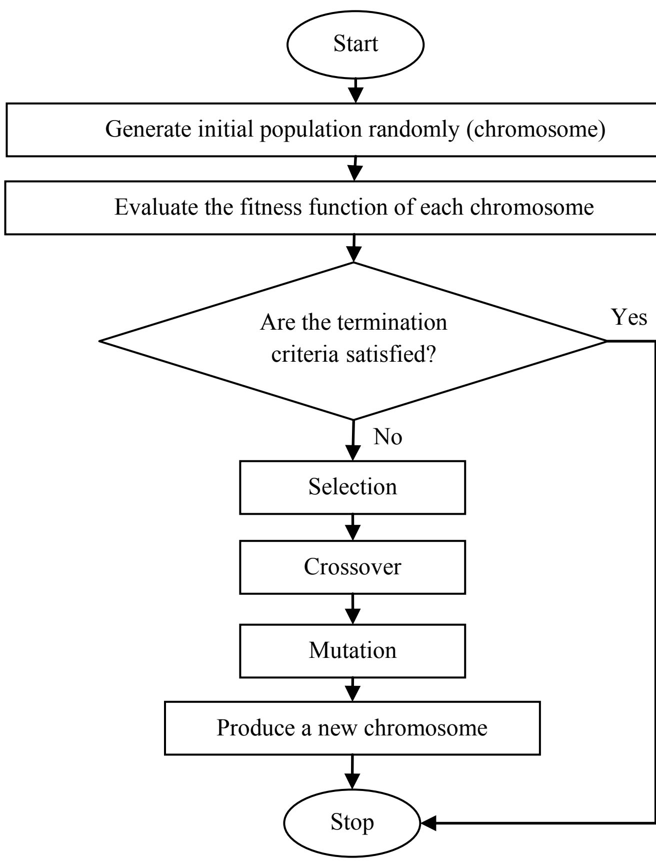

Fig.2
Flowchart of the proposed GA optimization procedure.

4.2 Design variables

In the most industrial applications, conical shells are used as components of a larger structure. For a shell to be well-matched with the mating components, overall dimensions such as the radii at two ends, length of the shell and the cone angle are fixed beforehand. Therefore, seven design variables including the shell thickness, widths and depths of rings and stringers and number/spacing of stiffeners are considered in the present work. The design vector is taken as:

$$X^T = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\} = \{t, b_s, d_s, b_r, d_r, N_s, N_r\} \tag{27}$$

4.3 Objective function

The objective function is the sum of the individual weights of shell wall (W_w), rings (W_r) and stringers (W_s). The weight as a function of the design variables, materials properties and geometry is given by

$$W_{tot} = W_w + W_s + W_r \tag{28}$$

where

$$W_w = \pi\rho Lt\left(a+b\right), \quad W_s = 2\rho_s b_s d_s LN_s, \quad W_r = 2\rho_r \pi\left(a+b\right) b_r d_r N_r \tag{29}$$

ρ, ρ_s and ρ_r represent the mass density of the shell, stringer and ring, and N_s and N_r denote the number of stringers and rings, respectively.

4.4 Design constraints

To avoid the failure mode of material by yielding, the induced stress in the shell given by Eq. (18) should be restricted to be less than the allowable stress. Thus

$$\sigma_{applied} - \sigma_y \le 0 \tag{30}$$

where σ_y is the yield stress and $\sigma_{applied}$ is defined by Eq. (18).

In order to avoid the possibility of failure of the shell due to global buckling under external loading, a constraint on the global buckling strength is necessary. Theoretically the conical shell has infinite global buckling load given by Eq. (19) for different combinations of integer values of *m* and *n*. If for any feasible starting design vector, the values of *m* and *n* corresponding to the minimum buckling load are m' and n', the buckling loads corresponding to different combinations of *m* and *n* in the neighborhood of m' and n' (namely, $m'-1,\ m',\ m'+1$ and $n'-1,\ n',\ n'+1$) are constrained as:

$$P_{applied} - P_{cr_i} \le 0 \ , \ i = 1,2,...,9 \tag{31}$$

where P_{cr_i} is defined by Eq. (19) $P_{applied}$ is the applied load intensity.

In addition, to avoid possibility of failure of the shell due to local buckling under external loading, three constraints on the local buckling strength is necessary. These constraints are represented in the following forms.

$$\sigma_{applied} - \sigma_{cr,l}^{p} \le 0 \tag{32}$$

$$\sigma_{applied} - \sigma_{cr,l}^{sp} \le 0 \tag{33}$$

$$\sigma_{applied} - \sigma^{st}_{cr,l} \le 0 \tag{34}$$

where $\sigma_{applied}$ is the applied stress and $\sigma^{p}_{cr,l}, \sigma^{sp}_{cr,l}$ and $\sigma^{st}_{cr,l}$ are the buckling stresses for panel instability, stiffener-skin instability and crippling of stiffener, respectively.

5 RESULTS AND DISCUSSION

According to the present optimization procedure, a computer code has been developed to obtain the optimal design variables and weight reduction percent based on the structural and material constraints. The material properties of the shell and stiffeners are the same as Aluminum, which modulus of elasticity *E*, Poisson's ratio *v* and yield stress of material are 72 *GPa*, 0.3 and 170 *MPa*, respectively. In all calculations, the length and mean radius at the small end of the conical shells are $L = 3m, a = 0.5m$, respectively. This section includes three issues as validation of the present stress and buckling results, study on the optimization procedure and finally, presentation of a graphical software package based on the optimization method.

5.1 Validation

In order to use the stress and buckling formulations discussed in Section 3, in the optimal procedure, at the first, the accuracy of these formulations should be examined. Buckling loads and maximum Von Mises stresses of internal pressurized stiffened conical shells under axial compression are displayed in Table 1. The results are obtained based on the present energy method and a finite element analysis in ABAQUS software. Calculations are carried out for the stiffened conical shells ($\alpha = 30^\circ$) subjected to axial compression as $F = 2000kN$ and various internal pressures as $P_i = (0, 2\times10^5, 4\times10^5)Pa$ with ten stringers ($b_s = 5mm, d_s = 20mm$) and five rings ($b_r = 6mm, d_r = 30mm$). The results shown in Table 1. report a good agreement between the present results and those obtained by the finite element analysis. The discrepancy is defined as:

$$Dicrepancy\,(\%) = \frac{FEM - \Pr esent}{FEM} \times 100 \tag{35}$$

It can be observed from Table 1. that the shell thickness has more effects on the stress values and buckling loads. It is also seen that increasing the internal pressure and shell thickness, the buckling loads enhance due to increasing the stiffness of the stiffened conical shell. The maximum Von Mises stress decreases when the shell thickness increases. Increasing of the internal pressure causes a reduction of the maximum Von Mises stress, firstly. But, due to enhancing the meridional stress and decreasing the circumferential one, the maximum Von Mises stress increases.

Table 1
Buckling loads and maximum von misses stresses of internal pressurized stiffened conical shells under axial compression ($N_s = 10, N_r = 5, \alpha = 30^\circ, b_s = 5mm, d_s = 20mm$, $b_r = 6mm, d_r = 30mm$, $F = 2000kN$).

		Buckling Analysis (*kN*)			Stress Analysis (*MPa*)		
P_i *(Pa)*	*t (mm)*	Present	FEM	Discrepancy (%)	Present	FEM	Discrepancy (%)
0	1	1591.0	1662.3	4.29	220.53	231.24	4.63
	2	2116.3	2204.2	3.99	115.27	122.11	5.60
	4	3303.9	3417.8	3.33	52.133	55.452	5.99
2×10^5	1	1983.7	2051.1	3.29	210.17	217.45	3.35
	2	2509.0	2582.0	2.83	102.09	109.78	7.00
	4	3696.6	3772.2	2.00	56.544	60.258	6.16
4×10^5	1	2376.4	2431.4	2.26	376.25	382.57	1.65
	2	2901.7	2955.4	1.82	178.12	183.55	2.96
	4	4089.3	4205.7	2.77	92.062	97.129	5.22

5.2 Study on the optimization procedure

5.2.1 The convergence study on the GA results

To study the convergence and the accuracy of the optimum solution, fitness function (total objective function) versus number of generations (GN) is plotted in Fig. 3 for two population sizes i.e. PS. The results reveal the fact that increasing the PS, the minimum required GN decreases while the run time for the convergence is lower.

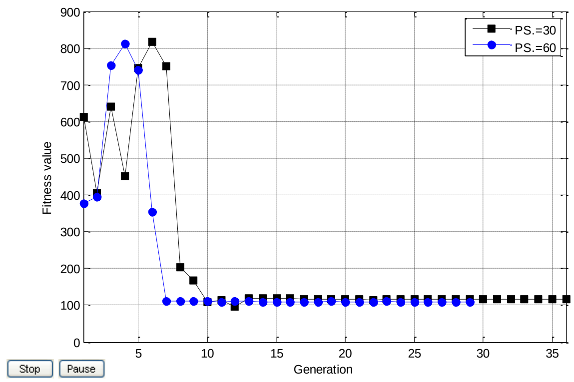

Fig.3
Convergence of fitness value for two different population sizes (PS.) for a stiffened conical shell.

5.2.2 The effect of axial compression force on the optimal results

Table 2. exhibits the optimum values of seven design variables as well as weight reduction percent (WRP) for axially compressive stiffened conical shells ($\alpha = 15^\circ, P_i = 0$) while various values of axial compression forces were used as 1000, 1500 and 2000 (*kN*). Three types of stiffened conical shells are considered which are stiffened by ring, stringer and ring-stringer stiffeners. It is seen from Table 2. that in all cases, the weight reduction percent (WRP) decreases with an increase in the axial force. Such behavior is due to the fact that enhancing the axial force, the local buckling of panel between the stiffeners can be critical. The GA optimum procedure increases the shell thickness to avoid the local buckling of panel between the stiffeners, leading to an increase in the shell thickness and the weight of the shell. Therefore, the weight reduction percent decreases. In the ring stiffened conical shell case, when the axial force is 2000 *kN*, the optimum procedure cannot be able to find an optimum model in the practical bound of the shell thickness. Inspection of the results presented in Table 2. reveals the fact that the WRP of the optimum conical shells stiffened by ring-stringer stiffeners has larger values. This observation shows that the unpressurized ring-stringer stiffened conical shells are very useful for engineering applications whose require minimum structural weight. On the other hand, for ring-stringer stiffened shells, the effect of rings consists in a reduction of the optimum shell thickness leading to an increase of WRP value.

Table 2
The optimal objective values of axially compressive stiffened conical shells (α=15°, P_i=0).

Type of stiffeners	F (*kN*)	The design variables								
			t (mm)	N_s	b_s *(mm)*	d_s (mm)	N_r	b_r (mm)	d_r (mm)	*WRP*[d] *(%)*
Ring	1000	(LB[a],UB[b],OV[c])	(1,4,3.4)	-	-	-	(2,30,2)	(5,20,5.00)	(10,50,10.0)	+7.58
	1500	(LB,UB,OV)	(1,4,4.0)	-	-	-	(2,30,2)	(5,20,11.9)	(10,50,26.8)	+2.65
	2000	(LB,UB,OV)	(1,4,----)	-	-	-	(2,30,-)	(5,20,-----)	(10,50,-----)	-----
Stringer	1000	(LB,UB,OV)	(1,4,3.3)	(3,50,42)	(2,15,2.0)	(10,50,10.0)	-	-	-	+8.79
	1500	(LB,UB,OV)	(1,4,4.0)	(3,50,46)	(2,15,2.1)	(10,50,12.9)	-	-	-	+4.59
	2000	(LB,UB,OV)	(1,4,4.0)	(3,50,50)	(2,15,7.3)	(10,50,10.2)	-	-	-	+1.45
Ring and stringer	1000	(LB,UB,OV)	(1,4,2.6)	(3,50,30)	(2,15,2.0)	(10,50,10.0)	(2,30,2)	(5,20,5.00)	(10,50,10.0)	+22.36
	1500	(LB,UB,OV)	(1,4,3.2)	(3,50,28)	(2,15,2.0)	(10,50,15.1)	(2,30,2)	(5,20,5.00)	(10,50,10.0)	+18.52
	2000	(LB,UB,OV)	(1,4,4.0)	(3,50,20)	(2,15,5.9)	(10,50,17.0)	(2,30,2)	(5,20,5.00)	(10,50,10.0)	+10.53

[a]LB=Lower Bound of the variable design
[b]UB=Upper Bound of the variable design
[c]OV=Optimum value
[d]WRP=Weight reduction percent

5.2.3 The effect of internal pressure on the optimal results

In Table 3. the effects of internal pressure on the optimum design variables are investigated. The small WRP values in Table 3. for all optimal cases having different types of stiffeners seem to be due to the presence of the stiffeners. In other words, the conical shells do not need ring and stringer stiffeners when subjected to internal pressure, especially for high pressure value and therefore an unstiffened (simple) conical shell has smaller structural weight than the stiffened one. The results reveal that for the stringer stiffened conical shells, the WRP values are negative. This is because the practical bound of number of stringers ($3<N_s<50$), causing the existence of at least three stringers in the optimum shell configurations. On the other side, the stringers do not resist internal pressure. Therefore, the weight of stringer stiffened conical shell is larger than unstiffened (simple) one leading to the negative WRP value. For ring and ring-stringer stiffened conical shells, enhancing internal pressure, depth and width of the rings are increased instead of the increase in the number of rings. This is due the fact that the stress failure mode in rings is more critical than different types of buckling failure modes, especially for high pressure value cases. Hence, increasing the depth and width of the rings, the optimization procedure avoids the possibility of stress failure in the rings due to internal pressure.

Table 3
The optimal objective values of internal pressurized stiffened conical shells subjected to axially compressive load (α=10^o, F=100kN).

Type of stiffeners	P_i (Pa)		The design variables							
			t (mm)	N_s	b_s (mm)	d_s (mm)	N_r	b_r (mm)	d_r (mm)	WRP^d(%)
Ring	4×10^5	(LB[a],UB[b],OV[c])	(1,6,1.40)	-	-	-	(2,30,2)	(3,20,7.00)	(5,50,22.6)	+2.00
	6×10^5	(LB,UB,OV)	(1,6,2.29)	-	-	-	(2,30,2)	(3,20,19.2)	(5,50,50.0)	+4.64
	8×10^5	(LB,UB,OV)	(1,6,3.51)	-	-	-	(2,30,2)	(3,20,20.0)	(5,50,50.0)	+4.01
Stringer	4×10^5	(LB,UB,OV)	(1,6,2.89)	(3,50,41)	(2,15,2)	(3,50,3)	-	-	-	+5.86
	6×10^5	(LB,UB,OV)	(1,6,3.75)	(3,50,35)	(2,15,2)	(3,50,3)	-	-	-	+4.79
	8×10^5	(LB,UB,OV)	(1,6,5.04)	(3,50,26)	(2,15,2)	(3,50,3)	-	-	-	+4.13
Ring and stringer	4×10^5	(LB,UB,OV)	(1,6,2.21)	(3,50,3)	(2,15,2)	(3,50,3)	(2,30,2)	(3,20,14.4)	(5,50,48.44)	-15.0
	6×10^5	(LB,UB,OV)	(1,6,2.30)	(3,50,3)	(2,15,2)	(3,50,3)	(2,30,2)	(3,20,19.2)	(5,50,49.94)	-2.01
	8×10^5	(LB,UB,OV)	(1,6,3.51)	(3,50,3)	(2,15,2)	(3,50,3)	(2,30,2)	(3,20,20.0)	(5,50,50.00)	-1.10

[a]LB=Lower Bound of the variable design
[b]UB= Upper Bound of the variable design
[c]OV=Optimum value
[d]WRP= Weight reduction percent

5.2.4 The effect of the cone angle α on the optimal results

The optimal design variables of axially compressive loaded stringer stiffened conical shells are listed in Table 4 for different values of the cone angle α ($F=1500kN, P_i=0$). It can be concluded from Table 4. that the number of stringers is increased by increasing the cone angle leading to an increase in the WRP and at the same time, the depth and width of the stringers decrease. This is because the skin local buckling and skin-stringer local buckling failure modes are more critical when the cone angle increases. Therefore, the optimization procedure finds maximum value for optimum shell thickness as 3 *mm* and enhances the number of stringers to avoid these failure modes.

Table 4
The optimal design variables of stringer stiffened conical shells subjected to axially compressive load for different values of the cone angle α (F=1500 kN, P_i=0).

α (Degree)		The design variables				
		t (mm)	N_s	b_s (mm)	d_s (mm)	WRP (%)
0	(LB,UB,OV)	(1,3,3)	(3,50,18)	(3,20,6.56)	(5,50,9.650)	+13.05
5	(LB,UB,OV)	(1,3,3)	(3,50,24)	(3,20,6.00)	(5,50,8.210)	+13.74
10	(LB,UB,OV)	(1,3,3)	(3,50,28)	(3,20,3.82)	(5,50,18.67)	+15.77
15	(LB,UB,OV)	(1,3,3)	(3,50,41)	(3,20,3.48)	(5,50,10.33)	+19.10
20	(LB,UB,OV)	(1,3,3)	(3,50,48)	(3,20,3.00)	(5,50,9.050)	+23.61

5.3 A graphical software package

Most researchers develop analysis and design programs to solve particular problems, but many times these codes are often shelved due to their difficulty of use, cumbersome data input and output, and lack of expert users. Hence, a user friendly and graphical software is an essential tool for engineers optimizing and analyzing stiffened conical shells. The purpose of this subsection is to describe a user-friendly and graphical computer program that permit the optimum design of stiffened cylindrical and conical shells under axial compression and internal pressure. The graphical user interface (GUI) tool in MATLAB software is a method with great potential to develop this graphical computer program. Therefore, based on the optimization procedure as well as using the GUI, a commercial software package named as Optimal Sizer is presented to optimally size the stiffened conical shells. Schematic of the software is shown in Fig. 4. Optimal Sizer has helped customers achieve weight savings of 5 to 30% and rapid analysis. Optimal Sizer finds minimum weight designs of cylindrical and conical shells stiffened by ring, stringer and ring-stringer stiffeners. The shells can be loaded by two combinations of internal pressure and axial compression. The material properties of the skin and stiffeners can be different. Upper and lower practical bounds of the design variables should be entered into the Optimal Sizer GUI. Also, users can choose the type of the shell and enter internal pressure and axial force values. After calculations, Optimal Sizer presents seven optimum design variables and WRP. It is noticeable that this software is significant due to its generality and ability to be linked accurately with finite element analysis. This will be presented in future communications.

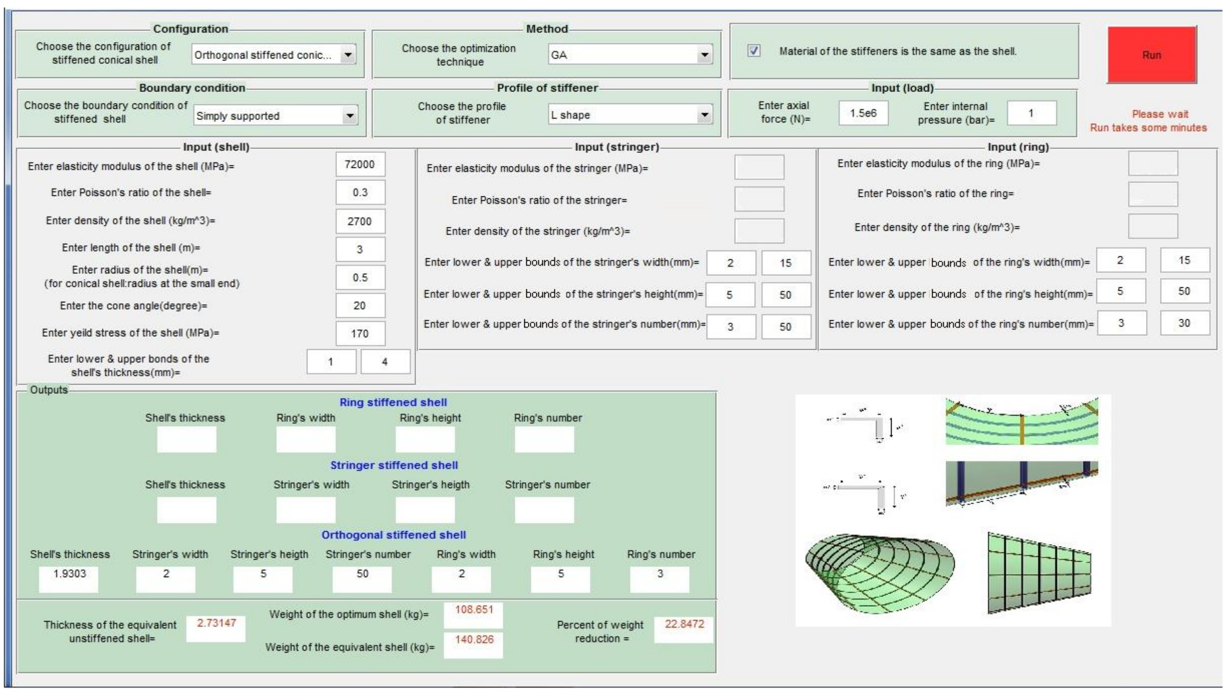

Fig.4
Schematic of the Optimal Sizer software.

6 CONCLUSIONS

The main purpose of this article is to present an optimum weight design of orthogonal stiffened conical shells under combined axial compression and internal pressure based on the genetic algorithm (GA). Twelve structural and material constraints including five stress and buckling failure modes and seven practical bounds of the design variables are considered. The energy method is employed to obtain the global buckling load and stress field in the stiffened shell. It is evident that there is an excellent agreement among the present results and the finite element analysis confirming the high accuracy of the current analytical approach for the global buckling load and stress field. A convergence study on the GA results is performed that shows a good ability of the GA to escape the local optimums and to find the global optimum rapidly. Considering the different types of stiffened conical shells, it is seen that the conical shells stiffened by ring-stringer stiffeners have larger WRP values and therefore are applicable for weight optimum designs when the shells are subjected to axial load and internal pressure. Based on the observations, it is worth noting that the skin local buckling, skin-stringer local buckling and ultimate collapse stress failure modes play a significant role in the weight optimum design of axially loaded stiffened conical shells whereas the effect of the crippling and global buckling failure modes is small. When a conical shell is subjected to high internal pressure value, the ring and stringer stiffeners are not required for corresponding optimum configuration and therefore an unstiffened (simple) conical shell has smaller structural weight than the stiffened one. According to the optimization procedure as well as using the GUI tool in MATLAB software, a user-friend graphical software package named as Optimal Sizer is proposed. The merit and convenience of the Optimal Sizer will enable every user to pursue the various steps of the solution and, therefore, he/she can easily compute the optimum configuration.

APPENDIX A

The operator matrix $\left[a_{ij}\right]\left(i,j=1,2,3\right)$ and C_{11} in Eq. (15) can be written as follows:

$$a_{11}=-A_{11}\lambda^2\left[\frac{1}{2\pi}\mathrm{L}(2\mathrm{a}S_\alpha\pi^2+\pi^2\mathrm{L})\right]-\left(A_{22}S_\alpha{}^2+A_{66}n^2\right)\int_0^{2\pi}\int_0^{L}\frac{C_x^2}{r_x{}^2}dxd\theta-E_sN_sA_s\lambda^2\int_0^{2\pi}\int_0^{L}C_x{}^2dxd\theta-\int_0^{2\pi}\left(A_rE_rS_\alpha{}^2\sum_{K=1}^{N_r}\frac{C_k{}^2}{r_{x_k}}\right) \tag{A.1}$$

where

$$\lambda=m\pi/L,S_\alpha=\sin\alpha,C_\alpha=\cos\alpha,S_x=\sin\left(\lambda x\right),C_x=\cos\left(\lambda x\right),S_k=\sin\left(\lambda x_k\right),C_k=\cos\left(\lambda x_k\right).$$
$$c_{11}=\int_0^{2\pi}\int_0^{L}r_xC_x\,dxd\theta \tag{A.2}$$

$$a_{12}=-\left[A_{22}+A_{66}\right]nS_\alpha\int_0^{2\pi}\int_0^{L}\frac{S_xC_x}{r_x}dxd\theta+\left[A_{66}+A_{12}\right]\lambda n\int_0^{L}C_x{}^2dx-2\pi A_rE_rnS_\alpha\sum_{K=1}^{N_r}\frac{S_kC_k}{r_{x_k}} \tag{A.3}$$

$$\begin{aligned}a_{13}=&-A_{22}C_\alpha S_\alpha\int_0^{2\pi}\int_0^{L}\frac{S_xC_x}{r_x}dxd\theta+2A_{12}\pi\lambda C_\alpha\int_0^{L}C_x{}^2dx+E_sN_sA_sZ_s\lambda^3\int_0^{L}C_x{}^2dx\\&-2\pi A_rE_rZ_rn^2S_\alpha\sum_{K=1}^{N_r}\frac{S_kC_k}{r_{x_k}{}^2}+2\pi A_rE_rZ_r\lambda S_\alpha{}^2\sum_{K=1}^{N_r}\frac{C_k{}^2}{r_{x_k}}-2\pi A_rE_rC_\alpha S_\alpha\sum_{K=1}^{N_r}\frac{S_kC_k}{r_{x_k}}\end{aligned} \tag{A.4}$$

$$a_{21}=-2\left[A_{22}+A_{66}\right]\pi nS_\alpha\int_0^{L}\frac{S_xC_x}{r_x}dx+2\left[A_{66}+A_{12}\right]\pi\lambda n\int_0^{L}C_x{}^2dx-2\pi A_rE_rnS_\alpha\sum_{K=1}^{N_r}\frac{S_kC_k}{r_{x_k}} \tag{A.5}$$

$$\begin{aligned}a_{22}=&-2\pi\left(A_{22}n^2+A_{66}S_\alpha{}^2+4D_{66}\lambda^2C_\alpha{}^2\right)\int_0^{L}\frac{S_x{}^2}{r_x}dx\\&-2\pi\left(D_{22}n^2C_\alpha{}^2+4D_{66}S_\alpha{}^2C_\alpha{}^2\right)\int_0^{L}\frac{S_x{}^2}{r_x{}^3}dx-A_{66}\lambda^2\int_0^{L}r_xS_x{}^2dx-2\pi A_rE_rn^2\sum_{K=1}^{N_r}\frac{S_k{}^2}{r_{x_k}}\end{aligned} \tag{A.6}$$

$$\begin{aligned}a_{23}=&-2\pi C_\alpha\left(A_{22}n+4D_{66}\lambda^2n+D_{12}\lambda^2n\right)\int_0^{L}\frac{S_x{}^2}{r_x}dx-2\pi C_\alpha\left(4D_{66}nS_\alpha{}^2+D_{22}n^3\right)\int_0^{L}\frac{S_x{}^2}{r_x{}^3}dx\\&+2\pi D_{22}\lambda nS_\alpha C_\alpha\int_0^{L}\frac{C_xS_x}{r_x{}^2}dx+2\pi A_rE_rnC_\alpha\int_0^{L}\frac{S_k{}^2}{r_x}dx+2\pi A_rE_rn^3Z_r\int_0^{L}\frac{S_k{}^2}{r_x{}^2}dx+2\pi A_rE_rZ_r\lambda nS_\alpha\sum_{K=1}^{N_r}\frac{C_kS_k}{r_{x_k}}\end{aligned} \tag{A.7}$$

$$
\begin{aligned}
a_{31} = & -A_{22}C_\alpha S_\alpha \int_0^L \frac{S_x C_x}{r_x}dx + 2A_{12}\pi\lambda C_\alpha \int_0^L S_x{}^2 dx + E_s N_s A_s Z_s \lambda^3 \int_0^L S_x{}^2 dx \\
& -2\pi A_r E_r Z_r n^2 S_\alpha \sum_{K=1}^{N_r} \frac{S_k C_k}{r_{x_k}{}^2} + 2\pi A_r E_r Z_r \lambda S_\alpha{}^2 \sum_{K=1}^{N_r} \frac{S_k{}^2}{r_{x_k}} - 2\pi A_r E_r C_\alpha S_\alpha \sum_{K=1}^{N_r} \frac{S_k C_k}{r_{x_k}}
\end{aligned} \tag{A.8}
$$

$$
\begin{aligned}
a_{32} = & -2\pi C_\alpha \left(\left(A_{22}n + 4D_{66}\lambda^2 n + D_{12}\lambda^2 n\right) \int_0^L \frac{S_x{}^2}{r_x}dx + \left(4D_{66}nS_\alpha{}^2 + D_{22}n^3\right) \int_0^L \frac{S_x{}^2}{r_x{}^3}dx \right) \\
& -2\pi D_{22}\lambda n S_\alpha C_\alpha \int_0^L \frac{C_x S_x}{r_x{}^2}dx - 2\pi A_r E_r n C_\alpha \sum_{K=1}^{N_r} \frac{S_k{}^2}{r_{x_k}} - 2\pi A_r E_r \left(n^3 Z_r \sum_{K=1}^{N_r} \frac{S_k{}^2}{r_{x_k}{}^2} + Z_r \lambda n S_\alpha \sum_{K=1}^{N_r} \frac{C_k S_k}{r_{x_k}} \right)
\end{aligned} \tag{A.9}
$$

$$
\begin{aligned}
a_{33} = & -2\pi\left(A_{22}C_\alpha{}^2 + D_{22}S_\alpha{}^2\lambda^2 + 4D_{66}\lambda^2 n^2 + 2D_{12}\lambda^2 n^2\right)\int_0^L \frac{S_x{}^2}{r_x}dx - 2\pi\left(D_{22}n^4 + 4D_{66}n^2 S_\alpha{}^2\right)\int_0^L \frac{S_x{}^2}{r_x{}^3}dx - 2\pi D_{11}\lambda^4 \int_0^L r_x S_x{}^2 dx \\
& -E_s N_s I_{os}\lambda^4 \int_0^L S_x{}^2 dx - G_s J_s N_s n^2 \lambda^2 \int_0^L \frac{S_x{}^2}{r_x{}^2}dx - 2\pi A_r E_r \left(Z_r^2 \lambda^2 S_\alpha{}^2 + C_\alpha{}^2\right) \sum_{K=1}^{N_r} \frac{S_k{}^2}{r_{x_k}} - 2\pi E_r I_{or} n^4 \sum_{K=1}^{N_r} \frac{S_k{}^2}{r_{x_k}{}^3} \\
& +4\pi A_r E_r Z_r n^2 C_\alpha \sum_{K=1}^{N_r} \frac{S_k{}^2}{r_{x_k}{}^2} + 2\pi G_r J_r \lambda^2 n^2 \sum_{K=1}^{N_r} \frac{S_k{}^2}{r_{x_k}}
\end{aligned} \tag{A.10}
$$

where $I_{os} = I_s + Z_s^2 A_s$, $I_{or} = I_r + Z_r^2 A_r$.

I_s and I_r are the moments of inertia of the stringers and rings about their centroidal axes, respectively, and J_s and J_r are the polar moments of inertia of the stringers and rings about their centroidal axes, respectively.

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