Variational Principle and Plane Wave Propagation in Thermoelastic Medium with Double Porosity Under Lord-Shulman Theory

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ABSTRACT

The present study is concerned with the variational principle and plane wave propagation in double porous thermoelastic infinite medium. Lord-Shulman theory [2] of thermoelasticity with one relaxation time has been used to investigate the problem. It is found that for two dimensional model, there exists four coupled longitudinal waves namely longitudinal wave (P), longitudinal thermal wave (T), longitudinal volume fractional wave corresponding to pores (PV1), and longitudinal volume fractional wave corresponding to fissures (PV2), in addition to, a transverse wave (S) which is not affected by the volume fraction fields and thermal properties. The different characteristics of the wave such as phase velocity and attenuation quality factor are computed numerically and depicted graphically. Some special cases are also deduced from the present investigation.

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Keywords : Double porosity; Lord-Shulman theory; Variational principle; Plane waves; Phase velocity; Attenuation quality factor.

1 INTRODUCTION

THE constitutive equations for thermoelastic material, which express the relations between the stress, the strain and the temperature change, were first introduced by Biot[1]. With Biot's theory, many solutions for thermal response caused by the change of temperature have been developed by numerous investigators. However, it involves a paradox that the thermal disturbances propagate at infinite speeds. In recent years increasing attention has been made to remove this paradox and to develop the generalized theory of thermoelasticity, which was found to give more realistic results than the coupled or uncoupled theories of thermoelasticity, especially when short time effects or step temperature gradients are considered. The theory of generalized thermoelasticity with one relaxation time was first introduced by Lord and Shulman [2], who obtained a wave-type heat equation by postulating a new law of heat conduction instead of the classical Fourier's law. Hetnarski and Ignaczak [3] have presented a review on the generalized theories of thermoelasticity. A comprehensive work has been done in the generalized theories of thermoelasticity with one relaxation time by different investigators by considering different problems Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Biot [4] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen [5],

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de Boer and Ehlers [6] in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modeled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al. [7], where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers.

The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super- saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system. Wilson and Aifanits [8] presented the theory of consolidation with the double porosity. Khaled, Beskos and Aifantis [9] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis[8]. Wilson and Aifantis [10] discussed the propagation of acoustics waves in a fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space. Beskos and Aifantis [11] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan [11] studied the unified theory of flow and deformation in double porous media. Aifantis [13-16] introduced a multi-porous system and studied the mechanics of diffusion in solids. Moutsopoulos et al. [17] obtained the numerical simulation of transport phenomena by using the double porosity/ diffusivity continuum model. Khalili and Selvadurai [18] presented a fully coupled constitutive model for thermo-hydro –mechanical analysis in elastic media with double porosity structure. Pride and Berryman [19] studied the linear dynamics of double –porosity dual-permeability materials. Straughan [20] studied the stability and uniqueness in double porous elastic media. Svanadze [21-25] investigated some problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity. Scarpetta et al. [26,27] proved the uniqueness theorems in the theory of thermoelasticity for solids with double porosity and also obtained the fundamental solutions in the theory of thermoelasticity for solids with double porosity. Kumar et al.[30-33] studied the plane wave propagation in different thermoelastic media.

In the present paper, we have derived the variational principle for thermoelastic material with double porosity structure for Lord-Shulman model. In addition to this, we have discussed the propagation of plane waves for thermoelastic material with double porosity structure with one relaxation time. Effect of porosity and relaxation time is shown graphically. Some special cases of interest are also deduced.

2   BASIC EQUATIONS

Following Lord and Shulman [2]; Iesan and Quintanilla [32], the field equations and the constitutive relations for isotropic homogeneous thermoelastic material with double porosity structure with one relaxation time can be written as:

2.1 Constitutive relations

$$t_{ij} = c_{ijrs} e_{rs} + B_{ij} \varphi + D_{ij} \psi - \beta_{ij} T \tag{1}$$

$$\sigma_i = \alpha_{ij} \varphi_{,j} + b_{ij} \psi_{,j} \tag{2}$$

$$x_i = b_{ji} \varphi_{,j} + \gamma_{ij} \psi_{,j} \tag{3}$$

$$\xi = -B_{ij} e_{ij} - \alpha_1 \varphi - \alpha_3 \psi + \gamma_1 T \tag{4}$$

$$\zeta = -D_{ij}e_{ij} - \alpha_3\varphi - \alpha_2\psi + \gamma_2 T \tag{5}$$

$$\rho T_0\dot{\eta} = -q_{i,i} \tag{6}$$

$$\rho\eta = \beta_{ij}e_{ij} + \gamma_1\varphi + \gamma_2\psi + aT \tag{7}$$

2.2 Equations of motion

$$t_{ji,j} + \rho f_i = \rho\ddot{u}_i \tag{8}$$

2.3 Equilibrated stress equations of motion

$$\sigma_{j,j} + \xi + \rho g = \kappa_1\ddot{\varphi} \tag{9}$$

$$x_{j,j} + \zeta + \rho l = \kappa_2\ddot{\psi} \tag{10}$$

2.4 Heat conduction equation

$$q_i + \tau_0\dot{q}_i = -K_{ij}T_{,j} \tag{11}$$

where λ and μ are Lame's constants, ρ is the mass density; $\beta = (3\lambda + 2\mu)\alpha_t$; α_t is the linear thermal expansion; C^* is the specific heat at constant strain, u_i is the displacement components; t_{ij} is the stress tensor; κ_1 and κ_2 are coefficients of equilibrated inertia; σ_i is the components of the equilibrated stress vector associated to pores; x_i is the components of the equilibrated stress vector associated to fissures; φ is the volume fraction field corresponding to pores and ψ is the volume fraction field corresponding to fissures; K_{ij} is the components of thermal conductivity, τ_0 is the thermal relaxation time, f_i is the body force per unit mass, κ_1 and κ_2 are coefficients of equilibrated inertia, g is the extrinsic equilibrated body force per unit mass associated to macro pores, l is the extrinsic equilibrated body force per unit mass associated to fissures, $c_{ijrs}(=c_{rsij}=c_{jirs}=c_{ijsr})$ is the tensor of elastic constants and $b,d,b_1,\gamma,\gamma_1,\gamma_2$ are constitutive coefficients; δ_{ij} is the Kronecker's delta; T is the temperature change measured form the absolute temperature $T_0(T_0 \neq 0)$; a superposed dot represents differentiation with respect to time variable t.

3 VARIATIONAL PRINCIPLE

The principal of virtual work with variation of displacements for elastic deformable body with double porosity is

$$\int_V [\rho(f_i - \ddot{u}_i)\delta u_i + (\rho g + \xi - \kappa_1\ddot{\varphi})\delta\varphi + (\rho l + \zeta - \kappa_2\ddot{\psi})\delta\psi]dV + \int_A [L\delta u_i + M\,\delta\varphi + N\,\delta\psi]dA$$
$$= \int_V [t_{ij}\delta u_{i,j} + \sigma_i\delta\varphi_{,i} + \chi_i\delta\psi_{,i}]dV \tag{12}$$

On the left hand side, we have the virtual work of body forces f_i, l, g ; inertial forces $\rho\ddot{u}_i, \kappa_1\ddot{\varphi}, \kappa_2\ddot{\psi}$; surface forces $L = t_{ji}n_j, M = \sigma_i n_i, N = x_i n_i$ whereas on the right hand side, we have the virtual work of internal forces. Here n_j is the outward unit normal of ∂V.

Using the symmetry of stress tensor and the definition of the strain tensor, the Eq. (12) can be written as:

$$\int_V \left[\rho(\mathrm{f}_i - \ddot{u}_i)\delta u_i + (\rho g + \xi - \kappa_1\ddot{\varphi})\delta\varphi + (\rho l + \zeta - \kappa_2\ddot{\psi})\delta\psi\right]dV + \int_A \left[L\delta u_i + M\,\delta\varphi + N\,\delta\psi\right]dA$$
$$= \int_V \left[t_{ij}\,\delta e_{ij} + \sigma_i\,\delta\varphi_{,i} + \chi_i\,\delta\psi_{,i}\right]dV \tag{13}$$

Substituting the value of t_{ij}, σ_i and x_i from Eqs.(1),(2)and (3) in Eq. (13), we obtain

$$\int_V \left[\rho(\mathrm{f}_i - \ddot{u}_i)\delta u_i + (\rho g + \xi - \kappa_1\ddot{\varphi})\delta\varphi + (\rho l + \zeta - \kappa_2\ddot{\psi})\delta\psi\right]dV + \int_A \left[L\delta u_i + M\,\delta\varphi + N\,\delta\psi\right]dA$$
$$= \delta W + \delta E + \delta F + \delta G + \delta H + \int_V B_{ij}\varphi\delta e_{ij}\,dV + \int_V D_{ij}\psi\delta e_{ij}\,dV - \int_V \beta_{ij}T\,\delta e_{ij}\,dV \tag{14}$$

where

$$W = \frac{1}{2}\int_V c_{ijrs}e_{ij}e_{rs}\,dV\,,\quad E = \frac{1}{2}\int_V \alpha_{ij}\varphi_{,i}^2\,dV\,,\quad F = \frac{1}{2}\int_V b_{ij}\psi_{,i}^2\,dV\,,\quad G = \frac{1}{2}\int_V b_{ij}\varphi_{,i}^2\,dV\,,\quad H = \frac{1}{2}\int_V \gamma_{ij}\psi_{,i}^2\,dV$$

We define a vector J (Biot [1]) connected with the entropy through the relation

$$\rho\eta = -J_{i,i} \tag{15}$$

On combining Eqs. (6), (7), (11) and (15), we obtain

$$T_0 L_{ij}\left(\frac{d}{dt} + \tau_0 \frac{d^2}{dt^2}\right)J_i + T_{,j} = 0 \tag{16}$$

$$-J_{i,i} = \beta_{ij}e_{ij} + \gamma_1\varphi + \gamma_2\psi + aT \tag{17}$$

where L_{ij}, the resistivity matrix, is the inverse of the thermal conductivity K_{ij}.

Multiplying both sides of Eq.(16) by δJ_i and integrating over the region of the body and using the divergence theorem with the aid of (17), we obtain

$$\int_A (T\,\delta J_i)n_j\,dA + \int_V \beta_{ij}T\,\delta e_{ij}\,dV + \gamma_1\int_V T\,\delta\varphi\,dV + \gamma_2\int_V T\,\delta\psi\,dV + \delta(P+Q) = 0 \tag{18}$$

where

$$P = \frac{a}{2}\int_V T^2\,dV\,,\qquad \delta P = a\int_V T\,\delta T\,dV\,,$$
$$Q = \frac{T_0}{2}\int_V L_{ij}\left(\frac{dJ_i}{dt} + \tau_0\frac{d^2 J_i}{dt^2}\right)J_j\,dV\,,\quad \delta Q = T_0\int_V L_{ij}\left(\frac{dJ_i}{dt} + \tau_0\frac{d^2 J_i}{dt^2}\right)\delta J_j\,dV \tag{19}$$

Substituting the value of ξ and ζ from (4) and (5) in relation (14), we obtain

$$\int_V \left[\rho(\mathrm{f}_i - \ddot{u}_i)\delta u_i + (\rho g - \kappa_1\ddot{\varphi})\delta\varphi + (\rho l - \kappa_2\ddot{\psi})\delta\psi\right]dV + \int_A \left[L\delta u_i + M\,\delta\varphi + N\,\delta\psi\right]dA$$
$$+ \int_V \beta_{ij}T\,\delta\mathrm{e}_{ij}\,dV + \gamma_1\int_V T\,\delta\varphi\,dV + \gamma_2\int_V T\,\delta\psi\,dV = \delta(\mathrm{W} + E + F + G + H + R + S + U + Y) \tag{20}$$

where

$$R = \int_V B_{ij} e_{ij} \varphi dV , \quad S = \int_V D_{ij} e_{ij} \psi dV , \quad U = \left(\frac{\alpha_1 + \alpha_3}{2}\right) \int_V \varphi^2 dV , \quad Y = \left(\frac{\alpha_3 + \alpha_2}{2}\right) \int_V \psi^2 dV \qquad (21)$$

Eliminating integrals $\int_V \beta_{ij} T \, \delta e_{ij} dV$, $\gamma_1 \int_V T \, \delta \varphi dV$ and $\gamma_2 \int_V T \, \delta \psi dV$ from Eqs.(18) and (20), we obtain the variational principle in the following form:

$$\delta(W + E + F + G + H + P + Q + R + S + U + Y) = \int_V \rho(f_i - \ddot{u}_i) \delta u_i dV + \int_V (\rho g - \kappa_1 \ddot{\varphi}) \delta \varphi dV$$
$$+ \int_V (\rho l - \kappa_2 \ddot{\psi}) \delta \psi dV + \int_A L \delta u_i dA + \int_A M \, \delta \varphi dA + \int_A N \, \delta \psi dA - \int_A T \, \delta J_{,i} n_i dA \qquad (22)$$

On the right-hand side of Eq. (22), we find the all the causes, the mass forces, inertial forces, the surface forces, the heating potential and equilibrated stress vectors on the surface A bounding the body.

4   PLANE WAVE PROPAGATION

We obtain equation of motion, equilibrated stress equations of motion and heat conduction equation, by making use of Eqs.(1)-(7) in Eqs.(8)-(11), without body forces, extrinsic equilibrated body forces and heat source as:

$$\mu \nabla^2 \vec{u} + (\lambda + \mu) \nabla \nabla \cdot \vec{u} + b \nabla \varphi + d \nabla \psi - \beta \nabla T = \rho \frac{\partial^2 u}{\partial t^2}, \qquad (23)$$

$$\alpha \nabla^2 \varphi + b_1 \nabla^2 \psi - b \nabla \cdot \vec{u} - \alpha_1 \varphi - \alpha_3 \psi + \gamma_1 T = \kappa_1 \frac{\partial^2 \varphi}{\partial t^2}, \qquad (24)$$

$$b_1 \nabla^2 \varphi + \gamma \nabla^2 \psi - d \nabla \cdot \vec{u} - \alpha_3 \varphi - \alpha_2 \psi + \gamma_2 T = \kappa_2 \frac{\partial^2 \psi}{\partial t^2}, \qquad (25)$$

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\left(\beta T_0 \nabla \dot{\vec{u}} + \gamma_1 T_0 \dot{\varphi} + \gamma_2 T_0 \dot{\psi} + \rho C^* \dot{T}\right) = K^* \nabla^2 T \qquad (26)$$

where K^* is the coefficient of thermal conductivity ; a superposed dot represents differentiation with respect to time variable t.

$$\nabla = \hat{i} \frac{\partial}{\partial x_1} + \hat{j} \frac{\partial}{\partial x_2} + \hat{k} \frac{\partial}{\partial x_3}, \qquad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

are the gradient and Laplacian operators, respectively.

For the two-dimensional problem, we take $u = (u_1, 0, u_3)$ and define the following non-dimensional quantities as:

$$x_1' = \frac{\omega_1}{c_1} x_1, \; x_3' = \frac{\omega_1}{c_1} x_3, u_1' = \frac{\omega_1}{c_1} u_1 \; , u_3' = \frac{\omega_1}{c_1} u_3, T' = \frac{T}{T_0}, \tau_0' = \omega_1 \tau_0,$$
$$\varphi' = \frac{\kappa_1 \omega_1^2}{\alpha_1} \varphi, \; \psi' = \frac{\kappa_1 \omega_1^2}{\alpha_1} \psi, t' = \omega_1 t, \sigma_1' = \left(\frac{c_1}{\alpha \omega_1}\right) \sigma_1, \tau_1' = \left(\frac{c_1}{\alpha \omega_1}\right) \tau_1 \qquad (27)$$

where $c_1^2 = \dfrac{\lambda + 2\mu}{\rho}, \omega_1 = \dfrac{\rho C^* c_1^2}{K^*}$. Here ω_1 and c_1 are the constants having the dimension of frequency and velocity in the medium respectively.

The displacement components u_1 and u_3 are related by potential functions φ_1 and ψ_1 as:

$$u_1 = \frac{\partial \varphi_1}{\partial x_1} - \frac{\partial \psi_1}{\partial x_3}, \qquad u_3 = \frac{\partial \varphi_1}{\partial x_3} + \frac{\partial \psi_1}{\partial x_1} \tag{28}$$

For plane harmonic wave, we assume the wave solution as:

$$\left(\varphi_1, \varphi, \psi, T, \psi_1\right) = \left(\bar{\varphi}_1, \bar{\varphi}, \bar{\psi}, \bar{T}, \bar{\psi}_1\right) \exp\left[i\left\{\xi(l_1 x_1 + l_3 x_3) - \omega t\right\}\right] \tag{29}$$

where ω is the angular frequency and ξ is the complex wave number. $\bar{\varphi}_1, \bar{\varphi}, \bar{\psi}, \bar{T}$ and $\bar{\psi}_1$ are undetermined amplitude vectors that are independent of time t and coordinates $x_1, x_3; l_1^2 + l_3^2 = 1$. Making use of (29) in (23)-(26) with the aid of (27) and (28), we obtain a linear system of four homogeneous equations in four unknowns $\bar{\varphi}_1, \bar{\varphi}, \bar{\psi}$ and $\bar{T}$. For non-trivial solution, the determinant of the coefficients $\left[\bar{\varphi}_1, \bar{\varphi}, \bar{\psi}, \bar{T}\right]^{tr}$ vanishes, yields to the characteristic polynomial equation in c as:

$$B_1 c^8 + B_2 c^6 + B_3 c^4 + B_4 c^2 + B_5 = 0 \tag{30}$$

The cofficients B_1, B_2, B_3, B_4 and B_5 are given in the Appendix B.

$$\left(\omega^2 - a_{19} \xi^2\right) \bar{\psi}_1 = 0 \tag{31}$$

Eq.(31) is uncoupled equation in terms of $\bar{\psi}_1$. The complex coefficients in (30) imply that four roots of the equation may be complex. The complex phase velocities of the longitudinal waves, given by c_i, $i = 1, 2, 3, 4$, will be varying with the direction of phase propagation. The complex velocity of the longitudinal waves, i.e. $c = c_R + i c_I$, defines the phase propagation velocity $V_i = \dfrac{c_R^2 + c_I^2}{c_R}$, and attenuation quality factor $Q_i^{-1} = \dfrac{\text{Img}\left(1/c_i^2\right)}{\text{Re}\left(1/c_i^2\right)}$ for the corresponding waves. Therefore the four waves in such a medium are attenuating. Corresponding to these roots, there exist four waves in descending order of their velocities, namely longitudinal wave (*P*), longitudinal thermal wave (*T*), longitudinal volume fractional wave (PV1) corresponding to pores and longitudinal volume fractional wave corresponding to fissures (PV2).

5   PARTICULAR CASES

Case (i) If $b_1 = \alpha_3 = \gamma = \alpha_2 = \gamma_2 = d \rightarrow 0$ in Eqs.(23)-(26), we obtain the corresponding expressions for thermoelastic medium with single porosity.

Case (i) If $\tau_0 = 0$, in Eqs. (23)-(26), yield the corresponding expressions for thermoelastic medium with double porosity in context of coupled theory of thermoelasticity.

6   NUMERICAL RESULTS AND DISCUSSION

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief and Saleh [33] as,

$$\lambda = 7.76 \times 10^{10} \, Nm^{-2}, \, C^* = 3.831 \times 10^3 \, m^2 s^{-2} K^{-1}, \mu = 3.86 \times 10^{10} \, Nm^{-2}, \, K^* = 3.86 \times 10^3 \, Ns^{-1} K^{-1},$$

$$\omega_1 = 1 \times 10^{11} s^{-1}, T_0 = 0.293 \times 10^3 \, K, \alpha_t = 1.78 \times 10^{-5} K^{-1}, t = 0.1s, \rho = 8.954 \times 10^3 \, Kgm^{-3}$$

Following Khalili [34], the double porous parameters are taken as,

$$\alpha_2 = 2.4 \times 10^{10} \, Nm^{-2}, \alpha_3 = 2.5 \times 10^{10} \, Nm^{-2}, \gamma = 1.1 \times 10^{-5} \, N, \alpha = 1.3 \times 10^{-5} \, N$$

$$\gamma_1 = 0.16 \times 10^5 \, Nm^{-2}, b_1 = 0.12 \times 10^{-5} \, N, d = 0.1 \times 10^{10} \, Nm^{-2}$$

$$\gamma_2 = 0.219 \times 10^5 \, Nm^{-2}, \kappa_1 = 0.1456 \times 10^{-12} \, Nm^{-2} s^2, b = 0.9 \times 10^{10} \, Nm^{-2}$$

$$\alpha_1 = 2.3 \times 10^{10} \, Nm^{-2}, \, \kappa_2 = 0.1546 \times 10^{-12} \, Nm^{-2} s^2, \tau_0 = 0.1s$$

The software MATLAB has been used to find the values of phase velocity and attenuation quality factor. The variations of these values with respect to angular frequency ω have been shown in Figs. (1)-(8) respectively. In all these figures, solid line and small dashes line without central symbols correspond to thermal double porous material (DP) for LS and CT theory respectively while solid line and small dashes line with central symbols correspond to thermal single porous material (SP) for LS and CT theory respectively.

Fig. 1 shows that values of phase velocity converges to boundary surface for all the values of angular frequency ω for DP in case of LS theory while in case of CT theory it increase slightly with the increase in angular frequency ω. For SP, it is found that, the values of phase velocity increases monotonically with the increase in angular frequency ω for both LS and CT theories of thermoelasticity. It is noticed that magnitude values are more for CT in comparison to LS theory for DP while a reverses behavior is noticed in case of SP.

From Fig. 2, it is clear that values of phase velocity increase monotonically with the increase in the value of angular frequency ω for both DP and SP model. It is noticed that the values of phase velocity are very close for the range $0 < \omega < 1.1$ for both models while for the remaining range the values of phase velocity are higher for LS as compared to CT theory of thermoelasticity in case of DP and it shows an opposite trend of variation, for SP.

Fig. 3 depicts that there is a similar trend of variation for both the models. It is found that the values of phase velocity decreases for $0 < \omega < 2$ and becomes almost stationary for $\omega \geq 2$. The difference in the magnitude values of phase velocity is very small for both the models.

Fig. 4 indicates that the trend and behavior of variation are similar for both LS and CT theories of thermoelasticity incase of DP, but the magnitude values are more in case of LS as compared to CT theory.

From Fig. 5, it is found that for DP, the values of attenuation quality factor decrease with the increase in angular frequency ω while for SP, it is found that, the values increase slowly as angular frequency ω increase. The magnitude values are more for CT in comparison to LS theory for SP, while the difference in magnitude values are small for $0 < \omega < 6.8$ and further the difference in magnitude values increases for $\omega > 6.8$ in case of DP.
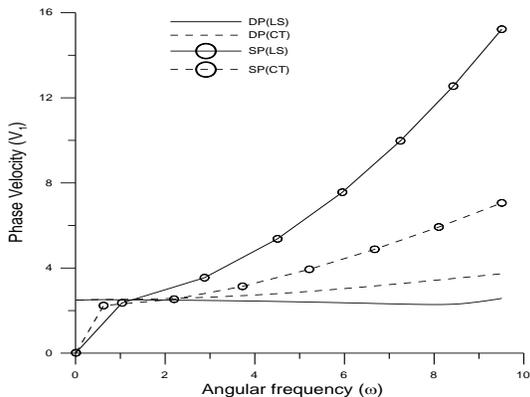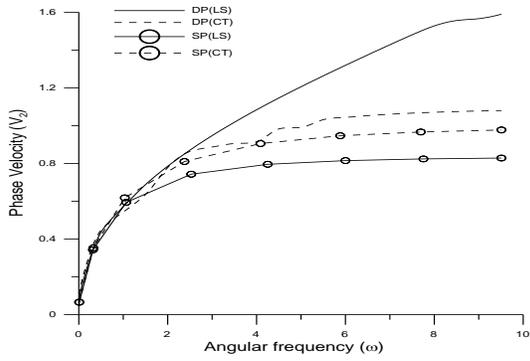

Fig.1
Variation of phase velocity w.r.t. angular frequency (P-wave).

Fig.2
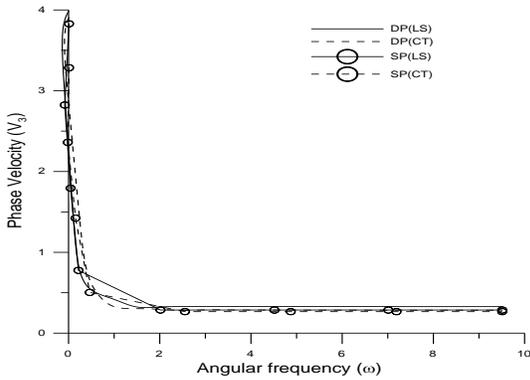Variation of phase velocity w.r.t. angular frequency (T-wave).

Fig.3
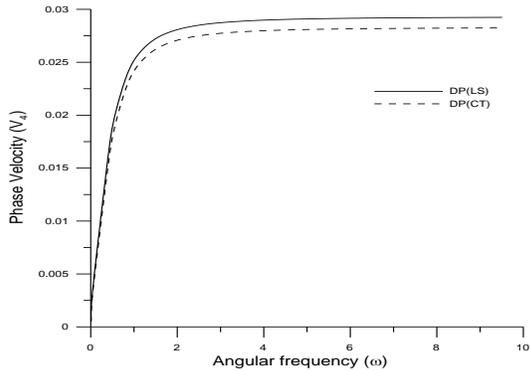Variation of phase velocity w.r.t. angular frequency (PV1-wave).

Fig.4
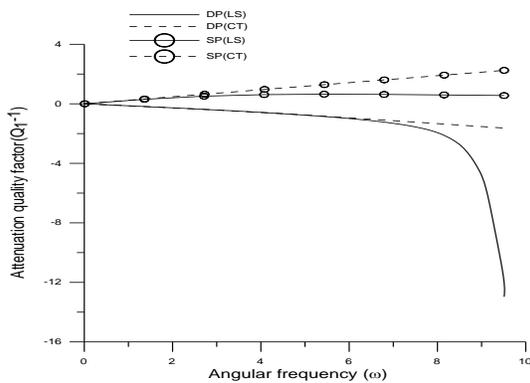Variation of phase velocity w.r.t. angular frequency (PV2-wave).

Fig.5
Variation of attenuation quality factor w.r.t. angular frequency (P-wave).

Fig. 6 depicts that for LS theory, the value of attenuation quality factor increases sharply for $0 < \omega < 1.0$, decreases sharply for $1.0 < \omega < 1.7$, then increases with small magnitude value for $1.7 < \omega < 2.2$ and converges to boundary surface for $\omega > 2.2$, while for CT theory, similar trend of variation is noticed but the magnitude values

are smaller for CT in comparison to LS theory . For SP, the value of attenuation quality factor decreases for $0 < \omega < 1.1$ and then converges to boundary surface for $\omega > 1.1$ for both the theories.

Fig.7 indicates that for DP, the value of attenuation quality factor decreases sharply for $0 < \omega < 0.5$, increases sharply for $0.5 < \omega < 1.7$ and then converges to boundary surface for the remaining range. For SP, the trend of variation is similar for both the theories and also the magnitude values are almost same for both the theories of thermoelasticity.

From Fig.8, it is clear that the values of attenuation quality factor decrease for $0 < \omega < 1.0$ and then again increase as the value of angular frequency increases in case of DP for both the theories of thermoelasticity.
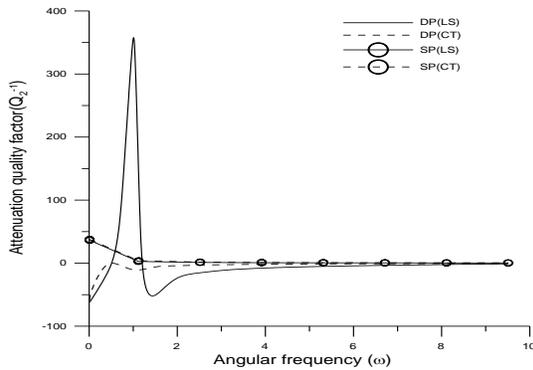
Fig.6
Variation of attenuation quality factor w.r.t. angular frequency (T-wave).
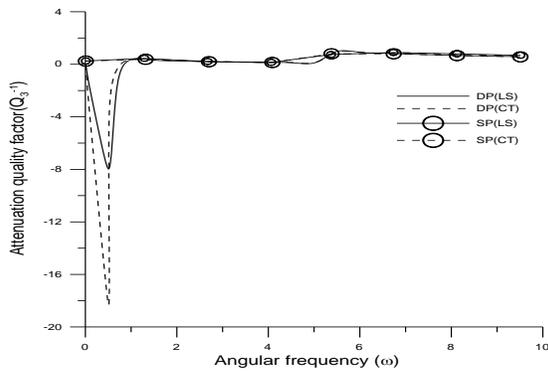
Fig.7
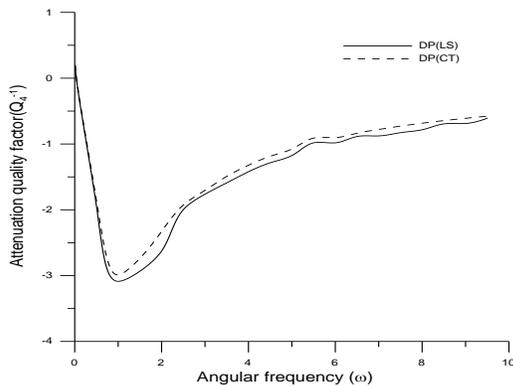Variation of attenuation quality factor w.r.t angular frequency (PV1-wave).

Fig.8
Variation of attenuation quality factor w.r.t. angular frequency (PV2-wave).

6   CONCLUSIONS

In this paper, we have derived the Variational principle and studied the plane wave propagation for thermoelastic medium with double porosity under Lord-Shulman theory. The results concluded from the above analysis can be summarized as:

i) Variational principle is formulated for thermoelastic material with double porosity for Lord-Shulman theory. The established results of the paper will be useful for numerical computation based on variational principle and provide theoretical basis for modern numerical techniques such as finite element and boundary element methods.

ii) It is found that there exist a set of four coupled longitudinal waves in thermoelastic medium with double porosity with one relaxation time. The phase velocities and attenuation quality factors of these plane waves are computed and presented graphically with respect to angular frequency. Appreciable effects of porosity and thermal relaxation time is observed on each set of the longitudinal waves.

iii) There also exists one transverse wave, which is not effected by volume fraction fields and thermal properties of the body, gets decoupled from the rest of motion.

The problem though theoretical, but it can provide useful information for experimental researchers working in the field of geophysics, earthquake engineering, along with seismologist working in the field of mining tremors and drilling into the crust of the earth.

APPENDIX A

$$a_1 = \frac{b\,\alpha_1}{\rho c_1^2 \kappa_1^2 \omega_1^2}, a_2 = \frac{d\,\alpha_1}{\rho c_1^2 \kappa_1^2 \omega_1^2}, a_3 = \frac{\beta T_0}{\rho c_1^2}, a_4 = \frac{\alpha}{\kappa_1 c_1^2}, a_5 = \frac{b_1}{\kappa_1 c_1^2}, a_6 = \frac{b}{\alpha_1}, a_7 = \frac{\alpha_1}{\kappa_1 \omega_1^2}, a_8 = \frac{\alpha_3}{\kappa_1 \omega_1^2}, a_9 = \frac{\gamma_1 T_0}{\alpha_1},$$

$$a_{10} = \frac{b_1}{\kappa_2 c_1^2}, a_{11} = \frac{\gamma}{\kappa_2 c_1^2}, a_{12} = \frac{d\,\kappa_1}{\kappa_2 \alpha_1}, a_{13} = \frac{\alpha_3}{\kappa_2 \omega_1^2}, a_{14} = \frac{\alpha_2}{\kappa_2 \omega_1^2}, a_{15} = \frac{\gamma_2 T_0 \kappa_1}{\alpha_1 \kappa_2}, a_{16} = \frac{\beta c_1^2}{K^* \omega_1}, a_{17} = \frac{\gamma_1 c_1^2}{\kappa_1 K^* \omega_1^3},$$

$$a_{18} = \frac{\gamma_2 c_1^2}{\kappa_1 K^* \omega_1^3}, a_{19} = \frac{\mu}{\rho c_1^2}, a_{20} = (1 - i\,\omega\tau_0) i\,\omega a_{16}, a_{21} = -i\,\omega(1 - i\,\omega\tau_0) a_{17}, a_{22} = -i\,\omega(1 - i\,\omega\tau_0) a_{18}, a_{23} = \omega^2 (1 - i\,\omega\tau_0)$$

$$g_1 = \omega^2 - a_7, g_2 = \omega^2 - a_{14}, e_1 = (a_3 a_5 + a_4 g_2 + a_8 a_{10}), e_2 = (e_1 + g_1 a_{11}), e_3 = (a_8 a_{21} + g_1 a_{22}), e_4 = (a_4 a_{11} - a_5 a_{10})$$

APPENDIX B

$$B_1 = \omega^2 \left[g_2 (g_1 a_{23} - a_9 a_{21}) - a_3 (a_8 a_{23} + a_9 a_{22}) - a_{15} e_3 \right]$$

$$B_2 = \omega^2 \begin{bmatrix} \omega^2 (a_3 a_8 - g_1 g_2) - \omega^2 a_{23} e_2 + \omega^2 a_{15} (a_4 a_{22} - a_5 a_{21}) \\ + \omega^2 a_9 (a_{11} a_{21} - a_{10} a_{22}) + (a_2 a_3 + a_1 g_2)(a_9 a_{20} - a_6 a_{23}) + a_3 g_2 (g_1 a_{20} - a_6 a_{21}) \\ + (a_1 a_{15} - a_3^2)(a_6 a_{22} + a_8 a_{20}) + (a_3 - a_1 a_{12})(a_8 a_{23} + a_9 a_{22}) + a_2 a_{21}(a_9 a_{12} - a_6 a_{15}) \\ + a_2 g_1 (a_{15} a_{20} - a_{12} a_{23}) + (a_{15} - a_3 a_{12}) e_3 + g_2 (a_9 a_{21} - g_1 a_{23}) \end{bmatrix}$$

$$B_3 = \omega^4 \begin{bmatrix} \omega^2 e_1 + \omega^2 a_{23}(a_4 a_{11} - a_5 a_{10}) + a_{15}(a_5 a_{21} - a_4 a_{22}) \\ + a_{11}(g_1 a_{23} - a_9 a_{21}) + a_{10}(a_8 a_{23} + a_9 a_{22}) - a_3 a_{20}(a_8 a_{10} + a_4 g_2) \\ + a_3 a_6 (a_{11} a_{21} - a_{10} a_{22}) + g_2 (a_4 a_{23} + a_1 a_6) + a_2 (a_3 a_6 + g_1 a_{12}) \\ + a_3 a_5 (a_{23} - a_3 a_{20}) + a_{12}(a_1 a_8 + a_2 g_1) + a_6 (a_2 a_3 + a_1 g_2) + (g_1 g_2 - a_3 a_8) \end{bmatrix}$$

$$B_4 = \omega^6 \left[(a_3 a_{20} - a_{23} - \omega^2) e_4 + a_1 (a_5 a_{12} - a_6 a_{11}) + a_2 (a_6 a_{10} - a_4 a_{12}) - e_2 \right]$$

$$B_5 = \omega^8 e_4$$

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