

# Transient Nonlinear Vibration of Randomly Excited Cylindrical Shallow Panels in Non Aging Viscous Medium

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## ABSTRACT

In this paper, the nonlinear transient vibration of a cylindrical shallow panel under lateral white noise excitation is studied. The panel is in contact with a non aging viscoelastic medium. Since the external load is a time varying random wide band process, deterministic and conventional approaches cannot be used. Instead, the evolution of the probability density function of the response is investigated. To compute the density function, the famous Monte Carlo simulation is employed while its correctness for this specific application is validated with another work in literature. The governing equation is rewritten to a non dimensional format; so that the results can be applied to a wide range of panels. Specifically, the transient behavior is investigated with respect to the quasi slenderness ratio and the non dimensional mean value of lateral load. As expected, both the simple damped oscillation and unstable jumping phenomenon are seen relative to the values of prescribed parameters. Finally, the joint probability density function of the response is drawn that give someone an idea about the quality of the response in the phase plane.

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**Keywords :** Circular panels; Shallow shells, Monte carlo simulation; Random vibration; Non aging materials.

## 1 INTRODUCTION

THE cylindrical shallow panel is one of the most basic ingredients in mechanical, aero-space and civil structures. The fuselages of some these engineering applications such as coastal buildings and smart structures are in contact with viscous media such as soil, clay, resin, mud, rubbers and recently smart isolating materials. These materials show viscoelastic response due to even constant loads. Of course, many interaction loads applied to these prescribed structures are inherently changing in time and space randomly. Fluid solid interaction loads, seismic and transport loads are some examples of these essential random sources of forces. In many of the cases mentioned above, it is essential to consider both the transient and steady state accurate vibration analysis of panels and shells to investigate the dynamic response, stability and performance of the whole structure.

Linear and nonlinear vibrations of thin walled structures such as shells, panels and plates have been vastly studied in literature for two past decades. A complete study on the geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction was done by Amabili et al. [1]. Par'doussis [2] worked on the stability of the fluid filled shells. In this area and more recently, an investigation was done by Kubenko and Koval'chuk [3]. A review on the nonlinear investigation of plates, curved panels and shells in

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contact with aerodynamic forces was done by Mei et al. [4]. In above references, all the studies are analytic or numeric conventional deterministic investigations.

In the case of transient response analysis of thin walled structures, several works can be also seen in the literature. For example, using the modal expansion technique, Abrade [5] studied the transient response of the beams, plates and shells to certain impulsive loads. More recently, Bodaghi and Shakeri [6] studied the transient response of functionally graded piezoelectric cylindrical panels subjected to impulsive loads. Şenyer and Kazancı [7] investigated the dynamic analysis of a laminated hybrid composite plate subjected to time-dependent external pulses. Many of the above articles and the others in the literature, used impulsive loads to investigate the transient vibration of such structures.

The method of solution for stochastic systems is completely different with the ones usually used in deterministic ones. Generally, the probability density of a variable is studied rather than the variable itself [8]. This density gives somebody an idea about the possible presence of a random variable in the working space. Monte Carlo simulation [9] as one of the famous numeric methods and the Fokker Planck Kolmogorov equation [10] as one of the powerful analytic approaches have been used to obtain the transient and stationary probability density functions. As the semi analytic methods, the evolution of statistical moments [11] and the averaging method [12] are other methodologies to study the behavior of such systems. The sources of random force may vary from a wide band stationary process with several excitation frequencies to a narrow band one with limited specific frequencies [11, 13]. The system under study may be also linear or nonlinear; conservative or non conservative [13].

It is noted that the most experimental studies of the white noise excitation have been seen in the behavior investigation of structures against seismic loads such as earthquakes. This is because, in a real earthquake, there is a wide range of excitation frequencies. Of course, in practice, these experiments are done with a wide specific range of frequencies. For example, in [14] the range of 0.5 to 50 Hz was selected while in [15] the range was assumed to be 1 to 100 Hz.

Other parameters such as the nonlinearity that may come from the large deformations or the material properties complicate the problem [16]. Although, the developments in numerical simulations in recent decades simplify the process of modeling of precise and applicable systems, the nonlinear analysis of randomly excited shallow shells and circular panels in viscous medium has been less considered in the literature. As a recent work, Asnafi [17] used a semi static approach to obtain the stability conditions of cylindrical shallow shells in non aging viscous medium. In this paper, it is tried to evaluate the transient probability density function of the response for a randomly excited cylindrical shallow panel. Once the probability density function is obtained, other statistical properties of response can be achieved. With reference to this density, the stable damped behavior and unstable jumping behavior are identified. Despite many articles, here, the transient vibration is investigated for a general white noise excitation with a wide range of exciting frequencies rather than a deterministic impulsive load. A study on the joint probability density function is also made somebody see the posture of the transient behavior in the state space.

## 2 THEORY OF THE PROBLEM

### 2.1 The governing equation of a cylindrical shallow panel in a non-aging viscous medium

The typical diagram of a simply supported cylindrical shallow panel in a non aging viscoelastic medium is drawn in Fig. 1. The panel is elongated along the  $y$  direction while the lateral load  $q$ , is uniformly distributed over the panel surface. Due to symmetry and above assumptions, the deflection of the panel is a function of  $x$  only. We borrowed the governing equation of vibration along the  $z$  axis from [18] and rewrote it by assuming general viscous medium as:

$$\rho h \frac{\partial^2 W}{\partial t^2} + D \frac{\partial^4 W}{\partial x^4} - N \frac{\partial^2 W}{\partial x^2} - \frac{N}{r} + cO\{W\} = q \quad (1)$$

where  $\rho$  and  $h$  are the density and thickness of the panel respectively,  $r$  is the radius of curvature for the middle plane of the panel,  $N$  is the normal in-plane force per unit length of the panel along  $y$  direction,  $D$  is the panel cylindrical stiffness,  $c$  is the bed constant and  $O(\ )$  is the operator that demonstrates the viscoelastic behavior of the surroundings. In other words, the operator explains the constitutive equation between the force and deformation in the assumed medium. In a classic linear viscoelastic medium, the operator gives the resultant force in terms of the deformation and its rate while in other media it may take more complicated form.

For constant in-plane force  $N$  and due to the simply supported constraints, one can compute the in-plane force as [18]:

$$N = \frac{Eh}{(1-\nu^2)l} \left( \frac{1}{2} \int_0^l \left( \frac{dW}{dx} \right)^2 dx - \frac{1}{r} \int_0^l W dx \right) \quad (2)$$

where  $\nu$  and  $E$  are the poisson's ratio and the modulus of elasticity respectively.

To Solve Eq. (1), the relation of the in-plane force  $N$ , must be first identified. This is because, this term relates to the function of the deflection  $W$  itself. Here, the Galerkin's method is employed to solve this coupled nonlinear partial differential equation. The method is among the best to prepare a weak formula for the partial differential equation of the continuous vibratory system [19]. Based on the prescribed method, the deflection  $W$  can be written by a series of normal modes obtained from the free vibration analysis of the system. The first and of course the most effective normal mode is selected and picked up; thus, the deflection  $W$  is approximated by:

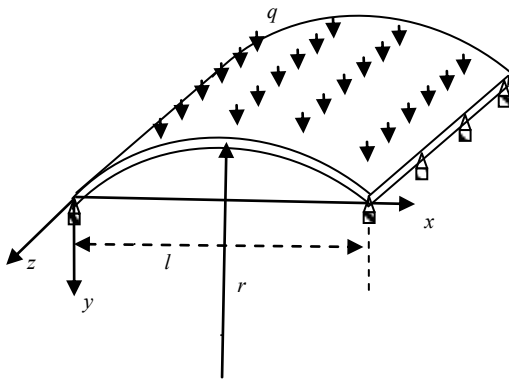
$$W(x,t) = \tilde{w}(t) \sin \frac{\pi x}{l} \quad (3)$$

with reference to Eq. (3), the approximated formula also satisfies the boundary conditions. Substitute (3) in (2) results in:

$$N = \frac{Eh\pi^2}{4(1-\nu^2)l^2} \left( \tilde{w}^2 - \frac{8l^2}{\pi^3 r} \tilde{w} \right) \quad (4)$$

By substituting the obtained in-plane force  $N$  (Eq. (4)) and the suggested formula for the deflection  $W$  (Eq. (3)) into Eq. (1) and then integration over the panel geometrical domain with emphasis on the properties of orthogonal functions; one can reach such the following governing equation:

$$\ddot{\tilde{w}} + \frac{\pi^4 D}{l^4 \rho h} \tilde{w} + \frac{3\pi^4 D}{l^4 \rho h} \left( \tilde{w}^3 - \frac{12l^2}{\pi^3 r} \tilde{w}^2 + \frac{32l^4}{\pi^6 r^2} \tilde{w} \right) + \frac{c}{\rho h} O\{\tilde{w}\} = \frac{4}{\pi \rho h} q \quad (5)$$



**Fig.1**  
The typical geometry of the cylindrical shallow panel.

## 2.2 Relaxation kernel and the boltzmann superposition principle

Several models including the famous Kelvin-Voigt, Maxwell, Buggers etc. have been presented in the literature for viscoelastic media [20, 21]. In past two decades, some researchers have tried to introduce a general relation between the stress and strain in viscoelastic materials using the Boltzmann superposition principle [22, 23].

Based on the Boltzmann superposition principle, for initially free stress rectangular bar under the uniaxial loading, the stress at any moment of loading time is a function of the history of produced strain [22, 23], i.e.

$$\sigma(t) = \int_0^t k^*(t, t^*) d\varepsilon(t^*) \quad (6)$$

where  $k^*(t, \tau)$  is an integrable function of  $t^*$  in any fixed value of time  $t$ . For smooth enough functions of stress and strain, one can use the integration by parts technique to rewrite the Rel. 6 as:

$$\sigma(t) = k^*(t, t)\varepsilon(t) - \int_0^t \frac{\partial k^*}{\partial \tau}(t, t^*)\varepsilon(t^*) dt^* \quad (7)$$

A general form for  $k^*(t, t^*)$  is defined by using the current Young's modulus  $E(t^*)$  and the relaxation measure  $\mathcal{G}(t, t^*)$  as [22, 23]:

$$k^*(t, t^*) = E(t^*) + \mathcal{G}(t, t^*) \quad (8)$$

where

$$\begin{aligned} E(t^*) &= k^*(t^*, t^*) \\ \mathcal{G}(t, \tau) &= k^*(t, t^*) - k^*(t^*, t^*) \end{aligned} \quad (9)$$

Also the relaxation kernel is defined as [22, 23]:

$$R^*(t, t^*) = \frac{1}{E(t)} \frac{\partial k^*(t, t^*)}{\partial t^*} \quad (10)$$

Using Eqs. (6-10), one can reach such the following general constitutive equation for a linear viscoelastic material:

$$\sigma(t) = E(t) \left[ \varepsilon(t) - \int_0^t R^*(t, t^*) \varepsilon(t^*) dt^* \right] \quad (11)$$

The relaxation kernel which is a function of both  $t$  and  $t^*$ , is a property of aging materials. Aging is understood as a time dependent variation of mechanical properties, which are not caused by stresses [24]. Temperature effects in general or time-deteriorating effects, damage caused by melting and solidification of concrete, are some practical examples of aging. In contrast, plasticity or cracking which are stress dependent effects cannot be included in this definition. A good relaxation (or inverse of creep) function can describe the behavior of the aging materials. For some materials like polymers, it depends on the load duration time ( $t-t^*$ ) rather than both ( $t, t^*$ ). For these materials, the relaxation kernel is a function of ( $t-t^*$ ) which means that the material properties do no influenced from the reference time of experiment. Therefore, for such materials we have:

$$\sigma(t) = E(t) \left[ \varepsilon(t) - \int_0^t \tilde{R}(t-t^*) \varepsilon(t^*) dt^* \right] \quad (12)$$

Eq. (12) is the important general constitutive equation for linear viscoelastic non-aging materials when the relaxation kernel is given. Other models of linear viscoelasticity such as stress relaxation, strain creep, hysteresis and famous models can be extracted from this general relation [23]. Developments of semi-linear and nonlinear viscoelastic models have been also done using this general model [23].

### 2.3 Non dimensional nonlinear integro-differential oscillator

Now, we must rewrite Eq. (5) using the relaxation kernel introduced in Eq. (12). Based on Eq. (12), the operator  $O(\bullet)$  becomes:

$$\begin{aligned} O\{\bullet\} &= (1 - \tilde{R})(\bullet) \\ \tilde{R}(\bullet) &= \int_0^t R(t-t^*) \times \bullet(\tau) d\tau \end{aligned} \quad (13)$$

Inserting Eq. (13) into Eq. (5) and some simple calculations, one can reach such the following non-dimensional integro-differential nonlinear oscillator that demonstrates the behavior of the panel transverse vibration:

$$w'' + \left(1 + \frac{96s^2}{\pi^6}\right)w - \frac{36s}{\pi^3}w^2 + 3w^3 + \beta \left(w - \int_0^{t_1} R(t-t^*)w(t^*)dt^*\right) = F \quad (14)$$

where

$$(\bullet)' = \frac{\partial}{\partial \tau}, \quad \tau = \omega t, \quad \omega = \sqrt{\frac{\pi^4 D}{l^4 \rho h}}, \quad w = \frac{\tilde{w}}{h}, \quad s = \frac{1}{\sqrt{rh}}, \quad \beta = \frac{c}{\rho h \omega^2}, \quad F = \frac{4}{\pi \rho h^2 \omega^2} q \quad (15)$$

In these above two equations,  $\omega$  is the appropriate frequency of the panel oscillation.  $\tau$ ,  $w$ ,  $F$  and  $\beta$  are the non-dimensional time of oscillation, the non-dimensional deflection, the non-dimensional transverse load and the non-dimensional bed coefficient respectively.  $s$  is also introduced as the quasi slenderness ratio. In this research, the non-dimensional lateral load  $F$  is assumed to be a random white noise excitation with a form of Eq. (16). It demonstrates a unit white noise excitation  $\xi(t)$ , multiplied by the intensity of  $k$  and varied about a mean value  $F_0$  i.e.

$$F = F_0 + k \xi(t) \quad (16)$$

### 2.4 The state space formulation

In what follows and in order to better result investigation, a state space formulation is made. To construct any type of simulation and numerical solution; first, the relaxation operator  $R$  in Eq. (14) should be specified. With reference to many works in the literature (see for example [18, 22, 23], the exponential form is selected for the relaxation operator, i.e.

$$R = x \zeta e^{-x(t-t^*)} \quad (17)$$

Now, the state space variables are defined as:

$$x_1 = w \quad x_2 = w' \quad x_3 = \int_0^{t_1} x \zeta e^{-x(t-t^*)} w(t^*) dt^* \quad (18)$$

and finally the state space formulation becomes:

$$x_1' = x_2 \quad x_2' = -\left(1 + \frac{96s^2}{\pi^6}\right)x_1 + \frac{36s}{\pi^3}x_1^2 - 3x_1^3 - \beta(x_1 - x_3) + F_0 + k \xi(t) \quad x_3' = x \zeta x_1 - x_3 \quad (19)$$

### 3 METHOD OF SOLUTION

#### 3.1 Probability and joint probability density functions

Since all the statistical parameters of a random variable can be achieved from the probability density function [11, 25], it plays an essential role in studying the behavior of randomly excited systems. Simply if  $X$  be a continuous random variable, the corresponding probability density function of  $X$  is a function  $f(X)$  such that for any two numbers  $a$  and  $b$  with  $a \leq b$  [26],

$$P(a \leq X \leq b) = \int_a^b f(X) dx \quad (20)$$

In other words, it is the probability that  $X$  takes on a value in the interval  $[a, b]$ . The graph of  $f(X)$  is often referred to as the density curve. Now, the expected value and other statistical moments can be obtained using this probability density function as [26]:

$$E\{X\} = \int_a^b Xf(X) dx \quad \text{meanvalue} \quad (21)$$

$$E\{X^n\} = \int_a^b X^n f(X) dx \quad n^{\text{th}} \text{ order moment}$$

The probability density function for any general random variable  $X$ , must satisfy the following conditions [26]:

$$P \geq 0$$

$$\int_{-\infty}^{+\infty} P(X) dX = 1 \quad (22)$$

$$\frac{\partial P}{\partial X} = 0, P(X) \rightarrow 0 \quad \text{when } X \rightarrow \infty$$

The local peaks of the probability density play the essential role in studying the behavior of the panel. They demonstrate the number of presence of a random variable in a specific location for an assumed interval of time. In other words it lets somebody see and track the stable and unstable equilibrium and stationary points of a system. Bigger and sharper peak values mean more accumulated stable points in the probability density curve.

For more than one random variable, the joint probability density function must be computed and investigated. Simply, a generalization is made and a probability density surface is drawn and studied instead.

#### 3.2 The monte carlo simulation

Monte Carlo simulation is an efficient technique in quantitative analysis and decision making. It carries out a probability distribution for any factor that has inherent uncertainty. After that, it calculates results over and over, each time using a different set of random values from the probability functions. Depending upon the number of uncertainties and their ranges, it may occupy several recalculations before stopping. Finally it produces the distributions of possible outcome values.

The state space representation of a general nonlinear stochastic differential equation has such the following form [27, 28]:

$$dX_t = f(X_t, t)dt + \sigma(X_t, t)dB_t \quad (23)$$

where  $f \in \mathbb{R}^n$ ,  $\sigma$  is a proper real matrix and  $B_t$  is an  $m$ -dimensional Wiener process. The first term in the right-hand side provides the deterministic drift while the second is the random noise or diffusion. The Euler-Maruyama discretization [28] is now employed to reach Eq. (24):

$$X_{n+1} = X_n + f(X_n, n\Delta t)\Delta t + \sigma(X_n, n\Delta t)\sqrt{\Delta t}Z_n \quad (24)$$

where  $Z_n$  is the independent zero mean initial normal distribution. The equation is being solved and simulated for several initial conditions and many trajectories have been obtained. The outcome distribution is calculated and the simulation will be repeated again and again such that a stable outcome probability density will be achieved.

#### 4 RESULTS AND DISCUSSION

In this section, applying the Monte Carlo simulation to Eq. (19), the transient joint probability density function of the panel is obtained and discussed. Since other statistical properties can be extracted from the probability density function, the transient vibration behavior relates completely to this function.

The parameters of Eq. (19) are all non-dimensional; therefore, the results obtained from this equation can be applied to a wide range of panels. In our simulation, the parameters of the viscous medium i.e.  $\beta$ ,  $x$  and  $\zeta$  are assumed to be constant due to their small variations. The intensity of the white noise excitation,  $k$  is also considered to be fixed in all simulations. In Table 1. the values of these parameters in our simulations are specified.

In actual and practical situations, most of the changes and variations are related to the value of the quasi slenderness ratio  $s$ , and the non-dimensional mean value of lateral load. The former may vary due to any changes in the geometry of the panel while the latter fluctuate with respect to the mean magnitude of external loads. Thus, the behaviors of panels in our simulations are studied with respect to these two non-dimensional parameters mentioned above.

We face with two types of behaviors in the transient response of our system. Relative to the values of quasi slenderness ratio and the mean value of lateral load, the deflection may approach to a stable equilibrium point or bifurcate to two stable zones (similar to simple buckling in stability analysis of columns). To study, investigate and draw the transient vibration, a verification on our simulation must also be done first.

**Table 1**

The values of parameter used in our simulations.

$k$	$\beta$	$\zeta$	$x$
1	0.001	0.4	0.3

##### 4.1 Validation of the simulation

As indicated previously, the steady state semi-dynamic stability of a shallow shell with the same conditions was studied in [17] recently. In what follows, to validate our simulation, a comparison is made between the steady state response of the shell studied in [17] and that is investigated in present article. Once the validation is satisfied, the simulation can be used for the transient analysis also.

Fig. 2 lets somebody see the stable, unstable and the border curve of instability with respect to  $s$  (quasi slenderness ratio) and  $F_0$  (mean value of lateral load). The figure was obtained via the exact stationary solution of the Fokker Planck Kolmogorov equation when the inertial force was neglected [17]. See [17] for more details, assumptions and the method. Based on this figure, everybody can realize the stability of response once the parameters  $s$  and  $F_0$  were specified. Of course, there are two differences between our investigations and those were done in [17]. First, the investigation in [17] is stationary while it is transient in this work. To make a proper analogy, it is sufficient to compare the transient response for largely enough iterations with previously obtained steady state solution. Second, the inertial force was neglected in [17] while it contributes in the response in present article. Although, the inertial force can influence on the probability density function; but, cannot change the location of the equilibrium points. This is due to the equilibrium points computed when all the dynamic forces vanish. If the peaks loci of the probability density function which relate to the equilibrium points, are the same for both methods, it means that both of them predict same behavior for the system and consequently, the code is validated.

Fig. 3 illustrates some comparisons between the probability density functions of the response obtained from our numerical simulation and the exact method presented in [17]. Two stable dissipative vibrations and one unstable jumping phenomenon are drawn and compared. As shown in the figure, the peak locations are completely the same for both methods; whereas, some differences in magnitude due to neglecting the inertial force in [17] are also seen.

4.2 Transient probability density functions of the cylindrical panel

Using the Monte Carlo simulation (see Sec. 3.2), the transient behavior of the panel with respect to two parameters  $s$  (quasi slenderness ratio) and  $F_0$  (mean value of lateral load) is studied in this section. Fig. 4 shows the time variation of the probability density of the non-dimensional deflection when  $s=5$  and  $F_0=5$ . As this figure displays, with the proper passage of the non-dimensional time, the transient PDF of the non-dimensional deflection approaches to the steady status. Figs. 5 and 6 demonstrate same investigations with other values of  $s$  and  $F_0$ . The results presented in these figures are obtained for a series of initial panel conditions. In other words, The PDFs for several initial conditions are obtained, superimposed and normalized in order to reach more accurate results.

A simple analogy indicates that for small enough quasi slenderness ratios, a stable behavior (the one peak PDF) is seen while the jumping and instability (the two peaks PDF) is unavoidable for larger values of this parameter. Besides, the location of the dominant peak of PDF relates to the sign of the mean value of lateral load  $F_0$ . Positive  $F_0$  results in positive deflection and vice versa. Besides, it is noted that the instability may begin earlier for small values of lateral load mean.

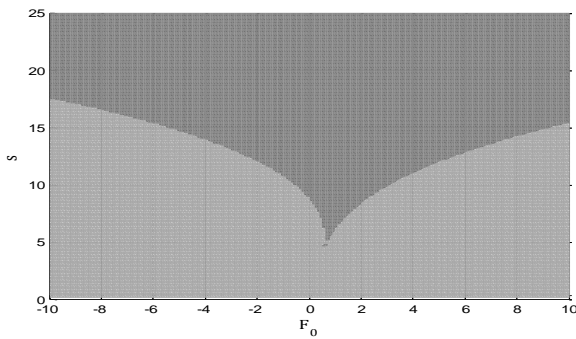


Fig.2 The stable (light) and unstable (dark) areas from Ref. [17]

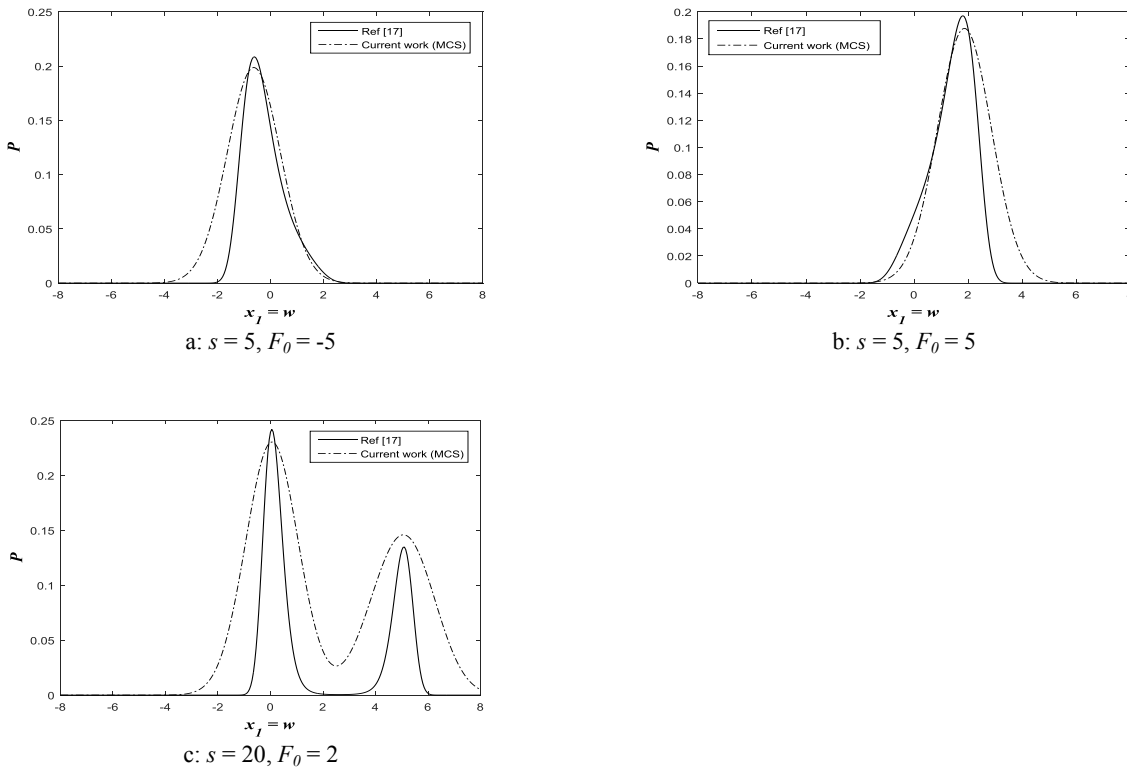
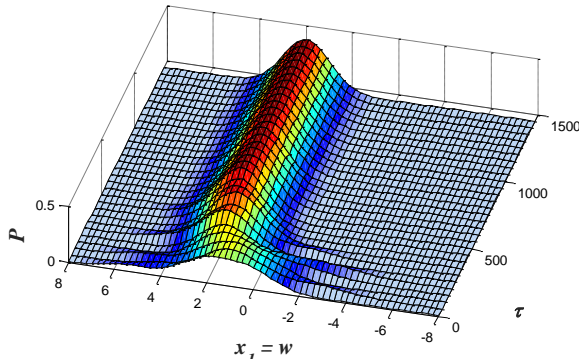
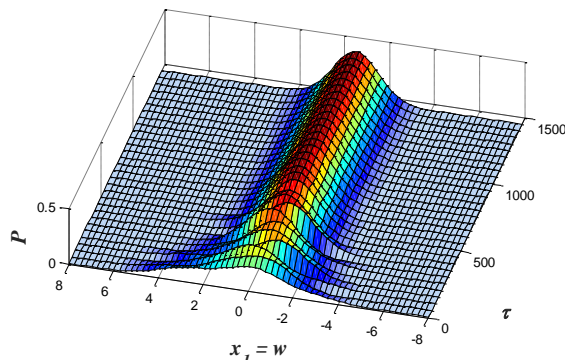


Fig.3 The comparison between the steady state probability density functions of the deflection obtained in this work and that were previously evaluated in [17].

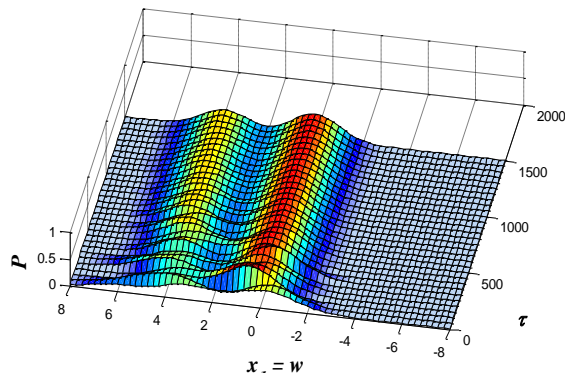


**Fig.4**

The evolution of the probability density of the non dimensional deflection when  $s = 5, F_0 = 5$ .

**Fig.5**

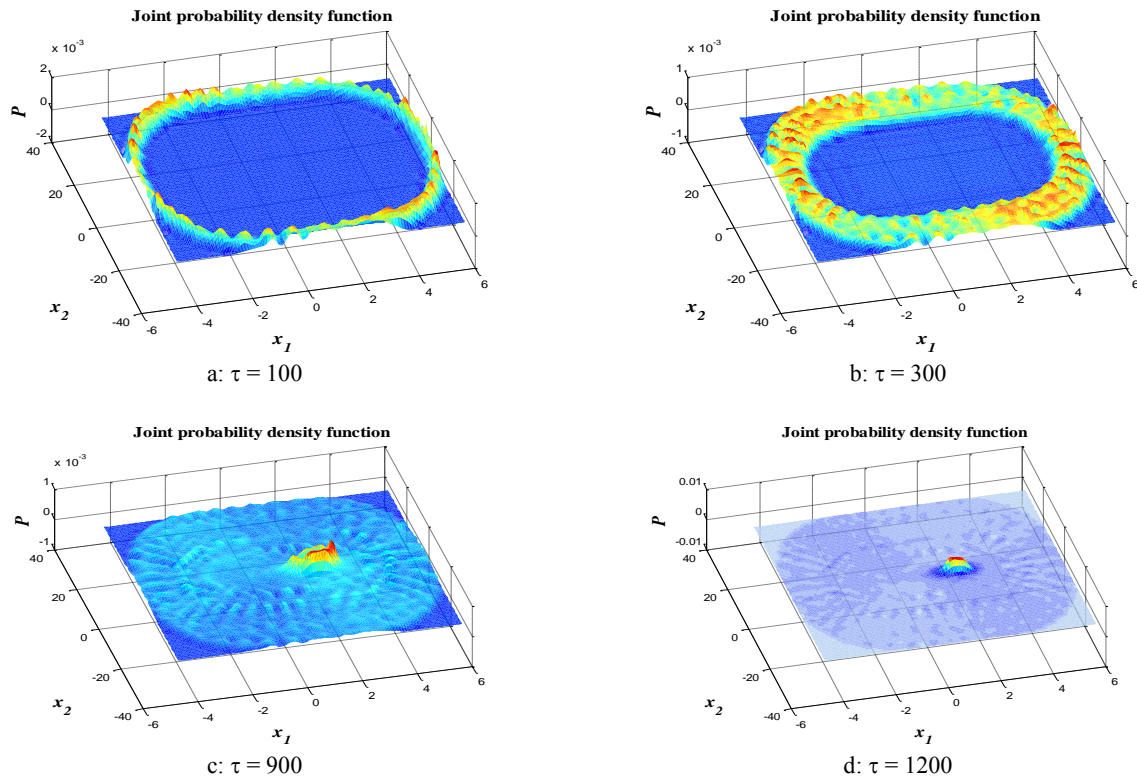
The evolution of the probability density of the non dimensional deflection when  $s = 5, F_0 = -5$ .

**Fig.6**

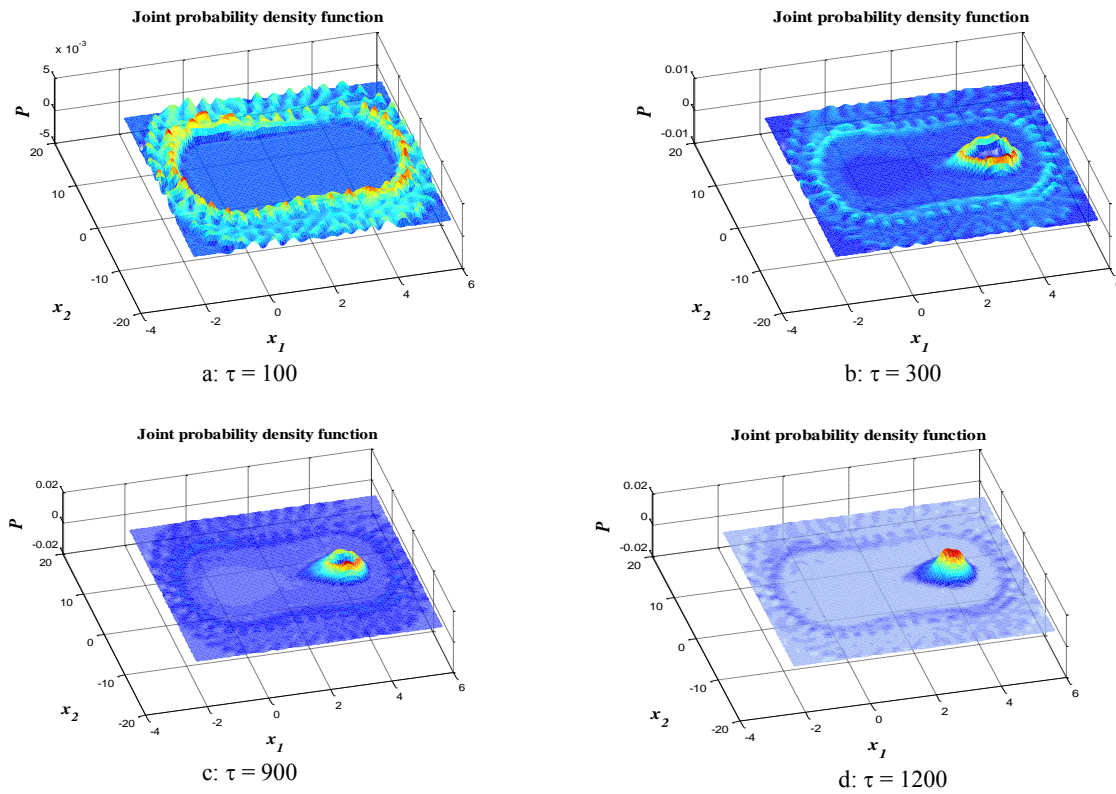
The evolution of the probability density of the non dimensional deflection when  $s = 15, F_0 = 0$ .

#### 4.3 A study on the joint transient probability density functions of the panel

The joint transient behavior of the first and second state variables (non-dimensional deflection and deflection rate) in the phase space is studied in this section. Phase space investigations play a crucial role in any behavior analysis of nonlinear systems. Using a time consuming simulation code, the transient joint probability density functions in the state space are obtained and drawn in Figs. 7 and 8. Fig. 7 makes somebody see the evolution of the JPDF of the response for a stable simple damped behavior while Fig. 8 illustrates the same for an unstable one with several jumping. These figures give an idea about the type of response convergence to dominant peaks in phase plane. Similar to the previous results, the JPDFs are obtained, superimposed and normalized for several initial conditions in order to reach more accurate results. To have a more clear realization, the locations of the statistical modes of the non-dimensional deflection (peaks of the probability density function) with respect to time are drawn in Fig. 9.



**Fig.7**  
The evolution of the joint probability density function of the response when  $s = 5, F_0 = 5$ .



**Fig.8**  
The evolution of the joint probability density function of the response when  $s = 15, F_0 = 0$ .

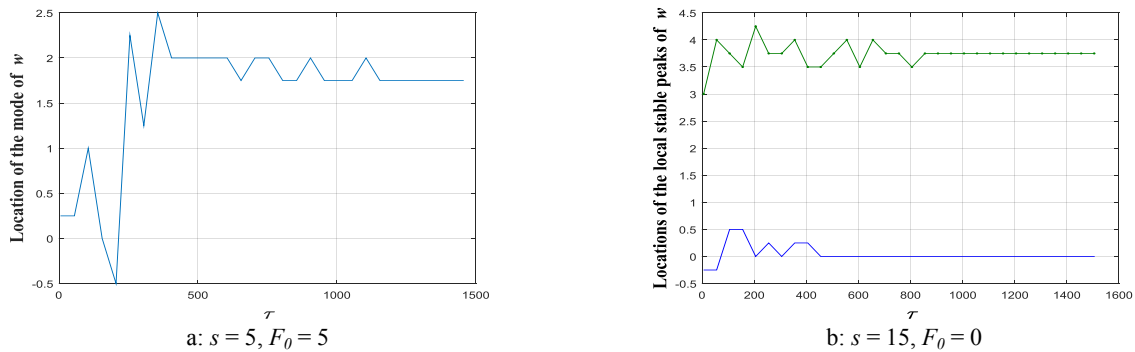


Fig.9

The location(s) of the statistical modes of the response (peaks of the PDF).

## 5 CONCLUSIONS

In this article, first the nonlinear integro-differential equation of a randomly excited circular panel in non aging viscoelastic medium is rearranged using Galerkin's method. The Monte Carlo simulation is validated and then employed to obtain the probability density function of the response. It is shown that the smaller values of the quasi slenderness ratio and mean magnitude of lateral load can stabilize the behavior while unstable jumping is seen for largely enough values of these parameters. The evolution of the probability density function of the deflection is drawn and studied for some values of quasi slenderness ratio and the mean magnitude of lateral load. Besides, the quality of the behavior in the phase plane is examined by evaluation and drawing the joint probability density function of the response. The results may help somebody to compare the behavior of a stochastically driven structure with the same deterministic one.

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