

# Theory of Generalized Piezoporo Thermoelasticity

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## ABSTRACT

In this paper, the basic constitutive equations and equations of motion are derived to describe the behavior of thermoelastic porous piezoelectric medium by using Biot's theory and the theory of generalized thermoelasticity with on relaxation time (Lord-Shulman). The electrical enthalpy density function is derived in the general coordinates. Also, clear definitions for the poroelastic modulus, electrical, thermal and additional mixed coefficients are embedded. The uniqueness of the solution for the complete system of equations is presented.

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**Keywords:** Porous piezo materials; Generalized thermoelasticity; Biot's theory; Two phase; Electro-dynamic Maxwell's equation

## 1 INTRODUCTION

THE porous media is made of a solid fully saturated with a fluid. There are solid media perforated by a multitude of small holes (called pores) filled with a fluid. When the wavelength is large in comparison with the size of the pores, rather than regarding such a medium as a heterogeneous medium, it is legitimate to use, at least locally, the theory of homogenization. This leads to Biot's model [1, 2, and 3] which involves as unknown not only the displacement field in the solid but also the displacement field in the fluid. Porous materials are widely used in different categories such as the sound and vibration absorption, the field of geo-mechanics and biological systems such as brain, lungs and bones.

Piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. Piezoelectric materials are integrated with structural systems to form a class of smart structures and they have wide-spread applications in many areas such as electronic technology, control of mechanical systems, medical appliance and other modern industrial fields.

Despite the significant progress made in enhancing the coupling characteristics between the electrical and mechanical properties in piezoelectric materials, monolithic piezoelectric materials generally exhibit limitation such as brittleness. Due to brittleness nature of the piezoelectric ceramics and possible defects of impurity, cavities and microcracks, failure of devices take place easily under mechanical or electrical loading. In order to overcome this limitation, material density is reduced through the addition of controlled porosity, where the resulting porous piezoelectric materials (PPM) are widely used for applications such as low frequency hydrophones, miniature, accelerometers, vibratory sensors and contact microphones. Due to lower acoustic impedance, these materials can be used to improve mismatch of acoustic impedances at the interfaces of medical ultrasonic imaging devices or underwater sonar detectors. Several piezoceramic constructive elements are porous especially when they are hot pressured or cast under pressure [4].

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A number of papers are published by Biot to define the concepts of porous materials in decades 40 to 60 [1, 2, 3 and 5]. In 1964, Mandl presented skeletal volume change in fluid-filled porous body under stress [6]. The dependence of the elastic properties of a porous rock on the compressibility of the pore fluid was investigated by Brown and Korringa [7]. Some studies about compressibility of porous rocks and compression between the grain and void in fractured and porous rock have been made by Zimmerman et al. [8, 9]. Garg and Nayfeh developed constitutive relations for liquid and/or gas saturated elastic porous media using the theory of interacting continua [10]. Pride et al. derived some equations for porous isotropic media and Thompson and Willis reformulated the equations of anisotropic poroelasticity [11, 12]. In 1993, Albert compared the propagation characteristics within water-filled and air-filled materials through the use of Biot's theory [13]. A survey article about the bone poroelasticity was published by Cowin [14]. In 2003, Tod developed a theory which described an anisotropic fracture in poroelastic effective medium [15]. Some characteristics of porous piezoelectric ceramics such as electromechanical properties were proposed by Wersing et al. [16] and Dunn and Taya [17]. Also, thermoelastic theory of the above-mentioned materials was stated by Kurashige [18] and Ciarletta [19]. Ghassemi and Diek derived the equations of porothermoelasticity for swelling shale in 2002 [20]. A study with finite element approach on piezo-poroelastic sound package was performed by Batifol et al. to show the active control on acoustic insulation at low frequencies [21]. Theory of generalized porothermoelasticity was presented by Youssef in 2007, which described the generalized thermoelasticity in a porous materials with one relaxation time, the so-called "Lord-Shulman" model [22]. Few years later, Vashishth and Gupta investigated vibrations of porous piezoelectric ceramic plates [4]. Recently, new model for piezoelectric porous medium is presented by Nasedkin [23], and Zielinski stated the fundamentals of multiphysics modelling of piezo-poro-elastic structures [24]. In 2010, Sharma presented a paper which described the piezoelectric effect on the velocities of waves in an anisotropic piezo poroelastic medium [25]. Also some text books pertinent to these studies such as "Theory of linear poroelasticity" [26], "Poromechanics" [27], "Mechanics and physics of porous solids" [28], "Fundamentals of Piezoelectricity" [29] and "Thermal stresses: advanced theory and applications" [30] were published.

The motivation for this study comes from the lack of analytical approach in piezo poro materials in other studies published before, so an attempt is made to formulate the theory of generalized piezo-poro-thermoelasticity and provide some expressions for characteristics of phases in PPM materials.

## 2 BASIC EQUATIONS AND DEFINITIONS OF THE THEORY

In the present study, the poroelastic medium is assumed to consist of a porous solid matrix with its interconnected pores filled with a single compressible fluid, which is assumed to be without viscosity. The deformations are assumed small enough, which guarantee the linearity of the mechanical processes. It also represents a case of no relative motion of fluid in the pores to the solid frame (skeleton) such that dissipation due to fluid friction disappears. The state of stress at a point of the bulk material is [22]

$$\tau_{ij} = (1 - \beta)\tau_{ij}^* - \beta p \delta_{ij} \quad (1)$$

where  $\beta$  is the porosity of the material. Now let

$$\begin{aligned} \sigma_{ij} &= (1 - \beta)\tau_{ij}^* \\ \sigma^f &= -\beta p \end{aligned} \quad (2)$$

From which it follows that

$$\tau_{ij} = \sigma_{ij} + \sigma^f \delta_{ij} \quad (3)$$

The displacements of the phases are  $u_i^s$  for the skeleton and  $u_i^f$  for the fluid. For the skeleton, we have [22]

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j}^s + u_{j,i}^s) \quad \varepsilon_{ij} = \varepsilon = u_{i,i}^s \quad (4)$$

and for the fluid phase, we have

$$\varepsilon^f = u_{i,i}^f \tag{5}$$

The summation convention is valid for repeated indices in the above equations. Let  $\rho^{s*}$  and  $\rho^{f*}$  denote the densities of the solid particles and the interstitial fluid. Then, the density of aggregate is given by  $\rho = (1-\beta)\rho^{s*} + \beta\rho^{f*}$  or  $\rho = \rho^s + \rho^f$  where  $\rho^s = (1-\beta)\rho^{s*}$  and  $\rho^f = \beta\rho^{f*}$  represent the densities of the two phases per unit volume of bulk. The mass coefficients are expressed in terms of the densities by means of  $\rho_{11} = \rho^s - \rho_{12}$  and  $\rho_{22} = \rho^f - \rho_{12}$ , where the dynamic coupling coefficient  $\rho_{12}$  is expressed as  $\rho_{12} = \beta\rho^{f*}(1-\alpha_\infty)$  which  $\alpha_\infty$  is tortuosity of pores [22, 31, and 32].

Omitting the displacement electric current, the governing electro-dynamic Maxwell equations for a piezoelectric elastic material are given by [33]

$$\begin{aligned} E_i &= -\psi_{,i} \\ D_{i,i} &= 0 \end{aligned} \tag{6}$$

The kinetic energy takes the form [22]

$$KE = \frac{1}{2}(\rho_{11}u_i^s u_i^s + \rho_{22}u_i^f u_i^f + 2\rho_{12}u_i^s u_i^f) \tag{7}$$

By Lagrange’s equations, we obtain [1]

$$\rho_{11}u_i^s + \rho_{12}u_i^f = \sigma_{ij,j} + \rho^s b_i^s \tag{8}$$

$$\rho_{12}u_i^s + \rho_{22}u_i^f = \sigma_{,j}^f + \rho^f b_i^f \tag{9}$$

where Eqs. (8) and (9) are written in the absence of dissipation term related to Darcy’s law. The absence of dissipation in Eqs. (8) and (9) implies that the skeletal frame is elastic and the pore-fluid is non-viscous or there is no fluid motion in the pores relative to the frame. The  $b_i^s$  and  $b_i^f$  are the body forces.

The first law of thermodynamics states that

$$\begin{aligned} \frac{d}{dt} \int_V \rho \xi dV + \frac{d}{dt} \int_V \frac{1}{2}(\rho_{11}u_i^s u_i^s + \rho_{22}u_i^f u_i^f + 2\rho_{12}u_i^s u_i^f) dV &= \int_V (\rho_1 b_i^s u_i^s + \rho_2 b_i^f u_i^f) dV - \\ \int_V (q_{i,i}^s + q_{i,i}^f) dV + \int_V \rho (h_s + h_f) dV - \int_S (D_i^s n_i \psi_i^s + D_i^f n_i \psi_i^f) dS + \int_S (\sigma_{ij} n_j u_i^s + \sigma n_i u_i^f) dS \end{aligned} \tag{10}$$

where  $\xi$  is the internal energy per unit mass of the bulk,  $q_i^s, q_i^f$  are the heat fluxes,  $h_s, h_f$  are the heat sources in the solid and the liquid, and  $n_i$  is the normal to the surface  $S$  that bounds the volume bulk  $V$  [22, 30].

Using Gauss’s integral theorem with Eqs. (8) and (9), we get

$$\rho \xi = \sigma_{ij} \varepsilon_{ij} + \sigma \varepsilon + \rho (h_s + h_f) - q_{i,i}^s - q_{i,i}^f - \psi_{i,i}^s D_i^s - \psi_{i,i}^f D_i^f \tag{11}$$

Let  $\phi$  be the combination of the internal energy and electrical energy and the entropy (Helmholtz’s function):

$$\phi = \xi - (T^s S^s + T^f S^f) - \frac{1}{\rho} (E_i^s D_i^s + E_i^f D_i^f) \tag{12}$$

Differentiating Eq. (12) with respect to time gives

$$\rho\phi = \rho\zeta - \rho T^s S^s - \rho T^f S^f - \rho T^f S^f - E_i^s D_i^s - E_i^f D_i^f - E_i^f D_i^f - E_i^f D_i^f \quad (13)$$

where  $T^n, S^n, E_i^n, D_i^n$  ( $n = s, f$ ) are the temperatures, the entropy, the electric field and the electric displacement for each phase per unit mass of aggregate, respectively.

By substituting Eq. (11) into (13) and using the Maxwell equations we obtain:

$$\rho\phi = \sigma_{ij}\varepsilon_{ij} + \sigma^f \varepsilon + \rho(h_s + h_f) - q_{i,i}^s - q_{i,i}^f - \rho T^s S^s - \rho T^f S^f - \rho T^f S^f - \rho T^f S^f - E_i^s D_i^s - E_i^f D_i^f \quad (14)$$

Now we will assume that  $\phi$  depends on the variables  $(\varepsilon_{ij}, \varepsilon^f, E_i^s, E_i^f, T^s, T^f)$ , i.e.

$$\phi = \phi(\varepsilon_{ij}, \varepsilon^f, E_i^s, E_i^f, T^s, T^f) \quad (15)$$

If we differentiate the above relation with respect to time and using the chain rule, we obtain

$$\dot{\phi} = \frac{\partial\phi}{\partial\varepsilon_{ij}} \frac{\partial\varepsilon_{ij}}{\partial t} + \frac{\partial\phi}{\partial\varepsilon^f} \frac{\partial\varepsilon^f}{\partial t} + \frac{\partial\phi}{\partial E_i^s} \frac{\partial E_i^s}{\partial t} + \frac{\partial\phi}{\partial E_i^f} \frac{\partial E_i^f}{\partial t} + \frac{\partial\phi}{\partial T^s} \frac{\partial T^s}{\partial t} + \frac{\partial\phi}{\partial T^f} \frac{\partial T^f}{\partial t} \quad (16)$$

If we compare Eqs. (16) and (14) with each other, we find

$$\sigma_{ij} = \rho \frac{\partial\phi}{\partial\varepsilon_{ij}} \quad (17)$$

$$\sigma^f = \rho \frac{\partial\phi}{\partial\varepsilon^f} \quad (18)$$

$$D_i^s = -\rho \frac{\partial\phi}{\partial E_i^s} \quad (19)$$

$$D_i^f = -\rho \frac{\partial\phi}{\partial E_i^f} \quad (20)$$

$$S^s = -\frac{\partial\phi}{\partial T^s} \quad (21)$$

$$S^f = -\frac{\partial\phi}{\partial T^f} \quad (22)$$

$$q_{i,i}^f - \rho h^f + \rho T^f S^f - K(T^s - T^f) = 0 \quad (23)$$

$$q_{i,i}^s - \rho h^s + \rho T^s S^s + K(T^s - T^f) = 0 \quad (24)$$

and for linearity, we let  $T^s \approx T^f \approx T_0$ . Hence, we get

$$q_{i,i}^s - \rho h^s + \rho T_0 S^s + K(T^s - T^f) = 0$$

$$q_{i,i}^f - \rho h^f + \rho T_0 S^f - K(T^s - T^f) = 0$$

where the term  $K(T^s - T^f)$  represents the inter phase heat transfer resulting from the unequal temperature of the phases at every point of the medium, and  $K$  is the inter phase thermal conductivity [22].

The generalized Fourier's law of heat conduction is [22]

$$q_i^s + \tau_0^s q_i^s = -(1-\beta)k^{s*}T_i^s = -k^s T_i^s \quad (25)$$

$$q_i^f + \tau_0^f q_i^f = -\beta k^{f*}T_i^f = -k^f T_i^f \quad (26)$$

where  $k^{s*}$  and  $k^{f*}$  are the thermal conductivity of the solid particles and the interstitial fluid and  $k^s$  and  $k^f$  are the thermal conductivity of the phases. Also  $\tau_0^s$  and  $\tau_0^f$  are the solid and liquid relaxation times, respectively.

The following potential function  $\psi$  is the most general form of Gibbs function which can be written as:

$$\begin{aligned} \psi = \rho\phi = & \frac{1}{2}[A]\varepsilon_{ij}\varepsilon_{kl} + \frac{1}{2}\{B\}\varepsilon^f\varepsilon^f + [C]\varepsilon_{ij}\varepsilon^f - \{D\}E_k^s\varepsilon_{ij} - \{E\}E_k^f\varepsilon_{ij} - \{F\}E_k^s\varepsilon^f - \\ & \{G\}E_k^f\varepsilon^f - \frac{1}{2}[H]E_i^sE_j^s - \frac{1}{2}[I]E_i^fE_j^f - [J]E_i^sE_j^f - [K]\varepsilon_{ij}\theta^s\delta_{ij} - [L]\varepsilon_{ij}\theta^f\delta_{ij} - \{M\}\varepsilon^f\theta^s \\ & - \{N\}\varepsilon^f\theta^f - \{O\}E_k^s\theta^s - \{P\}E_k^s\theta^f - \{Q\}E_k^f\theta^s - \{R\}E_k^f\theta^f - \frac{1}{2}\{S\}\theta^s\theta^s - \frac{1}{2}\{T\}\theta^f\theta^f - \{U\}\theta^s\theta^f \end{aligned} \quad (27)$$

Those above matrix coefficients are poroelastic modulus, electrical, thermal and additional mixed coefficients which are introduced below in details:

$[A] = C_{ijkl}$  : Elastic stiffness constants. [22, 30]

$[B] = b$  : The pressure to be exerted on fluid to push its unit volume into the porous matrix. [4, 27, 28]

$[C] = m_{ij}$  : Elastic coupling between the two phases of porous aggregate. [4]

$[D] = e_{kij}$  : Piezoelectric constants of solid phase.

$[E] = \varsigma_{kij}$  : Piezoelectric coupling of solid phase on fluid phase in porous aggregate.

$\{F\} = \eta_i$  : Piezoelectric coupling of fluid phase on solid phase in porous aggregate.

$\{G\} = \zeta_i$  : Piezoelectric constant of fluid phase.

$[H] = \chi_{ij}$  : Dielectric constant of solid phase.

$[I] = v_{ij}$  : Dielectric constant of fluid phase.

$[J] = \xi_{ij}$  : Dielectric coupling between the two phases of porous aggregate.

$[K] = B_{ij}$  : Thermoelastic coefficient of solid phase in existence of fluid phase.

$[L] = C_{ij}$  : Thermoelastic coefficient of fluid phase in existence of solid phase.

$[M] = M$  : Thermoelastic coupling of solid phase on fluid phase.

$[N] = N$  : Thermoelastic coupling of fluid phase on solid phase.

$\{O\} = P_i^s$  : Pyroelectric coefficient of solid phase.

$\{P\} = P_i^{fs}$  : Pyroelectric coupling of fluid phase on solid phase.

$\{Q\} = P_i^{sf}$  : Pyroelectric coupling of solid phase on fluid phase.

$\{R\} = P_i^f$  : Pyroelectric coefficient of fluid phase.

$[S] = S$  : Thermal coefficient for solid phase

$[T] = T$  : Thermal coefficient for fluid phase

$[U] = U$  : Thermal coupling of phases [22]

We should notice in Eq. (27)  $\theta^s = T^s - T_0$  and  $\theta^f = T^f - T_0$  such that  $\left|\frac{\theta^s}{T_0}\right| \ll 1$  and  $\left|\frac{\theta^f}{T_0}\right| \ll 1$  and in the

reference state we have  $T^s = T^f = T_0$  [22].

The constants for a transversely isotropic Pyroelectric porous medium are expressed as:

$$[A] = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda \\ \lambda & \lambda + 2\mu & \lambda \\ \lambda & \lambda & \lambda + 2\mu \end{bmatrix} \quad [B] = b \quad [C] = q$$

$$[D] = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} \end{bmatrix} \quad [E] = \begin{bmatrix} \varsigma_{11} & \varsigma_{12} & \varsigma_{13} & \varsigma_{14} & \varsigma_{15} & \varsigma_{16} \\ \varsigma_{21} & \varsigma_{22} & \varsigma_{23} & \varsigma_{24} & \varsigma_{25} & \varsigma_{26} \\ \varsigma_{31} & \varsigma_{32} & \varsigma_{33} & \varsigma_{34} & \varsigma_{35} & \varsigma_{36} \end{bmatrix}$$

$$\{F\} = [\eta_1 \quad \eta_2 \quad \eta_3]^T \quad \{G\} = [\zeta_1 \quad \zeta_2 \quad \zeta_3]^T$$

$$[H] = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{bmatrix} \quad [I] = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{bmatrix} \quad [J] = \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} \\ \xi_{21} & \xi_{22} & \xi_{23} \\ \xi_{31} & \xi_{32} & \xi_{33} \end{bmatrix}$$

$$[K] = \begin{bmatrix} B_{11} \\ B_{22} \\ B_{33} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda \\ \lambda & \lambda + 2\mu & \lambda \\ \lambda & \lambda & \lambda + 2\mu \end{bmatrix} \begin{bmatrix} \alpha_1^s \\ \alpha_2^s \\ \alpha_\varepsilon^s \end{bmatrix} + \alpha^{fs} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

If  $(\alpha_1^s = \alpha_2^s = \alpha_\varepsilon^s)$  then  $B_{ii} = \alpha^s (2\mu + 3\lambda) + \alpha^{fs} q$

$$[L] = \begin{bmatrix} C_{11} \\ C_{22} \\ C_{33} \end{bmatrix} = \alpha^f \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} + \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda \\ \lambda & \lambda + 2\mu & \lambda \\ \lambda & \lambda & \lambda + 2\mu \end{bmatrix} \begin{bmatrix} \alpha_1^{sf} \\ \alpha_2^{sf} \\ \alpha_3^{sf} \end{bmatrix}$$

If  $(\alpha_1^{sf} = \alpha_2^{sf} = \alpha_3^{sf})$  then  $C_{ii} = \alpha^f Q + \alpha^{sf} (2\mu + 3\lambda)$

$$[M] = 3\alpha^3 q + \alpha^{fs} b \quad [N] = \alpha^f b + 3\alpha^{sf} q \quad \{O\} = [P_1^s \quad P_2^s \quad P_3^s]^T$$

$$\{P\} = [P_1^{fs} \quad P_2^{fs} \quad P_3^{fs}]^T \quad \{Q\} = [P_1^{sf} \quad P_2^{sf} \quad P_3^{sf}]^T \quad \{R\} = [P_1^f \quad P_2^f \quad P_3^f]^T$$

$$[S] = \frac{\rho C_E^s}{T_0} \quad [T] = \frac{\rho C_E^f}{T_0} \quad [U] = -(3\alpha^s C_{ii} + \alpha^{fs} (\alpha^f b + 3\alpha^{sf} q)) = -(3\alpha^{sf} B_{ii} + \alpha^f (3\alpha^s q + \alpha^{fs} b))$$

It is useful to remember  $\alpha^s$  and  $\alpha^f$  are thermal expansion of solid and fluid phase, respectively and  $\alpha^{sf}$  and  $\alpha^{fs}$  are thermal expansion coefficient coupling between two phases.

By using Eqs. (17) - (22) and (27) we can express the constitutive equations such as stress, electrical displacement and entropy for each phase.

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} + m_{ij} \varepsilon^f - e_{ijk} E_i^s - \varsigma_{ijk} E_i^f + (-B_{ij} \theta^s - C_{ij} \theta^f) \delta_{ij} \quad (28)$$

$$\sigma^f = b \varepsilon^f + m_{ij} \varepsilon_{ij} - \eta_i E_i^s - \zeta_i E_i^f - M \theta^s - N \theta^f \quad (29)$$

$$D_i^s = e_{ijk} \varepsilon_{jk} + \eta_i \varepsilon^f + \chi_{ij} E_i^s + \xi_{ij} E_j^f + P_i^s \theta^s + P_i^{fs} \theta^f \quad (30)$$

$$D_i^f = \varsigma_{ijk} \varepsilon_{jk} + \zeta_i \varepsilon^f + \xi_{ij} E_j^s + v_{ij} E_j^f + P_i^{sf} \theta^s + P_i^f \theta^f \quad (31)$$

$$\rho S^s = B_{ii} \varepsilon_{kk} + M \varepsilon^f + P_i^s E_i^s + P_i^{sf} E_i^f + S \theta^s + U \theta^f \quad (32)$$

$$\rho S^f = C_{ii} \varepsilon_{kk} + N \varepsilon^f + P_i^{fs} E_i^s + P_i^f E_i^f + U \theta^s + T \theta^f \quad (33)$$

By using Eqs. (8), (9) and differentiate  $\sigma_{ij}$  and  $\sigma^f$  with respect to space variables we can obtain a set of two equations of motion which describes dynamical movement of particles of porous piezoelectric media.

$$(C_{ijkl} \varepsilon_{kl} + m_{ij} \varepsilon^f - e_{ijk} E_i^s - \varsigma_{ijk} E_j^f + (-B_{ij} \theta^s - C_{ij} \theta^f) \delta_{ij})_{,i} + \rho^s b_i^s = \rho_{11} u_i^s + \rho_{12} u_i^f \quad (34)$$

$$(b \varepsilon^f + m_{ij} \varepsilon_{ij} + \eta_i E_i^s - \zeta_i E_i^f - M \theta^s - N \theta^f)_{,i} + \rho^f b_i^f = \rho_{12} u_i^s + \rho_{22} u_i^f \quad (35)$$

Now we can substitute  $\varepsilon_{ij} = \frac{1}{2}(u_{i,j}^s + u_{j,i}^s)$  and  $\varepsilon^f = u_{i,i}^f$  and  $E_i = -\psi_{,i}^n$  ( $n = s, f$ ) in equations of motion to obtain Navier's equation for solid and interstitial fluid phases:

$$\frac{1}{2} C_{ijkl} (u_{i,jj}^s + u_{j,ii}^s) + m_{ij} u_{i,ii}^f + e_{ijk} \psi_{,ii}^s + \varsigma_{ijk} \psi_{,ii}^f - B_{ii} \theta_i^s - C_{ii} \theta_i^s - \rho^s b_i^s = \rho_{11} u_i^s + \rho_{12} u_i^f \quad (36)$$

$$b u_{i,ii}^f + \frac{1}{2} m_{ij} (u_{i,jj}^s + u_{j,ii}^s) - \eta_i \psi_{,ii}^s - \zeta_i \psi_{,ii}^f - M \theta_{,i}^s - N \theta_{,i}^f - \rho^f b_i^f = \rho_{12} u_i^s + \rho_{22} u_i^f \quad (37)$$

By using of the second part of Eq. (6) the electric field displacements in these phases of the piezo-poroelastic medium are expressed as follows:

$$\frac{1}{2} e_{ijk} (u_{i,jj}^s + u_{j,ii}^s) + \eta_i u_{i,ii}^f + \chi_{ij} \psi_{,ii}^s + \xi_{ij} \psi_{,ii}^f + P_i^s \theta_{,i}^s + P_i^{fs} \theta_{,i}^f = 0 \quad (38)$$

$$\frac{1}{2} \varsigma_{ijk} (u_{i,jj}^s + u_{j,ii}^s) + \zeta_i u_{i,ii}^f + \xi_{ij} \psi_{,ii}^s + v_{ij} \psi_{,ii}^f + P_i^{sf} \theta_{,i}^s + P_i^f \theta_{,i}^f = 0 \quad (39)$$

The heat equations can be written as follows:

$$k^s \theta_{,ii}^s = \left( \frac{\partial}{\partial t} + \tau_0^s \frac{\partial^2}{\partial t^2} \right) \{ (\rho C_E^s) \theta^s - T_0 (3\alpha^s (\alpha^f Q + \alpha^{sf} (2\mu + 3\lambda)) + \alpha^{fs} (2\mu + 3\lambda)) + \alpha^{fs} (\alpha^f b + 3\alpha^{sf} Q) \} \theta^f \quad (40)$$

$$T_0 (\alpha^s (2\mu + 3\lambda) + \alpha^{fs} Q) \varepsilon_{ii} + T_0 (3\alpha^s Q + \alpha^{fs} b) \varepsilon^f - \rho (1 + \tau_0^s \frac{\partial}{\partial t}) h^s + k (\theta^s - \theta^f)$$

$$k^f \theta_{,ii}^f = \left( \frac{\partial}{\partial t} + \tau_0^f \frac{\partial^2}{\partial t^2} \right) \{ -T_0 (3\alpha^{sf} (\alpha^s (2\mu + 3\lambda) + \alpha^{fs} Q) + \alpha^f (3\alpha^s Q + \alpha^{fs} b)) \theta^s + (\rho C_E^f) \theta^f + T_0 (\alpha^f Q + \alpha^{sf} (2\mu + 3\lambda)) \varepsilon_{ii} + T_0 (\alpha^f b + 3\alpha^{sf} Q) \varepsilon^f \} - \rho (1 + \tau_0^f \frac{\partial}{\partial t}) h^f - K (\theta^s - \theta^f) \quad (41)$$

### 3 UNIQUENESS THEOREM

Let  $V$  be an open regular region of space with boundary  $S$  occupied by the reference configuration of a homogeneous isotropic linear piezothermoelastic solid, where  $S$  is assumed closed and bounded [34]. The physical example of this

kind of boundary condition is a layer of soil which consists of liquid-crystal molecules. This layer can be influenced by earthquake or it may have heat exchange with surrounding. Acoustic panels which can be used for absorbing sounds and vibrations may be another example for this kind of boundary condition. In sound detector appliances we can measure the displacement of diaphragm and then by using this theory other physical specifications can be achieved. We supplement the equations of piezo-poro-thermoelasticity Eqs. (8), (9), (32) – (41) by the prescribed boundary conditions.

$$\begin{cases} u_i^s = \bar{u}_i^s, & u_i^f = \bar{u}_i^f \text{ on } S_1 \times [0, \infty) \\ E_i^s = \bar{E}_i^s, & E_i^f = \bar{E}_i^f \text{ on } S_2 \times [0, \infty) \end{cases} \quad (42)$$

$$\begin{cases} t_i^s = \sigma_{ij} n_j = \bar{t}_i^s, t_i^f = \sigma^f = \bar{t}_i^f \text{ on } S - S_1 \times [0, \infty) \\ C^s = D_i^s n_i = \bar{C}^s, C^f = D_i^f n_i = \bar{C}^f \text{ on } S - S_2 \times [0, \infty) \end{cases} \quad (43)$$

$$\theta_i^s = \bar{\theta}_i^s, \theta_i^f = \bar{\theta}_i^f \text{ on } S \quad (44)$$

where  $S_1 \subset S$  and  $S_2 \subset S$ . Also, we prescribed the following initial conditions within  $V$  at  $t = 0$  :

$$\begin{aligned} u_i^s &= u_{i0}^s, u_i^f &= u_{i0}^f, u_i^s &= u_{i0}^s, u_i^f &= u_{i0}^f, \theta^s &= \theta_0^s, \theta^f &= \theta_0^f, \theta^s &= \theta_0^s, \theta^f &= \theta_0^f, \\ E_i^s &= E_{i0}^s, E_i^f &= E_{i0}^f, E_i^s &= E_{i0}^s, E_i^f &= E_{i0}^f \end{aligned} \quad (45)$$

We now state the uniqueness theorem: Given a regular region of space  $V + S$  with boundary  $S$ , there exists at most one set of single-valued functions  $\sigma_{ij}(x_k, t) - \sigma^f(x_k, t) - \varepsilon_{ij}(x_k, t) - \varepsilon^f(x_k, t) - E_i^s(x_k, t) - E_i^f(x_k, t) - D_i^s(x_k, t) - D_i^f(x_k, t)$  of class  $C^{(1)}$ , and  $u_i(x_k, t) - u_i^f(x_k, t) - \theta_i^s(x_k, t) - \theta_i^f(x_k, t)$  of class  $C^{(2)}$  in  $V + S$ ,  $t \geq 0$  that satisfy Eqs.(8), (9), (32) – (41) and conditions (42) – (45) where all the parameters are all positive.[22]

The proof proceeds as follows. Let there be two sets of functions  $\sigma_{ij}^{(I)}$  and  $\sigma_{ij}^{(II)}$ ,  $\varepsilon_{ij}^{(I)}$  and  $\varepsilon_{ij}^{(II)}$ ,  $\sigma^{f(I)}$  and  $\sigma^{f(II)}$ ,  $\varepsilon^{f(I)}$  and  $\varepsilon^{f(II)}$ ,  $D_i^{s(I)}$  and  $D_i^{s(II)}$ ,  $D_i^{f(I)}$  and  $D_i^{f(II)}$ , etc. and let  $\sigma_{ij} = \sigma_{ij}^{(I)} - \sigma_{ij}^{(II)}$  and  $\sigma^f = \sigma^{f(I)} - \sigma^{f(II)}$ , etc. By virtue of the linearity of the problem, it is clear that these differences also satisfy the above-mentioned equations (without loss of generality we neglect the terms  $K(T^s - T^f)$ , the body forces and the heat fluxes) and the homogeneous counterparts of conditions (42) – (45), namely they satisfy the following field equations in  $V \times (0, \infty)$  [22, 34]:

$$\sigma_{ij,j} = \rho_{11} u_i^s + \rho_{12} u_i^f \quad (46)$$

$$\sigma_{,i}^f = \rho_{12} u_i^s + \rho_{22} u_i^f \quad (47)$$

$$\frac{1}{2} C_{ijkl} (u_{i,jj}^s + u_{j,ii}^s) + Q u_{i,ii}^f + e_{ijk} \psi_{,ii}^f - B_{ii} \theta_{,i}^s - C_{ii} \theta_{,i}^f = \rho_{11} u_i^s + \rho_{12} u_i^f \quad (48)$$

$$b u_{i,ii}^f + \frac{1}{2} Q (u_{i,jj}^s + u_{j,ii}^s) - \eta_i \psi_{,ii}^s - \zeta_i \psi_{,ii}^f - M \theta_{,i}^s - N \theta_{,i}^f + \rho^f b_i^f = \rho_{12} u_i^s + \rho_{22} u_i^f \quad (49)$$

$$q_{i,i}^s = -\rho T_0 S^s \quad (50)$$

$$q_{i,i}^f = -\rho T_0 S^f \quad (51)$$

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} + Q \varepsilon_{ij}^f - e_{ijk} E_i^s - \zeta_{ijk} E_i^f + (-B_{ij} \theta^s - C_{ij} \theta^f) \delta_{ij} \quad (52)$$

$$\sigma^f = b \varepsilon^f + Q \varepsilon_{ij} - \eta_i E_i^s - \zeta_i E_i^f - M \theta^s - N \theta^f \quad (53)$$

$$D_i^s = e_{ijk} \varepsilon_{jk} + \eta_i \varepsilon^f + \chi_{ij} E_j^s + \xi_{ij} E_j^f + P_i^s \theta^s + P_i^{fs} \theta^f \quad (54)$$

$$D_i^f = \zeta_{ijk} \varepsilon_{jk} + \zeta_j \varepsilon^f + \xi_{ij} E_j^s + v_{ij} E_j^f + P_i^{sf} \theta^s + P_i^f \theta^f \quad (55)$$

$$q_i^s + \tau_0^s q_i^s = -k^s \theta_{,i}^s \quad (56)$$

$$q_i^f + \tau_0^f q_i^f = -k^f \theta_{,i}^f \quad (57)$$

$$\rho S^s = B_{ii} \varepsilon_{kk} + M \varepsilon^f + P_i^s E_i^s + P_i^{sf} E_i^f + S \theta^s + U \theta^f \quad (58)$$

$$\rho S^f = C_{ii} \varepsilon_{kk} + N \varepsilon^f + P_i^{fs} E_i^s + P_i^f E_i^f + U \theta^s + T \theta^f \quad (59)$$

where

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j}^s + u_{j,i}^s) \quad \text{and} \quad \varepsilon^f = u_{i,i}^f \quad (60)$$

$$E_i^s = -\psi_{,i}^s \quad \text{and} \quad E_i^f = -\psi_{,i}^f \quad (61)$$

Together with the following boundary conditions:

$$\begin{cases} u_i^s = 0, & u_i^f = 0 & \text{on } S_1 \times [0, \infty) \\ E_i^s = 0, & E_i^f = 0 & \text{on } S_2 \times [0, \infty) \end{cases} \quad (62)$$

$$\begin{cases} t_i^s = 0, & t_i^f = 0 & \text{on } S - S_1 \times [0, \infty) \\ C^s = 0, & C^s = 0 & \text{on } S - S_2 \times [0, \infty) \end{cases} \quad (63)$$

$$\theta_i^s = 0, \theta_i^f = 0 \quad \text{on } S \quad (64)$$

where  $S_1 \subset S$  and  $S_2 \subset S$ . Also, we have the following initial conditions within  $V$  at  $t = 0$ :

$$u_i^s = 0, u_i^f = 0, u_i^s = 0, u_i^f = 0, \theta^s = 0, \theta^f = 0, \theta^s = 0, \theta^f = 0, E_i^s = 0, E_i^f = 0, E_i^s = 0, E_i^f = 0 \quad (65)$$

Now consider the following integral:

$$\int_V \sigma_{ij} \varepsilon_{jg} dV + \int_V \sigma^f \varepsilon^f \delta_{ij} dV = \int_V \sigma_{ij} u_{i,j}^s dV + \int_V \sigma_i^f u_{i,i}^f \delta_{ij} dV \quad (66)$$

Using integrating by parts, we obtain:

$$\int_V \sigma_{ij} \varepsilon_{ij} dV + \int_V \sigma^f \varepsilon^f \delta_{ij} dV = -\int_V \sigma_{ij,j} u_i^s dV - \int_V \sigma_i^f u_i^f \delta_{ij} dV \quad (67)$$

So we get:

$$\int_V \sigma_{ij} \varepsilon_{ij} dV + \int_V \sigma^f \varepsilon^f \delta_{ij} dV + \int_V \sigma_{i,j,j} u_i^s dV + \int_V \sigma_i^f u_i^f \delta_{ij} dV = 0 \quad (68)$$

If Eqs. (46), (47), (52) and (53) are inserted in the Eq. (68) then we have:

$$\begin{aligned} & \int_V (C_{ijkl} \varepsilon_{kl} + Q \varepsilon^f \delta_{ij} - e_{ijk} E_i^s - \varsigma_{ijk} E_i^f + (-B_{ij} \theta^s - C_{ij} \theta^f) \delta_{ij}) \varepsilon_{ij} dV + \\ & \int_V (b \varepsilon^f + Q \varepsilon_{ij} - \eta_i E_i^s - \zeta_i E_i^f - M \theta^s - N \theta^f) \varepsilon^f \delta_{ij} dV + \int_V (\rho_{11} u_i^s + \rho_{12} u_i^f) u_i^s dV + \\ & \int_V (\rho_{12} u_i^s + \rho_{22} u_i^f) u_i^f \delta_{ij} dV = 0 \end{aligned} \quad (69)$$

Now we can write Eq. (69) as follows:

$$\begin{aligned} & \int_V (C_{ijkl} \varepsilon_{kl} \varepsilon_{ij} + Q \varepsilon^f \varepsilon_{ij} \delta_{ij} - e_{ijk} E_i^s \varepsilon_{ij} - \varsigma_{ijk} E_i^f \varepsilon_{ij} + b \varepsilon^f \varepsilon^f + Q \varepsilon_{kk} \varepsilon^f - \eta_i E_i^s \varepsilon^f - \zeta_i E_i^f \varepsilon^f + \\ & \rho_{11} u_i^s u_i^s + \rho_{12} u_i^f u_i^s + \rho_{12} u_i^s u_i^f + \rho_{22} u_i^f u_i^f) dV - \int_V [(B_{ii} \varepsilon_{ij} + M \varepsilon^f) \theta^s + (C_{ii} \varepsilon_{ij} + N \varepsilon^f) \theta^f] dV = 0 \end{aligned} \quad (70)$$

If we differentiate the heat conduction equation of each phases Eqs. (56), (57) with respect to space variables and compare the results with Eq. (50) and (51), we find

$$T_0 \frac{d}{dt} (B_{ii} \varepsilon_{kk} + M \varepsilon^f + P_i^s E_i^s + P_i^{sf} E_i^f + S \theta^s + U \theta^f) = k^s \theta_{,ii}^s \quad (71)$$

$$T_0 \frac{d}{dt} (C_{ii} \varepsilon_{kk} + N \varepsilon^f + P_i^{fs} E_i^s + P_i^f E_i^f + U \theta^s + T \theta^f) = k^f \theta_{,ii}^f \quad (72)$$

Now if we substitute the upper relations in Eq. (70), we obtain:

$$\begin{aligned} & T_0 \frac{dz}{dt} + \int_V [k^s \theta_{,i}^s \theta_{,i}^s + k^f \theta_{,i}^f \theta_{,i}^f + \theta^s \frac{\partial}{\partial t^2} \{(\rho C_E^s) \theta^s - T_0 (3\alpha^s (\alpha^f Q + \alpha^{sf} (2\mu + 3\lambda)) + \alpha^{fs} (\alpha^f b + 3\alpha^{sf} Q)) \theta^f \\ & + T_0 (\alpha^s (2\mu + 3\lambda) + \alpha^{fs} Q) \varepsilon_{ii} + T_0 (3\alpha^s Q + \alpha^{fs} b) \varepsilon^f \} + \theta^f \frac{\partial}{\partial t^2} \{-T_0 (3\alpha^{sf} (\alpha^s (2\mu + 3\lambda) + \alpha^{fs} Q) + \\ & \alpha^f (3\alpha^s Q + \alpha^s Q + \alpha^{fs} b)) \theta^s + (\rho C_E^f) \theta^f + T_0 (\alpha^f Q + \alpha^{sf} (2\mu + 3\lambda) \varepsilon_{ii} + T_0 (\alpha^f b + 3\alpha^{sf} Q) \varepsilon^f \}] dV = 0 \end{aligned} \quad (73)$$

$$\begin{aligned} Z = & \int_V [\frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} + Q \varepsilon_{kk} \varepsilon^f - e_{ijk} E_i^s \varepsilon_{ij} - \varsigma_{ijk} E_i^f \varepsilon_{ij} + \frac{b}{2} \varepsilon^f{}^2 - \eta_i E_i^s \varepsilon^f + \rho_{11} u_i^s u_i^s + \rho_{22} u_i^f u_i^f + \rho_{12} u_i^s u_i^f + \\ & \frac{P_i^s}{T_0} E_i^s \theta^s + \frac{P_i^{sf}}{T_0} E_i^f \theta^s + \frac{P_i^{fs}}{T_0} E_i^s \theta^f + (\frac{s}{2T_0}) \theta^{s2} + (\frac{T}{2T_0}) \theta^{f2} + (\frac{U}{T_0}) \theta^s \theta^f] dV \end{aligned} \quad (74)$$

We are familiar with these two inequalities,

$$-q_i^s \theta_{,i}^s \geq 0 \quad -q_i^f \theta_{,i}^f \geq 0 \quad (75)$$

If we multiply  $\theta_{,i}^s$  and  $\theta_{,i}^f$  in heat conduction equation for phases and contrast the results with Eq. (74) we find:

$$\begin{aligned} & \int_V [k^s \theta_{,i}^s \theta_{,i}^s + k^f \theta_{,i}^f \theta_{,i}^f + \theta^s \frac{\partial}{\partial t^2} \{(\rho C_E^s) \theta^s - T_0 (3\alpha^s (\alpha^f Q + \alpha^{sf} (2\mu + 3\lambda)) + \alpha^{fs} (\alpha^f b + 3\alpha^{sf} Q)) \theta^f + \\ & T_0 (\alpha^s (2\mu + 3\lambda) + \alpha^{fs} Q) \varepsilon_{ii} + T_0 (3\alpha^s Q + \alpha^{fs} b) \varepsilon^f \} + \theta^f \frac{\partial}{\partial t^2} \{-T_0 (3\alpha^{sf} (\alpha^s (2\mu + 3\lambda) + \alpha^{fs} Q) + \\ & \alpha^f (3\alpha^s Q + \alpha^{fs} b)) \theta^s + (\rho C_E^f) \theta^f + T_0 (\alpha^f Q + \alpha^{sf} (2\mu + 3\lambda) \varepsilon_{ii} + T_0 (\alpha^f b + 3\alpha^{sf} Q) \varepsilon^f \}] dV \geq 0 \end{aligned} \quad (76)$$

Hence, we have

$$\begin{aligned}
T_0 \frac{dz}{dt} = & T_0 \frac{d}{dt} \int_V \left[ \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} + Q \varepsilon_{kk} \varepsilon^f - e_{ijk} E_i^s \varepsilon_{ij} - \zeta_{ijk} E_i^f \varepsilon_{ij} + \frac{b}{2} \varepsilon^{f2} - \eta_i E_i^s \varepsilon^f - \zeta_i E_i^f \varepsilon^f + \rho_{11} u_i^s u_i^s + \right. \\
& \rho_{22} u_i^f u_i^f + \rho_{12} u_i^s u_i^f + \frac{P_i^s}{T_0} E_i^s \theta^s + \frac{P_i^{sf}}{T_0} E_i^f \theta^s + \frac{P_i^{fs}}{T_0} E_i^s \theta^f + \frac{P_i^f}{T_0} E_i^f \theta^f + \left( \frac{S}{2T_0} \right) \theta^{s2} + \left( \frac{T}{2T_0} \right) \theta^{f2} + \left( \frac{U}{T_0} \right) \theta^{sf} + \\
& \left. \left( \frac{U}{T_0} \right) \theta^s \theta^f \right] dV \leq 0
\end{aligned} \quad (77)$$

Since the aforementioned difference functions satisfy homogeneous initial conditions, Eq. (65), the integral in the left-hand side of Eq. (77) is initially zero. By inequality (76), this integral has two statuses, it equals to zero or decreases to become negative. Only the first status is possible because the integral is the sum of squares and positive terms, i.e.

$$\begin{aligned}
& \int_V \left[ \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} + Q \varepsilon_{kk} \varepsilon^f - e_{ijk} E_i^s \varepsilon_{ij} - \zeta_{ijk} E_i^f \varepsilon_{ij} + \frac{b}{2} \varepsilon^{f2} - \eta_i E_i^s \varepsilon^f - \zeta_i E_i^f \varepsilon^f + \rho_{11} u_i^s u_i^s + \rho_{22} u_i^f u_i^f + \rho_{12} u_i^s u_i^f + \right. \\
& \left. \frac{P_i^s}{T_0} E_i^s \theta^s + \frac{P_i^{sf}}{T_0} E_i^f \theta^s + \frac{P_i^{fs}}{T_0} E_i^s \theta^f + \frac{P_i^f}{T_0} E_i^f \theta^f + \left( \frac{S}{2T_0} \right) \theta^{s2} + \left( \frac{T}{2T_0} \right) \theta^{f2} + \left( \frac{U}{T_0} \right) \theta^s \theta^f \right] dV = 0
\end{aligned} \quad (78)$$

Equating to zero of upper equation, the difference functions are identically zero throughout the body, for all times. Hence, in fact the two assumed solutions were the same, and therefore only one solution exists. This completes the proof of the theorem [22].

#### 4 CONCLUSIONS

Using Biot's theory and the theory of generalized thermoelasticity with on relaxation time (Lord-Shulman), the constitutive equations and equations of motion for describing the behavior of PPM hollow cylinder subjected to electric, thermal and mechanical loads are obtained. All material parameters for both solid and fluid phases are introduced. These parameters can be used to clarify characteristics of skeleton and interstitial fluid. Using the results of this paper, one is able to design and optimize piezo porous cylinder specification such as displacements, stresses, electric potentials and so on.

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#### REFERENCES

- [1] Biot M.A., 1941, General theory of three –dimensional consolidation, *Journal of Applied Physics* **12**:155-164.
- [2] Biot M.A., 1955, Theory of elasticity and consolidation for a porous anisotropic solid, *Journal of Applied Physics* **26**:182-185.
- [3] Biot M.A., Willis D.G., 1954, The elastic coefficients of the theory of consolidation, *Journal of Applied Physics* **24**:594-601.
- [4] Vashishth A.K., Gupta V., 2009, Vibrations of porous piezoelectric ceramic plates, *Journal of Sound and Vibration* **325**:781-797.
- [5] Biot M.A., 1962, Mechanics of deformation and acoustic propagation in porous media, *Journal of Applied Physics* **33**:1482-1498.
- [6] Mandl G., 1964, Change in skeletal volume of a fluid-filled porous body under stress, *Journal of the Mechanics and Physics of Solids* **12**:299–315.
- [7] Brown R.J.S, Korringa J., 1975, The dependence of the elastic properties of a porous rock on the compressibility of the pore fluid, *Geophysics* **40**:608-616.
- [8] Zimmerman R.W., Somerton W.H., King M.S., 1986, Compressibility of porous rocks , *Journal of Geophysical Research* **91**:12765–12777.

- [9] Zimmerman R.W., Myer L.R., Cook N.G.W., 1994, Grain and void compression in fractured and porous rock, *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts* **31**:179–184.
- [10] Garg S. K., Nayfeh A. H., 1986, Compressional wave propagation in liquid and/or gas saturated elastic porous media, *Journal of Applied Physics* **60**:3045-3055.
- [11] Pride S.R., Gangi A.F., Morgan F.D., 1992, Deriving the equations of motion for porous isotropic media, *Journal of the Acoustical Society of America* **92**:3278-3290.
- [12] Thompson M., Willis J.R., 1991, A reformulation of the equations of anisotropic poroelasticity, *Journal of Applied Mechanics* **58**:612–616.
- [13] Albert D. G., 1993, A comparison between wave propagation in water - saturated and air - saturated porous materials, *Journal of Applied Physics* **73**:28-36.
- [14] Cowin S., 1999, Bone poroelasticity, *Journal of Biomechanics* **32**:217-238.
- [15] Tod S.R., 2003, An anisotropic fractured poroelastic effective medium theory, *Geophysical Journal International* **155**:1006–1020.
- [16] Wersing W., Lubitz K., Moliaupt J., 1986, Dielectric elastic and piezoelectric properties of porous PZT ceramics, *Ferroelectrics* **68**:77-97.
- [17] Dunn H., Taya M., 1993, Electromechanical properties of porous piezoelectric ceramics, *Journal of the American Ceramic Society* **76**:1697-1706.
- [18] Kurashige M., 1989, A thermoelastic theory of fluid-filled porous materials, *International Journal of Solids and Structures* **25**: 1039-1052.
- [19] Ciarletta M., Scarpetta E., 1996, Some results on thermoelasticity for porous piezoelectric materials, *Mechanics Research Communications* **23**:1-10.
- [20] Ghassemi A., Diek A., 2002, Porothermoelasticity for swelling shales, *Journal of Petroleum Science and Engineering* **34**:123 –135.
- [21] Batifol C., Zielinski T. G., Galland M.A., Ichchou M. N., 2006, Hybrid piezo-poroelastic sound package concept: numerical/experimental validations, *Proceedings of International Symposium on Active Control of Sound and Vibration, Adelaide, Australia*.
- [22] Youssef H.M., 2007, Theory of generalized porothermoelasticity, *International Journal of Rock Mechanics and Mining Sciences* **44**:222-227.
- [23] Nasedkin A., 2009, New model for piezoelectric porous medium with application to analysis of ultrasonic piezoelectric transducers, *Proceedings of the 7th Euromech Solid Mechanics Conference, Lisbon, Portugal*.
- [24] Zielinski T.G., 2010, Fundamentals of multiphysics modelling of piezo-poro-elastic structures, *Archives of Mechanics* **62**:343-378.
- [25] Sharma M.D., 2010, Piezoelectric effect on the velocities of waves in an anisotropic piezo-poroelastic medium, *Proceedings the Royal of Society A Mathematical Physical and Engineering sciences* **466**:1977-1992.
- [26] Wang H.F., 2000, *Theory of Linear Poroelasticity: with Application to Geomechanics and Hydrogeology*, USA, Princeton University Press.
- [27] Coussy O., 2004, *Poromechanics*, New York, Wiley, First edition.
- [28] Coussy O., 2010, *Mechanics and Physics of Porous Solids*, New York, Wiley, First edition.
- [29] Ikeda T., 1990, *Fundamentals of Piezoelectricity*, Oxford University Press.
- [30] Hetnarski B., Eslami M.R., 2009, *Thermal Stresses: Advanced Theory and Applications*, USA, Springer, First edition.
- [31] Kidner M. R. F., Hansen C. H., 2008, A comparison and review of theories of the acoustics of porous materials, *International Journal Acoustics and Vibration* **13**:112-119.
- [32] Yang D., Zhang Z., 2002, Poroelastic wave equation including the Biot/squirt mechanism and the solid/fluid coupling anisotropy, *Wave Motion* **35**:223–245.
- [33] Kraus J.D., 1984, *Electromagnetic*, USA, McGraw-Hill Inc, Second edition.
- [34] Deresiewicz H., Skalak R., 1963, On uniqueness in dynamic poroelasticity, *Bulletin of the Seismological Society of America* **53**:783-788.