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Damping Ratio in Micro-Beam Resonators Based on Magneto-Thermo-Elasticity

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ABSTRACT

This paper investigates damping ratio in micro-beam resonators based on magneto-thermo-elasticity. A unique aspect of the present study is the effect of permanent magnetic field on the stiffness and thermo-elastic damping of the micro resonators. In our modeling the theory of thermo-elasticity with interacting of an externally applied permanent magnetic field is taken into account. Combined theoretical and numerical studies investigate the permanent magnetic field effect on the damping ratio in clamped-clamped and cantilever micro-beams. Furthermore, the influence of the magnetic field intensity on the frequency of the micro-beams with thermo-elastic damping effect is evaluated. Such evaluations are used to determine the influence of magnetic field on the vibration amplitude of the resonators. The meaningful conclusion is that the magnetic field increases the equivalent stiffness and thermo-elastic damping and consequently the energy consumption of the resonators.

Keywords : Thermo-elasticity; MEMS; Maxwell stress tensor; Lorentz force; Magnetic field.

1 INTRODUCTION

MICRO-ELECTRO-MECHANICAL-SYSTEMS (MEMS) technology has been rapidly growing since its beginning in 1980s. The mechanical elements that are used in MEMS devices are typically simple elements, including micro-beams, plates and membranes. On account of their versatility and economy, MEMS devices are finding numerous applications. Cantilever and fixed-fixed micro-beams are used in many MEMS devices such as MEMS resonators and resonant sensors. With the development of MEMS technology, thousands of micro-sensors can be fabricated together on a single silicon wafer and this leads to the production of the sensors with lower cost and smaller size [1, 2]. MEMS resonators have been studied for more than 30 years by many groups, with steady interest in their potential for frequency control and sensing applications in electronic systems. It is desired to design MEMS and NEMS resonators with very little loss of energy or very high performance. To obtain high performance of resonators, it is necessary to build the resonator that works with high quality factor. The quality factor of the resonator is a measure of the amount of energy loss. There are different types of dissipation mechanisms, which lower the quality factors of micro-structures. In order to enhance damping characteristics, numerous papers reported internal and external energy loss mechanisms. These loss mechanisms can be classified into thermo-elastic damping (TED), air damping, clamping loss and thermo-diffusive-elastic-damping [3, 4]. The most important intrinsic loss mechanism is the TED that was first explained by Zener [5]. The main motivation for this research is that the MEMS resonators are often set in permanent magnetic fields, the influence of the magnetic

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field on the attenuation mechanism and mechanical performance of these instruments and also the effect of Lorentz forces resulting from the applied magnetic field on the thermo-elastic damping of the resonators has not been investigated. A permanent magnet is a magnet that does not lose its magnetic field and remains over time without weakening.

In the previous investigations, the researchers were focused more on the attenuation effect of eddy currents and magnetic force in the resonators and magnetic field sensors, but in this study the effect of a permanent magnetic field in the MEMS resonators based on the theory of Magneto-Thermo-Elasticity (MTE) is discussed. Theory of MTE is concerned with interacting effects of an externally applied permanent magnetic field on the thermo-elasticity. If an electrically conducting elastic micro-beam resonator is deflected while immersed in a magnetic field, the laws of Hooke and Maxwell will still determine the elastic field and electro-magnetic field respectively, but superposition of these two fields results in from the fact that electro-magnetic field influences the elastic field by entering the elastic stress equations of motion as a force called as the Lorentz's force and Maxwell stress equation. Inspections on the behavior of interaction between magnetic, thermal, concentration and strain fields in solid mechanics have been conducted by many researchers. Investigation of this interaction is available in many papers such as Nowacki [6], sherief et al. [7], Ezzat and Awad [8], Abd-Alla et al. [9], Othman [10], Singh and Kumar [11], Aouadi [12], Sharma et al. [13], Kumar and Deswal [14], Moon and Pao [15], Shih et al. [16], Sharma [17] and Liu and Chang [18].

In our model, we consider the magneto-thermo-elastic damping effect based on Euler-Bernoulli beam assumptions. Governing equation of motion has been obtained by Hamilton's principle including thermo-elastic and magnetic field couplings. In this paper Maxwell's stress tensor and Lorentz force are used in order to formulate the effect of permanent magnetic field on the vibrations of the micro-beams. Vibration of the clamped-clamped and cantilever micro-beam resonators with isothermal boundary conditions at the ends are studied using Galerkin reduced order model formulation for the first mode of vibration. Using the Runge-Kutta method, the amplitude versus time diagram for the first mode is determined. The obtained results are compared with the numerical results of the thermo-elastic models. The effects of magnetic field intensity, thermo-elastic damping and ambient temperature on the damping ratio, frequency and amplitude of the micro-beams are discussed.

2 PROBLEM FORMULATIONS

The proposed models are two micro-beams (Cantilever and Clamped-Clamped) that are under the influence of a permanent magnetic field. Fig. 1 shows the schematic diagram of the physical system. In the present investigation small amplitudes of vibrations of a thin elastic micro-beam with dimensions of length L, width b and thickness h, is considered and the permanent magnetic field is acting towards the positive direction of the y axis $(\vec{H}_0 = (0, H, 0))$.

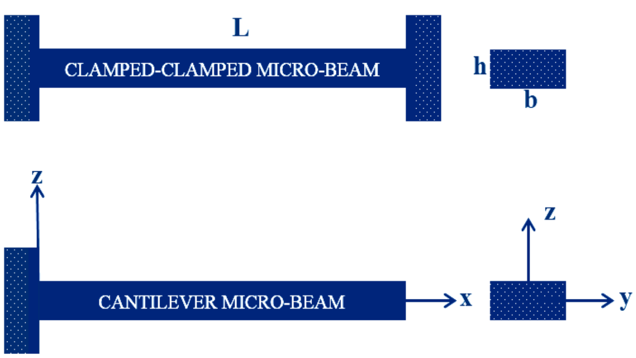

Fig.1
Schematic diagram of the physical system.

The displacements based on the Euler–Bernoulli beam assumption are given by:

$$\vec{u} = (u, v, w), \qquad u = -z\frac{\partial w}{\partial x}, \tag{1}$$

where $\vec{u}$ is the mechanical displacement vector. For a narrow beam (plane stress condition) the trace of the strain tensor and the component of the stress tensor in the x direction are as follows [3, 4, 21, 22]:

$$e=-(1-2v)z\frac{\partial^2 w}{\partial x^2}+2(1+v)\alpha_t T$$
$$\sigma_{xx}=-Ez\frac{\partial^2 w}{\partial x^2}-E\alpha_t T \tag{2}$$

For a wide beam (plane strain condition) the trace of the strain tensor and the component of the stress tensor in the *x* direction are as follows:

$$e=-\left(\frac{1-2v}{1-v}\right)z\frac{\partial^2 w}{\partial x^2}+\frac{1+v}{1-v}\alpha_t T$$
$$\sigma_{xx}=-\left(\frac{E}{1-v^2}\right)z\frac{\partial^2 w}{\partial x^2}-\frac{E\alpha_t}{1-v}T \tag{3}$$

where $T=T-T_0, T$ is the absolute temperature and T_0 is the temperature of the micro-beam in the natural state assumed to be equal to the ambient temperature, α_t is the coefficient of the linear thermal expansion, σ_{ij} is the components of the stress tensor, u_i is the components of the displacement vector, e_{ij} is the components of the strain tensor, *e* is the trace of the strain tensor, *v* is Poisson ratio and δ_{ij} is the Kronecker delta.

The micro-beam is a perfect electric conductor, and the linearized Maxwell equations are governing the magnetic field. In simple situations, such as a point charge moving in a permanent magnetic field, it is easy to calculate the forces on the point charge from the Lorentz force law. When the situation becomes more complicated, this ordinary procedure can become difficult. Therefore, it is convenient to calculate magnetic stresses from the Maxwell stress tensor [8, 10]:

$$\tau_{ij}=\mu_0\left[H_i h_j+H_j h_i-H_k h_k\delta_{ij}\right], \tag{4}$$

where μ_0 is magnetic permeability, h_i is the components of induced magnetic field vector over the primary magnetic field vector $\vec{H}_0, H_i$ is the components of the total magnetic field vector, τ_{ij} is the components of the Maxwell stress tensor and δ_{ij} is the Kronecker delta. The variations of the magnetic and electric fields for a perfectly conducting micro-beam resonator are given by Maxwell's equations [8, 10]:

$$div\vec{h}=0,\quad curl\ \vec{h}=\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t},\qquad div\vec{E}=0,\quad curl\ \vec{E}=-\mu_0\frac{\partial\vec{h}}{\partial t}, \tag{5}$$

where

$$\vec{h}=curl\left(\vec{u}\times\vec{H}_0\right)=\left(0,-H(e_x+e_z),0\right)$$
$$\vec{H}=\vec{H}_0+\vec{h}=\left(0,H(1-e_x-e_z),0\right) \tag{6}$$
$$\vec{E}=-\mu_0\left(\frac{\partial\vec{u}}{\partial t}\times\vec{H}\right)=\left(\mu_0\dot{w}H,0,-\mu_0\dot{u}H\right)$$

$\vec{E}$ is the induced electric field vector, $\vec{J}$ is the current density vector, ε_0 is the electric permeability, $\vec{H}_0$ and $\vec{H}$ are the initial magnetic vector and total magnetic field vector respectively. The initial magnetic field vector $\vec{H}_0$ is:

$$\vec{H}_0=\left(0,H,0\right) \tag{7}$$

H is the primary permanent magnetic field intensity. The nonzero components of the Maxwell stress tensor are as follows (after linearization):

$$\tau = \mu_0 H^2 \begin{bmatrix} (e_x + e_z) & 0 & 0 \\ 0 & -(e_x + e_z) & 0 \\ 0 & 0 & (e_x + e_z) \end{bmatrix} \tag{8}$$

In all previous papers it was assumed that the interactions between the elastic and magnetic fields take place by means of the Lorentz forces appearing in the equation of motion. $\vec{F}$ is the Lorentz force given by [8, 10]:

$$\begin{aligned} &\vec{F} = \mu_0 \left(\vec{J} \times \vec{H} \right) \\ &\vec{J} = \left(-\left[\frac{\partial h_y}{\partial z} + \varepsilon_0 \mu_0 (\ddot{w} H_y + \dot{w} \dot{H}_y \right], 0, \left[\frac{\partial h_y}{\partial x} + \varepsilon_0 \mu_0 (\ddot{u} H_y + \dot{u} \dot{H}_y \right] \right) \end{aligned} \tag{9}$$

From the above equations, we can obtain the components of the Lorentz force vector (after linearization):

$$\begin{aligned} &F_x = -\mu_0 H^2 z \frac{\partial^3 w}{\partial x^3} + \varepsilon_0 \mu_0^2 H^2 z \frac{\partial^3 w}{\partial x \, \partial t^2} \\ &F_y = 0 \\ &F_z = -\mu_0 H^2 \frac{\partial^2 w}{\partial x^2} - \varepsilon_0 \mu_0^2 H^2 \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{10}$$

Total stress tensor is as follows:

$$\sigma_{Tij} = \sigma_{ij} + \tau_{ij} \tag{11}$$

Substituting the τ_x in the Eq. (11), the total stress tensor for a narrow beam takes the following form:

$$\sigma_{Txx} = -Bz \frac{\partial^2 w}{\partial x^2} - B_t T \tag{12}$$

where

$$B = E + (1-v)\mu_0 H^2, \qquad B_t = E \alpha_t - \mu_0 H^2 (1+v)\alpha_t , \tag{13}$$

The total stress tensor for a wide beam takes the following form:

$$\sigma_{Txx} = -Bz \frac{\partial^2 w}{\partial x^2} - B_t T \tag{14}$$

where

$$B = \frac{E}{(1-v^2)} + \left(\frac{1-2v}{1-v} \right) \mu_0 H^2, \qquad B_t = \frac{E \alpha_t}{1-v} - \frac{1+v}{1-v} \alpha_t \mu_0 H^2 , \tag{15}$$

3 MOTION EQUATION OF THE MICRO-BEAM RESONATOR

The mechanical bending strain energy, U_m of the beam in terms of heat conduction is given by:

$$U_m = \int_0^L \int_A \left(\frac{1}{2} B e_x{}^2 - B_t T e_x \right) dA\, dx \tag{16}$$

$$U_m = \frac{1}{2} \int_0^L BI \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \int_0^L \frac{\partial^2 w}{\partial x^2} M_T dx \tag{17}$$

where

$$M_T = \int_{-h/2}^{h/2} b B_t T z dz \ ; \tag{18}$$

M_T is the thermal moment. $I = bh^3/12$ is the moment of inertia of the cross-sectional area *A*. On the other hand, the work done by the components of the Lorentz forces U_L in the form of transverse loading and axial loading reads:

$$\begin{aligned} &U_L = \int_0^L \int_A F_z w \text{ dAdx} + \frac{1}{2} \int_0^L \int_A \mathrm{F}_x w'^2 \text{ dAdx} \ ; \\ &\mathrm{U} = \mathrm{U}_m + \mathrm{U}_L \end{aligned} \tag{19}$$

The kinetic energy of the micro-beam can be written as:

$$K = \frac{1}{2} \int_0^L \rho A \left(\frac{\partial w}{\partial t} \right)^2 dx \tag{20}$$

By presenting Lagrangian $\mathcal{L}$ and extremizing it the equation of motion will be derived [19].

$$\begin{aligned} &\mathcal{L} = K - U = \int_0^L F dx \ ; \\ &F = \frac{1}{2} \rho A \dot{w}^2 - A F_z w - \frac{1}{2} A F_x w'^2 - M_T w'' - \frac{1}{2} BI w''^2 \end{aligned} \tag{21}$$

According to the calculus of variation the following condition should be satisfied:

$$\delta \int_{t_1}^{t_2} \mathcal{L} dt = \int_{t_1}^{t_2} \int_0^L \delta F(\mathrm{w}'', \mathrm{w}', \mathrm{w}, \dot{\mathrm{w}}) dx dt = o \ ; \tag{22}$$

$$\begin{aligned} &\int_{t_1}^{t_2} \int_0^L \left[\frac{\partial^2}{\partial x^2} \left(\frac{\partial F}{\partial w''} \right) - \frac{\partial}{\partial x} \left(\frac{\partial F}{\partial w'} \right) + \frac{\partial F}{\partial w} - \frac{\partial}{\partial t} \left(\frac{\partial F}{\partial \dot{w}} \right) \right] \delta w dx dt - \int_{t_1}^{t_2} \frac{\partial}{\partial x} \left(\frac{\partial F}{\partial w''} \right) \delta w \Big|_0^L dt + \int_{t_1}^{t_2} \frac{\partial F}{\partial w'} \delta w \Big|_0^L dt \\ &+ \int_{t_1}^{t_2} \frac{\partial F}{\partial w''} \delta w' \Big|_0^L dt + \int_0^L \left(\frac{\partial F}{\partial \dot{w}} \right) \delta w \Big|_{t_1}^{t_2} dx = 0 \end{aligned} \tag{23}$$

The dynamic governing equation of the beam in terms of magnetic field and heat conduction:

$$\frac{\partial^2}{\partial x^2}\left(\frac{\partial F}{\partial w''}\right)-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial w'}\right)+\frac{\partial F}{\partial w}-\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial \dot{w}}\right)=0$$
$$BI\,\frac{\partial^4 w}{\partial x^4}+\frac{\partial^2 M_T}{\partial x^2}+\frac{\partial}{\partial x}\left(\mathrm{A}F_x\,\frac{\partial w}{\partial x}\right)+\rho\mathrm{A}\frac{\partial^2 w}{\partial t^2}-AF_z=0 \tag{24}$$

After linearization:

$$BI\,\frac{\partial^4 w}{\partial x^4}+\frac{\partial^2 M_T}{\partial x^2}+\mu_0 H^2 A\,\frac{\partial^2 w}{\partial x^2}+(\rho A+\varepsilon_0\mu_0^2 H^2 A)\frac{\partial^2 w}{\partial t^2}=0 \tag{25}$$

The initial conditions:

$$\left(\frac{\partial F}{\partial \dot{w}}\right)\delta w\,\Big|_{t_1}^{t_2}=0 \tag{26}$$

The other equations prescribed at $x=0$ and $x=L$ as the boundary conditions:

$$\frac{\partial F}{\partial w''}\delta w'\,\Big|_0^L=0 \tag{27}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial w''}\right)\delta w\,\Big|_0^L=0 \tag{28}$$

$$\frac{\partial F}{\partial w'}\delta w\,\Big|_0^L=0 \tag{29}$$

4 THERMO-ELASTIC DAMPING

When the resonator is set in motion, it is taken out of equilibrium, having an excess of kinetic and potential energy. In the micro-beam resonator the coupling of the strain field to a temperature field provides an energy dissipation mechanism that this process of energy dissipation is called thermo-elastic damping. TED acts as a source of mechanical thermal noise which results in diminished quality factor and consequently higher energy consumption. Since Energy loss resulting from the TED is the main intrinsic mechanism of energy dissipation, finding a way to reduce TED, is a serious affair. The equation of coupled thermo-elasticity is as follows [3, 4]:

$$-k\,\frac{\partial^2 T}{\partial x^2}-k\,\frac{\partial^2 T}{\partial z^2}+\left(\rho C_v+\gamma E\,\alpha_t^2 T_0\right)\frac{\partial T}{\partial t}+\left(\rho C_v\,\tau_{0t}+\gamma E\,\alpha_t^2 T_0\tau_{0t}\right)\frac{\partial^2 T}{\partial t^2}-\left(\beta_t T_0\right)z\,\frac{\partial^3 w}{\partial x^2\partial t}-\left(\beta_t T_0\tau_{0t}\right)z\,\frac{\partial^4 w}{\partial x^2\partial t^2}=0 \tag{30}$$

where k is the thermal conductivity, τ_{0t} is thermal relaxation time, ρ is the density, C_v is the specific heat at constant strain and γ for the plane stress condition (narrow beam) is $2(1+v)/(1-2v)$ and for the plane strain condition (wide beam) is $(1+v)/(1-2v)(1-v)$. Here β_t and β_c equal to $E\alpha_t$ and $E\alpha_c$ for a narrow beam and also respectively equal to $E\alpha_t/(1-v)$ and $E\alpha_c/(1-v)$ for a wide beam. Note that for a wide beam, for which $b\geq 5h$ [3]. As the thermal boundary condition assumptions there is no heat flow through the free surfaces of the micro-beam. In this article vibration of a thin elastic micro-beam without considering the stretching (because of small amplitudes) is studied.

Following dimensionless quantities are defined to transform Eqs. (25) and (30) into non-dimensional forms:

$$\widehat{w}=\frac{w}{h},\widehat{x}=\frac{x}{L},\widehat{z}=\frac{z}{h},\widehat{T}=\frac{T}{T_0},\widehat{t}=\frac{t}{t^*},t^*=L\sqrt{\frac{\rho+\varepsilon_0\mu_0^2H^2}{E}},\widehat{M}_T=\frac{M_T}{T_0B_tbh^2}, \tag{31}$$

Applying these dimensionless quantities equations will take the following forms:

$$S_1\frac{\partial^4\widehat{w}}{\partial\widehat{x}^4}+S_2\frac{\partial^2\widehat{M}_T}{\partial\widehat{x}^2}+S_3\frac{\partial^2\widehat{w}}{\partial\widehat{x}^2}+\frac{\partial^2\widehat{w}}{\partial\widehat{t}^2}=0 \tag{32}$$

$$\frac{\partial^2\widehat{T}}{\partial\widehat{x}^2}+S_4\frac{\partial^2\widehat{T}}{\partial\widehat{z}^2}-S_5\frac{\partial\widehat{T}}{\partial\widehat{t}}+S_6\widehat{z}\frac{\partial^3\widehat{w}}{\partial\widehat{x}^2\partial\widehat{t}}-S_7\frac{\partial^2\widehat{T}}{\partial\widehat{t}^2}+S_8\widehat{z}\frac{\partial^4\widehat{w}}{\partial\widehat{x}^2\partial\widehat{t}^2}=0 \tag{33}$$

In which

$$S_1=\frac{bh^2}{12EL^2};\quad S_2=\frac{T_0B_t}{E};\quad S_3=\frac{\mu_0H^2}{E};\quad S_4=\frac{L^2}{h^2};\quad S_5=\frac{\left(\rho C_v+\gamma E\alpha_t^2T_0\right)L^2}{kt^*};\quad S_6=\frac{h^2\beta_t}{kt^*};$$
$$S_7=\frac{\left(\rho C_v\tau_{0t}+\gamma E\alpha_t^2T_0\tau_{0t}\right)L^2}{kt^{*2}};\quad S_8=\frac{\tau_{0t}\beta_th^2}{kt^{*2}}; \tag{34}$$

5 NUMERICAL SOLUTIONS

In order to analyze the frequency of the resonator a Galerkin based reduced order method is used:

$$\hat{w}\left(\hat{x},\hat{t}\right)=\sum_{k=1}^{p}\varpi_k\left(\hat{t}\right)\psi_k\left(\hat{x}\right) \tag{35}$$

$$\hat{T}\left(\hat{x},\hat{z},\hat{t}\right)=\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}\left(\hat{t}\right)\varphi_i\left(\hat{x}\right)\phi_j\left(\hat{z}\right) \tag{36}$$

M_T can be represented in non-dimensional form as follows:

$$\hat{M}_T=\int_{-0.5}^{0.5}\hat{T}\hat{z}d\hat{z}=\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i(\hat{x})\int_{-0.5}^{0.5}\hat{z}\,\phi_j(\hat{z})d\hat{z} \tag{37}$$

Substituting Eqs. (35-37) into Eqs. (32) and (33) leads to the following equations:

$$S_1\sum_{k=1}^{p}\varpi_k(\hat{t})\psi_k^{(IV)}(\hat{x})+S_2\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i''(\hat{x})\int_{-0.5}^{0.5}\hat{z}\,\phi_j(\hat{z})d\hat{z}+S_3\sum_{k=1}^{p}\varpi_k(\hat{t})\psi_k'(\hat{x})+\sum_{k=1}^{p}\ddot{\varpi}_k(\hat{t})\psi_k(\hat{x})=\varepsilon_1 \tag{38}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i''(\hat{x})\phi_j(\hat{z})+S_4\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j^{\circ\circ}(\hat{z})-S_5\sum_{i=1}^{n}\sum_{j=1}^{m}\dot{u}_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j(\hat{z})+S_6\hat{z}\sum_{k=1}^{p}\dot{\varpi}_k(\hat{t})\psi_k''(\hat{x})$$
$$-S_7\sum_{i=1}^{n}\sum_{j=1}^{m}\ddot{u}_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j(\hat{z})+S_8\hat{z}\sum_{k=1}^{p}\ddot{\varpi}_k(\hat{t})\psi_k''(\hat{x})=\varepsilon_2 \tag{39}$$

According to Galerkin method the following conditions should be satisfied:

$$\int_0^1 \psi_f(\hat{x})\varepsilon_1 d\hat{x} = 0 \qquad f = 1,...,p \tag{40}$$

$$\int_0^1 \int_{-0.5}^{0.5} \varphi_q(\hat{x})\phi_g(\hat{z})\varepsilon_2 d\hat{z}d\hat{x} = 0 \ ; \quad q = 1,...,n \ , \ g = 1,...,m \tag{41}$$

Now applying Eqs. (40) and (41) to Eqs. (38) and (39) leads to the following equations:

$$S_1\sum_{k=1}^{p}\varpi_k K_{fk}^{(1)} + S_2\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}K_{fi}^{(2)}K_j^{(3)} + \sum_{k=1}^{p}\ddot{\varpi}_k K_{fk}^{(4)} + S_3\sum_{k=1}^{p}\varpi_k K_{fk}^{(5)} = 0 \tag{42}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}G_{qi}^{(2)}G_{gj}^{(4)} + S_4\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}G_{qi}^{(1)}G_{gj}^{(5)} - S_5\sum_{i=1}^{n}\sum_{j=1}^{m}\dot{u}_{ij}G_{qi}^{(1)}G_{gj}^{(4)} + S_6\sum_{k=1}^{p}\dot{\varpi}_k G_{qk}^{(3)}G_g^{(6)} - S_7\sum_{i=1}^{n}\sum_{j=1}^{m}\ddot{u}_{ij}G_{qi}^{(1)}G_{gj}^{(4)}\\&+S_8\sum_{k=1}^{p}\ddot{\varpi}_k G_{qk}^{(3)}G_g^{(6)} = 0\end{aligned} \tag{43}$$

In which

$$\begin{aligned}&K_{fk}^{(1)} = \int_0^1 \psi_f(\hat{x})\psi_k^{(IV)}(\hat{x})d\hat{x} \ ; \ K_{fi}^{(2)} = \int_0^1 \psi_f(\hat{x})\varphi_i''(\hat{x})d\hat{x} \ ; \ K_j^{(3)} = \int_{-0.5}^{0.5} \hat{z}\,\phi_j(\hat{z})d\hat{z} \ ;\\&K_{fk}^{(4)} = \int_0^1 \psi_f(\hat{x})\psi_k(\hat{x})d\hat{x} \ ; \ K_{fk}^{(5)} = \int_0^1 \psi_f(\hat{x})\psi_k''(\hat{x})d\hat{x}\end{aligned} \tag{44}$$

For the second coupled equation:

$$\begin{aligned}&G_{qi}^{(1)} = \int_0^1 \varphi_q(\hat{x})\varphi_i(\hat{x})d\hat{x} \ ; \ G_{qi}^{(2)} = \int_0^1 \varphi_q(\hat{x})\varphi_i''(\hat{x})d\hat{x} \ ; \ G_{qk}^{(3)} = \int_0^1 \varphi_q(\hat{x})\psi_k''(\hat{x})d\hat{x} \ ;\\&G_{gj}^{(4)} = \int_{-0.5}^{0.5} \phi_g(\hat{z})\phi_j(\hat{z})d\hat{z} \ ; \ G_{gj}^{(5)} = \int_{-0.5}^{0.5} \phi_g(\hat{z})\phi_j^{\circ\circ}(\hat{z})d\hat{z} \ ; \ G_g^{(6)} = \int_{-0.5}^{0.5} \hat{z}\,\phi_g(\hat{z})d\hat{z}\end{aligned} \tag{45}$$

By solving Eqs. (42) and (43) in which:

$$\varpi_k = \bar{\varpi}_k e^{i\Omega_k t} ; u_{ij} = \bar{u}_{ij}e^{i\Omega_{ij} t} \tag{46}$$

Frequencies are obtained ($\hat{T}$ and $\hat{w}$ vibrate at the same frequency, $\Omega_k = \Omega_{ij} = \Omega$ [20]). Damping ratio can be calculated as [3]:

$$\zeta = \left|\frac{\Im(\Omega)}{\sqrt{\Re^2(\Omega) + \Im^2(\Omega)}}\right| \tag{47}$$

where $\Re(\Omega)$ and $\Im(\Omega)$ are the real and imaginary part of the ζ respectively.

6 NUMERICAL RESULTS

The proposed models have the following properties as shown in Table 1. The copper material (a diamagnetic metal) was chosen for purposes of numerical evaluations [8, 10]. Assume coefficient of linear thermal expansion, magnetic permeability, electric permeability, thermal conductivity and the other material properties of the model are constant.

Table 1
Geometrical and material properties of the proposed model.

Symbols	Parameters	Values
L	Length	100 μm
b	Width	5 μm
h	Thickness (Wherever not mentioned)	10 μm
E	Young's modulus	120 GPa
v	Poisson's ratio	0.34
K	Thermal conductivity	383 $W\,m^{-1}\,K^{-1}$
ρ	Density	8954 $kg\,m^{-3}$
C_v	Specific heat at constant volume	383.1 $J\,kg^{-1}\,K^{-1}$
α_t	Coefficient of linear thermal expansion	$1.78\times10^{-5}\,K^{-1}$
T_0	Ambient Temperature	300 K
μ_0	Magnetic permeability	$4\pi\times10^{-7}\,N\,ms^2C^{-2}$
ε_0	Electric permeability	$10^{-9}/36\pi\;C^2N^{-1}m^{-2}$

Figs. 2 and 3 are the comparison of calculated results of damping ratio to magneto-thermo-elastic damping model versus beam thicknesses for various values of ambient temperature for cantilever and clamped-clamped micro-beam resonators respectively. In these figures the value of the permanent magnetic field intensity is $H_0 = 1\times10^5 A/m$. The related thickness to the maximum value of the damping ratio can be known as the magneto-thermo-elastic damping critical thickness. It must be noted that the critical thickness takes place when the thermal characteristic time (The time necessary for temperature gradients to relax) is equal to the inverse of the beam fundamental frequency [20]. One can understand from these figures that the damping ratio increases with the increasing of the ambient temperature and this increment are shown clearly in Figs. 6 and 7.

Figs. 4 and 5 are the comparison of calculated results of damping ratio versus beam thicknesses for various values of permanent magnetic field intensity for cantilever and clamped-clamped micro-beam resonators respectively. In these figures solid lines represent the solution of the problem in the absence of a magnetic field, while dashed lines represent the solution when the permanent magnetic field is under consideration. It is seen from these figures that the magnetic field acts to increase the magnitude of the damping ratio and this is usually known as the magnetic damping.

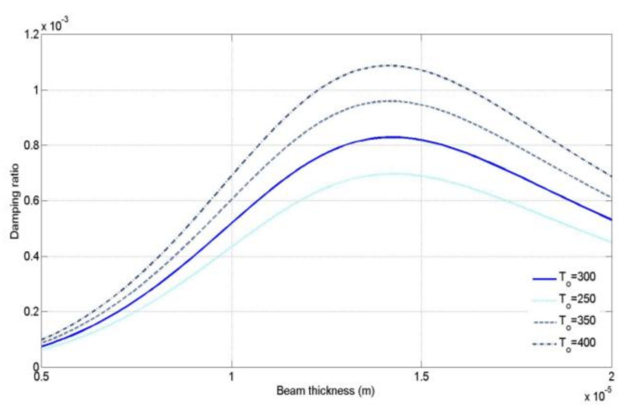

Fig.2
MTE damping ratio versus beam thickness (cantilever beam).

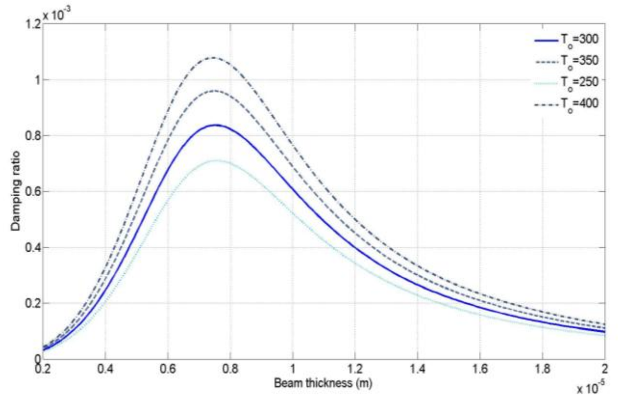

Fig.3
MTE damping ratio versus beam thickness (clamped-clamped beam).

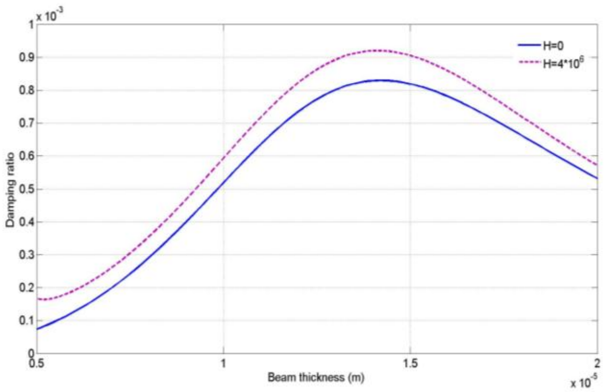

Fig.4
Damping ratio versus beam thickness (cantilever beam).

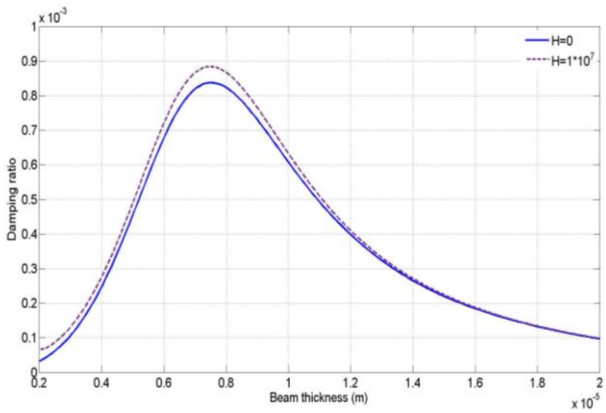

Fig.5
Damping ratio versus beam thickness (clamped-clamped beam).

We have plotted damping ratio, in Figs. 6 and 7 as a function of ambient temperatures for constants magnetic intensity $H_0 = 0$ and $H_0 = 4\times10^6 A/m$ for the cantilever micro-beam and for constants magnetic intensity $H_0 = 0$ and $H_0 = 1\times10^7 A/m$ for the clamped-clamped micro-beam respectively. Note that when the magnetic intensity is equal to zero, it is the same to the TDED model. As it is obvious from these figures the damping ratio increases with the increasing of the ambient temperature and magnetic field intensity. This increment of damping ratio in cantilever beam is more than the clamped-clamped beam.

In Figs. 8 and 9 damping ratio for various values of magnetic field intensity are represented for cantilever and clamped-clamped micro beams respectively. As is evident, in the presence of the permanent magnetic field the damping of the micro-beams increases, in other word energy consumption of the resonators increases. Therefore, magnetic field has an important effect on the damping ratio of the micro resonators. Thus, in designing of the micro resonators, with high quality factor and consequently to minimizing energy consumption the effect of the permanent magnetic field on the damping ratio must be taken into account.

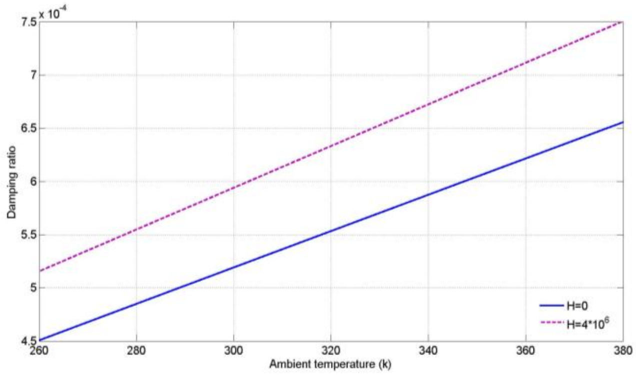

Fig.6
Damping ratio versus ambient temperature (cantilever beam).

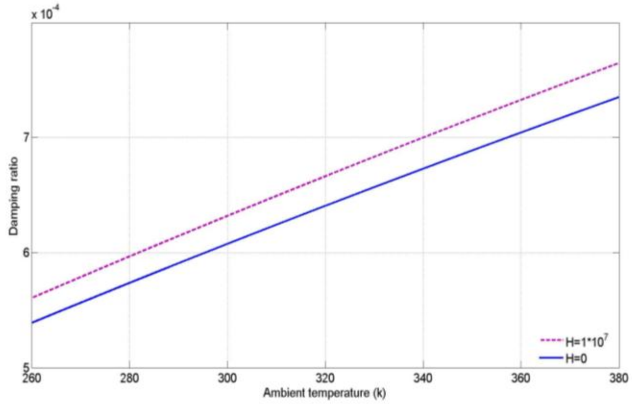

Fig.7
Damping ratio versus ambient temperature (clamped-clamped beam).

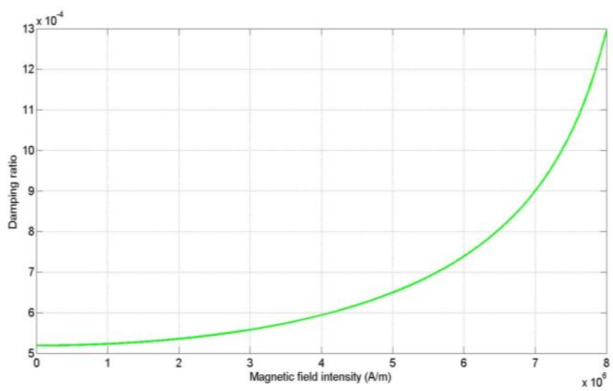

Fig.8
Damping ratio versus magnetic intensity (cantilever beam).

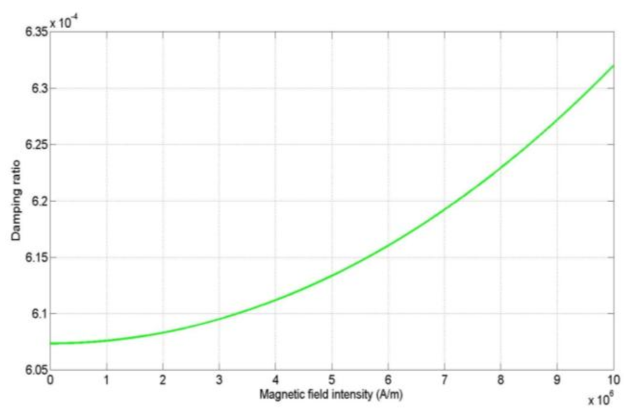

Fig.9
Damping ratio versus magnetic intensity (clamped-clamped beam).

Dimensionless frequency for the both cantilever and clamped-clamped beams versus magnetic intensity is illustrated in Figs. 10 and 11 respectively. As it can be obtained from these figures, by increasing the permanent magnetic field intensity, frequency of the micro-beams decreases and this is due to the influence of the Lorentz force and Maxwell stresses on the vibrations of the micro-beams.

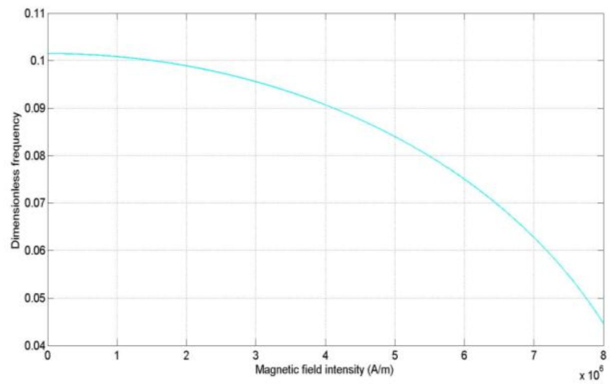

Fig.10
Dimensionless frequency versus magnetic intensity (cantilever beam).

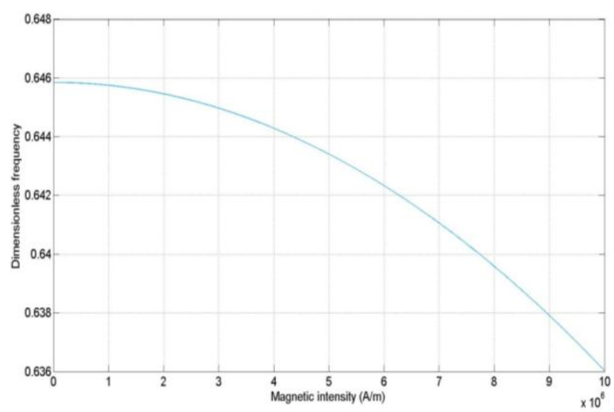

Fig.11
Dimensionless frequency versus magnetic intensity (clamped-clamped beam).

Figs. 12 and 13 respectively, are the dynamic vibrations of the free end of the cantilever beam and the midpoint of the clamped-clamped micro-beam in the presence and absence of the magnetic field. Vibration of the beams decays with time increasing when the coupling between the strain and temperature fields is taken into account. This decrement of the amplitude increases when the permanent magnetic field is considered. Since the vibration of the beams weakens so slowly that it is hardly distinguished from the curve in these figures, the difference between the vibrations of the steady state and MTDE mode is shown in Figs. 14 and 15 to see the vibration decay caused by magneto-thermo-elastic damping. These comparisons are made for different values of the magnetic intensity.

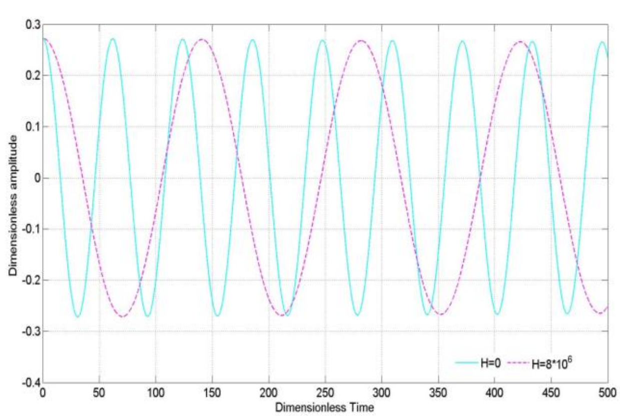

Fig.12
Dimensionless amplitude versus dimensionless time (cantilever beam).

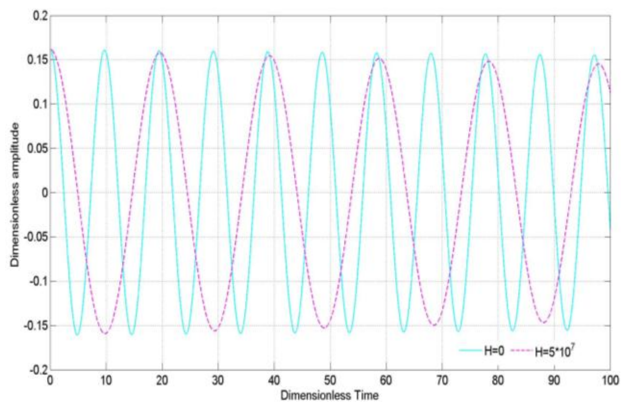

Fig.13
Dimensionless amplitude versus dimensionless time (clamped-clamped beam).

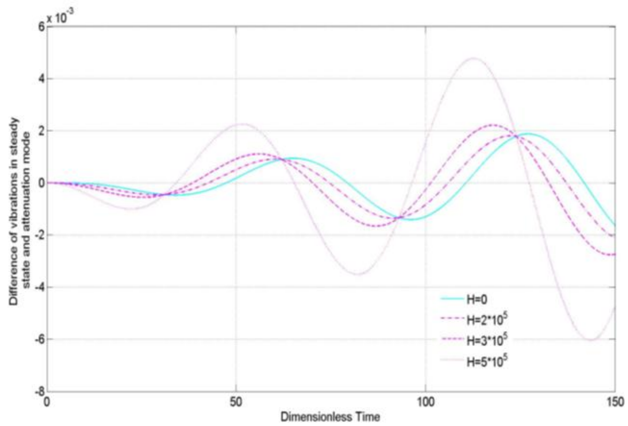

Fig.14
Effect of MTE on the vibration for various values of magnetic intensity (cantilever beam).

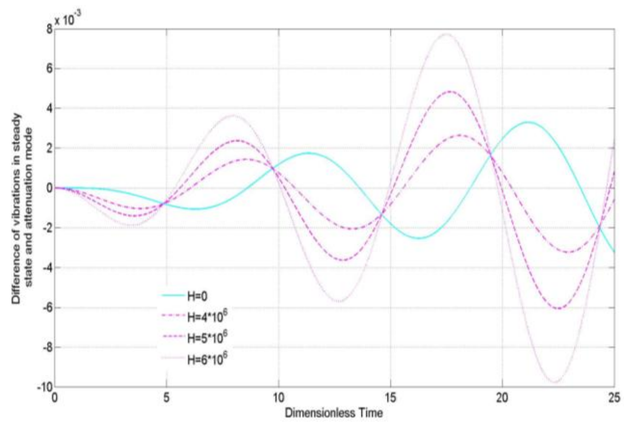

Fig.15
Effect of MTE on the vibration for various values of magnetic intensity (clamped-clamped beam).

7 CONCLUSIONS

In the present work, the damping ratio in the both cantilever and clamped-clamped micro-beam resonators based on magneto-thermo-elasticity was studied. A unique aspect of the present study is the effect of permanent magnetic field on the attenuation mechanism, thermo-elastic damping and frequency of the micro resonators. Using Hamilton's principle, governing equations of the problem based on Euler–Bernoulli beam assumptions by considering thermo-elastic damping effect were derived. Maxwell's stress tensor and Lorentz force were used in order to formulate the effect of the permanent magnetic field on the vibrations of the micro-beams. In order to dynamic analysis, Galerkin reduced order model was employed.

From the numerical results, following conclusions can be drawn: The magnetic field acts to increase the magnitude of the damping ratio and this is usually known as the magnetic damping and also the damping ratio increases with the increasing of the ambient temperature. This increment in cantilever beams is more than the clamped-clamped beam model. The increasing of the damping ratio due to the magnetic field decreases the quality factor and consequently increases the energy consumption of the resonators. Therefore, the magnetic field has an important effect on the damping ratio of the micro resonators. Thus, in designing of the micro resonators, with high quality factor and less energy consumption the effect of the permanent magnetic field on the damping ratio must be taken into account.

In addition, our investigations unveil that by increasing the permanent magnetic field intensity, frequency of the micro-beams decreases and this is due to the influence of the Lorentz force and Maxwell stresses on the vibrations of the micro-beams. It is worth to mention that the effects of permanent magnetic field on the dynamic behavior of the cantilever and clamped-clamped micro-beams were investigated in detail by plotting time histories and difference of attenuation modes and steady state models.

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