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Reflection and Transmission of Longitudinal Wave at Micropolar Viscoelastic Solid/Fluid Saturated Incompressible Porous Solid Interface

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ABSTRACT

In this paper, the reflection and refraction of longitudinal wave from a plane surface separating a micropolar viscoelastic solid half space and a fluid saturated incompressible half space is studied. A longitudinal wave (P-wave) impinges obliquely at the interface. Amplitude ratios for various reflected and transmitted waves have been obtained. Then these amplitude ratios have been computed numerically for a specific model and results thus obtained are shown graphically with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave as well as on the properties of media. A particular case when longitudinal wave reflects at free surface of micropolar viscoelastic solid has been deduced and discussed. From the present investigation, a special case when fluid saturated porous half space reduces to empty porous solid has also been deduced and discussed with the help of graphs.

Keywords: Micropolar viscoelastic solid; Porous; Reflection; Longitudinal wave; Amplitude ratios

1 INTRODUCTION

MOST of natural and man-made materials, including engineering, geological and biological media, possess a microstructure. The ordinary classical theory of elasticity fails to describe the microstructure of the material. To overcome this problem, Suhubi and Eringen [28] , Eringen and Suhubi [12] developed a theory in which they considered the microstructure of the material and they showed that the motion in a granular structure material is characterized not by a displacement vector but also by a rotation vector. Eringen [11] developed the linear theory of micropolar viscoelasticity. Many researchers like Kumar et al. [19], Singh [26], Singh [27], discussed the problems of waves and vibrations in micropolar viscoelastic solids.

Based on the work of Fillunger model [13], Bowen [2] and de Boer and Ehlers [5-6] developed an interesting theory for porous medium having all constituents to be incompressible. There are sufficient reasons for considering the fluid saturated porous constituents as incompressible. For example, consider the composition of soil in which the solid constituents as well as liquid constituents which are generally water or oils are incompressible. Therefore, the assumption of incompressible constituents meet the properties appearing in many branches of engineering and avoids the introduction of many complicated material parameters as considered in the Biot theory. Based on this theory, many researchers like de Boer and Liu [8-9], de Boer and Liu [10], Liu [22], Yan et al. [31], Kumar and Hundal [18], de Boer and Didwania [4], Tajuddin and Hussaini [29], Kumar et al. [20] studied some problems of

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wave propagation in fluid saturated porous media. Based on this theory, many researchers like de Boer and Liu [8-9], de Boer and Liu [10], Liu [22], Yan et al. [31], Kumar and Hundal [18], de Boer and Didwania [4], Tajuddin and Hussaini [29], Kumar et al. [20] studied some problems of wave propagation in fluid saturated porous media.

Using the theory of de Boer and Ehlers [5] for fluid saturated porous medium and Eringen [11] for micropolar viscoelastic solid, the reflection and transmission phenomenon of longitudinal wave at an interface between micropolar viscoelastic solid half space and fluid saturated porous half space is studied. The reflection coefficient of reflected waves at the free surface has also been obtained. A special case when fluid saturated porous half space reduces to empty porous solid has been deduced and discussed. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically and discussed accordingly.

2 BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

2.1 For medium M_1 (Micropolar viscoelastic solid)

Following Eringen [11], the constitutive and field equations of a micropolar viscoelastic solid in the absence of body forces and body couples, are as:

$$t_{kl} = \lambda u_{r,r}\delta_{kl} + \mu\left(u_{k,l} + u_{l,k}\right) + k\left(u_{l,k} - \epsilon_{klr}\,\phi_r\right), \tag{1}$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k}\,, \tag{2}$$

$$\left(c_1^2 + c_3^2\right)\nabla\left(\nabla\cdot u\right) - \left(c_2^2 + c_3^2\right)\nabla\times\left(\nabla\times u\right) + c_3^2\nabla\times\phi = \ddot{u}\,, \tag{3}$$

$$\left(c_4^2 + c_5^2\right)\nabla\left(\nabla\cdot\phi\right) - c_4^2\nabla\times\left(\nabla\times\phi\right) + \omega_0^2\nabla\times u - 2\omega_0^2\phi = \ddot{\phi}\,, \tag{4}$$

where

$$c_1^2 = \frac{\left(\lambda + 2\mu\right)}{\rho}, c_2^2 = \frac{\mu}{\rho}, c_3^2 = \frac{k}{\rho}, c_4^2 = \frac{\gamma}{\rho j}, c_5^2 = \frac{\left(\alpha + \beta\right)}{\rho j}, \omega_0^2 = \frac{k}{\rho j}, \lambda = \lambda^* + \lambda_\upsilon^*\left(\frac{\partial}{\partial t}\right), \mu = \mu^* + \mu_\upsilon^*\left(\frac{\partial}{\partial t}\right),$$
$$k = k^* + k_\upsilon^*\left(\frac{\partial}{\partial t}\right), \alpha = \alpha^* + \alpha_\upsilon^*\left(\frac{\partial}{\partial t}\right), \beta = \beta^* + \beta_\upsilon^*\left(\frac{\partial}{\partial t}\right), \gamma = \gamma^* + \gamma_\upsilon^*\left(\frac{\partial}{\partial t}\right), \nabla = i\left(\frac{\partial}{\partial x}\right) + k\left(\frac{\partial}{\partial z}\right). \tag{5}$$

λ^*, μ^*, k^*, α^*, β^*, γ^*, λ_υ^*, μ_υ^*, k_υ^*, α_υ^*, β_υ^* and γ_υ^* are material constants, ρ is the density and j the rotational inertia. u and ϕ are displacement and microrotation vectors respectively. Superposed dots on right hand side of Eqs. (3) and (4) represent the second order partial derivative with respect to time.

Taking $u = \left(u_1, 0, u_3\right)$ and $\phi = \left(0, \phi_2, 0\right)$ and introducing potentials $\phi\left(x,z,t\right)$ and $\psi\left(x,z,t\right)$ which are related to displacement components as:

$$u_1 = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial z}, u_3 = \frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial x}. \tag{6}$$

with the help of displacement components given by (6) in (3) and (4), we get

$$\left(\nabla^2 - \frac{1}{\left(c_1^2 + c_3^2\right)}\frac{\partial^2}{\partial t^2}\right)\phi = 0, \tag{7}$$

$$\left(\nabla^2 - \frac{1}{\left(c_2^2 + c_3^2\right)}\frac{\partial^2}{\partial t^2}\right)\psi - p\phi_2 = 0, \tag{8}$$

$$\left(\nabla^2 - 2q - \frac{1}{c_4^2}\frac{\partial^2}{\partial t^2}\right)\phi_2 + q\nabla^2\psi = 0, \tag{9}$$

where

$$p = \frac{\mu}{\mu + k}, \quad q = \frac{k}{\gamma}. \tag{10}$$

Assuming the time variation as:

$$\phi(x,z,t) = \bar{\phi}(x,z)\exp(i\omega t), \qquad \psi(x,z,t) = \bar{\psi}(x,z)\exp(i\omega t), \qquad \phi_2(x,z,t) = \bar{\phi}_2(x,z)\exp(i\omega t). \tag{11}$$

Using (11) in (7) to (9), we obtain

$$\left(\nabla^2 + \left(\omega^2 / V_1^2\right)\right)\bar{\phi} = 0, \tag{12}$$

$$\left(\nabla^4 + \omega^2 B\nabla^2 + \omega^4 C\right)\left(\bar{\psi}, \bar{\phi}_2\right) = 0, \tag{13}$$

where

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{\left(c_2^2 + c_3^2\right)} + \frac{1}{c_4^2}, \qquad C = \frac{1}{\left(c_2^2 + c_3^2\right)}\left(\frac{1}{c_4^2} - \frac{2q}{\omega^2}\right), \tag{14}$$

and

$$V_1^2 = c_1^2 + c_3^2. \tag{15}$$

In an unbounded medium, the solution of (12) corresponds to modified longitudinal displacement wave (LD wave) propagating with velocity V_1.Writing the solution of (13) as:

$$\bar{\psi} = \bar{\psi}_1 + \bar{\psi}_2, \tag{16}$$

where $\bar{\psi}_1$ and $\bar{\psi}_2$ satisfy

$$\left(\nabla^2 + \delta_1^2\right)\bar{\psi}_1 = 0, \tag{17}$$

$$\left(\nabla^2 + \delta_2^2\right)\bar{\psi}_2 = 0, \tag{18}$$

and

$$\delta_1^2 = \lambda_1^2\omega^2, \ \delta_2^2 = \lambda_2^2\omega^2 \tag{19}$$

$$\lambda_1^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \ \lambda_2^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right]. \tag{20}$$

From (8) we obtain $\bar{\phi}_2 = E\bar{\psi}_1 + F\bar{\psi}_2$,where

$$E=\frac{\left(\dfrac{\omega^2}{c_2^2+c_3^2}-\delta_1^2\right)}{p},\quad F=\frac{\left(\dfrac{\omega^2}{c_2^2+c_3^2}-\delta_2^2\right)}{p}. \tag{21}$$

Thus there are two waves propagating with velocities λ_1^{-1} and λ_2^{-1}, each consisting of transverse displacement ψ and transverse microrotation ϕ_2. Following Parfitt and Eringen [23], these waves are modified coupled transverse displacement wave and transverse microrotational waves (CD I and CD II waves) respectively.

2.2 For medium M2 (Fluid saturated incompressible porous medium)

Following de Boer and Ehlers [6] , the governing equations in a fluid-saturated incompressible porous medium are

$$\mathrm{div}\left(\eta^s\dot{\boldsymbol{x}}_s+\eta^F\dot{\boldsymbol{x}}_F\right)=0, \tag{22}$$

$$\mathrm{div}\boldsymbol{T}_{\boldsymbol{E}}^{\boldsymbol{S}}-\eta^S\mathrm{grad}\ p+\rho^S\left(\boldsymbol{b}-\ddot{\boldsymbol{x}}_S\right)-\boldsymbol{P}_{\boldsymbol{E}}^{\boldsymbol{F}}=0, \tag{23}$$

$$\mathrm{div}\boldsymbol{T}_{\boldsymbol{E}}^{\boldsymbol{F}}-\eta^F\mathrm{grad}\ p+\rho^F\left(\boldsymbol{b}-\ddot{\boldsymbol{x}}_F\right)+\boldsymbol{P}_{\boldsymbol{E}}^{\boldsymbol{F}}=0, \tag{24}$$

where $\dot{\boldsymbol{x}}_i$ and $\ddot{\boldsymbol{x}}_i\left(i=S,F\right)$ denote the velocities and accelerations, respectively of solid (S) and fluid (F) phases of the porous aggregate and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid phases respectively and b is the body force per unit volume. $\boldsymbol{T}_{\boldsymbol{E}}^{\boldsymbol{S}}$ and $\boldsymbol{T}_{\boldsymbol{E}}^{\boldsymbol{F}}$ are the effective stress in the solid and fluid phases respectively, $\boldsymbol{P}_{\boldsymbol{E}}^{\boldsymbol{F}}$ is the effective quantity of momentum supply and η^S and η^F are the volume fractions satisfying

$$\eta^S+\eta^F=1. \tag{25}$$

If $\boldsymbol{u}_S$ and $\boldsymbol{u}_F$ are the displacement vectors for solid and fluid phases, then

$$\dot{\boldsymbol{x}}_S=\dot{\boldsymbol{u}}_S,\quad \ddot{\boldsymbol{x}}_S=\ddot{\boldsymbol{u}}_S,\quad \dot{\boldsymbol{x}}_F=\dot{\boldsymbol{u}}_F,\quad \ddot{\boldsymbol{x}}_F=\ddot{\boldsymbol{u}}_F. \tag{26}$$

The constitutive equations for linear isotropic, elastic incompressible porous medium are given by de Boer, Ehlers and Liu [7] as:

$$\boldsymbol{T}_{\boldsymbol{E}}^{\boldsymbol{S}}=2\mu^S\boldsymbol{E}_S+\lambda^S\left(E_S\cdot\boldsymbol{I}\right)\boldsymbol{I}, \tag{27}$$

$$\boldsymbol{T}_{\boldsymbol{E}}^{\boldsymbol{F}}=0, \tag{28}$$

$$\boldsymbol{P}_{\boldsymbol{E}}^{\boldsymbol{F}}=-\boldsymbol{S}_v\left(\dot{\boldsymbol{u}}_F-\dot{\boldsymbol{u}}_S\right), \tag{29}$$

where λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and $\boldsymbol{E}_S$ is the linearized Langrangian strain tensor defined as:

$$\boldsymbol{E}_S=\frac{1}{2}\left(\mathrm{grad}\,\boldsymbol{u}_S+\mathrm{grad}^T\boldsymbol{u}_S\right). \tag{30}$$

In the case of isotropic permeability, the tensor $\boldsymbol{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers [6] as:

$$\boldsymbol{S}_v = \frac{\left(\eta^F\right)^2 \gamma^{FR}}{K^F}\boldsymbol{I} , \tag{31}$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

Making the use of (26) in Eqs. (22)-(24), and with the help of (27)-(30), we obtain

$$\operatorname{div}\left(\eta^S \dot{\boldsymbol{u}}_S + \eta^F \dot{\boldsymbol{u}}_F\right) = 0 , \tag{32}$$

$$\left(\lambda^S + \mu^S\right)\operatorname{grad}\operatorname{div}\boldsymbol{u}_S + \mu^S \operatorname{div}\operatorname{grad}\boldsymbol{u}_S - \eta^S \operatorname{grad} p + \rho^S\left(\boldsymbol{b} - \ddot{\boldsymbol{u}}_S\right) + S_v\left(\dot{\boldsymbol{u}}_F - \dot{\boldsymbol{u}}_S\right) = 0 \tag{33}$$

$$-\eta^F \operatorname{grad} p + \rho^F\left(\boldsymbol{b} - \ddot{\boldsymbol{u}}_F\right) - S_v\left(\dot{\boldsymbol{u}}_F - \dot{\boldsymbol{u}}_S\right) = 0 . \tag{34}$$

For the two dimensional problem, we assume the displacement vector $\boldsymbol{u}_i\left(i = F, S\right)$ as:

$$\boldsymbol{u}_i = \left(u^i, 0, w^i\right) \text{ where } i = F,\ S. \tag{35}$$

Eqs .(32)-(34) with the help of Eq. (35) in the absence of body forces take the form

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x \partial t} + \frac{\partial^2 w^S}{\partial z \partial t}\right] + \eta^F\left[\frac{\partial^2 u^F}{\partial x \partial t} + \frac{\partial^2 w^F}{\partial z \partial t}\right] = 0, \tag{36}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{37}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{38}$$

$$\left(\lambda^S + \mu^S\right)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{39}$$

$$\left(\lambda^S + \mu^S\right)\frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{40}$$

where

$$\theta^S = \frac{\partial\left(u^S\right)}{\partial x} + \frac{\partial\left(w^S\right)}{\partial z}, \tag{41}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \tag{42}$$

Also, t_{zz}^S and t_{zx}^S the normal and tangential stresses in the solid phase are as under

$$t_{zz}^S = \lambda^S\left(\frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}\right) + 2\mu^S \frac{\partial w^S}{\partial z}, \tag{43}$$

$$t_{zx}^S = \mu^S\left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x}\right). \tag{44}$$

The displacement components u^j and w^j are related to the dimensional potential ϕ^j and ψ^j as :

$$u^j = \frac{\partial \phi^j}{\partial x} + \frac{\partial \psi^j}{\partial z},\ w^j = \frac{\partial \phi^j}{\partial z} - \frac{\partial \psi^j}{\partial x}, j = S,\ F. \tag{45}$$

Using Eq. (45) in Eqs . (36)-(40), we obtain the following equations determining ϕ^S, ϕ^F, ψ^S, ψ^F and p as:

$$\nabla^2 \phi^S - \frac{1}{C_1^2}\frac{\partial^2 \phi^S}{\partial t^2} - \frac{S_v}{\left(\lambda^S + 2\mu^S\right)\left(\eta^F\right)^2}\frac{\partial \phi^S}{\partial t} = 0, \tag{46}$$

$$\phi^F = -\frac{\eta^S}{\eta^F}\phi^S, \tag{47}$$

$$\mu^S \nabla^2 \psi^S - \rho^S \frac{\partial^2 \psi^S}{\partial t^2} + S_v\left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{48}$$

$$\rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v\left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{49}$$

$$\left(\eta^F\right)^2 p - \eta^S \rho^F \frac{\partial^2 \phi^S}{\partial t^2} - S_v \frac{\partial \phi^S}{\partial t} = 0, \tag{50}$$

where

$$C_1 = \sqrt{\frac{\left(\eta^F\right)^2\left(\lambda^S + 2\mu^S\right)}{\left(\eta^F\right)^2 \rho^S + \left(\eta^S\right)^2 \rho^F}}. \tag{51}$$

Assuming the solution of the system of Eqs. (46)-(50) in the form

$$\left(\phi^S, \phi^F, \psi^S, \psi^F, p\right) = \left(\phi_1^S, \phi_1^F, \psi_1^S, \psi_1^F, p_1\right)\exp\left(i\omega t\right), \tag{52}$$

where ω is the complex circular frequency.

Making the use of (52) in Eqs. (46)-(50), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{C_1^2} - \frac{i\omega S_v}{\left(\lambda^S + 2\mu^S\right)\left(\eta^F\right)^2}\right]\phi_1^S = 0, \tag{53}$$

$$\left[\mu^S \nabla^2 + \rho^S \omega^2 - i\omega S_v\right]\psi_1^S = -i\omega S_v \psi_1^F, \tag{54}$$

$$\left[-\omega^2 \rho^F + i\omega S_v\right]\psi_1^F - i\omega S_v \psi_1^S = 0, \tag{55}$$

$$\left(\eta^F\right)^2 p_1 + \eta^S \rho^F \omega^2 \phi_1^S - i\omega S_v \phi_1^S = 0, \tag{56}$$

$$\phi_1^F = -\frac{\eta^S}{\eta^F}\phi_1^S. \tag{57}$$

Eq. (53) corresponds to longitudinal wave propagating with velocity $\bar{V}_1$, given by

$$\bar{V}_1^2 = \frac{1}{G_1}, \tag{58}$$

where

$$G_1 = \left[\frac{1}{c_1^2} - \frac{iS_v}{\omega\left(\lambda^S + 2\mu^S\right)\left(\eta^F\right)^2} \right]. \tag{59}$$

From Eqs. (54) and (55), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{\bar{V}_2^2} \right] \psi_1^S = 0. \tag{60}$$

Eq. (60) corresponds to transverse wave propagating with velocity $\bar{V}_2$, given by $\bar{V}_2^2 = 1/G_2$ where

$$G_2 = \left\{ \frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S \omega} - \frac{S_v^2}{\mu^S\left(-\rho^S\omega^2 + i\omega S_v\right)} \right\}. \tag{61}$$

3 FORMULATION OF THE PROBLEM

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface $z = 0$ separating the uniform micropolar viscoelastic solid half space medium M_1 ($z > 0$) and fluid saturated porous half space medium M_2 ($z < 0$). Consider a longitudinal wave propagating through the medium M_1, incident at the plane $z = 0$ and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal wave, we get three reflected waves in the medium M_1 and two transmitted waves in medium M_2 as shown in Fig. 1.

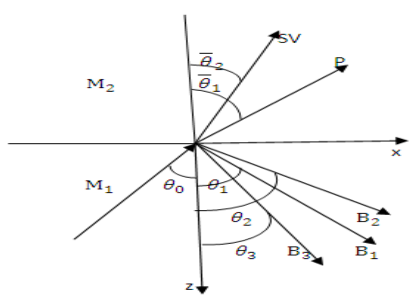

Fig. 1
Geometry of the problem.

In medium M1

$$\phi = B_0 \exp\left\{ik_0\left(x\sin\theta_0 - z\cos\theta_0\right) + i\omega_1 t\right\} + B_1 \exp\left\{ik_0\left(x\sin\theta_1 + z\cos\theta_1\right) + i\omega_1 t\right\}, \tag{62}$$

$$\psi = B_2 \exp\left\{i\delta_1\left(x\sin\theta_2 + z\cos\theta_2\right) + i\omega_2 t\right\} + B_3 \exp\left\{i\delta_2\left(x\sin\theta_3 + z\cos\theta_3\right) + i\omega_3 t\right\}, \tag{63}$$

$$\phi_2 = EB_2 \exp\left\{i\delta_1\left(x\sin\theta_2 + z\cos\theta_2\right) + i\omega_2 t\right\} + FB_3 \exp\left\{i\delta_2\left(x\sin\theta_3 + z\cos\theta_3\right) + i\omega_3 t\right\}, \tag{64}$$

In medium M_2

$$\left\{\phi^S,\phi^F,p\right\}=\left\{1,m_1,m_2\right\}\left[A_1\exp\left\{i\bar{k}_1\left(x\sin\bar{\theta}_1-z\cos\bar{\theta}_1\right)+i\bar{\omega}_1 t\right\}\right], \tag{65}$$

$$\left\{\psi^S,\psi^F\right\}=\left\{1,m_3\right\}\left[A_2\exp\left\{i\bar{k}_2\left(x\sin\bar{\theta}_2-z\cos\bar{\theta}_2\right)+i\bar{\omega}_2 t\right\}\right], \tag{66}$$

where

$$m_1=-\frac{\eta^S}{\eta^F},\ m_2=-\left[\frac{\eta^S\omega_1^2\rho^F-i\omega_1 S_v}{\left(\eta^F\right)^2}\right],\ m_3=\frac{i\omega_2 S_v}{i\omega_2 S_v-\omega_2^2\rho^F} \tag{67}$$

and B_0,B_1,B_2,B_3 are amplitudes of incident P-wave, reflected P-wave, reflected CDI and reflected CDII waves respectively, A_1 and A_2 are amplitudes of transverse P-wave and SV-wave, respectively and all these unknowns are to be determined from boundary conditions.

4 BOUNDARY CONDITIONS

The appropriate boundary conditions are the continuity of displacement, micro rotation and stresses at the interface separating media M_1 and M_2. Mathematically, these boundary conditions at $z=0$ can be written as:

$$t_{zz}=t_{zz}^S-p,\ \ t_{zx}=t_{zx}^S,\ \ m_{zy}=0,\ \ u_1=u^S,\ \ u_3=w^S. \tag{68}$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0}=\frac{\sin\theta_1}{V_1}=\frac{\sin\theta_2}{\lambda_1^{-1}}=\frac{\sin\theta_3}{\lambda_2^{-1}}=\frac{\sin\bar{\theta}_1}{\bar{V}_1}=\frac{\sin\bar{\theta}_2}{\bar{V}_2}. \tag{69}$$

For longitudinal wave,

$$V_0=V_1,\ \ \theta_0=\theta_1. \tag{70}$$

Also

$$k_0V_1=\delta_1\lambda_1^{-1}=\delta_2\lambda_2^{-1}=\bar{k}_1\bar{V}_1=\bar{k}_2\bar{V}_2=\omega. \tag{71}$$

Making the use of potentials given by Eqs. (62)-(66) in Eqs. (1)-(2) and (6) and (43)-(45) and (65) and then using the boundary conditions given by Eq.(68) and using (69)-(71), we get a system of five non homogeneous which can be written as:

$$\sum_{j=0}^{5}a_{ij}Z_j=Y_i,\qquad \left(i=1,2,3,4,5\right) \tag{72}$$

where

$$Z_1=\frac{B_1}{B_0},\ Z_2=\frac{B_2}{B_0},\ Z_3=\frac{B_3}{B_0},\ Z_4=\frac{A_1}{B_0},\ Z_5=\frac{A_2}{B_0}, \tag{73}$$

i.e. Z_1 to Z_5 be the amplitude ratios of reflected modified longitudinal displacement wave, reflected CD I wave at an angle θ_2 reflected CD II wave at an angle θ_3 refracted P-wave and refracted SV-wave, respectively and a_{ij} in non-dimensional form are as:

$$
\begin{aligned}
&a_{11}=\left[\frac{\lambda}{\mu}+D_2\cos^2\theta_1\right],\ a_{12}=-D_2\frac{\delta_1^2}{k_0^2}\sin\theta_2\cos\theta_2,\ a_{13}=-D_2\sin\theta_3\cos\theta_3\frac{\delta_2^2}{k_0^2},a_{14}=\frac{-\bar{k}_1^2\left(\lambda^S+2\mu^S\cos^2\bar{\theta}_1\right)-m_2}{\mu k_0^2},\\
&a_{15}=\frac{-2\mu^S\bar{k}_2^2\sin\bar{\theta}_2\cos\bar{\theta}_2}{\mu k_0^2},\ Y_1=-a_{11},a_{21}=D_2\sin\theta_1\cos\theta_1,\ a_{22}=\frac{\delta_1^2}{k_0^2}\left[\left(D_1\cos^2\theta_2-\sin^2\theta_2\right)+\frac{k}{\mu}\frac{E}{\delta_1^2}\right],\\
&a_{23}=\frac{\delta_2^2}{k_0^2}\left[\left(D_1\cos^2\theta_3-\sin^2\theta_3\right)+\frac{k}{\mu}\frac{F}{\delta_2^2}\right],\ a_{24}=\frac{\mu^S\bar{k}_1^2\sin 2\bar{\theta}_1}{\mu k_0^2},a_{25}=\frac{\mu^S\bar{k}_2^2\left(\sin^2\bar{\theta}_2-\cos^2\bar{\theta}_2\right)}{\mu k_0^2},Y_2=a_{21},\\
&a_{31}=\sin\theta_1,\ a_{32}=\frac{\delta_1}{k_0}\cos\theta_2,\ a_{33}=\frac{\delta_2}{k_0}\cos\theta_3,\ a_{34}=-\frac{\bar{k}_1}{k_0}\sin\bar{\theta}_1,a_{35}=\frac{\bar{k}_2}{k_0}\cos\bar{\theta}_2,\ Y_3=-a_{31},\\
&a_{41}=\cos\theta_1,\ a_{42}=-\frac{\delta_1}{k_0}\sin\theta_2,\ a_{43}=-\frac{\delta_2}{k_0}\sin\theta_3,\ a_{44}=\frac{\bar{k}_1}{k_0}\cos\bar{\theta}_1,a_{45}=\frac{\bar{k}_2}{k_0}\sin\bar{\theta}_2,\ Y_4=a_{41},\\
&a_{51}=0,\ a_{52}=\cos\theta_2,\ a_{53}=\frac{F\delta_2}{E\delta_1}\cos\theta_3,\ a_{54}=0,\ a_{55}=0,\ Y_5=0.
\end{aligned}
\tag{74}
$$

5 PARTICULAR CASES

Case1

If pore is absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and can be neglected, so the relation (53) reduces to

$$C_0=\sqrt{\frac{\lambda^S+2\mu^S}{\rho^S}}. \tag{75}$$

Then fluid saturated incompressible porous medium reduces to empty porous solid.

Case2

When upper half space is not present in the given formulation. Considering a micropolar viscoelastic solid with free boundary surface, i.e. upper half space is not present in the given formulation. A plane wave (P-wave) propagating through the micropolar viscoelastic solid making an angle θ_0 with z-axis at the free surface $z=0$. Corresponding to each incident wave we get three reflected waves. Boundary conditions for this case reduces to

$$t_{zz}=t_{zz}^S-p,\ \ t_{zx}=t_{zx}^S,\ \ m_{zy}=0 \tag{76}$$

And hence we obtain a system of three non-homogeneous equations which can be written as:

$$\sum_{j=1}^{3}a_{ij}Z_j=Y_i,\qquad \left(i=1,2,3\right) \tag{77}$$

where $a_{11},a_{12},a_{13},a_{21},a_{22},a_{23},a_{31},a_{32}$ and a_{33} are given by Eq. (74)

6 NUMERICAL RESULTS AND DISCUSSION

The theoretical results obtained above indicate that the amplitude ratios $Z_i\left(i=1,2,3,4,5\right)$ depend on the angle of incidence of incident wave and material properties of half spaces. In order to study in more detail the behaviour of various amplitude ratios, we have computed them numerically for a particular model for which the values of various physical parameters are as under. In medium M_1, the physical parameters for micropolar viscoelastic elastic solid are taken from Gauthier [14] as:

$$\lambda^* = 7.59\times10^{11}\,\text{dyne/cm}^2, \qquad \mu^* = 1.89\times10^{11}\,\text{dyne/cm}^2$$
$$k^* = 0.0149\times10^{11}\,\text{dyne/cm}^2, \qquad \rho = 2.19\,\text{gm/cm}^3$$
$$\gamma^* = 0.0268\times10^{11}\,\text{dyne}, \qquad j = 0.0196\,\text{cm}^2$$

$$\lambda=\lambda^*\left(1+\frac{i}{Q_1}\right),\qquad \mu=\mu^*\left(1+\frac{i}{Q_2}\right),\qquad k=k^*\left(1+\frac{i}{Q_3}\right),\qquad \gamma=\gamma^*\left(1+\frac{i}{Q_4}\right), \tag{78}$$

where the quality factors $Q_i\left(i=1,2,3,4\right)$ are taken arbitrarily as:

$$Q_1=5,\ Q_2=10,\ Q_3=15,\ Q_4=13$$

In medium M_2, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu [7] as:

$$\eta^S=0.67,\ \ \eta^F=0.33,\ \ \rho^S=1.34\text{Mg/m}^3,\ \rho^F=0.33\text{Mg/m}^3,\ \lambda^S=5.5833\text{MN/m}^2,\ \ K^F=0.01\text{m/s},$$
$$\gamma^{FR}=10.00\text{KN/m}^3,\ \ \mu^S=8.3750\text{N/m}^2. \tag{79}$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and transmitted waves for the particular model and to depict graphically. In Figs (2)-(6) solid lines show the variations of amplitude ratios when medium-I is micropolar viscoelastic solid (MVES) and medium-II is incompressible fluid saturated porous medium (FS) whereas dashed lines show the variations of amplitude ratios when medium-II becomes incompressible empty porous solid (EPS). Figs. (2)-(6) indicate the effect of pores fluid.

In Figs. 7-11 solid lines show the variations of amplitude ratios when medium-I is micropolar viscoelastic solid (MVES) and medium-II is incompressible fluid saturated porous medium (FS) whereas dashed lines show the variations of amplitude ratios when medium-I becomes micropolar elastic solid(MES). In this case the modulus of amplitude ratios changes slightly due to effect of viscosity. Figs. (12)-(14) show the variation of the modulus of the amplitude ratios of various reflected waves at free surface of micropolar viscoelastic solid (MVES).In these figures solid lines show the variations of amplitude ratios when medium is micropolar viscoelastic solid (MVES) whereas dashed lines show the variations of amplitude ratios when the medium becomes micropolar elastic solid(MES) . These figures show that the effect of viscosity is significant in the range $52^\circ \le \theta \le 74^\circ$ approx.

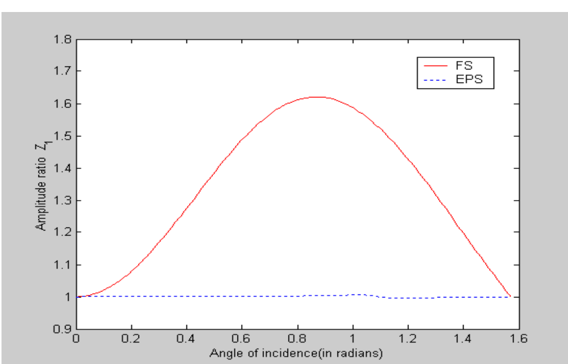

Fig. 2
Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident longitudinal wave.

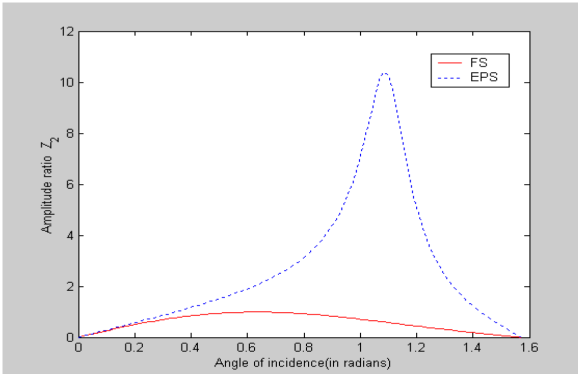

Fig. 3
Variation of the amplitude ratio $|Z_2|$ with angle of incidence of the incident longitudinal wave.

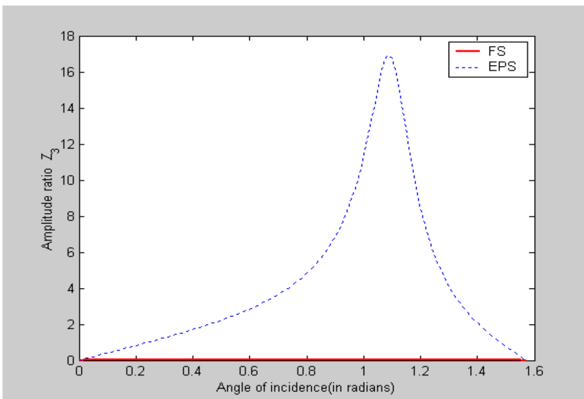

Fig. 4
Variation of the amplitude ratio $|Z_3|$ with angle of incidence of the incident longitudinal wave.

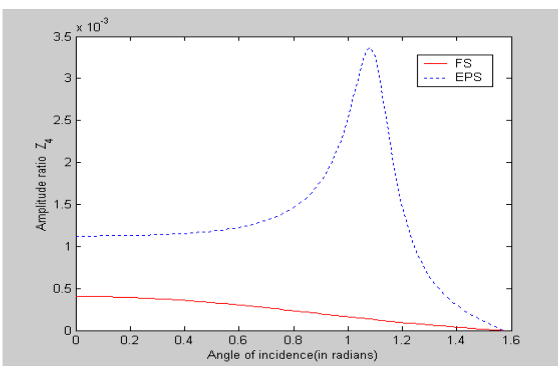

Fig. 5
Variation of the amplitude ratio $|Z_4|$ with angle of incidence of the incident longitudinal wave.

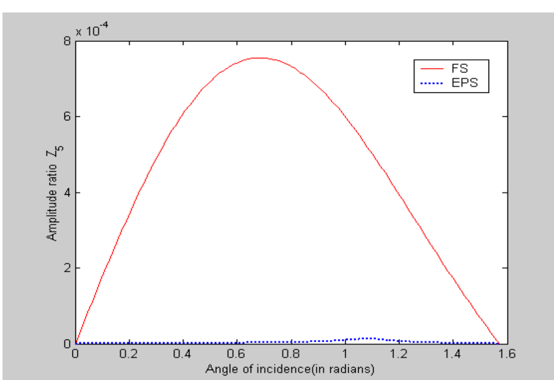

Fig. 6
Variation of the amplitude ratio $|Z_5|$ with angle of incidence of the incident longitudinal wave.

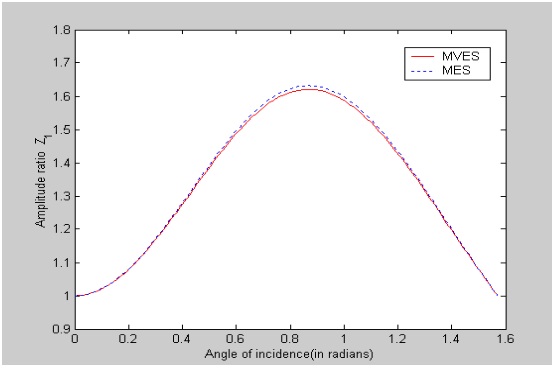

Fig. 7
Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident longitudinal wave.

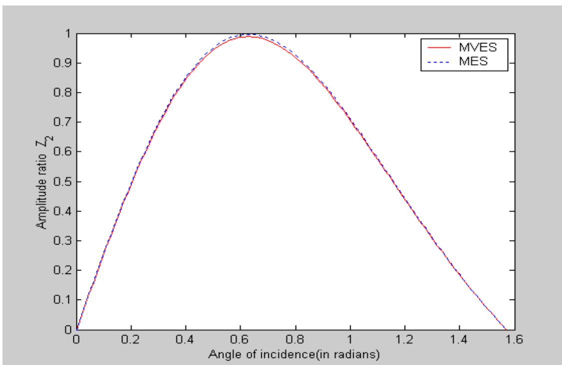

Fig. 8
Variation of the amplitude ratio $|Z_2|$ with angle of incidence of the incident longitudinal wave.

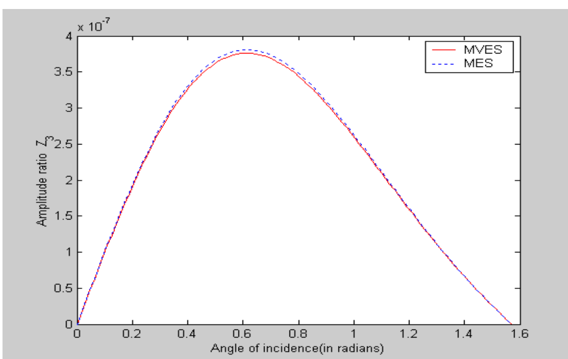

Fig. 9
Variation of the amplitude ratio $|Z_3|$ with angle of incidence of the incident longitudinal wave.

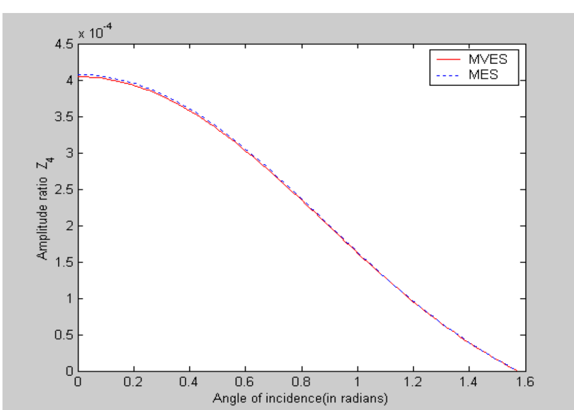

Fig. 10
Variation of the amplitude ratio $|Z_4|$ with angle of incidence of the incident longitudinal wave.

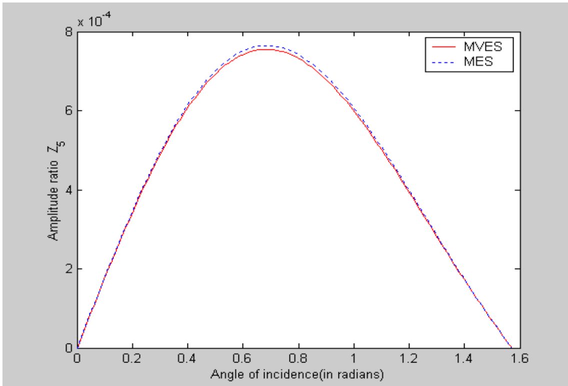

Fig. 11
Variation of the amplitude ratio $|Z_5|$ with angle of incidence of the incident longitudinal wave.

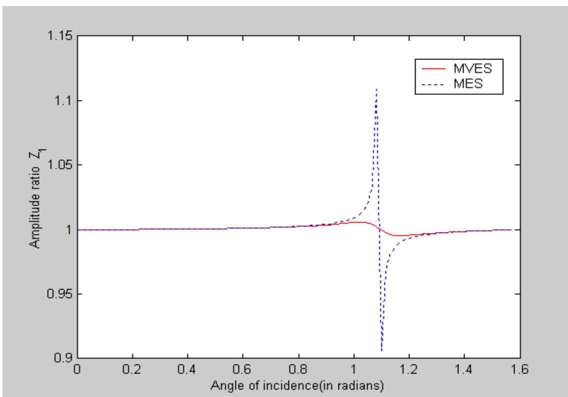

Fig. 12
Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident longitudinal wave (free surface).

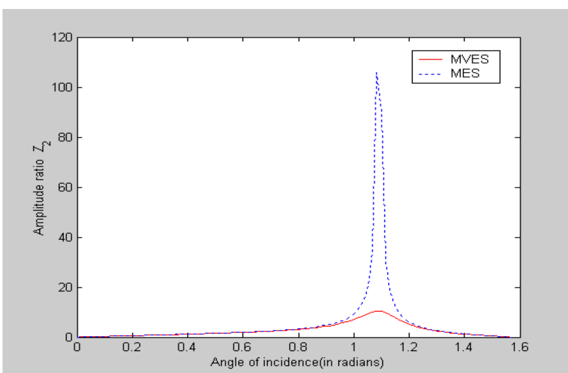

Fig. 13
Variation of the amplitude ratio $|Z_2|$ with angle of incidence of the incident longitudinal wave (free surface).

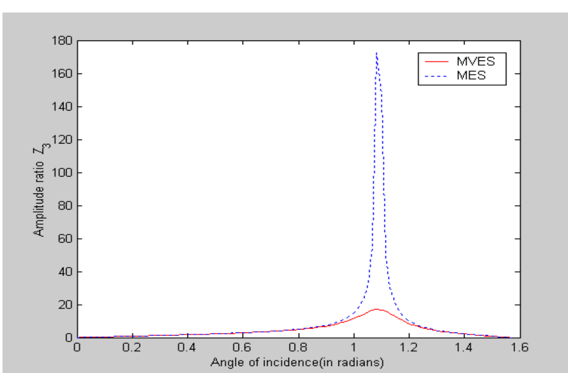

Fig. 14
Variation of the amplitude ratio $|Z_3|$ with angle of incidence of the incident longitudinal wave (free surface).

7 CONCLUSIONS

In conclusion, a mathematical study of reflection and refraction coefficients at an interface separating micropolar viscoelastic solid half space and fluid saturated incompressible porous half space is made when longitudinal wave is incident. It is observed that

(i). The amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties of half spaces.
(ii). The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on the amplitudes ratios.
(iii). If we neglect the viscous effect of micropolar viscoelastic solid then the variations in the amplitude ratios of various reflected and refracted waves have been affected but not significantly.
(iv). There is significant difference in the values of modulus of amplitudes ratios for reflected waves in both the cases (i) when upper half space is present (ii) when upper half space is not present.
(v). Appreciable effect of viscosity has been observed on the amplitudes ratios for the reflected waves in case of free surface boundary.

The model presented in this paper is one of the more realistic forms of the earth models. It may be of some use in engineering, seismology and geophysics etc.

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