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# Mechanical Behavior of a FGM Capacitive Micro-Beam Subjected to a Heat Source

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**ABSTRACT**

This paper presents mechanical behavior of a functionally graded (FG) cantilever micro-beam subjected to a nonlinear electrostatic pressure and thermal moment considering effects of material length scale parameters. Material properties through the beam thickness direction are graded. The top surface of the micro-beam is made of pure metal and the bottom surface from a mixture of metal and ceramic. The material properties through the thickness direction follow the volume fraction of the constitutive materials in exponential function form. The governing nonlinear thermo-electro-mechanical differential equation based on *Euler-Bernoulli* beam theory assumptions is derived using modified couple stress theory (MCST) and is solved using the Galerkin based weighted residual method. The effects of the electrostatic pressure and temperature changes on the deflection and stability of the FGM micro-beam, having various ceramic constituent percents, are studied. The obtained results are compared with the results predicted by classic theory (CT) and for some cases are verified with those reported in the literature.

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**Keywords:** FGM Cantilever micro-beam; Euler–Bernoulli; Pull-in voltage; Size-dependent; Exponential volume fraction law

## 1 INTRODUCTION

THE study and researches about behavior of functionally graded materials (FGMs) have been an interesting topic during the past decades. FGMs are special composites whose composition varies continuously as a function of position along thickness of a structure to achieve an appropriate function. This is obtained by gradually varying volume fraction of the constituent materials. FGMs are generally made of a mixture of ceramic and metal to satisfy the demand of ultra-high-temperature environment and to eliminate the interface problems, as ceramic has a low thermal conductivity and thus excellent temperature residence. The continuous change in the compositions leads to a smoothing variation in mechanical property, which has many advantages over the laminated composite materials. For example, the cracking and delaminating phenomenon which often are observed in conventional multi-layered systems are avoided due to the smooth transition between the properties of the components in FGMs. The area of smart materials and structures has experienced rapid growth. The mechanics of smart structures involve coupling between the mechanical, electrical or thermal effects.

Due to what above-mentioned favorable features, many studies have been conducted on the static and dynamic behavior of FGM structures [1-5]. Suresh and Mortensen [1] provide an excellent introduction to the fundamentals of FGM. Sankar and his co-workers [5–7] solved the elastic problem of FGM beam and computed the thermal

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stresses in an FGM Euler-beam based on the uncoupled thermo-elasticity assumption. The analytical solution of the coupled thermoelasticity of beams with the *Euler-Bernoulli* assumption is given by Massalas and Kalpakidis [8]. Chakraborty et al. [9] developed a new beam element based on the first order shear deformation theory to study the thermo elastic behavior of FGM beam structures. Alibeigloo [10] presented analytical solution for functionally graded material (FGM) beams integrated with piezoelectric actuator and sensor under an applied electric field and thermo-mechanical load. Babaei et al. [11] studied the thermoelastic vibration of FG beams under lateral thermal shock with the *Euler-Bernoulli* beam assumptions.

Thanks to the increased awareness among the research community, innovative technological breakthroughs and increased market demand micromechanics has made rapid progress in recent years [12]. Perhaps most widely known nonlinear phenomenon in micro-electro-mechanical systems (MEMS) is pull-in or jump-to-contact instability [13]. Hasanyan et al. [14] studied for the first time pull-in instability in functionally graded MEMS due to the heat producted by the electric current. Rezazadeh et al. [15] studied Thermo-mechanical behavior of a bilayer micro-beam subjected to nonlinear electrostatic pressure. A novel remote temperature sensor based on a bi-layer micro cantilever beam has been proposed by Rezazadeh et al. [16]. Mohammadi-Alasti et al. [17] have studied the static behavior of the functionally graded cantilever micro-beam and its static instability, subjected to a nonlinear electrostatic pressure and temperature changes.They derived nonlinear integral-differential thermo-electro mechanical equation based on *Euler-Bernoulli* beam theory. In their study, the FGM beam material properties considered vary continuously through the thickness according to an exponential distribution law.

According to the vast and increasing applications of MEMS, and also the fact that the classical theory of elasticity is unable to predict size-dependent mechanical behaviors of microstructures, applying a non-classical theory of elasticity to the microstructures seems to have great merits. Size dependent behavior is an inborn property of materials and when the characteristic size such as thickness or diameter is close to the internal material length scale parameter must be considered, Kong et al. [18]. The size-dependent static and vibration behavior in micro scale beam had been experimentally observed in metals [19-21]. Because of difficulties of determining length scale parameters, Yang et al [22] introduced the modified couple stress theory (MCST) which only one length scale parameter is in its equation and of course its results have acceptable accurate. Lacking intrinsic length scale parameters of the materials, the scale free classic theories of mechanics cannot give sufficient predication of the behavior of the materials. Recently basis on the MCST, a FG micro-beam under electrostatic forces is studied by Abbasnejad et al. [23] and Asghari et al. [24] have investigated the size-dependent static and vibration behavior of micro-beams made of functionally graded materials (FGMs). They have shown that the static and dynamic pull-in voltage for some materials cannot be obtained using classic theory and components of couple stress theory must be taken into account.

It is found out from the contents as mentioned above that there are no studies on the stability of electrostatically and thermally actuated micro-structures especially FG micro-beams considering length scale parameters. In this study utilizing MCST, the size dependent static behavior of a FG *Euler-Bernoulli* micro-beam is investigated. Moreover, the effects of the electrostatic pressure and temperature changes on the deflection and stability of the FG beam, having various values of the ceramic constituent percent, are studied. Results of pull-in voltages and pull-in temperatures based on MCST and CT compared.

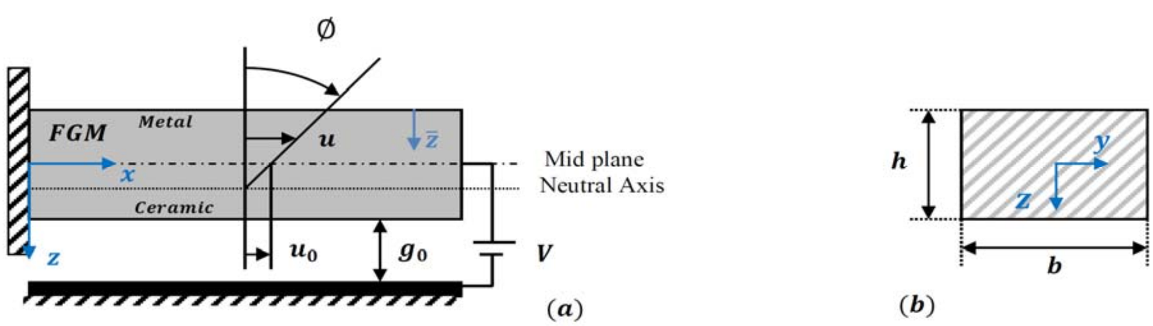


**Fig. 1**
The schematic view of a FGM capacitive cantilever beam and its coordinates, (a) side view; (b) section view.

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## 2 MODEL DESCRIPTION AND PROBLEM FORMULATION

### *2.1 Functionally graded materials*

Typically, the FGMs are made of a mixture of two materials, namely, the metals and ceramics. We assume that the material properties of FGM micro-beam are varying along its thickness, and the top surface is metal rich and the bottom surface consists of material blended with both of them. Subscript ''*c*'' for ceramic and ''*m*'' for metal is used. We can consider exponential functions for representation of continuous gradation of the material properties in analytical solution of FGM problems as [17]:

$$E(\bar{z}) = E_m e^{\gamma \bar{z}}, \qquad G(\bar{z}) = G_m e^{\mu \bar{z}}, \qquad \alpha(\bar{z}) = \alpha_m e^{\beta \bar{z}}, \qquad K(\bar{z}) = K_m e^{\lambda \bar{z}}, \qquad l(\bar{z}) = l_m e^{\tau \bar{z}} \tag{1}$$

where $E_m$, $G_m$, $\alpha_m$, $K_m$ and $l_m$ are the Young's modulus of elasticity , shear modulus, thermal expansion coefficient, thermal conductivity and length scale parameter of metal component of the FGM micro-beam, respectively. Parameters $\gamma$, $\mu$, $\beta$, $\lambda$ and $\tau$ are values depend on the dispersion of the ceramic into the metal. Considering the relation between $z$ and $\bar{z}$ :

$$\bar{z} = z + \frac{h}{2} \tag{2}$$

Now assume that ceramic constituent percent of the bottom surface varies from 0% to 100%. In order to determine material properties of the bottom surface $(P_h)$, volume fraction of material is used as follow [17]:

$$P_h = P_{(\bar{z}=h)} = V_c P_c + V_m P_m \tag{3}$$

where $V_c$ and $V_m$ are the ceramic and metal volume fractions and $P_c$ and $P_m$ are the ceramic and metal properties. Therefore, using Eqs. (1), (2) and (3), parameters $\gamma$, $\mu$, $\beta$, $\lambda$ and $\tau$ can be specified as:

$$\gamma = \frac{1}{h}\ln\left(\frac{E_h}{E_m}\right), \qquad \mu = \frac{1}{h}\ln\left(\frac{G_h}{G_m}\right), \qquad \beta = \frac{1}{h}\ln\left(\frac{\alpha_h}{\alpha_m}\right), \qquad \lambda = \frac{1}{h}\ln\left(\frac{K_h}{K_m}\right), \qquad \tau = \frac{1}{h}\ln\left(\frac{l_h}{l_m}\right) \tag{4}$$

Considering the ceramic constituent percent of the bottom surface, five different types of FGM micro-beams was investigated.

### *2.2 Formulation of beam equation*

Assume an elastic beam with rectangular cross section with dimensions of length $L\ (0 \le x \le L)$, width $b\ (-b/2 \le y \le b/2)$ and thickness $h\ (-h/2 \le z \le h/2)$ as shown in Fig.1. The *h*/*L* ratio is assumed to be small enough to eliminate the shear deformation effects. We will follow the *Euler-Bernoulli* beam theory assumption that plane sections normal to the beam axis (*x* axis) remain plane and normal after deformation. Hence, the axial strain and displacement is written as follow [25]:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad u = u_0 - z\frac{\partial w}{\partial x} \tag{5}$$

where $u_0$ is the displacement or extension of the mid plane in *x* direction and *u* is the total extension along *x*-axis of the layer located at a given distance *z*. Assuming that the beam material is linear elastic, the resultant stress considering temperature changes for the *Euler-Bernoulli* beam in a cross section area of the micro-beam using Hooke's law can be presented as [25]:

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$$\sigma_x = \overline{E}(\varepsilon_x - \overline{\alpha}\theta) = \overline{E}\left(\frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right) - \overline{E}\overline{\alpha}\theta \tag{6}$$

In which $\theta$ is the temperature change, measured with respect to a reference or environment temperature $(T_\infty)$. For plain stress conditions $\overline{E}$ and $\overline{\alpha}$ are equal to $E$ and $\alpha$, respectively and for plain strain condition are $E/(1-\nu^2)$ and $\alpha(1+\nu)$, respectively [26]. Due to free end of the beam, the axial force along the $x$ axis is zero:

$$\int_A \sigma_x \mathrm{d}A = 0 \tag{7}$$

As a result, $\partial u_0 / \partial x$ has been produced from substituting Eq. (6) into Eq. (7)

$$\frac{\partial u_0}{\partial x} = \frac{\overline{B}}{\overline{A}}\frac{\partial^2 w}{\partial x^2} + \frac{1}{\overline{A}}F_T \tag{8}$$

The parameters appeared in Eq. (8) are:

$$\overline{B} = \frac{h}{2\gamma}(e^{h\gamma}+1) - \frac{1}{\gamma^2}(e^{h\gamma}-1), \quad \overline{A} = \frac{e^{h\gamma}-1}{\gamma} \tag{9}$$

and

$$F_T = \int_{-h/2}^{h/2} \alpha_m \theta(z) e^{\left(z+\frac{h}{2}\right)(\gamma+\beta)} \mathrm{d}z \tag{10}$$

There exist many classical couple stress theories such as Toupin [27], Mindlin and Tiersten [28], Koiter [29], Mindlin [30] and [31], Eringen [32], which are constructed based on Cosserat or Micro-polar Continuum Mechanics theory with different constitutive equations. For example, in Koiter's couple stress model (1964) constitutive equation is:

$$m_{ij} = 4GL^2(\chi_{ij} + \eta\chi_{ji}) \tag{11}$$

where $m_{ij}$ is the deviatoric part of the couple stress tensor and $\eta$ is often omitted. Therefore, by neglecting $\eta$ Koiter's couple stress model is reduced to the Yang et al. [22] modified couple stress theory model in the beam analysis, which is used in the present manuscript:

$$m_{ij} = 2l^2 G\chi_{ij} \tag{12}$$

The couple stress tensor $\mu_{ij}$ can be decomposited into the spherical $(\mu_s)$ and deviatoric part $(m_{ij})$ as following:

$$\mu_{ij} = \mu_s\delta_{ij} + m_{ij}, \qquad \mu_s = \frac{1}{3}\mu_{ii}, \qquad \mu_{ij} = m_{ij}, \qquad i \neq j \tag{13}$$

Therefore, to satisfy the equilibrium condition in a beam section not only moment owing to the classic stress tensor must be considered but also owing to the couple stress tensor.

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$$M = M_{\sigma} + M_m = \int_A \sigma_{xx} z dA + \int_A \mu_{xy} dA = \int_A \sigma_{xx} z dA + \int_A m_{xy} dA \tag{14}$$

Finally, the static deflection equation considering material length scale parameter is derived as:

$$((EI)_{eq} + (GAl^2)_{eq}) \frac{\partial^2 w}{\partial x^2} = M - M_T \tag{15}$$

where $M_T$ is the thermal moment and for the plane stress condition equals to:

$$M_T = \frac{bE_m}{\bar{A}} \int_{-h/2}^{h/2} \left[ z\mathrm{e}^{\gamma\left(z+\frac{h}{2}\right)} F_T - \bar{A}\alpha_m z\theta(z) \mathrm{e}^{(\gamma+\beta)\left(z+\frac{h}{2}\right)} \right] \mathrm{d}z \tag{16}$$

For the plain stress condition, $(EI)_{eq}$ is the equivalent bending stiffness and specified as:

$$(EI)_{eq} = \int_A \left\{ zE(z) \left( \frac{\int_A zE(z)\mathrm{d}A}{\int_A E(z)\mathrm{d}A} \right) - z^2 E(z) \right\} \mathrm{d}A \tag{17}$$

and $(GAl^2)_{eq}$ obtained from:

$$(GAl^2)_{eq} = \int_A G(z)(l(z))^2 \,\mathrm{d}A \tag{18}$$

*2.3 Classical coupled thermo elasticity*

In the present study, it is assumed that the variation of temperature only occurs along the thickness direction. The thermal analysis is conducted by solving a simple steady state heat transfer equation in the thickness direction [11]:

$$\frac{\partial}{\partial z}\left( -K(z) \frac{\partial \theta(z)}{\partial z} \right) = 0 \tag{19}$$

Solving this one-dimensional second-order differential equation, thermal gradient along the thickness $\theta(z)$ is acquired as:

$$\theta(z) = C_1 e^{-\lambda z} + C_2 \tag{20}$$

where

$$\theta = T - T_{\infty} \tag{21}$$

As the thermal boundary conditions, assume the upper surface of beam is imposed by constant temperatures due to a heat source and the lower surface has a convectional boundary condition with environment. By substituting the boundary conditions $C_1$, $C_2$ and $T_1$ (lower surface temperature) are specified. As regards the obtained value of $C_1$ (for the properties given in Table 1), which has a very small value, we can conclude that the temperature distribution can be considered uniformly through the beam thickness, and the thermal moment (Eq. (16)) in a beam section is only due to the difference between the material properties of the constituents in the thickness direction.

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*2.4 Static deflection of beam subjected to a DC voltage*

The equation of static deflection of the FGM micro-beam subjected to a bias DC voltage considering length scale parameters is represented by [24]:

$$((EI)_{eq}+(GAl^2)_{eq})\frac{\mathrm{d}^4 w_s(x)}{\mathrm{d}x^4}=\frac{\varepsilon_0 bV^2}{2(g_0-w_s(x))^2} \tag{22}$$

where $\varepsilon_0$ is the dielectric coefficient (permittivity) of the air, $V$ is the applied voltage and $g_0$ is the initial gap between the micro-beam and the ground electrode, and $w_s$ is the transverse static deflection of the micro-beam, defined to be positive downward. Following dimensionless quantities are defined to rearrange Eq. (22) into a non-dimensional form:

$$\bar{w}_s=\frac{w_s}{g_0},\qquad \bar{x}=\frac{x}{L},\qquad \bar{z}=\frac{z}{h},\qquad \zeta=\frac{(EI)_{eq}+(GAl^2)_{eq}}{(EI)_m} \tag{23}$$

Then the non-dimensional static equation derived as:

$$\zeta\frac{\partial^4 \bar{w}_s(\bar{x})}{\partial \bar{x}^4}=D_1\frac{V^2}{(1-\bar{w}_s(\bar{x}))^2} \tag{24}$$

where

$$D_1=\frac{L^4\varepsilon_0 b}{2(EI)_m g_0^3} \tag{25}$$

The corresponding boundary conditions for the beam static deflection are:

$$\bar{w}(0)=0,\qquad \frac{\partial\bar{w}(0)}{\partial\bar{x}}=0,\qquad \frac{\partial\bar{w}^2(1)}{\partial\bar{x}^2}=0,\qquad \frac{\partial\bar{w}^3(1)}{\partial\bar{x}^3}=0 \tag{26}$$

*2.5 Application of a step DC voltage on thermally deflected beam*

The total deflection of the beam, caused by thermal moment and a DC voltage introduce as:

$$w_s(x)=w_{0T}(x)+\varpi_s(x) \tag{27}$$

where $w_{0T}(x)$ is primal static deflection of the micro-beam owing to the temperature rise and $\varpi_s(x)$ is the deflection due to the applied voltage, substituting Eq. (27) into Eq. (22), results in following equation:

$$((EI)_{eq}+(GAl^2)_{eq})\frac{\mathrm{d}^4\varpi_s(x)}{\mathrm{d}x^4}+((EI)_{eq}+(GAl^2)_{eq})\frac{\mathrm{d}^4 w_{0T}(x)}{\mathrm{d}x^4}=\frac{\varepsilon_0 bV^2}{2(g_0-w_{0T}(x)-\varpi_s(x))^2} \tag{28}$$

Following dimensionless quantities and Eq. (23) are presented to transform Eq. (28) into non-dimensional forms:

$$\bar{w}_{0T}=\frac{w_{0T}}{g_0},\qquad \bar{\varpi}_s=\frac{\varpi_s}{g_0} \tag{29}$$

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The non-dimensional static deflection equation for a thermally deflected micro-beam is presented as:

$$\zeta \frac{d^4 \bar{\varpi}_s(\bar{x})}{d\bar{x}^4} + \zeta \frac{d^4 \bar{w}_{0T}(\bar{x})}{d\bar{x}^4} = D_1 \frac{V^2}{(1 - \bar{w}_{0T}(\bar{x}) - \bar{\varpi}_s(\bar{x}))^2} \tag{30}$$

The corresponding boundary conditions for the thermally deflected beam are:

$$\bar{w}_s(0) = 0, \qquad \frac{\partial \bar{w}_s(0)}{\partial \bar{x}} = 0, \qquad \zeta \frac{\partial \bar{w}_s^{\,2}(1)}{\partial \bar{x}^2} = \frac{M_T L^2}{(EI)_m g_0}, \qquad \frac{\partial \bar{w}_s^{\,3}(1)}{\partial \bar{x}^3} = 0 \tag{31}$$

## 3 NUMERICAL SOLUTION

*3.1. Static deflection of the beam due to an applied DC voltage*

Due to the non-linearity of the static equation (Eq. (24)), the solution is complicated and time consuming. One method for resolving the problems associated with the non-linearity of the equation involve a linearization technique for changing the governing equations into linear equations. A step-by-step increasing of applied DC voltage is used and the governing equation is linearized at each step [23]. To use this method, it is supposed that $\bar{w}_s^{\,i}$ is the displacement of the beam due to the applied DC voltage $V_i$. By increasing the applied voltage to a new value, the displacement can be written as:

$$\bar{w}_s^{\,i+1} = \bar{w}_s^{\,i} + \delta w = \bar{w}_s^{\,i} + \phi_s(\bar{x}) \tag{32}$$

when

$$V_{i+1} = V_i + \delta V \tag{33}$$

Using Calculus of Variation Theory and Taylor series expansion about $\bar{w}_s^{\,i}$ and $V_i$, and truncating its higher orders Eq. (24) is rearranged as follows:

$$\zeta \frac{\partial^4 \phi_s(\bar{x})}{\partial \bar{x}^4} - \frac{2V_i^2}{(1 - \bar{w}_s^{\,i})^3} \phi_s(\bar{x}) = D_1 \frac{2V_i}{(1 - \bar{w}_s^{\,i})^2} \delta V \tag{34}$$

The obtained linear differential equation can be solved using Galerkin weighted residual method [26]. Choosing suitable shape function $\psi_i(\bar{x})$, satisfying boundary conditions of the beam (Eq. (26)), $\phi_s(\bar{x})$ can be approximated by the following series:

$$\phi_s(\bar{x}) = \sum_{i=1}^{n} a_i \psi_i(\bar{x}) \tag{35}$$

where $a_i$ are constants coefficients, which are calculated in each step.

*3.2 Application of a step DC voltage on thermally deflected beam*

In this case as mentioned previousely, firstly $w_{0T}$ must be calculated. By double integrating of Eq. (15), $w_{0T}$ is determined. It is worth to pointig out that in the absence of electerostatic pressure value of bending moment casused by electrical load (M) equals to zero, therefore:

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$$((EI)_{eq}+(GAl^2)_{eq})w_{0T}(x)=-M_T\frac{x^2}{2}+c_1x+c_2 \tag{36}$$

Considering boundary conditions, (Eq.(31)) constants $c_1$ and $c_2$ are obtained zero and finally $\overline{w}_{0T}$ determined as follow:

$$\zeta\overline{w}_{0T}(\overline{x})=-\overline{M}_T\overline{x}^2,\qquad \overline{M}_T=\frac{M_T}{\Omega},\qquad \Omega=\frac{2g_0(EI)_m}{L^2} \tag{37}$$

Substituting $\overline{w}_{0T}$ into Eq. (30) then considering $\overline{\varpi}_s^{i+1}=\overline{\varpi}_s^{i}+\delta w=\overline{\varpi}_s^{i}+\varphi_s(\overline{x})$ when $V_{i+1}=V_i+\delta V$, by means of Calculus of Variation Theory and Taylor series expansion about $\overline{\varpi}_s^{i}$ and $V_i$, the non-dimensional static equation of FGM micro beam with primal temperature caused deflection derived as:

$$\zeta\frac{\partial^4\varphi_s(\overline{x})}{\partial\overline{x}^4}-D_1\frac{2V_i^2}{\left(1+\frac{\overline{M}_T}{\zeta}\overline{x}^2-\overline{w}_s^{i}\right)^3}\varphi_s(\overline{x})=\frac{D_1\delta V}{\left(1+\frac{\overline{M}_T}{\zeta}\overline{x}^2-\overline{w}_s^{i}\right)^2} \tag{38}$$

Identically the consequent linear differential equation is solved by Galerkin method, and $\varphi_s(\overline{x})$ based on function spaces can be expressed as:

$$\varphi_s(\overline{x})=\sum_{i=1}^{n}a_i\Omega_i(\overline{x}) \tag{39}$$

In which $\Omega_i(\overline{x})$ is suitable shape function satisfying accompanying boundary conditions and $a_i$ are constant coefficients is calculated in each step.

## 4 NUMERICAL RESULTS AND DISCUSSION

The considered material and geometrical properties of micro-beam and also the surrounding air characteristics are shown in Table 1. According to the different ceramic constituent percent of the bottom surface, five types of functionally graded (FG) beams are presented, with the characteristics shown in Table 2. Sadeghian et al. [33] experimentally showed that the size-dependent mechanical properties for a silicon cantilever are significant when the cantilever thickness approaches nano-meter scale. Therefore, the value of material length scale parameter of Silicon Nitride is considered equal to zero in the analysis. For a Nickel beam the size dependent behavior becomes important even for the micro-scale beams [23]. Figs. 2-4 illustrate the diagrams of Young's modulus, thermal expansion coefficient and material length scale parameter along z direction for the five types of FGM micro-beams in this analysis, respectively.

Figs. 5and 6 demonstrate the dimensionless end gap with respect to the applied DC voltage for five types of FGM micro-beams based on the classic beam theory (CBT) and modified couple stress beam theory (MCSBT), respectively, in absence of the temperature changes. Pull-in voltages obtained by the classic theory agree with the results reported in by Mohammadi-Alasti et al. [17], where it was shown that increasing the ceramic constituent percent causes an increase in the pull-in voltage by enhancing equivalent stiffness ($(EI)_{eq}$) of the FGM micro beam. However, Fig. 6 indicates that when the material length scale parameter is taken into account, enhancing ceramic constituent percent renders the beam more prone to pull-in instability. Values of $(EI)_{eq}$ and $(Gal^2)_{eq}$ in type 1 micro-beam (fully metallic) are the minimum and maximum respectively, amongst the presented types. It is worth to pointing out that with increasing the ceramic constituent percent $(Gal^2)_{eq}$ is decreased. Note that material length

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scale parameter has no effect on the stability of type 5 micro beam as $(Gal^2)_{eq}$ equals zero in this case and the results of modified couple stress theory and classic theory are the same.

Figs. 7 and 8 express graphically how the tip gaps of five types of FGM micro-beams using CT and MCST change with temperature rise, respectively. By enhancing ceramic constituent percent, the difference of the thermal expansion coefficient between the micro-beam surfaces is increased, so the thermal moment (MT) and the micro-beam deflection are increased versus the temperature changes.

**Table 1**
Geometrical and material properties [17, 23]

| Symbols | Parameters | Values |
|---|---|---|
| $L$ | Length | 500 μm |
| $b$ | Width | 90 μm |
| $h$ | Thickness | 6 $\mu m$ |
| $g_0$ | Initial gap | 2 μm |
| $E_m$ | Young's modulus of Nickel | 204 GPa |
| $G_m$ | Shear modulus of Nickel | 76 GPa |
| $\alpha_m$ | Thermal expansion of Nickel | $13.2\times10^{-6}$ °C$^{-1}$ |
| $K_m$ | Thermal conductivity of Nickel | $90.9\,Wm^{-1}$ °C$^{-1}$ |
| $l_m$ | Length scale parameter of Nickel | 5 μm |
| $E_c$ | Young's modulus of Silicon Nitride | 310 GPa |
| $G_c$ | Shear modulus of Silicon Nitride | 194 GPa |
| $\alpha_c$ | Thermal expansion of Silicon Nitride | $3.4\times10^{-6}$ °C$^{-1}$ |
| $K_c$ | Thermal conductivity of Silicon Nitride | 30 Wm$^{-1}$°C$^{-1}$ |
| $l_c$ | Length scale parameter of Silicon Nitride | ≃0 μm |
| $\varepsilon_0$ | Permittivity of air | 8.854 pF/m |
| $T_\infty$ | Temperature of air | 25 °C |
| $h_c$ | Convection Coefficient of air | 25 W/(m$^2$°C) |

**Table 2**
Characteristics of FGM micro-beams

| Type | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Ceramic percent of bottom surface | 0% (Metal-rich) | 25% | 50% | 75% | 100% |
| $E_h$ (Gpa) | 204 | 230.5 | 257 | 283.5 | 310 |
| $G_h$ (Gpa) | 76 | 105.5 | 135 | 164.5 | 194 |
| $\alpha_h\times10^{-6}$k$^{-1}$ | 13.2 | 10.75 | 8.3 | 5.85 | 3.4 |
| $K_h$ (wm$^{-1}$k$^{-1}$) | 90.9 | 75.65 | 60.5 | 45.2 | 30 |
| $l_h$ ($\mu$m) | 5 | 3.75 | 2.5 | 1.25 | 0 |
| $\gamma$ | 0 | 20355 | 48493 | 54849 | 69742 |
| $\mu$ | 0 | 54663 | 95757 | 128700 | 156190 |
| $\beta$ | 0 | -34218.5 | -77327 | -135629 | -226073 |
| $\lambda$ | 0 | -30552 | -67991 | -116352 | -187460 |
| $\tau$ | 0 | -47947 | -115520 | -231050 | -∞ |

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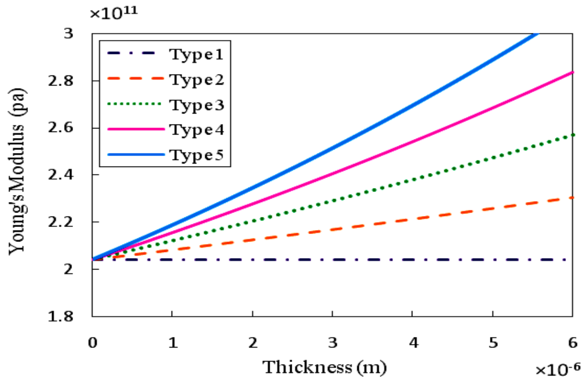


**Fig. 2**
Variation of the Young's modulus along thickness for five types of FGM micro-beams.

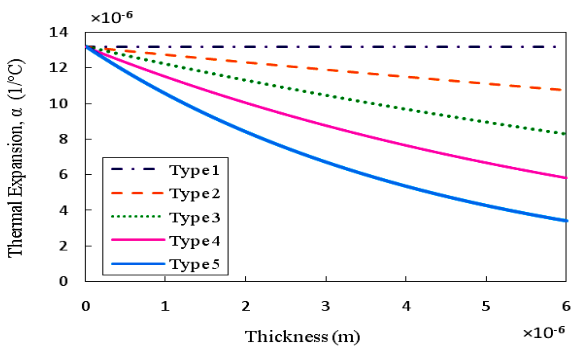


**Fig. 3**
Variation of the thermal conductivity along thickness for five types of FGM micro-beams.

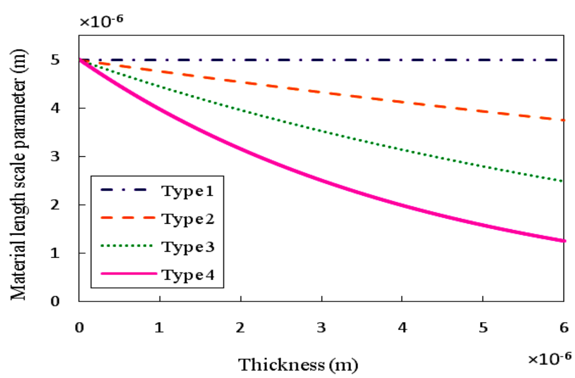


**Fig. 4**
Variation of the material length scale parameter along thickness for four types of FGM micro-beams.

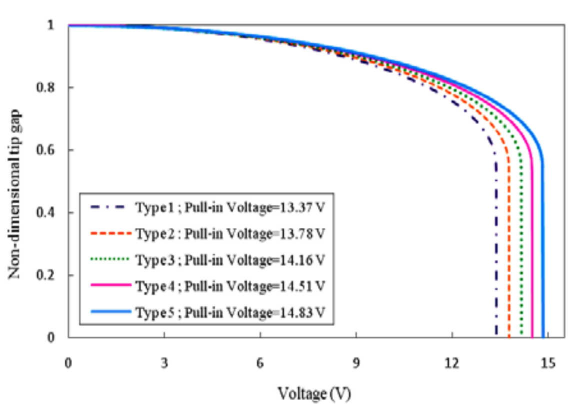


**Fig. 5**
Non-dimensional tip gap versus applied voltage for five types of FGM micro-beams based on CT for $\theta = 0.$ .

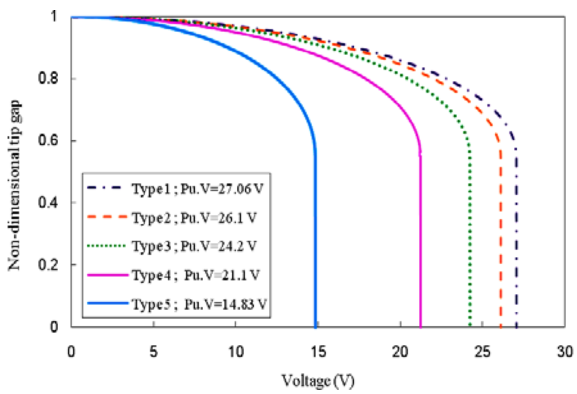


**Fig.6**
Non-dimensional tip gap versus applied voltage for five types of FGM micro-beams based on MCST for $\theta = 0$ (Pu.V=Pull-in voltage).

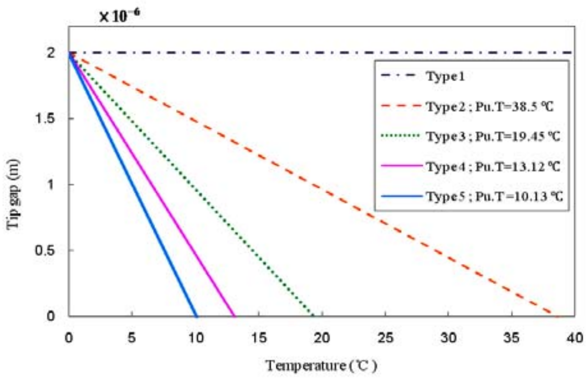


**Fig. 7**
The tip gap versus Temperature for five types of FGM micro-beams based on CT, for $V = 0$ (Pu.T = Pull-in Temperature).

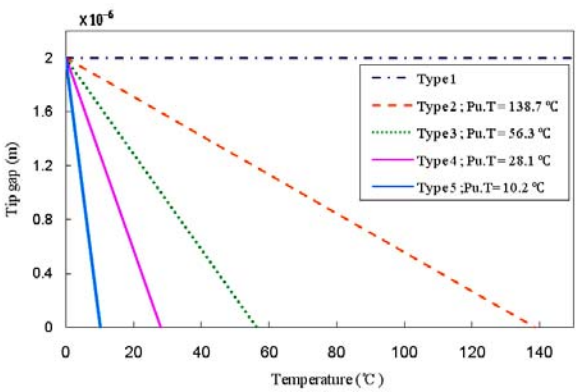


**Fig. 8**
The tip gap versus Temperature for five types of FGM micro-beams based on MCST, for $V$=0.

Note that ''pull-in temperature'' is the value of temperature which the tip of FGM micro-beam contacts with substrate (stationary electrode). According to reference [17], the calculated pull-in temperatures which is shown in Fig. 7 also, are less than the corresponding values when there is considered material length scale parameter, see Fig. 8. Due to lack of difference of thermal expansion coefficient between the beam surfaces, first type indicates no reflection to temperature change (because of $M_T$=0). Comparing Figs. 7 and 8 with each other it can be concluded, for types 1, 2, 3 and 4 considering length scale parameter exerts profound effect over our results and so applying MCST is necessary. As it obvious from Fig. 8, similar to results in pull-in voltages (Fig. 6), for type 5 material length scale has no effect on the contact temperature.

Figs .9 and 10 show the non-dimensional tip gap of the thermally deflected 4th and 5th type of the FGM micro-beam with respect to voltage when the top surface of beam is imposed to different primal temperatures, based on CT

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and MCST, respectively. Results show that the pull-in voltages decrease as the temperature of top surface increases. Comparison of Figs. 9 and 10 suggests that the obtained pull-in voltages which calculated by applying MCST are greater than the corresponding values ignoring material length scale parameters (CT).

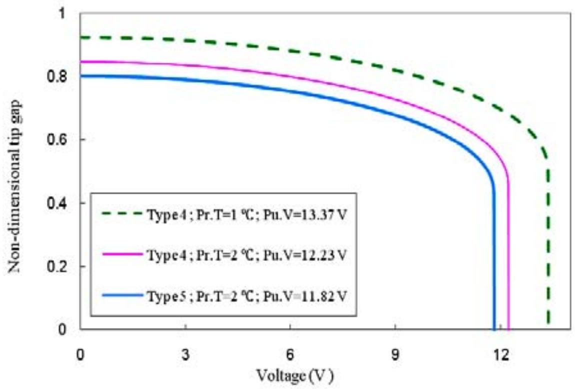


**Fig. 9**
Non-dimensional tip gap of types 4 and 5 of the FGM micro-beam versus voltage for thermally deflected beam based on CT. Pr.T=Primal Temperature; Pu.V=Pull-in Voltage.

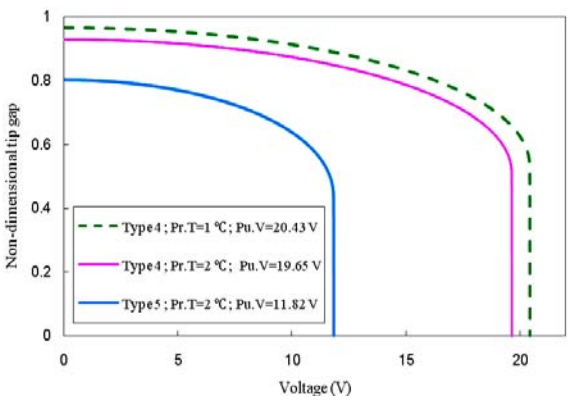


**Fig. 10**
Non-dimensional tip gap of types 4 and 5 of the FGM micro-beam versus voltage for thermally deflected beam based on MCST; Pr.T=Primal Temperature, Pu.V=Pull-in Voltage.

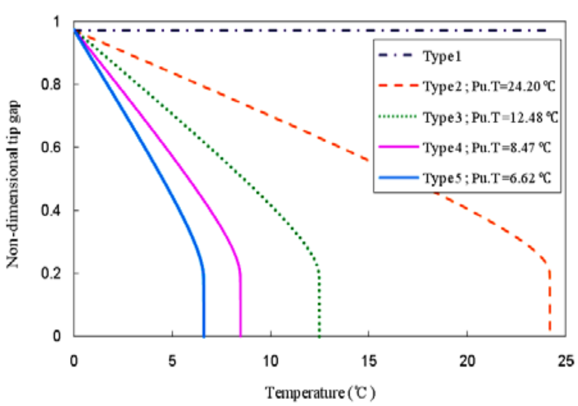


**Fig. 11**
Non-dimensional tip gap of the all types of the electrostatically deflected FGM micro-beams versus temperature rise based on CT. (5(*V*) primal voltage is applied to deflect micro-beam electrostatically).

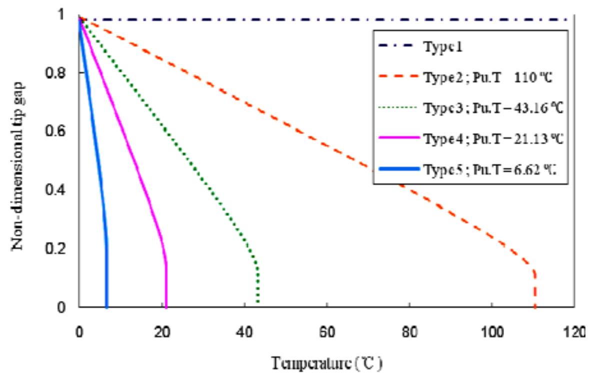


**Fig. 12**
Non-dimensional tip gap of the all types of the electrostatically deflected FGM micro-beams versus temperature rise based on MCST. (5(*V*) primal voltage is applied to deflect micro-beam electrostatically).

Figs .11 and 12 show the non-dimensional tip gap of the all types of electrostatically deflected FGM micro-beams versus temperature rise based on CT and MCST, respectively. 5 volt primal voltage is applied to deflect micro-beams electrostatically, then the temperature changes are applied and it causes to increase the micro-beam deflections up to the pull-in temperature. As displayed in diagram it can be found out that enhancing ceramic constituent percent causes to instability with a lesser voltage. When there is no applied voltage and micro beam deflects only by temperature changes, the tip deflection varies linearly with respect to temperature as far as contacting with the stationary electrode. Finally, as has been discussed in the previous sections, except type 5, with increasing ceramic constituent percent total stiffness ($(EI)_{eq}+(Gal^2)_{eq}$) of the beam is increased and it causes to increase of the temperature pull-in value. According to Fig. 11, the obtained value of pull-in temperature for type 5 (equals to 6.62 °C ) is in good agreement with the value predicted by Ref. [17].

## 5 CONCLUSIONS

In this paper, the size-dependent static behavior of a cantilever micro-beams made of functionally graded materials (FGMs) subjected to nonlinear electrostatic force and thermal moment was investigated. It was assumed that the top surface of the FGM micro-beam was made of pure metal but the bottom surface from a mixture of metal and ceramic. The governing nonlinear differential equations based on *Euler-Bernoulli* beam assumptions and using modified couple stress theory (MCST) were solved by Galerkin weighted residual method. The static instability of the FGM micro-beam was studied when the beam subjected to an electrostatic force and thermal moments, separated and simultaneously. Comparison of the obtained results with results presented in literature shows that when the length scale parameter is considered, pull-in voltages and pull-in temperatures have greater values than those predicted by classic theory of elasticity of beams. By increasing the ceramic constituent of the FG beam, the value of $(EI)_{eq}$ is increased but the value of $(Gal^2)_{eq}$ is decreased, because the length scale parameter of the ceramic material of the beam, is equal to zero. Therefore, it is concluded that the sum of $(EI)_{eq}$ and $(Gal^2)_{eq}$ as the equivalent stiffness of the FG beam in modified couple stress theory identify pull-in voltages and pull-in temperatures of the structure. The obtained results can be useful for MEMS designers to establish stable regions of micro-electro-mechanical (MEM) devices and to optimize their performance.

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