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A Mathematical Formulation to Estimate the Fundamental Period of High-Rise Buildings Including Flexural-Shear Behavior and Structural Interaction

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ABSTRACT

The objective of the current study is to develop a simple formula to estimate the fundamental vibration period of tall buildings for using in equivalent lateral force analysis specified in building codes. The method based on Sturm-Liouville differential equation is presented here for estimating the fundamental period of natural vibration. The resulting equation, based on the continuum representation of tall buildings with various lateral resisting systems for natural vibration of the buildings, is proved to be the forth-order Sturm-Liouville differential equation, and a quick method for determining the fundamental period of natural vibration of the building is presented. Making use of the coupled wall theory for natural vibration, the method is extended to deal with vibration problem of other buildings braced by frame, walls or/and tube. The proposed formulation will allow a more consistent and accurate use of code formulae for calculating the earthquake-induced maximum base shear in a building. Use of the method is economical with respect to both computer time and equipment and can be used to verify the results of the finite element analyses where the time-consuming procedure of handling all the data can always be a source of errors.

Keywords: Fundamental period; Continuum; FEA; 4th order sturm-liouville differential equation

1 INTRODUCTION

MULTI-STOREY buildings have become taller and more slender, and with this trend the analysis of tall buildings may emerge as a critical design item. When walls or/and core wall are situated in advantageous positions in the building, they can be very efficient in resisting lateral loads originating from wind or earthquakes. During an earthquake, ground motions occur in a random fashion in all directions. When a structure is subjected to ground motions in an earthquake, it responds in a vibratory fashion. Since it is impossible accurately to predict the characteristics of the ground motions that may occur at any given site, for example, it is impossible to evaluate the complete behavior of a structure when subjected to very large seismic disturbances. It is, however, possible to impart to the design features that will ensure the most desirable behavior. Free vibration analysis plays an important role in the structural design of buildings, especially for the fundamental mode because the fundamental mode shape is a dominant component in wind- and earthquake-induced vibrations of tall buildings.

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When the structure is behaving elastically, the maximum response acceleration will depend on the structural natural period of vibration and the magnitude of the damping present. The design seismic loading recommended by building codes is in the form of static lateral loading. Earthquake static lateral loading for tall buildings is normally applied with a near- triangular distribution to the structure, placing the greatest load at the top, thus simulating the deflected shape of the fundamental mode of vibration. Structural engineers have developed confidence in the design of the buildings. The practical use of this lies in the earthquake design of tall buildings for which the minimum base shear is usually expressed in codes of practice as a function of the periods of natural vibration.

Most seismic codes specify empirical formulae to estimate the fundamental vibration period of buildings. It is, however, shown that the code formulae provide periods that are generally shorter than measured periods (Goel and Chopra [1]), and current code formulae for estimating the fundamental period of buildings are grossly inadequate (Goel and Chopra [2]). The periods are usually estimated by simple but crude formulae that may be as much as 100% in error. These large errors would result in overly conservative values of base shear and, hence, overdesign of the structure. Of course, the static and dynamic characteristics of buildings regarded as a spatial structural system should be analyzed three-dimensionally by matrix techniques. The problem would, however, be too complex to solve for practical designs, and many uncertain factors in the spatial structural analysis, such as earthquake excitation and non-linear properties of the materials, usually are not determined exactly in the above analysis. For such building design, therefore, there must be an available, simple and more perfect method at least for a preliminary design because a computer run at this stage of design is neither feasible nor economical as even the structural member itself might be changed in further studies. To facilitate the practical application of such a procedure by engineers, it is necessary to reduce the complexity of each step and to simplify the technique.

Studies on the natural periods of buildings have been reported by many authors [1-7]. The fundamental vibration period of a tall building appears in the equation specified in building codes to calculate the design base shear and lateral forces. An objective of the current paper is to describe a simple method for the calculation of the fundamental period of tall buildings. Since seismic responses are usually not very sensitive to the accuracy of the natural periods, any method yielding natural periods within 8% accuracy shall be assumed adequate. To simplify their computation, a uniform shear- flexure cantilever beam model is used to perform a dynamic analysis of buildings. To present a simple technique for the calculation, a resulting equation for the free vibration of buildings is proved strictly to be the fourth-order Sturm- Liouville differential equation, and a formulation for calculating the fundamental natural period of the structure is derived. The theory used in the whole course is strict, and a simple approximate solution is obtained by operator. Finally, some engineering examples are worked out in detail to illustrate that the method provides results that compare closely with those obtained from more exact but lengthier methods of analysis. It can be seen that the proposed method is simple and accurate enough to be used at the concept design stage. The current paper is intended to be useful for the practicing engineer.

In preliminary seismic design and seismic assessment, it is important to evaluate accurately the natural fundamental frequency of vibration of a building structure, since it is directly related to the corresponding seismic forces and deformations. Traditionally, the fundamental frequency required in the analysis is generally estimated from empirical equations which are normally expressed in terms of the height of a building only. For example, the natural fundamental period of vibration for moment-resisting frames $T_1 = 0.1N$ sec., where N is the number of storeys, where the effects of the interaction of different structural forms in the building, rigidities of the structure and intensities of the gravity and applied lateral loads, etc., on the vibration characteristics are not considered in the calculation. This will clearly result in an inaccurate estimate of natural vibration frequency or period of a building.

Realizing the consequences of inaccurate estimate of the natural frequency, improved formulas have been derived. Rakesh et al. [8] calibrated the period data of both reinforced concrete and steel moment-resisting frames to produce two different sets of empirical formulas depending on the construction materials. Later Goel and Chopra [2] proposed another set of empirical period formulae, including the upper and lower bound values for concrete shear wall buildings, in which the influence of the height of building structure and the area of the structural walls at the ground floor are considered. To obtain a more accurate estimate of the vibration frequency, more structural parameters, such as the mass and the degree of structural interaction, have been implemented to the period formula based on rationale engineering models.

Building structures can be represented by engineering models developed from the continuum techniques for conducting simplified static and dynamic analyses. For instance, Rutenberg [9] derived an expression for the natural frequency of coupled shear walls and Wang et al. [10] developed earthquake design spectra for wall frame structures based on a flexural-shear cantilever. It has been recognized that the continuum modelling is a very useful tool for analysis of tall buildings (Smith and Coull [11]) and has been used for the development of various drift spectra for preliminary seismic assessment. This leads Iwan [12] to establish a drift spectrum demand measure for earthquake ground motions, and Miranda and Akkar [13] further developed the generalized interstorey drift spectrum approach.

Despite the rapid development of modelling techniques for buildings in structural engineering, the connection between the model and the corresponding building structure is rarely discussed, which sometimes leads to difficulties in practical use. Instead of using rigidity and structural interactions, which are the by-products of the system, more practical parameters that are design variables should be used in modelling. In this study, a theoretical, yet practical, model is developed for the quick prediction of the natural fundamental period of vibration for tall building structures.

The proposed method is formulated based on continuum modelling, where a tall-building structure is considered as a continuous flexural-shear cantilever. It is presented in a closed form mathematical expression for calculating the natural fundamental period T_1, which is a function of the interaction of different structural forms in the building, structural height, top and interstorey drifts, and the intensities of the applied loading. Numerical studies pertaining to determining the fundamental periods of moment-resisting frames, shear walls and a wall-frame structures show that the results from the proposed formula agree very well to the finite-element analysis (FEA). The proposed analysis has been shown to provide a simple and quick, yet accurate, means of determining the fundamental period of vibration for buildings behaving elastically.

1.1 Continuum flexural-shear cantilever

In the analysis, a tall-building structure may generally be modeled as a two-dimensional continuum model (Smith and Coull [11]) by assuming that the structural properties, including strength and stiffness, are uniform in height with negligible torsional effect. A typical model is the continuum flexural-shear cantilever as shown in Fig. 1. This continuum model consists of a flexural cantilever and a shear cantilever, where the flexural cantilever is a representation of the flexurally dominant systems, such as shear walls, while the shear cantilever represents shear dominant systems, such as moment-resisting frame structures. The two cantilevers are connected by axially rigid links, which may be represented by the axially rigid continuum that transmits horizontal forces only. Thus, the flexural and shear cantilevers will deflect identically under the action of horizontal loading.

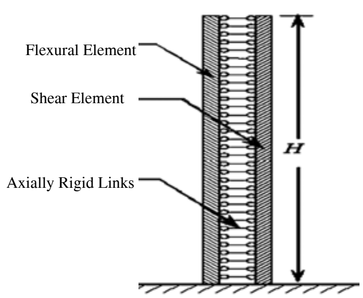


Fig. 1
Continuum flexural-shear cantilever.

The deformation characteristic of the continuum flexural-shear cantilever is mainly governed by the degree of structural interaction between the flexural and shear cantilevers, defined by

$$\alpha = H\sqrt{\frac{GA}{EI}} \tag{1}$$

where EI and GA are the flexural and shear rigidities of the building respectively, and H is the total height of the structure.

The deformation profile of the flexural-shear cantilever is significantly influenced by the degree of structural interaction α . The structure may deform with a pure flexural shape when $\alpha = 0$ and a pure shear shape when α becomes infinite, as shown in Fig. 2. In practice, the value of α generally ranges from 0 to 2 for pure shear-wall buildings, 5 to 20 for moment-resisting frames, and 1.5 to 6 for wall-frame structures (Miranda and Reyes [14]).

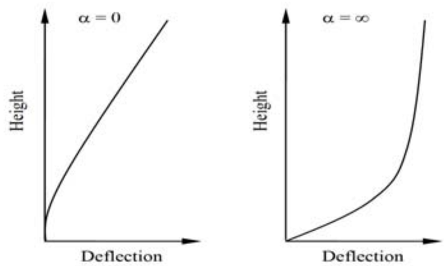


Fig. 2
Deformation shapes of flexural-shear cantilever (Miranda and Reyes [14]).

1.2 Dynamics properties of the model

A free-body diagram of the flexural-shear cantilever at time t under free vibration, with the mass replaced by its inertia force is shown in Fig. 3.

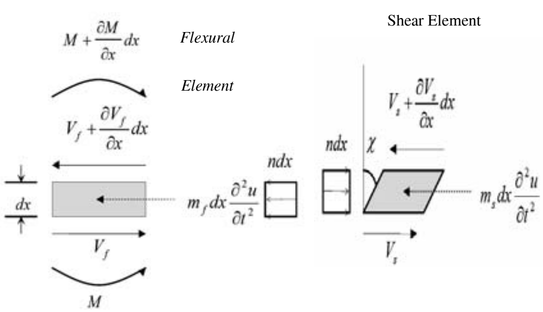


Fig. 3
Free body diagram of flexural-shear cantilever at time t under free vibration.

Consider the dynamic equilibrium of both flexural and shear elements in the flexural-shear cantilever, so the two equilibrium equations in horizontal direction can be developed. For the flexural element,

$$m_f\,dx\,\frac{\partial^2 u}{\partial t^2}+V_f+\frac{\partial V_f}{\partial x}dx-V_f+ndx=0 \tag{2}$$

and for the shear element,

$$m_s dx\,\frac{\partial^2 u}{\partial t^2}+V_s+\frac{\partial V_s}{\partial x}dx-V_s-ndx=0 \tag{3}$$

where m_f and m_s are the distributed masses of the flexural and shear elements respectively, u is the relative lateral displacement, n is the stress of the axially rigid continuum, and V_f and V_s are shear forces of the flexural and shear elements respectively given by

$$V_f=\frac{\partial M}{\partial x}=\frac{\partial}{\partial x}\left(EI\,\frac{\partial^2 u}{\partial x^2}\right) \tag{4}$$

$$V_s=-GA\,\chi=-GA\,\frac{\partial u}{\partial x} \tag{5}$$

in which M is the bending moment and x is the shear strain. By combining the equations of equilibrium of the elements represented in Eqs. (2) and (3) and then substituting the shear forces given by Eqs. (4) and (5), the governing equation of motion of the continuum flexural-shear model under free vibration can be derived and simplified as:

$$m\frac{\partial^2 u}{\partial t^2}+EI\frac{\partial^4 u}{\partial x^4}-GA\frac{\partial^2 u}{\partial x^2}=0 \tag{6}$$

where $m = mf + ms$ is the distributed mass. Let the relative lateral displacement be

$$u(x,t)=\phi(x)q(t) \tag{7}$$

where $\phi(x)$ is the shape function and $q(t)$ is the modal coordinate, with different independent variables. Eq. (6) can be solved by the technique of separation of variables, and thus decomposed into two equations with variables x and t respectively,

$$q''(t)+\omega^2 q(t)=0 \tag{8}$$

$$\frac{EI}{m}\phi^{(4)}(x)-\frac{GA}{m}\phi''(x)-\omega^2\phi(x)=0 \tag{9}$$

where ω is the natural vibration frequency of the continuum model. The fourth-order ordinary differential Eq. (9) can be solved by considering four appropriate boundary conditions, resulting in Eq.(10) that contains the natural vibration characteristics of the flexural-shear cantilever, has to be satisfied under the fixed-base condition,

$$2+\left(2+\frac{\alpha^4}{\beta_i^2\gamma_i^2}\right)\cos(\gamma_i)\cosh(\beta_i)+\frac{\alpha^2}{\beta_i\gamma_i}\sin(\gamma_i)sinh(\beta_i)=0 \tag{10}$$

where $\gamma_i^2=\frac{1}{2}\left[-\alpha^2+\sqrt{\alpha^2+4mH^4\omega_i^2/EI}\right]$ and $\beta_i^2=\alpha^2+\gamma_i^2$

The natural periods and shape functions corresponding to the characteristic Eq. (10) are then given by

$$T_i=\sqrt{\frac{m}{EI}}\frac{2\pi}{\sqrt{\gamma_i^2\left(\gamma_i^2+\alpha^2\right)}}H^2 \tag{11}$$

$$\phi_i(z)=C_{i1}\cos(\gamma_i)\left(\frac{x}{H}\right)+C_{i2}\sin(\gamma_i)\left(\frac{x}{H}\right)+C_{i3}\cos h(\beta_i)\left(\frac{x}{H}\right)+C_{i4}\sinh(\beta_i)\left(\frac{x}{H}\right) \tag{12}$$

in which C_i is a coefficient composed of α, β_i and γ .

2 THE PROPOSED MODEL FOR QUICK ESTIMATE OF FUNDAMENTAL PERIOD T_1

It is well known that the flexural rigidity of a building has a significant effect on the natural fundamental frequencies of vibration. A free body diagram of a flexural-shear cantilever subjected to an inverted triangular load with a maximum intensity of w_o, which gives the smallest practical lateral load, is shown in Fig. 4.

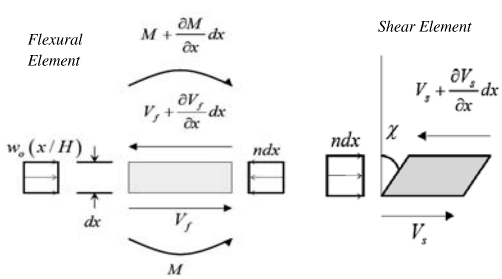


Fig. 4
Free body diagram of flexural-shear cantilever under lateral loading.

The governing equation with respect to lateral deflection can then be derived and is given in a similar form to that Eq.(6),

$$EI\frac{\partial^4 y}{\partial x^4} - GA\frac{\partial^2 y}{\partial x^2} = \frac{w_0 x}{H} \tag{13}$$

where y is the static lateral deflection. With four appropriate boundary conditions, Eq.(13) can be solved and the lateral deflection under the fixed-base condition is given by

$$y(x) = \frac{w_0 H^4}{EI\alpha^4}\left[\begin{array}{l}\left(\dfrac{\alpha\sinh(\alpha)+2}{2\cosh(\alpha)} - \dfrac{\sinh(\alpha)}{\alpha\cosh(\alpha)}\right)\left(cosh\dfrac{\alpha x}{H} - 1\right) \\ +\left(\dfrac{1}{\alpha} - \dfrac{\alpha}{2}\right)sinh\dfrac{\alpha x}{H} - \dfrac{\alpha^2}{6}\left(\dfrac{x}{H}\right)^3 + \left(\dfrac{\alpha^2}{2} - 1\right)\dfrac{x}{H}\end{array}\right] \tag{14}$$

The flexural rigidity EI is directly related to the design roof drift ratio R_Δ, which is the ratio of $y(H)/H$, and can be obtained by substituting $x = H$ into Eq.(14) for derivation,

$$EI = \frac{w_0 H^3}{R_\Delta}\frac{1}{\alpha^4}\left(\frac{\alpha^2}{3} + \frac{\sinh(\alpha)}{\alpha\cosh(\alpha)} - \frac{\alpha\sinh(\alpha)+2}{2\cosh(\alpha)}\right) \tag{15}$$

Substituting Eq.(15) into the equation of natural periods of vibration, Eq.(11), and only considering the first mode of vibration ($i = 1$) lead to the following formula of the natural fundamental period

$$T_1 = f(\alpha)|_{i=1}\sqrt{\frac{mH}{w_0}R_\Delta} \tag{16}$$

where

$$f(\alpha)|_{i=1} = \frac{2\pi\alpha^2}{\sqrt{\gamma_1^2\left(\gamma_1^2+\alpha^2\right)\left(\dfrac{\alpha^2}{3} + \dfrac{\sinh(\alpha)}{\alpha\cosh(\alpha)} - \dfrac{\alpha\sinh(\alpha)+2}{2\cosh(\alpha)}\right)}} \tag{17}$$

where, $f(\alpha)$ is a function of the degree of structural interaction between the flexural and shear cantilevers and considered as an index of structural interaction. The value of $f(\alpha)$ varies between 5.9 and 6.8 , as shown in Fig. 5, if only the first mode of vibration ($i = 1$) is considered, giving a maximum difference of 15%, which indicates the fundamental period T_1 can be changed up to 15% due to the structural interaction.

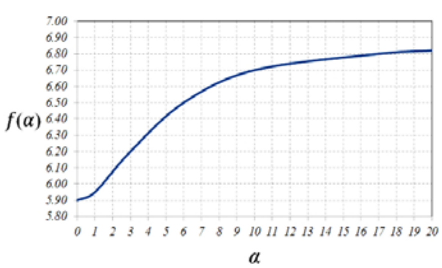


Fig. 5
Index of structural interaction.

2.1 Prediction of degree of structural interaction

The degree of structural interaction between the flexural and shear cantilevers, α, may be calculated using Eq. (1) by identifying the flexural and shear rigidities of structural systems.

A suggested approach was proposed by Smith and Coull [11] in their book "*Tall Building Structures: Analysis and Design*". The flexural rigidity of a building structure can be regarded as the total flexural rigidities of the shear walls and the related components, which is

$$EI=\sum_{i=1}^{n}E_iI_i \tag{18}$$

where n is the number of walls, E is the elastic modulus of the material and I is the moment of inertia of the section.

The shear rigidity of a building is contributed from the moment resisting frames constructed by columns and beams. The equation of shear rigidity is

$$GA=\frac{12E}{h\left(\frac{1}{B}+\frac{1}{C}\right)} \tag{19}$$

where h is the storey height, $B=\sum_{i=1}^{N}\left(\frac{I_b}{L}\right)_i$,$C=\sum_{i=1}^{M}\left(\frac{I_c}{h}\right)_i$ .Where N is the number of beams, M is the number of columns, L is the length of the beam. I_b and I_c are the moment of inertia of the beam's section and column's section, respectively. Normally, the exact different lateral systems in a building are very difficult to be identified due to structural couplings in the systems. Eq.(1) would not be recommended generally for determining the value of α. Deflection, which results from the action of external forces under the resultant lateral system, could be used to predict the degree of structural interaction.

The mode of deformation is governed by the degree of structural interaction. By increasing the degree of structural interaction, the model deforms from a flexural mode to a shear mode. Therefore, the degree of structural interaction can be predicted from the deformation characteristic. Under lateral loading, the model deforms and the deformation characteristic can be changed by the degree of structural interaction. By minimizing the difference in deflection between the building and the model, the degree of structural interaction corresponding to the building can be obtained.

The deflection of the continuum model under uniform load can be calculated by Eq.(20):

$$y(x)=\frac{wH^4}{EI\alpha^4}\left(\frac{\alpha\sinh(\alpha)+1}{\cosh(\alpha)}\left(cosh\frac{\alpha x}{H}-1\right)-\alpha sinh\frac{\alpha x}{H}+\alpha^2\left(\frac{x}{H}-\frac{x^2}{2H^2}\right)\right) \tag{20}$$

where w is the magnitude of the uniform load. The roof displacement is

$$y(H)=\frac{wH^4}{EI\alpha^4}\left(1+\frac{\alpha^2}{2}-\frac{\alpha\sinh(\alpha)+1}{\cosh(\alpha)}\right) \tag{21}$$

A dimensionless expression which is the ratio of deflection to the roof displacement is given by

$$\frac{y(x)}{y(H)}=\frac{\left(\frac{\alpha\sinh(\alpha)+1}{\cosh(\alpha)}\left(cosh\frac{\alpha x}{H}-1\right)-\alpha sinh\frac{\alpha x}{H}+\alpha^2\left(\frac{x}{H}-\frac{x^2}{2H^2}\right)\right)}{\left(1+\frac{\alpha^2}{2}-\frac{\alpha\sinh(\alpha)+1}{\cosh(\alpha)}\right)} \tag{22}$$

Eq. (22) is the deflection shape of the model where the roof displacement is adjusted to be unity. If the model is identical to the building, the difference in deflection shape between the model and the building should be zero. This is equivalent to the following mathematical expression

$$\Delta=\sum_{i=1}^{n}\left|\frac{y(x_i)}{y(H)}-\frac{Y(x_i)}{Y(H)}\right|=0 \tag{23}$$

where Δ is the sum of the absolute differences, Y is the deflection of the building and i is the floor number.

Since it is a very strict constraint where Δ is equal to zero, a more appropriate approach is to minimize Eq. (23) by changing the degree of structural interaction. Although this method would give a degree of structural interaction that can best describe the mode of deflection of the building, it is not possible to obtain displacement of every floor. An alternative is to obtain the displacement from the corresponding multi-degree-of-freedom (MDOF) model of the building.

Since buildings are usually designed in a finite-element-based program, the particular displacement can be extracted from the model for analysis. When the corresponding MDOF model of the building is not available, a more convenient way is to approximate the value of α from the deformation profile of the building, which is characterised by $R_\Delta/R_{\delta m}$, a ratio of the design roof drift ratio to the design maximum interstorey drift ratio of the building. When a building is subjected to an inverted triangular load w_o, the interstorey drift ratio can be obtained by differentiating Eq. (14),

$$R_\delta(x) = \frac{w_0 H^3}{EI\alpha^3}\left[\begin{array}{c} \left(\dfrac{\alpha\sinh(\alpha)+2}{2\cosh(\alpha)} - \dfrac{\sinh(\alpha)}{\alpha\cosh(\alpha)}\right) sinh\,\dfrac{\alpha x}{H} \\ +\left(\dfrac{1}{\alpha}-\dfrac{\alpha}{2}\right) cosh\,\dfrac{\alpha x}{H} - \dfrac{\alpha}{2}\left(\dfrac{x}{H}\right)^2 + \left(\dfrac{\alpha}{2}-\dfrac{1}{\alpha}\right)\end{array}\right] \tag{24}$$

The design maximum interstorey drift ratio $R_{\delta m}$ is obtained by substituting x_o into Eq.(24), where x_o is the height at which the maximum interstorey drift occurs. The variation of α against the drift ratio $R_\Delta/R_{\delta m}$ is plotted in Fig. 6.

At the preliminary design stage, the roof drift ratio R_Δ and the maximum interstorey drift ratio $R_{\delta m}$ may be assumed as $h/500$ and $h/350$ respectively, where h is the storey height, and are consistent with those recommended in design codes of practice. The value of α may then be quickly determined using Fig. 6. With the given value of α, the index of structural interaction, $f(\alpha)$ is found in Fig. 5. Hence, the natural fundamental period T_1 can be conveniently calculated using Eq. (16).

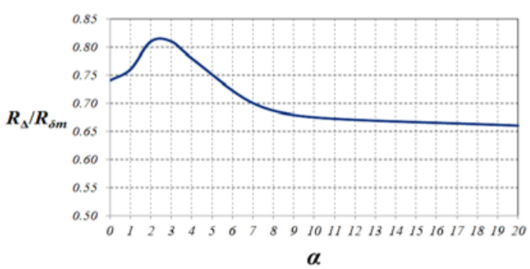


Fig. 6
Variation of α against drift ratio $R_\Delta/R_{\delta m}$.

It is seen from the equation of the natural fundamental period, Eq.(16), that when this equation is used to determine the value of T_1 of a building, the effects of the structural interaction, roof and interstorey drifts, gravity and lateral loads, and height of the structure on the fundamental period of vibration have been considered.

2.2 Degree of structural interaction in non-uniform buildings

A constant degree of structural interaction implies uniform rigidity along the height and it is an approximation to general buildings which are not uniform. The changes of stiffness along the height most of the time is not significant because the sizes of the structural components are restricted to provide minimum strength in the design. Significant change in stiffness only occurs in building structures with transfer storey. A 50-storey building with non-uniform stiffness and a transfer storey shown in Fig. 7 is analyzed to study the influence of a constant degree of structural interaction on the response. The building is constructed by three main floor plans. They are the floors below the transfer storey, the transfer storey and the floors above the transfer storey. The main differences between the three floor plans are the number of columns, the arrangement of columns, and dimensions of the columns and beams at the periphery.

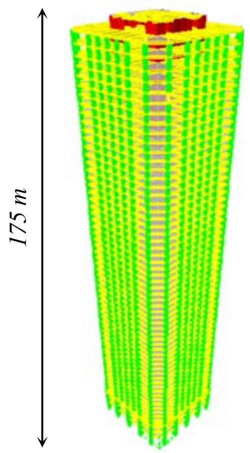


Fig. 7
Studied structure model in SAP2000 platform.

Since both fundamental period and the degree of structural interaction are the key parameters for a continuum model, they have to be similar to those of the equivalent MDOF model, so that they are equivalent to the MDOF model. A method for constructing a continuum model from an MDOF model is proposed. The flexural rigidity and shear rigidity, which construct the degree of structural interaction, are estimated from the deflection characteristic of the equivalent MDOF model. The deflection of the continuum model under a point load is calculated as:

$$y(x)=\frac{PH^3}{EI\alpha^3}\tanh(\alpha)-\frac{PH^2}{EI\alpha^2}x-\frac{PH^3}{EI\alpha^3}\tanh(\alpha)\cosh\frac{\alpha x}{H}+\frac{PH^3}{EI\alpha^3}\sinh\frac{\alpha x}{H} \tag{25}$$

where P is the magnitude of the point load at the roof.

Since the continuum model has to be equivalent to the MDOF model, the difference in deflection between the two models should be zero. This is equivalent to minimizing the differences in deflection between two models by changing two variables: the flexural rigidity (EI) and the degree of structural interaction (α).

$$\Delta=\sum_{i=1}^{n}\left|y(x_i)-Y_m(x_i)\right| \tag{26}$$

where Y_m is the deflection of the MDOF model. Since both models would have similar fundamental frequency, Eq. (10) of the characteristic equation containing the fundamental frequency should be held as a minimum, which is given by

$$2+\left(2+\frac{\alpha^4}{\left(\alpha^2+\gamma_1^2\right)\gamma_1^2}\right)\cos(\gamma_1)cosh\sqrt{\alpha^2+\gamma_1^2}+\frac{\alpha^2}{\gamma\sqrt{\alpha^2+\gamma_1^2}}\sin(\gamma_1)sinh\sqrt{\alpha^2+\gamma_1^2}=0 \tag{27}$$

where

$$\gamma_1^2=\frac{-\alpha^2+\sqrt{\alpha^4+\frac{4mH^4\omega_1^2}{EI}}}{2} \tag{28}$$

when the flexural rigidity (EI) and the degree of structural interaction (α) are obtained, the shear rigidity can be calculated from Eq.(1).

By the proposed method, the rigidities of different floor plans are estimated from uniform models made with the respective floor plans. The rigidity of the equivalent continuum model, which has a similar deflection and fundamental frequency, are also constructed. The variation of rigidity in the MDOF model and the equivalent continuum model are compared in Fig. 8. It can be seen that there is a significant change in rigidity at the transfer storey and the change is at least twice that of the other floors while the change of rigidity above the transfer storey is gradual. The rigidity of the continuum model is close to that of the floors above the transfer storey.

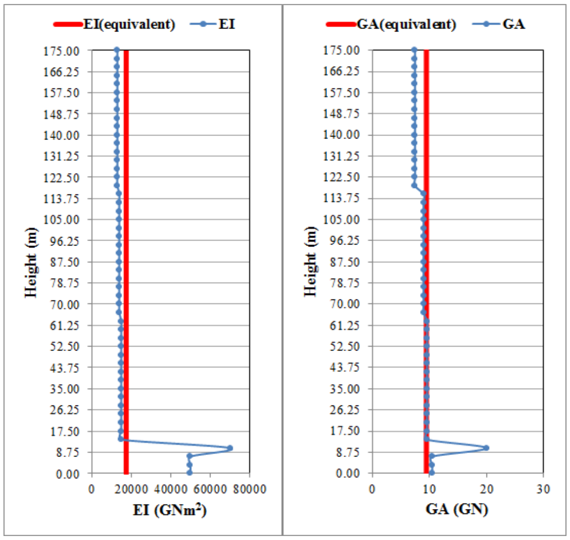
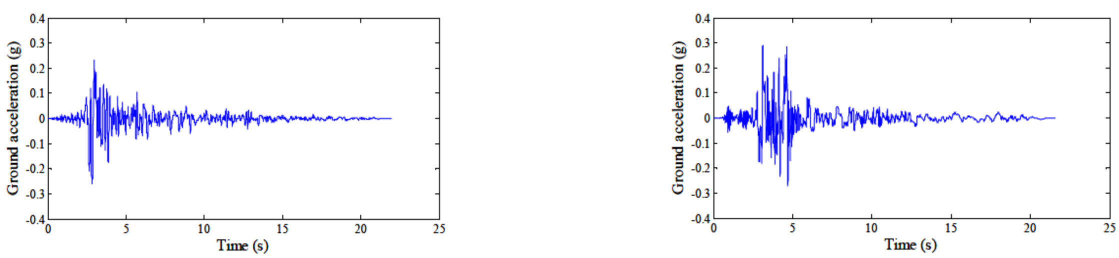


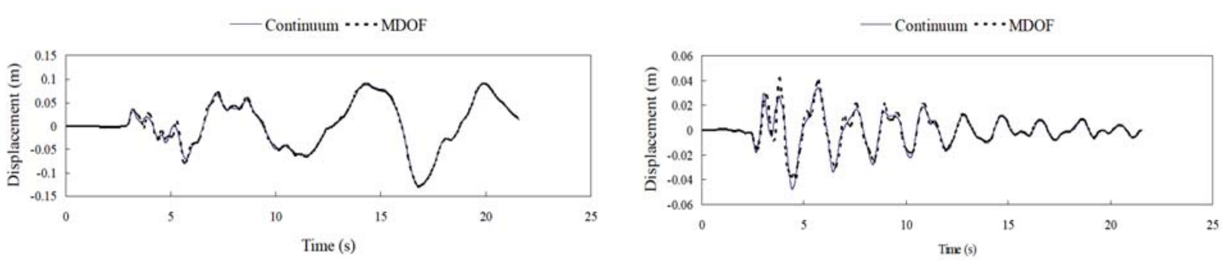
Fig. 8
The variation of stiffness and the equivalent stiffness of the building along the height.

Both the MDOF model and the equivalent continuum model are then subjected to two selected excitations in Fig. 9 from the "PEER strong Motion Database". The peak ground acceleration of both ground motion records is approximately 0.3g. The deflection history of both models is then observed at three different floor levels: 175.00 m (Fig. 10), 105.00 m (Fig. 11) and the transfer storey 12.25 m (Fig. 12).



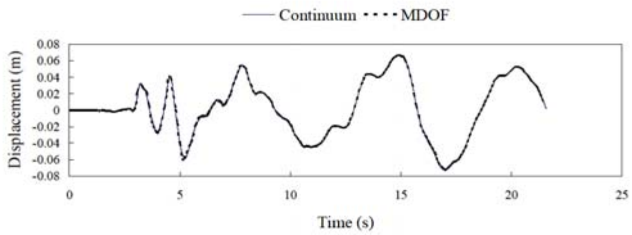
(a) (b)

Fig. 9
(a) From PEER with record ID: P0714. (b) From PEER with record ID:P0410.

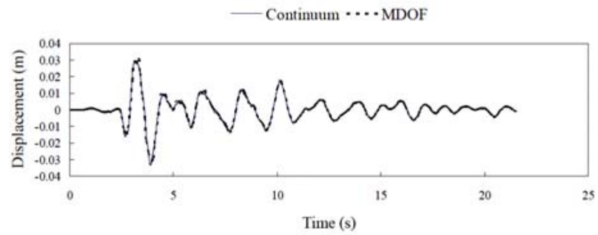


(a) (b)

Fig. 10
Time history response at height of 175.00 m.

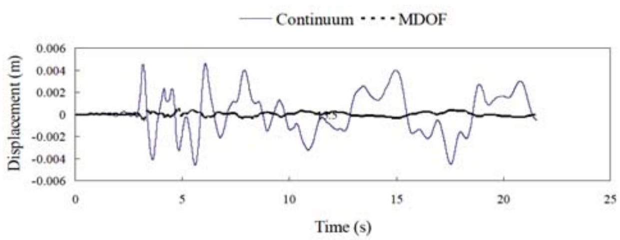


(a)

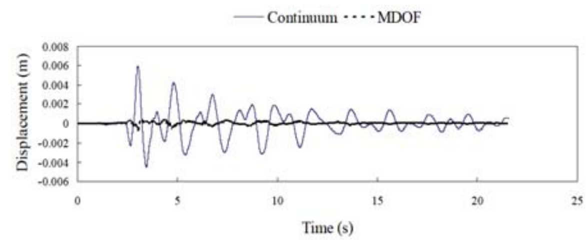


(b)

Fig. 11
Time history response at height of 105.00 m.



(a)



(b)

Fig. 12
Time history response at height of 12.25 m.

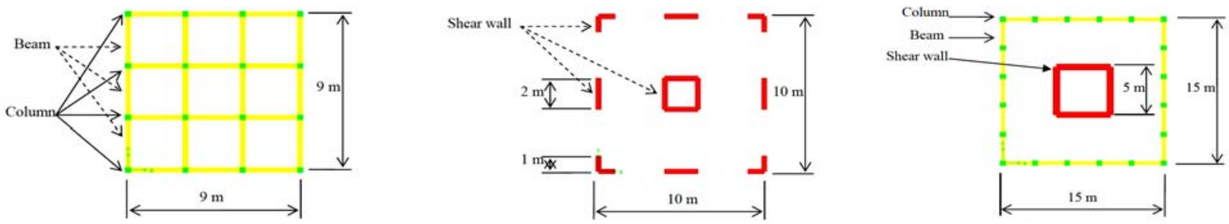
The comparisons in Figs. 10 and 11 show that the deflection of the equivalent continuum model is close to that of the MDOF model in both phase and magnitude. However at the transfer storey, the continuum model could not capture the response of the MDOF model as shown in Fig. 12. The reason for this difference is the abrupt change of the stiffness at the transfer storey. When a non-uniform MDOF model is replaced by its corresponding continuum model, the response of the equivalent continuum model is close to the MDOF model at the region where the rigidity is similar, but at the region where rigidity changes abruptly, the responses predicted by the continuum model may not be reliable.

Therefore, it is concluded that an equivalent continuum model could be used to predict the response of general buildings which do not have significant change in rigidity. For building structures with significant change in rigidity, the equivalent model could not give a reliable prediction at the location where rigidity changes abruptly. Thus a constant degree of structural interaction can be used to establish an equivalent continuum model that represents general building structures.

3 NUMERICAL STUDIES

The fundamental periods of vibration of three reinforced concrete tall buildings, including a 10-storey moment-resisting frame, a 20-storey shear-wall structure and a 30-storey wall-frame structure, are calculated using the proposed formula and analyzed by the finite element method, employing SAP2000 (Computer and Structures Inc.)[15]. The three buildings have the identical storey heights of 3 m, and the floor plans are shown in Fig. 13.

Structural members, as in the practical design, are given in Table 1. , while the reinforcement ratios of the members satisfy the strength and minimum reinforcement ratio requirements, which are 0.25% for both the beam and wall sections, and 0.4% for column sections. The design concrete cube strength is 35 MPa with density of 2400 kg/m^3, and the reinforcement yield strength is 450 MPa.



(a) 10-Storey MRF (b) 20-Storey SW (c) 30-Storey Wall-Frame

Fig. 13
Floor plans of example buildings.

Table 1
Dimensions of structural members

Floor	Column(m)		Beam(m)		Wall(m)	
	10-Storey	30-Storey	10-Storey	30-Storey	20-Storey	30-Storey
21st to 30th	-	0.30×0.30	-	0.40×0.25	-	0.40
11th to 20th	-	0.40×0.40	-	0.50×0.30	0.30	0.50
1st to 10th	0.30×0.30*	0.50×0.50	0.40×0.25*	0.60×0.30	0.40	0.60

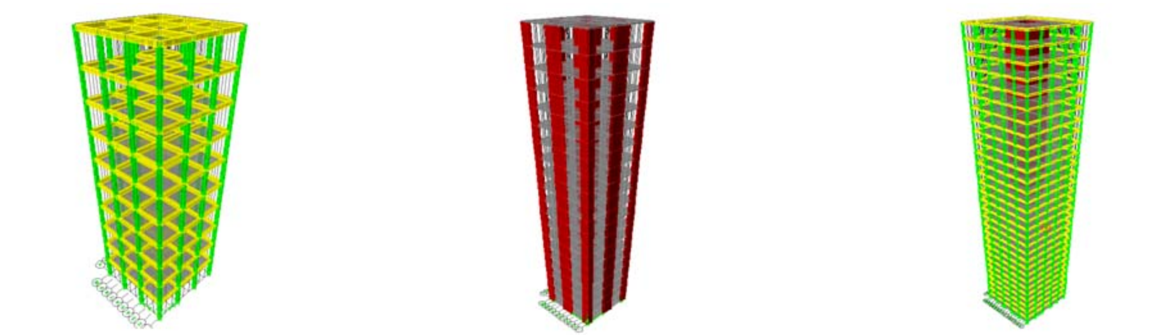
* In the 10-Storey frame, columns at 1st floor and central columns at 2nd floor are 0.35×0.35m.

The three example buildings are designed to resist a dead load (DL) of 4.5 kN/m^2, a live load (LL) of 8 kN/m^2, and lateral loads (WL) of 2.5 kN/m^2, 3.0 kN/m^2 and 3.5 kN/m^2 at the roof level respectively, which decrease linearly to 0 at the base level. Three load combinations are considered in the analysis:
(1) $1.4DL + 1.6LL$,
(2) $1.2DL + 1.2LL + 1.2WL$, and
(3) $1.0DL + 1.0LL + 1.0WL$.

To evaluate the fundamental period of vibration T_1 for the buildings using the proposed formula given by Eq.(16), the structural parameters, including the degree of structural interaction α and its corresponding index $f(\alpha)|_{i=1}$, design roof drift R_Δ and design maximum interstorey drift ratio $R_{\delta m}$, as well as the distributed mass m, are first calculated, as shown in Table 2. FEM models of the buildings are shown in Fig. 14.

Table 2
Structural parameters and applied lateral load

Buildings	m (kg/m)	H (m)	w_0 (kN/m)	R_Δ (10^{-3})	$R_{\delta m}$ (10^{-3})	α	$f(\alpha)\|_{i=1}$
10-Storey	21800	30	22500	0.96	1.35	7	6.59
20-Storey	37200	60	30000	0.34	0.40	2	6.08
30-Storey	90400	90	52500	1.12	1.31	2	6.10



a) 10-Storey moment-resisting (b) 20-Storey shear walls (c) 30-Storey-wall-frame

Fig.14
Finite element models in SAP2000 platform.

Table 3
Comparison of fundamental periods T_1 predicted by the proposed formula and FEA

Buildings	Proposed(sec)	FEA(sec)	Difference (%)
10-Storey	1.12	1.16	3.6%
20-Storey	0.92	0.96	4.3%
30-Storey	2.47	2.58	4.4%

Comparison of the results of the natural fundamental periods predicted by the proposed formula and the finite-element analysis by SAP2000 is made and presented in Table 3. It can be seen from Table 3. that the two sets of the results show very good agreement.

4 CONCLUSIONS

In this study, a model is developed for the quick estimate of the natural fundamental period of vibration for all types of building structures. The model is based on the continuum modelling, where a building structure is considered as a continuous, interactive flexural-shear cantilever. The proposed model is presented in a closed form mathematical expression for use of calculating the natural fundamental period T_1, in which the effects of the interaction of different structural forms in the building, design roof drift and maximum interstorey drift ratios, intensities of applied loading and height of the structure on the fundamental period of vibration have been considered. Numerical studies pertaining to predict the fundamental periods of the moment-resisting frames, shear walls and wall-frame structures show that the results from the proposed formula and finite-element analysis agree very well, which in the worst condition, the computed error would be less that 5%. The proposed formula has been shown to provide a simple and quick, yet accurate, means of estimating the fundamental period of vibration for building structures, in particular at the preliminary seismic design and seismic assessment stage.

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