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Wave Propagation in a Layer of Binary Mixture of Elastic Solids

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ABSTRACT

This paper concentrates on the propagation of waves in a layer of binary mixture of elastic solids subjected to stress free boundaries. Secular equations for the layer corresponding to symmetric and antisymmetric wave modes are derived in completely separate terms. The amplitudes of displacement components and specific loss for both symmetric and antisymmetric modes are obtained. The effect of mixtures on phase velocity, attenuation coefficient, specific loss and amplitude ratios for symmetric and antisymmetric modes is depicted graphically. A particular case of interest is also deduced from the present investigation.

Keywords: Mixture; Phase velocity; Attenuation coefficient; Specific loss; Amplitude ratios

1 INTRODUCTION

A mixture is a material composed of several distinct constituents. In the continuum theories of mixtures, the great abstraction that a material can be modeled as a continuum was extended by assuming that the constituents of a mixture could be modeled as superposed continua, so that each point in the mixture was simultaneously occupied by a material point of each constituent. The first continuum theory of mixtures was proposed by Truesdell [1] in terms of kinematic and thermodynamic variables associated with each constituent of the mixture. A brief description is contained in the article Truesdell and Toupin [2].

A survey of continuum theories that have been developed to model the thermomechanical behavior of mixtures consisting of various constituents is presented in detail in review articles by Bowen [3], Atkin and Craine [4, 5], Bedford and Drumheller [6] and in the books of Samohyl [7] and Rajagopal and Tao [8]. In the theories for a mixture of elastic solids presented in Bowen [3], Green and Steel [9] and Steel [10], the independent constituent variables are the displacement gradient and the relative velocity and the spatial description is used. The first theory for a mixture of elastic solids based on the Lagrangian description has been presented in Bedford and Stern [11, 12]. In this theory, the independent constituent variables are displacement gradient and the relative displacement. In the recent years, an increasing interest has been developed in the study of the qualitative properties of this theory (Iesan and Quintanilla [13]).

It is worth noting that a model of interpenetrating solid continua was applied in Tiersten and Jahanmir [14] to derive a theory of composites where the relative displacements of the individual constituents is infinitesimal. Iesan [15] derived a theory for binary mixtures of elastic solids in which the independent constituent variables are the displacement gradients, displacement fields, volume fractions, and volume fraction gradients. He also presented the linear constitutive equations in case of an isotropic body with a centre of symmetry and established a uniqueness theorem in the linear dynamic theory with no definiteness assumption on the elasticity and no restriction on the initial stress. Also Iesan [16] presented the boundary value problems of the linear theory for binary mixtures of elastic bodies and derived fundamental solutions in the equilibrium theory of homogeneous and isotropic mixtures. Ciarletta [17] presented non-linear theory for binary mixtures and developed basic equations of the linear theory.

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Later Ciarletta and Passarella [18] studied spatial behavior of dynamic processes in elastic mixtures and obtained a precise determination of the domain of influence and the spatial decay estimates with time dependent decay rate inside the domain of influence. Iesan [19] derived basic equations of a non-linear theory of heat conducting viscoelastic mixtures in Lagrangian description and established basic equations of linear theory. Quintanilla [20] considered theory of viscoelastic mixtures proposed by Iesan [19] and determined the dissipation effects by the viscosity of rate type of a constituent and relative velocity. Iesan [21] derived a continuum theory of viscoelastic composite which is modeled as a mixture of a micropolar elastic solid and a micropolar Kelvin-Voigt material.

In the present paper we have studied the propagation of waves in a layer of mixture of two elastic solids and the numerical results are illustrated graphically to study the behavior of curves for phase velocity, attenuation coefficient, specific loss and amplitudes of displacement components for various modes of wave propagation.

2 BASIC EQUATIONS

We consider the basic equations for the binary mixture in the framework of linearized theory and assume that the constituents s_1 and s_2 are each elastic bodies. Following Quintanilla [20], the equations of equilibrium in the absence of external body forces are

$$t_{ji,j} - p_i = \rho_1 \ddot{u}_i, \quad s_{ji,j} + p_i = \rho_2 \ddot{w}_i \tag{1}$$

where t_{ij} and s_{ij} are the components of stress tensor, ρ_1 and ρ_2 are the mass densities, u_i and w_i are the components of displacement associated with the constituents s_1 and s_2, respectively, p_i are the components of internal body force. The constitutive relations for a centrosymmetric homogenous and isotropic mixture are given by

$$\begin{aligned} t_{ji} &= (\lambda+\mu)e_{rr}\delta_{ij} + 2(\mu+\zeta)e_{ji} + (\alpha+\nu)g_{ss}\delta_{ji} + (2\beta+\zeta)g_{ji} + (2\gamma+\zeta)g_{ij} \\ s_{ji} &= \nu\, e_{rr}\delta_{ji} + 2\zeta e_{ij} + \alpha g_{rr}\delta_{ji} + 2\beta g_{ij} + 2\gamma g_{ji} \\ p_i &= \xi d_i \end{aligned} \tag{2}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad g_{ij} = u_{j,i} + w_{i,j}, \quad d_i = u_i - w_i \tag{3}$$

and $\alpha,\beta,\gamma,\lambda,\mu,\nu,\zeta,\xi$ are prescribed constants and δ_{ij} is Kronecker delta. Using (2) in (1) and with the help of (3), we obtain the field equations in terms of displacement fields as

$$\begin{aligned} &\alpha_1\Delta u_i + \alpha_2 u_{r,ri} + \beta_1\Delta w_i + \beta_2 w_{r,ri} - \xi(u_i - w_i) = \rho_1\ddot{u}_i \\ &\beta_1\Delta u_i + \beta_2 u_{r,ri} + \gamma_1\Delta w_i + \gamma_2 w_{r,ri} + \xi(u_i - w_i) = \rho_2\ddot{w}_i \end{aligned} \tag{4}$$

where Δ is Laplacian operator and

$$\alpha_1 = \mu+2\beta+2\zeta\,,\ \alpha_2 = \lambda+\mu+\alpha+2\nu+2\gamma+2\zeta\,,\ \beta_1 = 2\gamma+\zeta\,,\ \beta_2 = \alpha+\nu+\zeta+2\beta\,,\ \gamma_1 = 2\beta\,,\ \gamma_2 = 2\gamma+\alpha$$

3 PROBLEM FORMULATION AND SOLUTION

We consider an infinite layer of mixture of two elastic solids having thickness $2H$. For two dimensional problem taking origin of the coordinate system (x,y,z) on the middle surface of the layer. The $x-y$ plane is chosen to coincide with the middle surface and the z-axis normal to it along the thickness of the layer. The surfaces $z=\pm H$

are subjected to stress free boundaries. We take $x-z$ plane as the plane of incidence so that $\vec{u}=(u_1,0,u_3)$, $\vec{w}=(w_1,0,w_3)$, and assume that the solutions are explicitly independent of y i.e. $\frac{\partial}{\partial y}=0$. Thus the field equations reduce to

$$\alpha_1(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial z^2})+\alpha_2(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_3}{\partial x\partial z})+\beta_1(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial z^2})+\beta_2(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_3}{\partial x\partial z})-\xi(u_1-w_1)=\rho_1\ddot{u}_1 \tag{5}$$

$$\alpha_1(\frac{\partial^2 u_3}{\partial x^2}+\frac{\partial^2 u_3}{\partial z^2})+\alpha_2(\frac{\partial^2 u_1}{\partial x\partial z}+\frac{\partial^2 u_3}{\partial z^2})+\beta_1(\frac{\partial^2 w_3}{\partial x^2}+\frac{\partial^2 w_3}{\partial z^2})+\beta_2(\frac{\partial^2 w_1}{\partial x\partial z}+\frac{\partial^2 w_3}{\partial z^2})-\xi(u_3-w_3)=\rho_1\ddot{u}_3 \tag{6}$$

$$\beta_1(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial z^2})+\beta_2(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_3}{\partial x\partial z})+\gamma_1(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial z^2})+\gamma_2(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_3}{\partial x\partial z})+\xi(u_1-w_1)=\rho_2\ddot{w}_1 \tag{7}$$

$$\beta_1(\frac{\partial^2 u_3}{\partial x^2}+\frac{\partial^2 u_3}{\partial z^2})+\beta_2(\frac{\partial^2 u_1}{\partial x\partial z}+\frac{\partial^2 u_3}{\partial z^2})+\gamma_1(\frac{\partial^2 w_3}{\partial x^2}+\frac{\partial^2 w_3}{\partial z^2})+\gamma_2(\frac{\partial^2 w_1}{\partial x\partial z}+\frac{\partial^2 w_3}{\partial z^2})+\xi(u_3-w_3)=\rho_2\ddot{w}_3 \tag{8}$$

For further considerations, it is convenient to introduce the dimensionless variables defined by

$$x'=\frac{\omega^*}{c_1}x,\quad z'=\frac{\omega^*}{c_1}z,\quad u_1'=\frac{\omega^*}{c_1}u_1,\quad u_3'=\frac{\omega^*}{c_1}u_3,\quad w_1'=\frac{\omega^*}{c_1}w_1,\quad w_3'=\frac{\omega^*}{c_1}w_3,\quad t_{ij}'=\frac{t_{ij}}{\mu},\quad s_{ij}'=\frac{s_{ij}}{\mu},\quad t'=\omega^* t,$$
$$c_1^2=\frac{\lambda+2\mu}{\rho_1} \tag{9}$$

where ω^* is a constant having dimensions of circular frequency. Using the expressions relating displacement components u_i, w_i to the scalar potential functions ϕ,ψ and $\overline{\phi},\overline{\psi}$ in dimensionless form after suppressing the dashes as

$$u_1=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},\quad u_3=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}$$
$$w_1=\frac{\partial\overline{\phi}}{\partial x}-\frac{\partial\overline{\psi}}{\partial z}\quad u_3=\frac{\partial\overline{\phi}}{\partial z}+\frac{\partial\overline{\psi}}{\partial x} \tag{10}$$

The field equations reduce to

$$\{(1+a_1)\nabla^2-(a_4+a_5\frac{\partial^2}{\partial t^2})\}\phi+\{(a_2+a_3)\nabla^2+a_4\}\overline{\phi}=0 \tag{11}$$

$$\{(1+a_6)\nabla^2+a_9\}\phi+\{(a_7+a_8)\nabla^2-(a_9+a_{10}\frac{\partial^2}{\partial t^2})\}\overline{\phi}=0 \tag{12}$$

$$\{\nabla^2-(a_4+a_5\frac{\partial^2}{\partial t^2})\}\psi+\{a_2\nabla^2+a_4\}\overline{\psi}=0 \tag{13}$$

$$(\nabla^2+a_9)\psi+\{a_7\nabla^2-(a_9+a_{10}\frac{\partial^2}{\partial t^2})\}\overline{\psi}=0 \tag{14}$$

where

$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$, $a_1=\alpha_2/\alpha_1$, $a_2=\beta_1/\alpha_1$, $a_3=\beta_2/\alpha_1$, $a_4=\xi c_1^2/\alpha_1\omega^{*2}$, $a_5=\rho_1 c_1^2/\alpha_1$, $a_6=\beta_2/\beta_1$, $a_7=\gamma_1/\beta_1$, $a_8=\gamma_2/\beta_1$, $a_9=\xi c_1^2/\beta_1\omega^{*2}$, $a_{10}=\rho_2 c_1^2/\beta_1$

The solution of (8)-(11) for the waves propagating along positive x−direction are

$$\phi = \{A_1\cos(m_1 z) + B_1\sin(m_1 z) + A_2\cos(m_2 z) + B_2\sin(m_2 z)\}e^{ik(x-ct)}$$
$$\overline{\phi} = \{q_1(A_1\cos(m_1 z) + B_1\sin(m_1 z)) + q_2(A_2\cos(m_2 z) + B_2\sin(m_2 z))\}e^{ik(x-ct)}$$
$$\psi = \{A_3\cos(m_3 z) + B_3\sin(m_3 z) + A_4\cos(m_4 z) + B_4\sin(m_4 z)\}e^{ik(x-ct)}$$
$$\overline{\psi} = \{q_3(A_3\cos(m_3 z) + B_3\sin(m_3 z)) + q_4(A_4\cos(m_4 z) + B_4\sin(m_4 z))\}e^{ik(x-ct)} \quad (15)$$

where k and c are respectively, the wave number and phase velocity of wave and $A_1,...,A_4, B_1,...,B_4$ are arbitrary constants,

$$m_i^2 = k^2(\lambda_i^2 c^2 - 1), \qquad i = 1,...,4,$$
$$\lambda_i^2 = (A \mp \sqrt{A^2 - 4B})/2, \qquad i = 1,2,$$
$$\lambda_j^2 = (A' \mp \sqrt{A'^2 - 4B'})/2, \qquad j = 3,4,$$
$$A = \frac{M}{Lk^2c^2},\ B = \frac{N}{Lk^4c^4},\ A' = \frac{L'}{(a_7 - a_2)k^2c^2},\ B' = \frac{N}{(a_7 - a_2)k^4c^4}$$
$$L = (1 + a_1)(a_7 + a_8) - (1 + a_6)(a_2 + a_3)$$
$$M = \{(a_7 + a_8)(k^2c^2a_5 - a_4) + (1 + a_1)(k^2c^2a_{10} - a_9) - a_4(1 + a_6) - a_9(a_2 + a_3)\}$$
$$N = \{(k^2c^2a_{10} - a_9)(k^2c^2a_5 - a_4) - a_4a_9\},\ L' = \{a_7(k^2c^2a_5 - a_4) + k^2c^2a_{10} - a_9 - a_4 - a_2a_9\}$$
$$q_i = \frac{\{(m_i^2 + k^2)(1 + a_1) + a_4 - k^2c^2a_5\}}{\{a_4 - (a_2 + a_3)(m_i^2 + k^2)\}} \qquad i = 1,2,$$
$$q_j = \frac{m_j^2 - k^2c^2a_5 + a_4 + k^2}{a_4 - a_2(m_j^2 + k^2)} \qquad j = 3,4.$$

3.1 Derivation of frequency equation

At $z = \pm H$ the appropriate boundary conditions are:

$$t_{33} = 0,\ t_{31} = 0,\ s_{33} = 0,\ s_{31} = 0 \quad (16)$$

Using dimensionless variables defined by (9), in the expressions of stresses and then using boundary conditions given by (16) with the help of (2), (3) and (10) for $x - z$ plane, we obtain eight homogeneous equations in eight unknowns $A_1,...,A_4, B_1,...,B_4$. The condition for the existence of non-trivial solution of these equations gives the frequency equation

$$\frac{T_2^{\pm1}}{T_1^{\pm1}}\left(1 - \frac{L_2P_1}{L_1P_2}\right) + \frac{T_2^{\pm2}}{T_1^{\pm1}T_3^{\pm1}}\left(\frac{P_3}{P_3} - \frac{L_3P_1}{L_1P_2}\right)\left(\frac{H_2W_4 - H_4W_2}{H_3W_4 - H_4W_3}\right) + \frac{T_2^{\pm2}}{T_1^{\pm1}T_4^{\pm1}}\left(\frac{L_4P_1}{L_2P_2} - \frac{P_4}{P_2}\right)\left(\frac{H_2W_3 - H_3W_2}{H_3W_4 - H_4W_3}\right) +$$
$$\frac{T_2^{\pm1}}{T_4^{\pm1}}\left(\frac{L_2P_4}{L_1P_2} - \frac{L_4}{L_2}\right)\left(\frac{H_1W_3 - H_3W_1}{H_3W_4 - H_4W_3}\right) + \frac{T_2^{\pm2}}{T_3^{\pm1}T_4^{\pm1}}\left(\frac{L_3P_4}{L_1P_2} - \frac{L_4P_3}{L_2P_2}\right)\left(\frac{H_1W_2 - H_2W_1}{H_3W_4 - H_4W_3}\right) + \frac{T_2^{\pm1}}{T_3^{\pm1}}\left(\frac{L_2P_3}{L_1P_2} + \frac{L_3}{L_1}\right) \quad (17)$$
$$\left(\frac{H_1W_4 - H_4W_1}{H_3W_4 - H_4W_3}\right) = 0$$

where $T_i = \tan(m_iH) \qquad i = 1,...,4,$ and

$$L_i = -k^2(M_1 + q_{ii}M_3) - m_i^2(M_2 + (\beta_1+\beta_2)q_i),\ L_j = ikm_j(M_1 - M_2 + q_j(M_3 - \beta_1 - \beta_2))$$
$$H_i = -ikm_i(\alpha_1 + M_4 + q_i(\beta_1 + 2\beta + \zeta)),\quad H_j = m_j^2(\alpha_1 + q_j\beta_1) - k^2(M_4 + q_3(2\beta+\zeta))$$
$$P_i = -k^2(M_3 + \alpha q_i) - m_i^2(\beta_1 + \beta_2 + q_i(\gamma_1 + \gamma_2)),\quad P_j = ikm_j(M_3 - \beta_1 - \beta_2 + q_3(2\beta + \zeta)),$$
$$W_i = -ikm_i(\zeta + \beta_1 + \gamma_1 + q_i(\gamma_1 + 2\gamma)),\quad W_j = m_j^2(\beta_1 + q_j\gamma_1) - k^2(\zeta + \gamma_1 + 2\gamma q_j),\qquad i = 1,2;\quad j = 3,4, \tag{18}$$
$$M_1 = \lambda + 2\nu + \alpha,\ M_2 = \lambda + \alpha + 2(\mu + \nu + \beta + \gamma) + 4\zeta,\ M_3 = \alpha + \nu,\ M_4 = \mu + 2\gamma + 2\zeta$$

4 SPECIFIC LOSS

The specific loss is the ratio of energy (ΔW) dissipated in taking a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is maximum. The specific loss is the most direct method of defining the internal friction for a material. Kolsky [22] shows that specific loss $(\Delta W / W)$ is 4π times the absolute value of the ratio of the imaginary part of wave number to the real part of wave number i.e.

$$\frac{\Delta W}{W} = 4\pi\left|\frac{\mathrm{Im}\,(k)}{\mathrm{Re}\,(k)}\right|$$

5 AMPLITUDES OF DISPLACEMENTS

The amplitudes of displacement components of both the constituents of mixture for symmetric and skew-symmetric modes of plane waves can be obtained as:

$$(u_1)_{sym} = (\sum_{i=1}^{2} ik(A_i \cos(m_i z)) - \sum_{j=3}^{4} m_j B_j \cos(m_j z))e^{ik(x-ct)}$$
$$(u_1)_{asym} = (\sum_{i=1}^{2} ik(B_i \sin(m_i z)) + \sum_{j=3}^{4} m_j A_j \sin(m_j z))e^{ik(x-ct)}$$
$$(u_3)_{sym} = (-\sum_{i=1}^{2} m_i A_i \sin(m_i z) + \sum_{j=3}^{4} ik(B_j \sin(m_j z)))e^{ik(x-ct)}$$
$$(u_3)_{asym} = (\sum_{i=1}^{2} m_i B_i \cos(m_i z) + \sum_{j=3}^{4} ik(A_j \cos(m_j z)))e^{ik(x-ct)}$$
$$(w_1)_{sym} = (\sum_{i=1}^{2} ik(q_i A_i \cos(m_i z)) - \sum_{j=3}^{4} m_j q_j B_j \cos(m_j z))e^{ik(x-ct)}$$
$$(w_1)_{asym} = (\sum_{i=1}^{2} ik(q_i B_i \sin(m_i z)) + \sum_{j=3}^{4} m_j q_j A_j \sin(m_j z))e^{ik(x-ct)}$$
$$(w_3)_{sym} = (-\sum_{i=1}^{2} m_i q_i A_i \sin(m_i z) + \sum_{j=3}^{4} ik(q_j B_j \sin(m_j z)))e^{ik(x-ct)}$$
$$(w_3)_{asym} = (\sum_{i=1}^{2} m_i q_i B_i \cos(m_i z)) + \sum_{j=3}^{4} ik(q_j A_j \cos(m_j z)))e^{ik(x-ct)} \tag{19}$$

6 PARTICULAR CASE

In the limiting case if we neglect the presence of second constituent of the mixture and vanish the parameters corresponding to the coupling between the two constituents of mixture, then we will recover the results of classical theory of elasticity for isotropic elastic solid which are similar to those obtained in Graff [23] by changing dimensionless quantities into the physical quantities.

7 NUMERICAL RESULTS AND DISCUSSION

In order to illustrate theoretical results obtained in the preceding sections, we now present some numerical results taking

$$\lambda = 2.57\times10^{10}\,\text{Nm}^{-2},\ \nu = 0.79\times10^{10}\,\text{Nm}^{-2}, \mu = 2.278\times10^{10}\,\text{Nm}^{-2}, \alpha = 0.5\times10^{10}\,\text{Nm}^{-2}, \beta = 0.45\times10^{10}\,\text{Nm}^{-2},$$
$$\zeta = 0.5\times10^{10}\,\text{Nm}^{-2}, \xi = 2.17\times10^{12}\,\text{Nm}^{-2}, \rho_1 = 1.74\times10^{3}\,\text{Kgm}^{-3}, \rho_2 = 0.84\times10^{3}\,\text{Kgm}^{-3}$$

All numerical computations are carried out by taking $H = 1.34$. Fig. 1 depicts the variations of phase velocity with respect to R i.e. real part of wave number for symmetric and antisymmetric modes of wave propagation for mixture of two elastic solids whereas Fig. 2 represents the same situation for isotropic elastic solid. Fig. 3 depicts the variations of attenuation coefficient with respect to R for symmetric and antisymmetric modes of propagation for mixture of two elastic solids whereas Fig. 4 represents the same situation for isotropic elastic solid. Similarly, Figs. 5 and 6 represent the variations of specific loss w.r.t wave number for both symmetric and antisymmetric modes in case of mixture and elastic solid respectively.

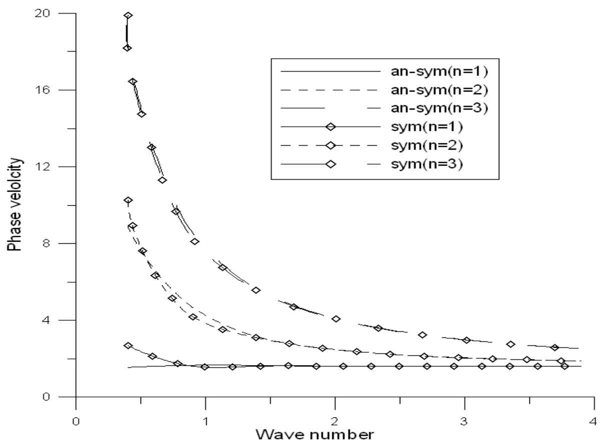

Fig. 1
Variation of phase velocity w.r.t wave number in mixture for symmetric and antisymmetric modes of propagation.

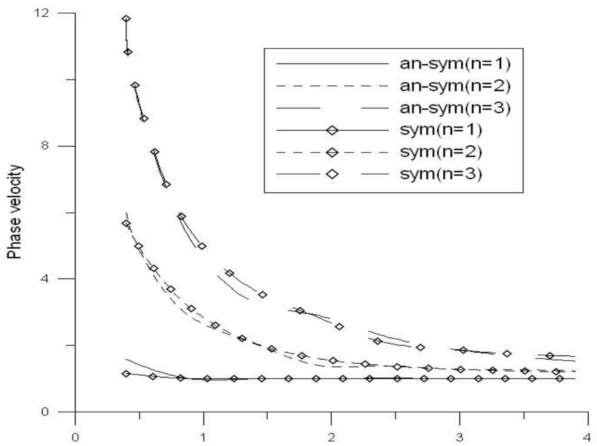

Fig. 2
Variation of phase velocity w.r.t wave number in elastic solid for symmetric and antisymmetric modes of propagation.

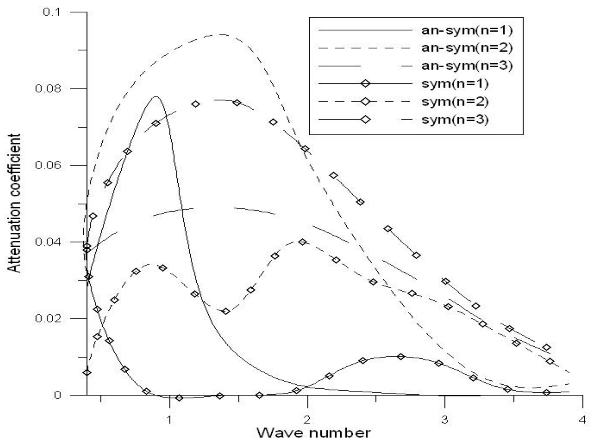

Fig. 3
Variation of attenuation coefficient w.r.t wave number in mixture for symmetric and antisymmetric modes of propagation.

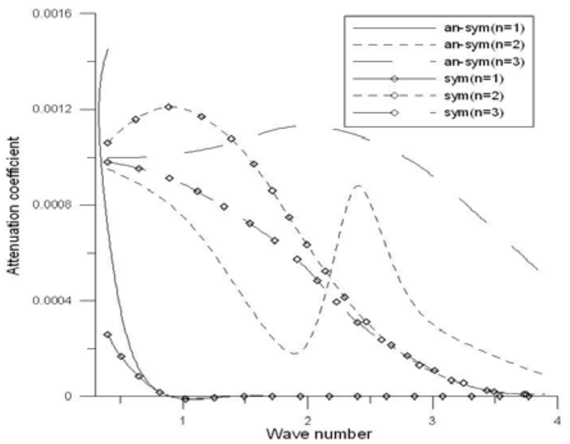

Fig. 4
Variation of attenuation coefficient w.r.t wave number in elastic solid for symmetric and antisymmetric modes of propagation.

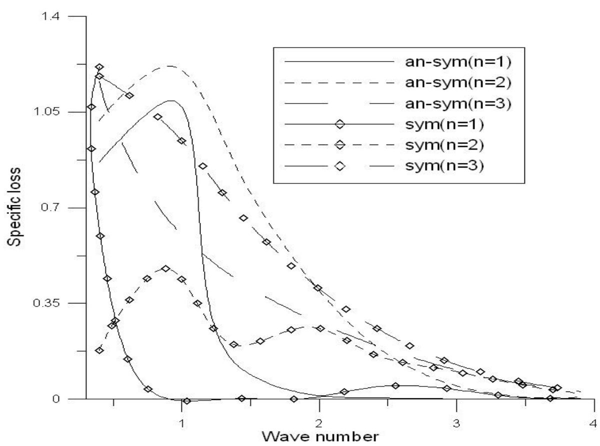

Fig. 5
Variation of specific loss w.r.t wave number in mixture for symmetric and antisymmetric modes of propagation.

In Figs. 1-6, the solid lines represent variations for first symmetric mode, small dashed lines represent variations for second symmetric mode and long dashed lines represent variations for third symmetric mode and the corresponding lines with central symbol represent variations for the respective antisymmetric modes. In Fig. 7 and 8, solid lines and dashed lines respectively represent variations for displacement component u_1 and u_3 whereas the corresponding lines with central symbols represent variations for w_1 and w_3, respectively. In Fig. 9, solid line and dashed line respectively represent variations for displacement component u_1 and u_3 for symmetric mode whereas the corresponding lines with central symbols represent variations for antisymmetric mode.

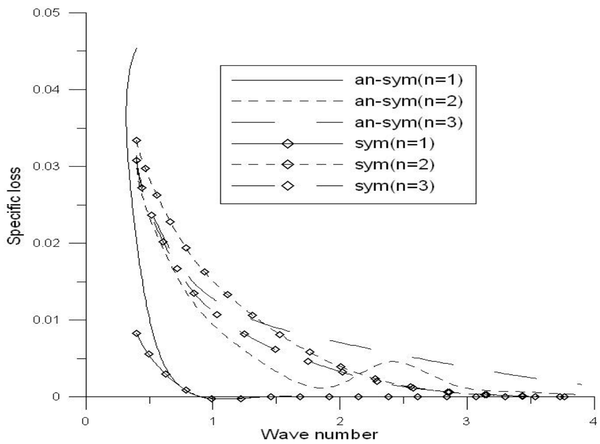

Fig. 6
Variation of specific loss w.r.t wave number in elastic solid for symmetric and antisymmetric modes of propagation.

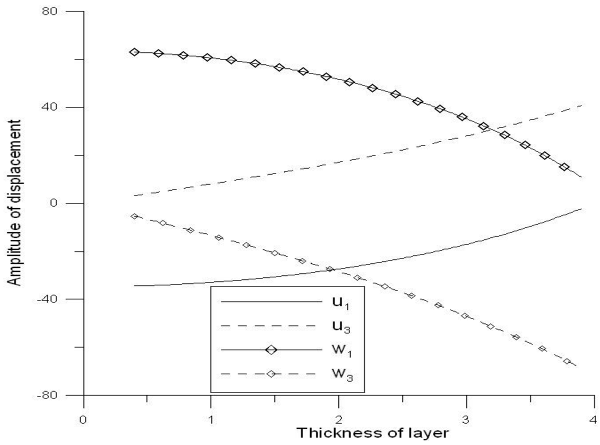

Fig. 7
Variation of amplitudes of displacement components in mixture for symmetric mode.

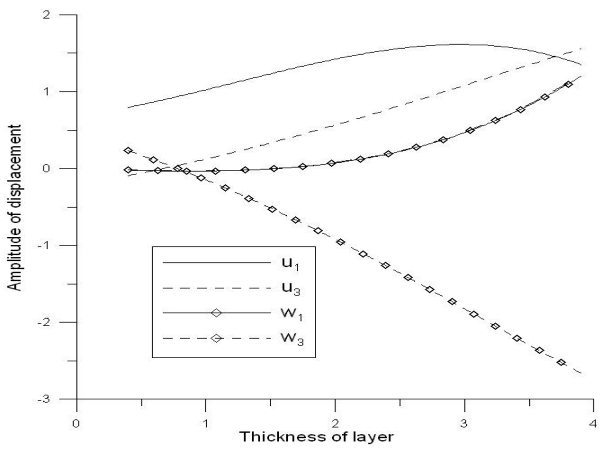

Fig. 8
Variation of amplitudes of displacement components in mixture for antisymmetric mode.

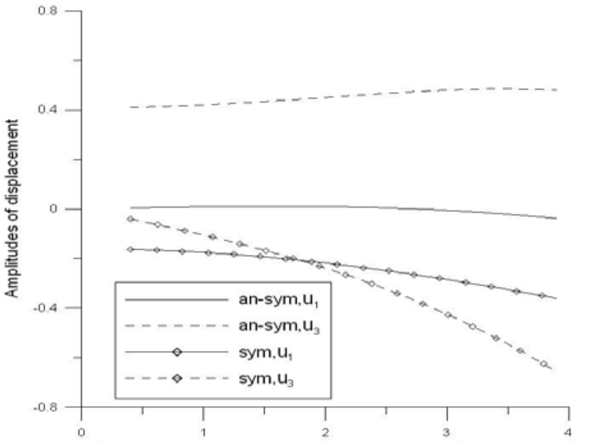

Fig. 9
Variation of amplitudes of displacement components in elastic solid for symmetric and antisymmetric modes of wave propagation.

The phase velocity, attenuation coefficient, and specific loss of various modes of propagation have been computed from secular equations for various values of wave number and for different boundary conditions. It is observed from Figs. 1 and 2 that the phase velocity of different modes of wave propagation starts from large values at vanishing wave number and then exhibits a strong dispersion until the velocity flattens out to the value of Rayleigh wave velocity of material at higher wave number. The reason for the asymptotic approach is that for short wavelengths (or high frequencies), the material plate behaves increasingly like a thick slab and hence the coupling between the upper and lower boundary surfaces is reduced and as a result the properties of symmetric and skew symmetric modes of wave propagation become more and more similar. In the limit for an infinite thick slab, the motion at the upper surface is not confined to the lower surface and the displacements become localized near the free boundaries, thus the Lamb wave dispersion curves asymptotically approach those for Rayleigh waves.

It is observed from Figs. 3 and 4 that for symmetric mode the values of attenuation coefficient first increase and then decrease for mixture of elastic solids, but the values of attenuation coefficient are higher for the second mode as compared to those for first mode. Also for antisymmetric mode, the values of attenuation coefficient increase with the increasing number of mode of propagation. For isotropic elastic solid the values of attenuation coefficient for the first mode of wave propagation exhibit a strong dispersion from higher values for both symmetric and antisymmetric modes. The values of attenuation coefficient of second mode of propagation are higher for antisymmetric mode of propagation within the range $0 < R < 2$ whereas the behavior is reverse beyond this range. For the third mode of wave propagation the values are higher for symmetric mode as compared to those for antisymmetric mode. Figs. 5 and 6 indicate that the values of specific loss decrease with wave number. Fig. 5 indicates that for the first two modes the values of specific loss are higher for symmetric mode as compared to those for antisymmetric mode but the behavior is reverse for the third mode. It is observed from Fig. 7 that the behavior and trend of variation of specific loss for first and third mode is similar for both symmetric and antisymmetric modes of wave propagation whereas for second mode the values are higher for antisymmetric mode within the range $0 < R < 2$ and the behavior is reverse beyond this range.

Figs. 7 and 8 represent the variations of amplitudes of displacement components of mixture w.r.t the thickness of plate. It is observed that for symmetric mode of propagation the amplitude of displacement components u_1, u_3 increase w.r.t the thickness of plate whereas the amplitudes of w_1 and w_3 decrease. For antisymmetric mode the variation of amplitude for u_3, w_3 show behavior similar to that for symmetric mode whereas the displacement component u_1 , w_1 show opposite behavior. It is observed from Fig. 9 that for elastic solid, the trend of variation of amplitudes of both displacement components for antisymmetric mode is similar with the exception that the values of displacement component u_3 are higher than those for w_1 but for symmetric mode the values of u_3 are greater than u_1 within the range $0 < z < 1.7$ whereas the behavior is reversed beyond this range.

8 CONCLUSION

The characteristics of the dispersion relations of plane waves propagating in a layer of binary mixture of elastic solids and in isotropic elastic solid are presented. After deriving secular equation, the behavior of dispersion curves of phase velocity and attenuation coefficient is studied graphically for various modes of wave propagation. The amplitudes of displacement and specific loss are also computed from the relative expressions and are shown graphically for both symmetric and antisymmetric modes of wave propagation. The numerically computed results are found to be in close agreement with the theoretical results.

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