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Thermomechanical Response in Thermoelastic Medium with Double Porosity

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ABSTRACT

A dynamic two dimensional problem of thermoelasticity with double porous structure has been considered to investigate the disturbance due to normal force and thermal source. Laplace and Fourier transform technique is applied to the governing equations to solve the problem. The transformed components of stress and temperature distribution are obtained .The resulting expressions are obtained in the physical domain by using numerical inversion technique. Numerically computed results for these quantities are depicted graphically to study the effect of porosity. Results of Kumar & Rani [42] and Kumar & Ailawalia [43] have also been deduced as special cases from the present investigation.

Keywords : Thermoelasticity; Thermomechanical sources; Double porosity; Laplace and Fourier transforms.

1 INTRODUCTION

POROUS media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Representation of a fluid saturated porous medium as a single phase material has been virtually discarded. The material with the pore spaces such as concrete can be treated easily because all concrete ingredients have the same motion if the concrete body is deformed. However the situation is more complicated if the pores are filled with liquid and in that case the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous thermoelastic medium becomes very difficult. So researchers from time to time, have tried to overcome this difficulty and we see many porous media in the literature. A brief historical background of these theories is given by de Boer [1,2].

As far as modern era is concerned Biot [3] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen [4], de Boer and Ehlers [5] in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modeled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of

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poroelasticity that has been studied extensively started with the works by Barenblatt et al. [6], where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifiers.

The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super- saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Wilson and Aifanits [7] presented the theory of consolidation with the double porosity. Khaled, Beskos and Aifantis [8] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis[7]. Wilson and Aifantis [9] discussed the propagation of acoustics waves in a fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space.

Beskos and Aifantis [10] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan [11] studied the unified theory of flow and deformation in double porous media. Aifantis [12-15] introduced a multi-porous system and studied the mechanics of diffusion in solids. Moutsopoulos et al. [16] obtained the numerical simulation of transport phenomena by using the double porosity/ diffusivity continuum model. Khalili and Selvadurai [17] presented a fully coupled constitutive model for thermo-hydro –mechanical analysis in elastic media with double porosity structure. Pride and Berryman [18] studied the linear dynamics of double –porosity dual-permeability materials. Straughan [19] studied the stability and uniqueness in double porous elastic media .

Svanadze [20-24] investigated some problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity. Scarpetta et al. [25, 26] proved the uniqueness theorems in the theory of thermoelasticity for solids with double porosity and also obtained the fundamental solutions in the theory of thermoelasticity for solids with double porosity.

A potential method is easy and common way to uncouple a set of coupled linear differential equation.The completeness of the potential function is a significant issue in the continuum mechanics. Many authors have presented the completeness of the potential functions for a different system of equations.

Nowacki [33] presented a unified potential representation in terms of two scalar potentials for three-dimensional classical elastodynamics and electrostatics and also degenerated a complete solution for the corresponding static problems.Wang and Wang [34] constructively proved the completeness of the general solutions in elastodynamics. Ghadi [35] presented a complete solution in terms of retarded potential functions for the wave equations in transversely isotropic media. Ghadi and Pak [36] presented a new general solution in terms of two scalar potential functions for classical elastodynamics of $x_3 -$ convex domains. Hayati et al. [37,38] derived the dynamic thermoelastic Green's functions of an axisymmetric linear elastic half-space concerned by using a method of potentials and gave an analytical formulation for an axisymmetric linear thermoelastic transversely isotropic half-space by using method of potentials. Ghadai et. al [39] gave an analytical formulation for an axisymetric linear thermoelastic transversely isotropic half-space y using method of potentials. Raoofian Naeeni et al. [40] derived the transient responses of an isotropic thermoelastic half-space subjected to time dependent tractions and half flux applied to a finite patch at an arbitrary depth below a free surface with the aid of a complete set of two scalar potential functions. Raoofian Naeeni et al. [47] presented a performance comparison among different numerical methods when they are applied to transformed functions related to actual engineering problems.

In the present investigation, we determine the components of stress and temperature distributrion in isotropic, homogeneous, thermoelastic medium with double porous structure due to thermomechanical sources.The solution is obtained by introducing potential functions after applying an integral transform technique. The integral transforms are inverted by using numerical inversion technique.

2 BASIC EQUATIONS

Following Iesan and Quintanilla [30], the constitutive relations and field equations for homogeneous thermoelastic material with double porosity structure without body forces, extrinsic equilibrated body forces and heat source can be written as:

Constitutive relations:

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi - \beta\delta_{ij}T \tag{1}$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i} \tag{2}$$

$$\tau_i = b_1\varphi_{,i} + \gamma\psi_{,i} \tag{3}$$

Equation of motion:

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla\cdot\vec{u} + b\nabla\varphi + d\nabla\psi - \beta\nabla T = \rho\frac{\partial^2 u}{\partial t^2} \tag{4}$$

Equilibrated stress equations of motion:

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - b\nabla\cdot\vec{u} - \alpha_1\varphi - \alpha_3\psi + \gamma_1 T = \kappa_1\frac{\partial^2\varphi}{\partial t^2}, \tag{5}$$

$$b_1\nabla^2\varphi + \gamma\nabla^2\psi - d\nabla\cdot\vec{u} - \alpha_3\varphi - \alpha_2\psi + \gamma_2 T = \kappa_2\frac{\partial^2\psi}{\partial t^2}, \tag{6}$$

Equation of heat conduction:

$$K^*\nabla^2 T - \beta T_0\nabla.\dot{\vec{u}} - \gamma_1 T_0\dot{\varphi} - \gamma_2 T_0\dot{\psi} - \rho C^*\dot{T} = 0 \tag{7}$$

where λ and μ are Lame's constants, ρ is the mass density; $\beta = (3\lambda + 2\mu)\alpha;\alpha$ is the linear thermal expansion; C^* is the specific heat at constant strain, u_i is the displacement components; t_{ij} is the stress tensor; κ_1 and κ_2 are coefficients of equilibrated inertia; v_1 is the volume fraction field corresponding to pores and v_2 is the volume fraction field corresponding to fissures ; φ and ψ are the volume fraction fields corresponding to v_1 and v_2 respectively ; σ_1 is the equilibrated stress corresponding to $v_1;\tau_1$ is the equilibrated stress corresponding to $v_2;K^*$ is the coefficient of thermal conductivity and $b,d,b_1,\gamma,\gamma_1,\gamma_2$ are constitutive coefficients; δ_{ij} is the Kronecker's delta; *T* is the temperature change measured form the absolute temperature $T_0\left(T_0 \neq 0\right)$; a superposed dot represents differentiation with respect to time variable *t*.

$$\nabla = \hat{i}\frac{\partial}{\partial x_1} + \hat{j}\frac{\partial}{\partial x_2} + \hat{k}\frac{\partial}{\partial x_3}, \qquad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

are the gradient and Laplacian operators, respectively.

3 FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, thermoelastic material with double porosity structure in the undeformed state at uniform temperature T_0. The rectangular Cartesian coordinate system (x_1, x_2, x_3) having origin on the surface $x_3 = 0$ with $x_3 -$axis pointing vertically into the medium is introduced. A concentrated normal force or thermal source is assumed to be acting at the origin of the rectangular Cartesian coordinates. It follows from the description of the problem that all the considered functions will depend upon (x_1, x_3, t). We thus obtain the displacement vector $\vec{u}$ of the form $\vec{u} = (u_1, 0, u_3)$.

The initial and regularity conditions are given by:

$$
\begin{aligned}
&u_1(x_1, x_3, 0) = \dot{u}_1(x_1, x_3, 0) = u_3(x_1, x_3, 0) = \dot{u}_3(x_1, x_3, 0) = 0 \\
&\varphi(x_1, x_3, 0) = \dot{\varphi}(x_1, x_3, 0) = \psi(x_1, x_3, 0) = \dot{\psi}(x_1, x_3, 0) = 0 \\
&T(x_1, x_3, 0) = 0 = \dot{T}(x_1, x_3, 0) \\
&\\
&\text{for} \quad x_3 \geq 0, -\infty < x_1 < \infty
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
&u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = \psi(x_1, x_3, t) = T(x_1, x_3, t) = 0 \\
&\\
&\text{for} \quad t > 0, \quad x_3 \to \infty
\end{aligned}
\tag{9}
$$

4 SOLUTION OF THE PROBLEM

To transform Eqs. (4)-(7) to non-dimensional form, we define the following non-dimensional constants :

$$
\begin{aligned}
&x_1' = \frac{\omega_1}{c_1} x_1,\ x_3' = \frac{\omega_1}{c_1} x_3, u_1' = \frac{\omega_1}{c_1} u_1\ , u_3' = \frac{\omega_1}{c_1} u_3, t_{ij}' = \frac{t_{ij}}{\beta t_0}, \varphi' = \frac{k_1 {\omega_1}^2}{\alpha_1} \varphi, \\
&\psi' = \frac{k_1 {\omega_1}^2}{\alpha_1}, T' = \frac{T}{T_0}, t' = \omega_1 t, \sigma_1' = \left(\frac{c_1}{\alpha \omega_1}\right) \sigma_1, \tau_1' = \left(\frac{c_1}{\alpha \omega_1}\right) \tau_1
\end{aligned}
\tag{10}
$$

where $c_1^2 = \dfrac{\lambda + 2\mu}{\rho}, \omega_1 = \dfrac{\rho C^* c_1^2}{K^*}$. Here ω_1 is the constant having the dimensions of frequency and c_1 is the velocity in the medium. Making use of dimensionless quantities given by (10) in Eqs. (4)-(7), we obtain (suppressing the primes for convenience)

$$
\left(\frac{\lambda + \mu}{\rho c_1^2}\right) \frac{\partial e}{\partial x_1} + \frac{\mu}{\rho c_1^2} \nabla^2 u_1 + a_1 \frac{\partial \phi}{\partial x_1} + a_2 \frac{\partial \psi}{\partial x_1} - a_3 \frac{\partial T}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2},
\tag{11}
$$

$$
\left(\frac{\lambda + \mu}{\rho c_1^2}\right) \frac{\partial e}{\partial x_3} + \frac{\mu}{\rho c_1^2} \nabla^2 u_3 + a_1 \frac{\partial \phi}{\partial x_3} + a_2 \frac{\partial \psi}{\partial x_3} - a_3 \frac{\partial T}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2},
\tag{12}
$$

$$
a_4 \nabla^2 \phi + a_5 \nabla^2 \psi - a_6 e - a_7 \phi - a_8 \psi + a_9 T = \frac{\partial^2 \varphi}{\partial t^2},
\tag{13}
$$

$$a_{10}\nabla^2\phi + a_{11}\nabla^2\psi - a_{12}e - a_{13}\phi - a_{14}\psi + a_{15}T = \frac{\partial^2\psi}{\partial t^2}, \tag{14}$$

$$a_{16}\nabla^2 T - a_{17}\frac{\partial e}{\partial t} - a_{18}\frac{\partial\varphi}{\partial t} - a_{19}\frac{\partial\psi}{\partial t} - \frac{\partial T}{\partial t} = 0, \tag{15}$$

where

$$a_1 = \frac{b\alpha_1}{\rho c_1^2 \kappa_1 \omega_1^{\,2}}, a_1 = \frac{d\alpha_1}{\rho c_1^2 \kappa_1 \omega_1^{\,2}}, a_3 = \frac{\beta T_0}{\rho c_1^2}, a_4 = \frac{\alpha}{c_1^2 \kappa_1}, a_5 = \frac{b_1}{c_1^2 \kappa_1}, a_6 = \frac{b}{\alpha_1}, a_7 = \frac{\alpha_1}{\kappa_1 \omega_1^{\,2}}, a_8 = \frac{\alpha_3}{\kappa_1 \omega_1^{\,2}}, a_9 = \frac{\gamma_1 T_0}{\alpha_1},$$
$$a_{10} = \frac{b_1}{c_1^2 \kappa_2}, a_{11} = \frac{\gamma}{c_1^2 \kappa_2}, a_{12} = \frac{d\kappa_1}{\alpha_1 \kappa_2}, a_{13} = \frac{\alpha_3}{\kappa_2 \omega_1^{\,2}}, a_{14} = \frac{\alpha_2}{\kappa_2 \omega_1^{\,2}}, a_{15} = \frac{\gamma_2 T_0 \kappa_1}{\alpha_1 \kappa_2}, a_{16} = \frac{\kappa\omega}{\rho C^* c_1^{\,2}}, a_{17} = \frac{\beta}{\rho C^*},$$
$$a_{18} = \frac{\gamma_1 \alpha_1}{\rho C^* \kappa_1 \omega_1^{\,2}}, a_{19} = \frac{\gamma_2 \alpha_1}{\rho C^* \kappa_1 \omega_1^{\,2}}$$

The displacement components u_1 and u_3 are related by potential functions φ_1 and ψ_1. As:

$$u_1 = \frac{\partial\varphi_1}{\partial x_1} - \frac{\partial\psi_1}{\partial x_3}, \qquad u_3 = \frac{\partial\varphi_1}{\partial x_3} + \frac{\partial\psi_1}{\partial x_1} \tag{16}$$

We define Laplace and Fourier transforms by

$$\bar{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t)e^{-st}dt \tag{17}$$

and

$$\tilde{f}(\xi, x_3, s) = \int_{-\infty}^{\infty} \bar{f}(x_1, x_3, s)e^{i\xi x}dx \tag{18}$$

Making use of (16) in (11)-(15) and applying (17) and (18), after simplification and assuming that $\tilde{\varphi}_1, \tilde{\varphi}, \tilde{\psi}, \tilde{T}, \tilde{\psi}_1 \to 0$ as $x_3 \to \infty$, we obtain

$$(\tilde{\varphi}_1, \tilde{\varphi}, \tilde{\psi}, \tilde{T}) = \sum_{i=1}^{4}(1, r_i, s_i, t_i)B_i e^{-m_i x_3} \tag{19}$$

$$\tilde{\psi}_1 = B_5 e^{-m_5 x_3} \tag{20}$$

where $m_l\,(l = 1,2,3,4)$ are the roots of the equation

$$E_1\frac{d^8}{dx_3^8} + E_2\frac{d^6}{dx_3^6} + E_3\frac{d^4}{dx_3^4} + E_4\frac{d^2}{dx_3^2} + E_5 = 0 \tag{21}$$

and

$$m_5=\sqrt{\xi^2+\frac{s^2}{r_5^2}}\ ,\ r_5^2=\frac{\mu}{\rho c_1^2} \tag{22}$$

where E_1,E_2,E_3,E_4 and E_5 are given in the Appendix A.

The coupling constants are given by $r_i=-\frac{D_{1i}}{D_{0i}},s_i=\frac{D_{2i}}{D_{0i}},t_i=-\frac{D_{3i}}{D_{0i}};i=1,2,3,4$ and $D_{0i},D_{1i},D_{2i},D_{3i}$ are given in the Appendix B.

5 BOUNDARY CONDITIONS

We consider a concentrated normal force/thermal source acting at $x_3=0$. Mathematically, the boundary conditions on the surface $x_3=0$ are

$$t_{33}=-F_1\delta(x)\delta(t) \tag{23}$$

$$t_{31}=0 \tag{24}$$

$$\sigma_3=0 \tag{25}$$

$$\tau_3=0 \tag{26}$$

$$T=F_2\delta(x)\delta(t) \tag{27}$$

where F_1 and F_2 are the magnitude of force and constant temperature applied on the boundary respectively and $\delta()$ is the Dirac delta function.

Substituting the values of $\tilde{\varphi}_1,\tilde{\varphi},\tilde{\psi},\tilde{T}$ and $\tilde{\psi}_1$ from (19) and (20) in (23)-(27) and with the aid of (1)-(3), (10), (17) and (18), we obtain the corresponding expressions for components of stress and temperature distribution as:

$$\tilde{t}_{33}=\frac{1}{\Delta}\left[Q_1\Delta_1e^{-m_1x_3}+Q_2\Delta_2e^{-m_2x_3}+Q_3\Delta_3e^{-m_3x_3}+Q_4\Delta_4e^{-m_4x_3}+Q_5\Delta_5e^{-m_5x_3}\right] \tag{28}$$

$$\tilde{t}_{31}=\frac{1}{\Delta}\left[R_1\Delta_1e^{-m_1x_3}+R_2\Delta_2e^{-m_2x_3}+R_3\Delta_3e^{-m_3x_3}+R_4\Delta_4e^{-m_4x_3}+R_5\Delta_5e^{-m_5x_3}\right] \tag{29}$$

$$\tilde{\sigma}_3=\frac{1}{\Delta}\left[U_1\Delta_1e^{-m_1x_3}+U_2\Delta_2e^{-m_2x_3}+U_3\Delta_3e^{-m_3x_3}+U_4\Delta_4e^{-m_4x_3}\right] \tag{30}$$

$$\tilde{\tau}_3=\frac{1}{\Delta}\left[V_1\Delta_1e^{-m_1x_3}+V_2\Delta_2e^{-m_2x_3}+V_3\Delta_3e^{-m_3x_3}+V_4\Delta_4e^{-m_4x_3}\right] \tag{31}$$

$$\tilde{T}=\frac{1}{\Delta}\left[t_1\Delta_1e^{-m_1x_3}+t_2\Delta_2e^{-m_2x_3}+t_3\Delta_3e^{-m_3x_3}+t_4\Delta_4e^{-m_4x_3}\right] \tag{32}$$

where

$$\Delta=\begin{vmatrix} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \\ R_1 & R_2 & R_3 & R_4 & R_5 \\ U_1 & U_2 & U_3 & U_4 & 0 \\ V_1 & V_2 & V_3 & V_4 & 0 \\ t_1 & t_2 & t_3 & t_4 & 0 \end{vmatrix} \tag{33}$$

$\Delta_i\left(i=1\text{to}5\right)$ are obtained by replacing i^{th} column of (33) with $\left[-F_1\ 0\ 0\ 0\ F_2\right]^T$ where

$$Q_1=P_1m_1^2-\xi^2P_2+P_3r_1+P_4s_1-t_1, Q_2=P_1m_2^2-\xi^2P_2+P_3r_2+P_4s_2-t_2, Q_3=P_1m_3^2-\xi^2P_2+P_3r_3+P_4s_3-t_3,$$
$$Q_4=P_1m_4^2-\xi^2P_2+P_3r_4+P_4s_4-t_4, Q_5=i\xi\left(P_1-P_2\right)m_5,$$
$$R_1=2m_1, R_2=2m_2, R_3=2m_3, R_4=2m_4, R_5=i\xi+\frac{m_5^2}{i\xi},$$
$$U_1=-\left(P_4r_1+P_5s_1\right)m_1, U_2=-\left(P_4r_2+P_5s_2\right)m_2, U_3=-\left(P_4r_3+P_5s_3\right)m_3, U_4=-\left(P_4r_4+P_5s_4\right)m_3,$$
$$V_1=-\left(P_5r_1+P_6s_1\right)m_1, V_2=-\left(P_5r_2+P_6s_2\right)m_2, V_3=-\left(P_5r_3+P_6s_3\right)m_3, V_4=-\left(P_5r_4+P_6s_4\right)m_4$$

5.1 Normal force acting on the surface

If $F_2=0$ in Eqs.(28)-(32), yields the resulting expressions for normal force.

5.2 Thermal source acting on the surface

If $F_1=0$ in Eqs.(28)-(32) , yields the resulting expressions for thermal source.

5.3 Particular cases

(i) If $b_1=\alpha_3=\gamma=\alpha_2=\gamma_2=d\to 0$ in Eqs. (28)-(32), we obtain the corresponding expressions for thermoelastic body with primary porosity for thermomechanical sources which are in agreement with the results obtained by Kumar and Rani [42].

(ii) If $\alpha=b=\gamma_1=\alpha_1=K^*=\beta=\rho\to 0$ in above case (i) yields the corresponding results for elastic body for thermomechanical sources which are similar as obtained by Kumar and Ailawalia [43] in absence of porous dissipation.

6 NUMERICAL RESULTS AND DISCUSSION

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief and Saleh [31] as,

$$\lambda=7.76\times10^{10}\,Nm^{-2}, \mathrm{C}^*=3.831\times10^3\,m^2s^{-2}K^{-1}, \mu=3.86\times10^{10}\,Nm^{-2}, \mathrm{K}^*=3.86\times10^3\,Ns^{-1}K^{-1},$$
$$\omega=1\times10^{11}\,\mathrm{s}^{-1}, T_0=0.293\times10^3\,K, \alpha_t=1.78\times10^{-5}\,K^{-1}\ , t=0.1s, \rho=8.954\times10^3\,Kgm^{-3}$$

Following Khalili [32], the double porous parameters are taken as,

$$\alpha_2 = 2.4\times10^{10}\,Nm^{-2}, \alpha_3 = 2.5\times10^{10}\,Nm^{-2}, \gamma = 1.1\times10^{-5}\,N, \alpha = 1.3\times10^{-5}\,N, \gamma_1 = 0.16\times10^{5}\,Nm^{-2}, b_1 = 0.12\times10^{-5}\,N,$$
$$d = 0.1\times10^{10}\,Nm^{-2}, \gamma_2 = 0.219\times10^{5}\,Nm^{-2}, \kappa_1 = 0.1456\times10^{-12}\,Nm^{-2}s^2, b = 0.9\times10^{10}\,Nm^{-2}, \alpha_1 = 2.3\times10^{10}\,Nm^{-2},$$
$$\kappa_2 = 0.1546\times10^{-12}\,Nm^{-2}s^2$$

The software MATLAB has been used to find the values of normal stress t_{33}, tangential stress t_{31}, equilibrated stresses σ_3, τ_3 and temperature distribution *T*. The variations of these values with respect to distance *x* have been shown in Figs.1-8 respectively. In all these figures, solid lines without and with central symbols correspond to thermal double porous material (TDP) for $t = 0.1, 0.5$ respectively and small dashes line without and with central symbols correspond to thermal porous material (TP) for $t = 0.1, 0.5$ respectively.

6.1 Normal force

Figs.1-4 depict the variation of normal stress t_{33}, tangential stress t_{31} equilibrated stress σ_3, temperature distribution *T* with respect to distance *x* due to normal force.

In Fig.1, the behavior and variation of t_{33} for TDP and TP is opposite to each other and also it is noticed that the porosity increases the value of t_{33} for the region $0 < x \le 1$ and $2.5 < x \le 3.8$ and decreases in the remaining region for *x*. It is also noticed that with the increase in the value of *t*, the magnitude values of t_{33} decrease although the variation remains the same.

In Fig.2, it is noticed that for TDP, the values of t_{31} are almost stationary for $0 < x \le 2$, then increase sharply in the range $2 < x < 3$ and again decrease sharply in the range $3 < x < 4$ and become constant afterwards while an oscillatory behavior is shown for all values of *x* in case of TP. Also the magnitude values of t_{31} decrease with the increase in value of *t* with the similar trend of variation .

In Fig. 3, it is found that the behavior is alike for both the materials for both values of *t* while as *t* increases, the magnitude values of σ_3 decreases. Also the variation of σ_3 is oscillatory in nature for both the materials for all values of *x* but for $0 < x \le 2.5$ and $x > 3.5$, the magnitude values are higher for TDP than that of TP while reverse behavior is noticed in the range $2.5 < x < 3.5$. Also, with the increase in the value of *t*, the magnitude values of σ_3 for both the materials decrease although the trend of variation remains the same.

In Fig.4, the trend of variation of *T* is oscillatory in nature for both TDP and TP while the magnitude values of *T* are higher for TP than that of TDP. It is found that with the increase in the value of *t*, the magnitude values of *T* increase and decrease for TDP and TP respectively.

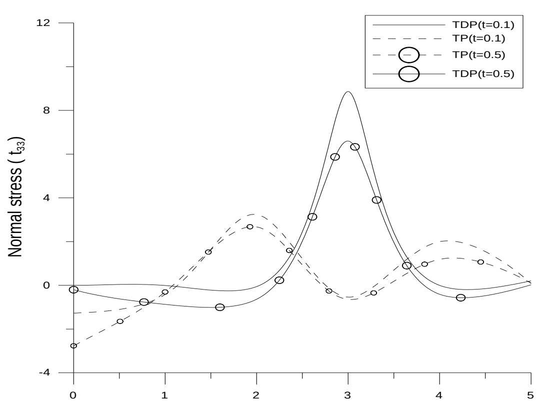

Fig.1
Variation of normal stress t_{33} *w.r.t. x* (Normal force).

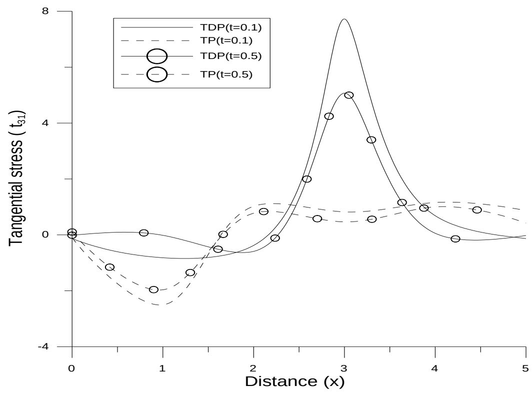

Fig.2
Variation of tangential stress t_{31} *w.r.t.x* (Normal force).

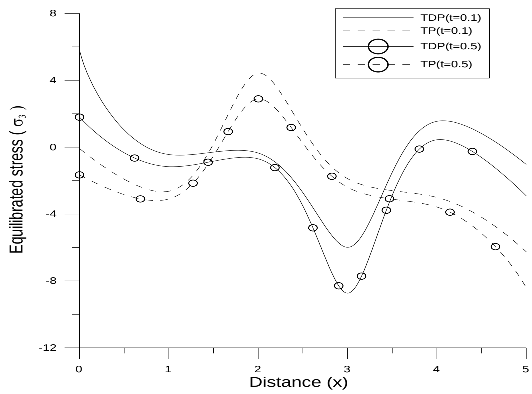

Fig.3
Variation of equilibrated stress σ_3 *w.r.t.x* (Normal force).

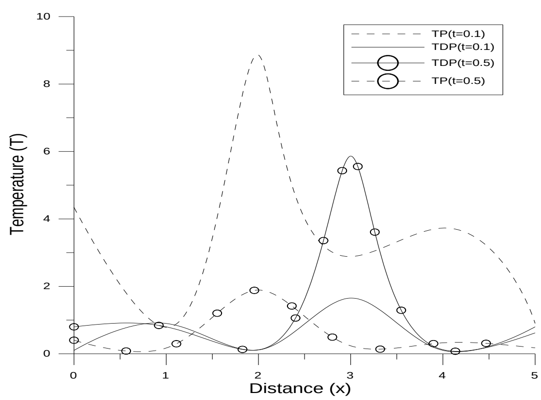

Fig.4
Variation of temperature distribution *T w.r.t.x*(Normal force).

6.2 Thermal source

Figs. 5-8 depict the variation of normal stress t_{33}, tangential stress t_{31}, equilibrated stresses σ_3 and temperature distribution *T* with respect to distance *x* due to thermal source.

In Fig. 5, it is noticed that the values of t_{33} are almost stationary for $0 < x \le 2$, then decrease sharply in the range $2 < x < 3$ and again increase sharply in the range $3 < x < 4$ and becomes constant afterwards for TDP. For TP, it increases for $0 < x < 1$, then decreases for $1 \le x < 3$ and becomes almost stationary with the increase in distance *x*. The values of t_{33} are more for TDP than that of TP near the application of source while trend gets reversed as moving away from source. It is also noticed that with the increase in the value of *t*, the magnitude values of t_{33} decrease although the trend of variation remains the same.

In Fig. 6, the variation of t_{31} is oscillatory in nature for both TDP and TP. The magnitude values of t_{31} are small for TP than TDP near the application of source whereas a reverse behavior is noticed away from source. It is also omitted that as the value of t increases, the magnitude values of t_{31} decrease for TDP while for TP it increase except for the region $1 < x < 2$.

In Fig. 7, the values of σ_3 remain almost constant for $0 < x < 3$, then decrease for $3 \le x < 4$ and increase for $x \ge 4$ for TDP whereas for TP, it increases for $0 < x < 2$, decreases for $2 \le x < 3$ and again increases sharply with the increases in distance *x*. Also the magnitude values of σ_3 decrease and increase for TDP and TP respectively with the increase in time t.

Fig.8. shows that the values of *T*, for TDP, decrease for the region $0 < x \le 2.5$, increase for $2.5 < x \le 4$ and again decrease with the increase in distance *x* whereas for TP, the value of *T* decreases for $0 < x \le 3$ and then start increasing for $x > 3$. The values of *T* are more for TDP as compared to TP. As the value of *t* increases, the magnitude values of *T* decrease for both TP and TDP.

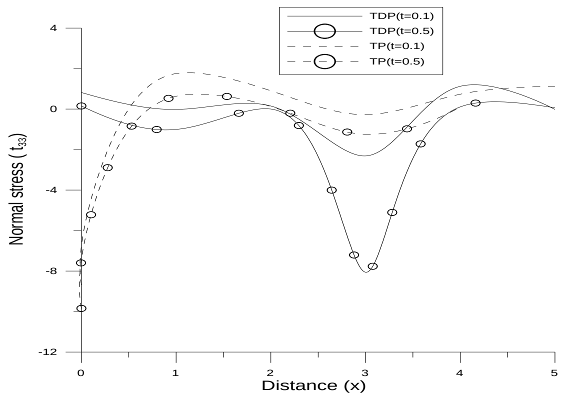

Fig.5
Variation of normal stress t_{33} *w.r.t. x* (Thermal source).

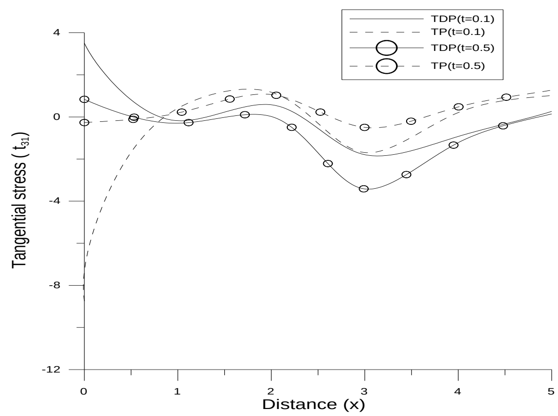

Fig.6
Variation of tangential stress t_{31} *w.r.t.x* (Thermal source).

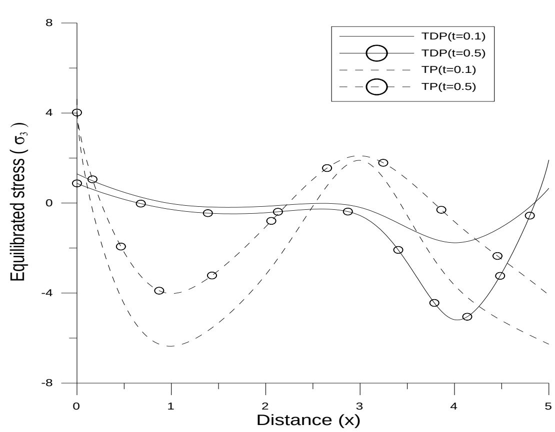

Fig.7
Variation of equilibrated stress σ_3 *w.r.t.x* (Thermal source).

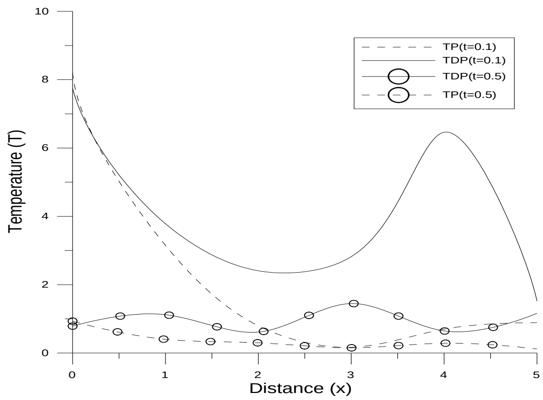

Fig.8
Variation of temperature distribution T *w.r.t.x* (Thermal source).

7 INVERSION OF THE TRANSFORMS

According to Bradie [49], the various quadrature formulae such as Newton-cotes, Romberg and Gaussian quadrature etc. can be used to approximate the value of an improper integral, provided the integral exists. However, some changes of variable must be made to achieve theoretical order of convergence, if required. Due to existence of damping terms, the dependence of characteristic roots $m_l\left(l=1,2,3,4,5\right)$ on s and ξ is very complicated and hence the inversion of integral transforms is quite difficult because the isolation of s is impossible. These difficulties, however, are reduced if we use some approximate or numerical methods as given by Sharma and Chauhan [48]. To obtain the solution of the problem in the physical domain, we must invert the transforms in Eqs. (28)-(32) for both the materials in case of normal force and thermal source applied. These expressions are functions of x_3, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $\hat{f}(\xi,x_3,s)$. To get the function $f(x_1,x_3,t)$ in the physical domain, first we invert the Fourier transform using

$$\begin{aligned}\overline{f}(x_1,x_3,s)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi x_1}\tilde{f}(\xi,x_3,s)d\xi,\\&=\frac{1}{\pi}\int_{0}^{\infty}\left\{\cos(\xi x)f_e-i\sin(\xi x)\,\mathrm{f}_0\right\}d\xi\end{aligned}\tag{34}$$

where f_e and f_0 are even and odd parts of the function $\tilde{f}(\xi,x_3,s)$ respectively. Thus, expression (34) gives us the Laplace transform $\overline{f}(x_1,x_3,s)$ of the function $f\,(x_1,x_3,t)$.

Now, for the fixed values of ξ,x_1 and $x_3,\overline{f}(x_1,x_3,s)$ in the expression (34) can be considered as the Laplace transform $\overline{g}(s)$ of some function $g(t)$. Following Honig and Hirdes [45], the Laplace transformed function $\overline{g}(s)$ can be inverted as follows:

The function $g(t)$ can be obtained by using

$$g(t)=\frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty}e^{st}\overline{g}(s)ds,\tag{35}$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\overline{g}(s)$. Taking $s=C+ix_3$, we get

$$g(s)=\frac{e^{Ct}}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi z}g(C+ix_3)dx_3,\tag{36}$$

Now, taking $e^{-Ct}g(t)$ as $h(t)$ and expanding it as Fourier series in $[0,2L]$, we obtain approximately the formula

$$\mathrm{g}(t)=\mathrm{g}_{\infty}(t)+E_{D'}, \tag{37}$$

where

$$\begin{aligned}\mathrm{g}_{\infty}(t)&=\frac{C_0}{2}+\sum_{k=1}^{\infty}C_k,\ \ 0\le t\le 2\pi,\\ C_k&=\frac{e^{Ct}}{L}\mathrm{Re}\left[e^{ik\pi/L}\bar{g}\left(C+\frac{ikL}{L}\right)\right],\end{aligned} \tag{38}$$

E_D is the discretization error and can be made arbitrarily small by choosing *C* large enough. The value of *C* and *L* are chosen according to the criteria outlined by Honig and Hirdes [45].

Since the infinite series in Eq. (38) can be summed up only to a finite number of *N* terms, so the approximate value of $\mathrm{g}(t)$ becomes

$$\mathrm{g}_N(t)=\frac{C_0}{2}+\sum_{k=1}^{N}C_k,\ 0\le t\le 2L, \tag{39}$$

Now we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error in evaluating $\mathrm{g}(t)$ using the above formula. Two methods are used to reduce the total error. The discretization error is reduced by using the 'Korrecktur'-method, Honig and Hirdes [45] and then 'ε-algorithm', is used to reduce the truncation error and hence to accelerate the convergence.

The 'Korrecktur' – method formula,to evaluate function $\mathrm{g}(t)$ is

$$\mathrm{g}(t)=g_{\infty}(t)-e^{-2CL}g_{\infty}(2L+t)+E_{D'}, \tag{40}$$

where

$$\left|E_{D'}\right|<<\left|E_D\right| \tag{41}$$

Thus, the approximate value of $\mathrm{g}(t)$ becomes

$$\mathrm{g}_{N_k}(t)=\mathrm{g}_N(t)-e^{-2CL}g_{N'}(2L+t), \tag{42}$$

where *N* is an integer such that $N'\prec N$.

We shall now describe the $\varepsilon-$algorithm which is used to accelerate the convergence of the series in Eq. (39). Let *N* be a natural number and $S_m=\sum_{k=1}^{m}C_k$ be the sequence of partial sums of Eq.(39).We define the $\varepsilon-$sequence by

$$\varepsilon_{0,m}=0,\varepsilon_{1,m}=S_m,\qquad \varepsilon_{n+1,m}=\varepsilon_{n-1,m+1}+\frac{1}{\varepsilon_{n,m+1}-\varepsilon_{n,m}};n,m=1,2,3,...$$

It can be shown (Honig and Hirdes [44]) that the sequence $\varepsilon_{1,1},\varepsilon_{3,1},...,\varepsilon_{N,1}$ converges to $g(t)+E_D-C_0/2$ faster than the sequence of partial sum $S_m, m=1,2,3,....$.The actual procedure to invert the Laplace transform consists of Eq.(42) together with $\varepsilon-$algorithm.

It should be noted that a good choice of the free parameters N is not only important for the accuracy of the results but also for the application of the Korrecktur method and $\varepsilon-$algorithm. The values are chosen according to the criterion outlined in Honig and Hirdes[45].

The last step is to evaluate the integral in Eq. (34). The method for evaluating this integral by Press et al. [46], which involves the use of Romberg's integration with adaptive step size. This also use the results from successive refinement of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8 CONCLUSIONS

It is concluded that analysis of elastodynamics deformation in thermoelastic materials with double porosity structure due to normal force and thermal source is a significant problem of mechanics. The behavior of components of stress and temperature distribution in an isotropic homogeneous thermoelastic material with double porosity structure has been investigated for thermoelastic interactions due to normal force and thermal source by using integral transform technique. It is observed that porosity effect increases the value of t_{33} and t_{31} near the application of the source while opposite trend is observed away from the source for both normal force and thermal source. Due to the effect of porosity, the magnitude values of temperature distribution decreases in case of application of normal force while reverse behavior is observed for thermal source. Figures indicate that double porosity has both increasing and decreasing effects on the numerical values of the physical quantity. It is found that as the value of t increases, the magnitude values of the normal and tangential stress decrease in case of normal force while for thermal source same behavior is observed near the application of source but reverse behavior is observed as moving away from the source. The magnitude values of σ_3 decrease for TDP for both the sources while for TP, it decreases for normal force and increases for thermal source as t increases. Also the magnitude values of T decrease for TP for both the sources while for TDP, it decreases for normal force and increases for thermal source as the value of t increases.

All the field quantities are observed to be very sensitive towards the porosity and thermal parameters. Graphical representation indicated that double porosity and single porosity have both the increasing and decreasing effects on the numerical values of the physical quantities. This type of study is useful due to its application in geophysics and rock mechanics. The results obtained in this investigation should prove to be beneficial for the researchers working on the theory of thermoelasticity with double porosity structure. The introduction of double porous parameter to the thermoelastic medium represents a more realistic model for these studies.

APPENDIX A

$$\delta_1=-\left(\xi^2+s^2\right),\delta_2=a_6\xi^2,\ \delta_3=-\left(a_4\xi^2+s^2+a_7\right),\delta_4=-\left(a_5\xi^2+a_8\right),\delta_5=a_{12}\xi^2,\delta_6=-\left(a_{10}\xi^2+a_{13}\right),$$

$$\delta_7=-\left(a_{11}\xi^2+s^2+a_{14}\right),\ \delta_8=-a_{20}\xi,\delta_9=-\left(a_{16}\xi^2+s^2\right),a_{20}=-sa_{17},a_{21}=-sa_{18},a_{22}=-sa_{19}$$

$$E_1=a_{16}(a_4a_{11}-a_5a_{10})$$

$$\begin{aligned}E_2=&\delta_9(a_4a_{11}-a_5a_{10})+a_{16}(\delta_7a_4-\delta_6a_5-a_1a_5a_{12}+a_1a_6a_{11}+a_2a_4a_{12}-a_2a_6a_{10}+\delta_1a_4a_{11}-\delta_1a_5a_{10}\\&+\delta_3a_{11}-\delta_4a_{10})+a_3a_{20}(a_4a_{11}-a_5a_{10})\end{aligned}$$

$$\begin{aligned}E_3=&a_{12}a_{16}(\delta_3a_2-\delta_4a_1)+a_1a_{20}(a_9a_{11}-a_5a_{15})+a_2a_{20}(a_4a_{15}-a_9a_{10})+(a_3a_{12}-a_{15})(a_4a_{22}-a_5a_{21})+\\&(a_3a_6-a_9)(a_{11}a_{21}-a_{10}a_{22})+\delta_2a_{16}(a_2a_{10}-a_1a_{11})+\delta_9(\delta_7a_4-\delta_6a_5+\delta_3a_{11}-\delta_4a_{10})+a_{16}(\delta_3\delta_7-\delta_4\delta_6)+\\&\delta_9a_2(a_4a_{12}-a_6a_{10})+\delta_9a_1(a_6a_{11}-a_5a_{12})+(\delta_8a_3+\delta_1\delta_9)(a_4a_{11}-a_5a_{10})+a_6a_{16}(\delta_7a_1-\delta_6a_2)+\\&(\delta_1a_{16}+a_3a_{20})(\delta_7a_4-\delta_6a_5+\delta_3a_{11}-\delta_4a_{10})+\delta_5a_{16}(a_1a_5-a_2a_4)\end{aligned}$$

$$\begin{aligned}E_4=&a_{16}(\delta_1\delta_3\delta_7-\delta_1\delta_4\delta_6+\delta_6\delta_2a_2-\delta_2\delta_7a_1-\delta_3\delta_5a_2+\delta_4\delta_5a_1)+a_{15}(\delta_4a_{21}-\delta_3a_{22})+\delta_5\delta_9(a_1a_5-a_2a_4)+\\&(\delta_8a_3+\delta_1\delta_9)(\delta_7a_4+\delta_3a_{11}-\delta_4a_{10}-\delta_6a_5)+a_9(\delta_6a_{22}-\delta_7a_{21})+\delta_9(\delta_3\delta_7-\delta_4\delta_6)+\delta_9a_6(\delta_7a_1-\delta_6a_2)+\\&\delta_2\delta_9(a_2a_{10}-a_1a_{11})+a_3a_{20}(\delta_3\delta_8-\delta_4\delta_6)+a_3\delta_8(\delta_3a_{11}-\delta_4a_{10})+a_{12}\delta_9(\delta_3a_2-\delta_4a_1)+a_3\delta_5(a_5a_{21}-a_4a_{22})\\&+a_9a_{20}(\delta_7a_1-\delta_6a_2)+a_3a_6(\delta_7a_{21}-\delta_6a_{22})+\delta_8a_1(a_9a_{11}-a_1a_{15})+\delta_8a_2(a_4a_{15}-a_9a_{10})+\delta_2a_3(a_{10}a_{22}-a_{11}a_{21})+\\&\delta_3a_2(a_{20}a_{15}-a_{12}a_{22})+\delta_1a_{15}(a_5a_{21}-a_4a_{22})+\delta_1a_9(a_{10}a_{22}-a_{11}a_{21})+a_2a_{21}(a_6a_{15}-a_9a_{12})+a_1a_{22}(a_9a_{12}-a_6a_{15})\end{aligned}$$

$$E_5=(\delta_5 a_9-\delta_2 a_{15})(a_2 a_{21}-a_1 a_{22})+(\delta_1\delta_9+\delta_8 a_3)(\delta_3\delta_7-\delta_4\delta_6)+\delta_2\delta_9(\delta_6 a_2-\delta_7 a_1)+\delta_5\delta_9(\delta_4 a_1-\delta_3 a_2)+$$
$$\delta_8 a_9(\delta_7 a_1-\delta_6 a_2)+(\delta_5 a_3+\delta_1 a_{15})(\delta_4 a_{21}-\delta_3 a_{22})+\delta_8 a_{15}(\delta_3 a_2-\delta_4 a_1)+(\delta_1 a_9+\delta_2 a_3)(\delta_6 a_{22}-\delta_7 a_{21})$$

APPENDIX B

$$D_{0i}=\begin{vmatrix} a_4 m_i^2+\delta_3 & a_5 m_i^2+\delta_4 & a_9 \\ a_{10} m_i^2+\delta_6 & a_{11} m_i^2+\delta_7 & a_{15} \\ a_{21} & a_{22} & a_{16} m_i^2+\delta_9 \end{vmatrix}, D_{1i}=\begin{vmatrix} -a_6 m_i^2+\delta_2 & a_5 m_i^2+\delta_4 & a_9 \\ -a_{12} m_i^2+\delta_5 & a_{11} m_i^2+\delta_7 & a_{15} \\ a_{20} m_i^2+\delta_8 & a_{22} & a_{16} m_i^2+\delta_9 \end{vmatrix},$$

$$D_{2i}=\begin{vmatrix} -a_6 m_i^2+\delta_2 & a_4 m_i^2+\delta_3 & a_9 \\ -a_{12} m_i^2+\delta_5 & a_{10} m_i^2+\delta_6 & a_{15} \\ a_{20} m_i^2+\delta_8 & a_{21} & a_{16} m_i^2+\delta_9 \end{vmatrix}, D_{3i}=\begin{vmatrix} -a_6 m_i^2+\delta_2 & a_4 m_i^2+\delta_3 & a_5 m_i^2+\delta_4 \\ -a_{12} m_i^2+\delta_5 & a_{10} m_i^2+\delta_6 & a_{11} m_i^2+\delta_7 \\ a_{20} m_i^2+\delta_8 & a_{21} & a_{22} \end{vmatrix}, \mathrm{i}=1 \text{ to } 4$$

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