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The Buckling of Non-Homogeneous Truncated Conical Shells under a Lateral Pressure and Resting on a Winkler Foundation

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ABSTRACT

In this paper, the buckling of non-homogeneous isotropic truncated conical shells under uniform lateral pressure and resting on a Winkler foundation is investigated. The basic relations and governing equations have been obtained for non-homogeneous truncated conical shells. The critical uniform lateral pressures of non-homogeneous isotropic truncated conical shells with or without a Winkler foundation are obtained. Finally, carrying out some computations, effects of the variations of truncated conical shell characteristics, the non-homogeneity and the Winkler foundation on the critical uniform lateral pressures have been studied. The results are compared with other works in open literature.

Keywords: Buckling; Non-homogeneous material; Truncated conical shell; Winkler foundation; Critical uniform lateral pressure

1 INTRODUCTION

HOMOGENEOUS and non-homogeneous shell structures are being widely used in aerospace, automotive, marine and other technical applications. The shell structures are often subjected to various uniform external pressures, and may be supported by an elastic foundation. There are different approaches to analyze the interaction between a structure and an ambient medium; see, for example, the models proposed in [1, 2]. In this study, response of an elastic medium is given by Winkler foundation model [3].

To a far lesser extent than beams and plates, the static and dynamic analyses of shells on elastic foundations have been studied. Most of researches have been limited to the stability and vibration analyses of cylindrical shells [4-8]. It is well known that real materials and structural components are often non-homogeneous, because of design, manufacturing process, production techniques, surface and thermal polishing processes or physical composition and imperfections in the underlying material. Thus, the physical properties of materials change from point to point as random, piecewise continuous or continuous functions of coordinates [9]. Continuous non-homogeneity is a direct generalization of homogeneity in theory; besides, material non-homogeneity becomes essential and must sufficiently be considered in a number of practical situations. In all the referenced works, and in most of available solutions to elastic non-homogeneity, it is assumed that the material is isotropic, the Poisson's ratio is constant, and the Young's modulus is either power or exponential functions of a spatial variable [10-14].

Considering the effects of non-homogeneity and an elastic foundation complicate the buckling problems considerably. Hence, the vibration and buckling problems of non-homogeneous structures resting on elastic foundations are rare in open literature [15-18]. However, there is no literature covering the buckling of homogeneous and non-homogeneous truncated conical shells subjected to uniform lateral pressure and resting on a Winkler foundation. This is the problem studied in the present paper, for the case when edges of the truncated conical shell are assumed to be simply supported.

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2 THE THEORETICAL FORMULATION

Fig. 1 shows the geometry of a thin truncated circular conical shell with semi-vertex cone angle γ, length L and thickness h. R_1 and R_2 are the radii at the two ends. A coordinate system (S,θ,ζ) is fixed on the mid-surface of the shell. The S - axis lies on the curvilinear middle surface of the cone, S_1 and S_2 being the coordinates of the points where this axis intersects the small and large bases, respectively. Furthermore, the ζ -axis is always normal to the moving S -axis, lying in the plane generated by the S -axis and the axis of the cone, and points inwards. The θ -axis is in the direction perpendicular to the $S-\zeta$ plane.

The non-homogeneous truncated conical shell is resting on a Winkler foundation. The foundation interface pressure N may be expressed as $N = K_w w$. Here $K_w(N/m^3)$ is the modulus of subgrade reaction for the foundation or foundation modulus [3].

The stress-strain relation for a non-homogeneous isotropic truncated conical shell is:

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{pmatrix} = \frac{E_0[1+\mu\varphi_0(\bar{\zeta})]}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & 1\text{-}\nu\end{bmatrix}\begin{bmatrix}\varepsilon_1^0 - \zeta\dfrac{\partial^2 w}{\partial S^2}\\ \varepsilon_2^0 - \zeta\left(\dfrac{1}{S^2}\dfrac{\partial^2 w}{\partial\varphi^2} + \dfrac{1}{S}\dfrac{\partial w}{\partial S}\right)\\ \varepsilon_{12}^0 - \zeta\left(\dfrac{1}{S}\dfrac{\partial^2 w}{\partial S\partial\varphi} - \dfrac{1}{S^2}\dfrac{\partial w}{\partial\varphi}\right)\end{bmatrix} \tag{1}$$

where $\sigma_1, \sigma_2, \sigma_{12}$ are the stresses, ε_1^0, ε_2^0, ε_{12}^0 are the strains on the middle surface, $\varphi = \theta\sin\gamma$, w is the displacement of the middle surface in the normal direction, positive towards the axis of the cone and assumed to be much smaller than the thickness, E_0 is the Young's moduli and υ is Poisson's ratio of homogeneous material, $\varphi_0(\bar{\zeta})$ is continuous function of non-homogeneity defining the variations of the Young's modulus, satisfying the condition $\left|\varphi_0(\bar{\zeta})\right| \le 1$, and μ is a non-homogeneity coefficient, satisfying $0 \le \mu \le 1$.

The force and moment resultants are expressed by

$$\left[(T_1, T_2, T_{12}), (M_1, M_2, M_{12})\right] = \int_{-h/2}^{h/2} (1,\zeta)(\sigma_1, \sigma_2, \sigma_{12})\,\mathrm{d}\zeta \tag{2}$$

The relations between the forces T_1, T_2 and T_{12} and the stress function Φ are given by

$$\left(T_1,\, T_2, T_{12}\right) = \left(\frac{1}{S^2}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{1}{S}\frac{\partial\Phi}{\partial S},\ \frac{\partial^2\Phi}{\partial S^2},\ -\frac{1}{S}\frac{\partial^2\Phi}{\partial S\partial\varphi} + \frac{1}{S^2}\frac{\partial\Phi}{\partial\varphi}\right) \tag{3}$$

The non-homogeneous truncated conical shell subjected to uniform lateral pressure:

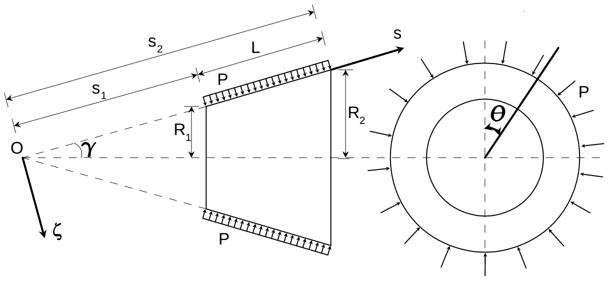

Fig. 1
Geometry of the truncated conical shell.

$$T_1^0 = 0;\ T_2^0 = -PS\tan\gamma;\ T_{12}^0 = 0 \tag{4}$$

where T_1^0, T_2^0 and T_{12}^0 are membrane forces for the condition with zero initial moments, P is uniform lateral pressure. Substituting Eq. (1) into (2) after some rearrangements, the relations found for moments and strains, being substituted into the stability and compatibility equations of the non-homogeneous truncated conical shell on a Winkler foundation [2, 20], together with relations (3) and (4), then considering the independent variables $S = S_1 e^z$ and $\Phi = \Phi_1 e^{2z}$, after lengthy computations, these equations for w and Φ_1 can be obtained as

$$\begin{aligned}
&c_{12}e^{2z}\frac{\partial^4\Phi_1}{\partial z^4} + 4c_{12}e^{2z}\frac{\partial^3\Phi_1}{\partial z^3} + (4c_{12} + S_1 e^z\cot\gamma)e^{2z}\frac{\partial^2\Phi_1}{\partial z^2} + 3S_1 e^{3z}\cot\gamma\frac{\partial\Phi_1}{\partial z} + \\
&+2S_1 e^{3z}\Phi_1\cot\gamma + c_{12}e^{2z}\frac{\partial^4\Phi_1}{\partial\varphi^4} + 2(c_{11} - c_{31})e^{2z}\frac{\partial^4\Phi_1}{\partial z^2\partial\varphi^2} + 4(c_{11} - c_{13})e^{2z}\frac{\partial^3\Phi_1}{\partial z\partial\varphi^2} + \\
&+2(c_{11} - c_{31} + c_{12})e^{2z}\frac{\partial^2\Phi_1}{\partial\varphi^2} - c_{13}\frac{\partial^4 w}{\partial\varphi^4} - 2(c_{14} + c_{32})\frac{\partial^4 w}{\partial z^2\partial\varphi^2} + 4(c_{14} + c_{32})\frac{\partial^3 w}{\partial z\partial\varphi^2} - \\
&-2(c_{14} + c_{32} + c_{13})\frac{\partial^2 w}{\partial\varphi^2} - c_{13}\frac{\partial^4 w}{\partial z^4} + 4c_{13}\frac{\partial^3 w}{\partial z^3} - 4c_{13}\frac{\partial^2 w}{\partial z^2} - \\
&-S_1^3 e^{3z}P\tan\gamma\left(\frac{\partial^2 w}{\partial\varphi^2} + \frac{\partial w}{\partial z}\right) - S_1^4 e^{4z}K_w w = 0
\end{aligned} \tag{5}$$

$$\begin{aligned}
&b_{11}e^{2z}\left(\frac{\partial^4\Phi_1}{\partial z^4} + 4\frac{\partial^3\Phi_1}{\partial z^3} + 4\frac{\partial^2\Phi_1}{\partial z^2} + \frac{\partial^4\Phi_1}{\partial\varphi^4}\right) + 2(b_{31} + b_{12})e^{2z}\left(\frac{\partial^4\Phi_1}{\partial z^2\partial\varphi^2} + 2\frac{\partial^3\Phi_1}{\partial z\partial\varphi^2}\right) + \\
&+2(b_{31} + b_{12} + b_{11})e^{2z}\frac{\partial^2\Phi_1}{\partial\varphi^2} - b_{14}\frac{\partial^4 w}{\partial\varphi^4} + 2(b_{32} - b_{13})\frac{\partial^4 w}{\partial z^2\partial\varphi^2} - \\
&-4(b_{32} - b_{13})\frac{\partial^3 w}{\partial z\partial\varphi^2} + 2(b_{32} - b_{13} - b_{14})\frac{\partial^2 w}{\partial\varphi^2} - b_{14}\frac{\partial^4 w}{\partial z^4} + 4b_{14}\frac{\partial^3 w}{\partial z^3} - \\
&-(4b_{14} - S_1 e^z\cot\gamma)\frac{\partial^2 w}{\partial z^2} - S_1 e^z\cot\gamma\frac{\partial w}{\partial z} = 0
\end{aligned} \tag{6}$$

in which expressions c_{ij}, b_{ij} $(i, j = 1,...,6)$ are defined as follows:

$$\begin{aligned}
&c_{11} = a_{11}^1 b_{11} + a_{12}^1 b_{12},\ c_{12} = a_{11}^1 b_{12} + a_{12}^1 b_{11},\ c_{13} = a_{11}^1 b_{13} + a_{12}^1 b_{23} + a_{11}^2, \\
&c_{14} = a_{11}^1 b_{14} + a_{12}^1 b_{13} + a_{12}^2,\ c_{31} = a_{66}^1 b_{31},\ \ c_{32} = a_{66}^1 b_{32} + a_{66}^2,\ b_{11} = a_{11}^0 L_0^{-1}, \\
&b_{12} = -a_{12}^0 L_0^{-1},\ b_{13} = (a_{12}^0 a_{12}^1 - a_{11}^1 a_{11}^0)L_0^{-1},\ b_{14} = (a_{12}^0 a_{11}^1 - a_{12}^1 a_{11}^0)L_0^{-1},\ b_{31} = 1/a_{66}^0, \\
&b_{32} = -a_{66}^1 / a_{66}^0,\ L_0 = a_{11}^0 a_{11}^0 - a_{12}^0 a_{12}^0;\ a_{11}^k = \frac{E_0 h^{k+1}}{1-\nu^2}\int_{-1/2}^{1/2}\bar{\zeta}^k\,[1 + \varphi_0(\bar{\zeta})]\,\mathrm{d}\bar{\zeta}, \\
&a_{12}^k = \nu a_{11}^k,\ a_{66}^k = (1-\nu)a_{11}^k,\ k = 0,1,2
\end{aligned} \tag{7}$$

3 THE SOLUTION OF BASIC EQUATIONS

Assuming that the truncated conical shell is simply supported at both ends and the boundary conditions are assumed as [19]:

$$w = \frac{\partial^2 w}{\partial S^2} = 0 \quad at \quad S = S_1 \quad and \qquad S = S_2 \tag{8}$$

The solution of the equation (6) is sought in the following form [20]:

$$w = f e^{z} \sin \beta_1 z \sin \beta_2 \varphi \tag{9}$$

where f is amplitude and the following definitions apply:

$$\beta_1 = \frac{m\pi}{z_0},\ \beta_2 = \frac{n}{\sin\gamma},\ z_0 = \ln\frac{S_2}{S_1},\ z = \ln\frac{S}{S_1} \tag{10}$$

Substituting Eq. (9) into Eq. (6) and applying the superposition method to the resulting equation, the particular solution is obtained as follows:

$$\Phi_1 = f(K_1 \sin \beta_1 z + K_2 \cos \beta_1 z + K_3 e^{-z} \sin \beta_1 z) \sin \beta_2 \varphi \tag{11}$$

where the following definitions apply:

$$\begin{aligned}
&K_1 = \frac{\beta_1(\beta_1 q_0 + q_2)}{q_0^2 + q_2^2} S_1 \cot\gamma,\ K_2 = \frac{\beta_1(\beta_1 q_2 - q_0)}{q_0^2 + q_2^2} S_1 \cot\gamma,\ K_3 = \frac{q_3}{q_1} \\
&q_0 = b_{11}(\beta_2^4 - 3\beta_1^2 - 2\beta_2^2) + 2(b_{31} + b_{12})\beta_2^2(\beta_1^2 - 1), \\
&q_1 = 2(b_{31} + b_{12})\beta_1^2\beta_2^2 + b_{11}(\beta_1^4 + \beta_2^4 + 2\beta_1^2 - 2\beta_2^2 + 1), \\
&q_2 = 4b_{11}\beta_1^3 + 4(b_{31} + b_{12})\beta_1\beta_2^2, \\
&q_3 = b_{14}[(\beta_2^2 - 1)^2 + \beta_1^2] + \beta_1^2[2(b_{13} - b_{32})\beta_2^2 + b_{14}(\beta_1^2 + 1)]
\end{aligned} \tag{12}$$

Substituting Eqs. (9) and (11) into Eq. (5) and applying Galerkin method, after integrating for the critical uniform lateral pressure of non-homogeneous truncated conical shells resting on a Winkler foundation, the following equation is obtained:

$$P_{Lcr}^{K_w} = \frac{U_1 + U_2 + U_3 + U_4 + K_w U_5}{U_6} \tag{13}$$

where the following definitions apply:

$$\begin{aligned}
&U_1 = \begin{bmatrix} c_{12}(K_1\beta_1^4 + 4K_2\beta_1^3 - 4K_1\beta_1^2 + K_1\beta_2^4 - 2K_1\beta_2^2) + \\ + 2(c_{11} - c_{31})\beta_2^2(2K_2\beta_1 - K_1 + K_1\beta_1^2) - K_3\beta_1^2 S_1 \cot\gamma \end{bmatrix} \frac{2\beta_1^2(1 - e^{3z_0})}{12\beta_1^2 + 27} \\
&U_2 = \begin{bmatrix} c_{12}(K_3\beta_1^4 + 2K_3\beta_1^2 + K_3 + K_3\beta_2^4 - 2K_3\beta_2^2) + 2K_3\beta_1^2\beta_2^2 \times \\ \times(c_{11} - c_{31}) - c_{13}(\beta_1^4 + 2\beta_1^2 + \beta_2^4 - 2\beta_2^2 + 1) - 2\beta_1^2\beta_2^2(c_{14} + c_{32}) \end{bmatrix} \frac{\beta_1^2(1 - e^{2z_0})}{4(\beta_1^2 + 1)} \\
&U_3 = \begin{bmatrix} c_{12}(K_2\beta_1^4 - 4K_1\beta_1^3 - 4K_2\beta_1^2 + K_2\beta_2^4 - 2K_2\beta_2^2) - \\ - 2(c_{11} - c_{31})\beta_2^2(-K_2\beta_1^2 + 2K_1\beta_1 + K_2) + K_3\beta_1 S_1 \cot\gamma \end{bmatrix} \frac{\beta_1(e^{3z_0} - 1)}{4\beta_1^2 + 9}; \\
&U_4 = 0.125(K_2 + K_1\beta_1)\beta_1(e^{4z_0} - 1)S_1 \cot\gamma\ ; \\
&U_5 = \frac{\beta_1^2(e^{6z_0} - 1)S_1^4}{12(\beta_1^2 + 9)};\ U_6 = \frac{S_1^3\beta_1^2(3 + 2\beta_2^2)\tan\gamma(e^{5z_0} - 1)}{5(25 + 4\beta_1^2)}
\end{aligned} \tag{14}$$

As $K_w = 0$, from Eq. (13), in special case for the critical uniform lateral pressure of non-homogeneous truncated conical shells without a Winkler foundation, the following equation is obtained:

$$P_{Lcr} = \frac{U_1 + U_2 + U_3 + U_4}{U_6} \tag{15}$$

The truncated conical shell is transformed into the cylindrical shell when $\gamma \to 0$. If $\gamma \to 0$ are substituted in Eqs. (13) and (15) corresponding formulas for simply supported cylindrical shells with or without a Winkler foundation are obtained. In this case, $P_{Lcr}^{K_w}$, P_{Lcr} in Eqs (13) and (15) are transformed into $P_{Lcyl}^{K_w}$, P_{Lcyl} respectively.

When $\mu = 0$, the appropriate formulas for the critical uniform lateral pressures of cylindrical and conical shells made of homogeneous materials with or without a Winkler foundation are found as a special case. The minimum values of lateral buckling pressure of truncated conical shells are obtained by minimizing Eqs. (13) and (15) with respect to *m* and *n*.

4 EXAMPLE PROBLEMS AND NUMERICAL STUDIES

4.1 Verification studies

To verify the present formulation and examine the accuracy of the present buckling analysis of truncated conical shells with or without a Winkler elastic foundation several comparisons are made with results available in open literature. The corresponding numerical results are tabulated in Tables 1–3.

In Table 1, the calculated critical uniform lateral pressure and corresponding circumferential wave numbers for simply supported, homogeneous, isotropic cylindrical shells without elastic foundations are compared with the results of Shen [11]. Our results are in good agreement with the results of Shen [11].

To validate the analysis, results for simply supported, homogeneous, isotropic truncated conical shells under uniform lateral pressure are compared with results of Singer [19] in Table 2. In the analysis, material and shell properties are as follows, $E_0 = 2.0104 \times 10^5$ MPa, $\nu = 0.32$, $R_2 / h = 500$; $L=0.25R_2\sin\gamma$. For comparison, the prediction for the critical uniform lateral pressure can be obtained using the approximate formula given by Singer [19]. As can be seen in Table 2, the difference between the values of the critical uniform lateral pressure is ignorable.

Table 1
Comparison of the critical uniform lateral pressure and corresponding wave numbers for homogeneous cylindrical shells without elastic foundations (E_0=2×10^{11} N/m^2, $\upsilon = 0.3$)

		$P_{Lcyl} \times 10^4$ (MPa) (n_{cr})	
L/R	*R/h*	Shen [11]	Present study
3	300	402.6 (7)	403.99(7)
	3000	1.251(12)	1.246(12)
5	300	239.10(5)	235.11(5)
	3000	0.748(9)	0.744(9)

Table 2
Comparison of critical load and corresponding wave numbers for isotropic truncated conical shells under uniform lateral pressure (E_0=2×10^{11} N/m^2, $\upsilon = 0.3$)

	$(P_{Lcyl} / E_0) \times 10^6$ and (n_{cr})						
γ	30°	35°	40°	45°	50°	55°	60°
Singer [19]	1.838(31)	1.504(29)	1.244(26)	1.009(24)	0.787(23)	0.605(21)	0.445(20)
Present study	1.979(31)	1.509(29)	1.184(26)	0.943(24)	0.753(23)	0.599(21)	0.470(20)

Table 3

Comparison of dimensionless frequency parameter $\omega_{1cyl}^{K_w}$ for a homogeneous cylindrical shell resting on a Winkler foundation ($R/h=100,\ L_1/R=2,\ K_w=10^{-4}$ N/m^3)

	$\omega_{1cyl}^{K_w}$	
(m,n)	Paliwal et al. [6]	Present study
(1,1)	0.6788227178	0.6792138004
(1,2)	0.3639407237	0.3646346369
(1,3)	0.2052558042	0.2080412884
(1,4)	0.1274543541	0.1382361504

As there are presently no results in open literature for the buckling of truncated conical shells under uniform lateral pressure and resting on elastic foundations, comparison of results in this study is made with those of Paliwal et al. [6] for free vibration analysis of cylindrical shells resting on a Winkler foundation. The comparison is shown in Table 3. When the inertial term is added into the left side of Eq. (5), after integrating and after some mathematical operations for the dimensionless frequency parameter ω_{1K_w} of free vibration of non-homogeneous truncated conical shells resting on a Winkler foundation, the following equation is obtained:

$$\omega_{1K_w}=\sqrt{\frac{(U_1+U_2+U_3+U_4+K_wU_5)(1-\nu^2)R_2^2\rho_0/E_0}{U_5\rho_1 h}} \tag{16}$$

Present results are obtained from Eq. (16) as $\mu=0,\ \gamma\to 0,\ R_2=R_1=R,\ L=L_1.$ This comparison is to ensure that elastic foundation effects have been correctly integrated into the present formulation.

4.2 Numerical results

Numerical computations, for homogeneous and non-homogeneous isotropic truncated conical shells with and without a Winkler foundation have been carried out using expressions (13) and (15). The results are presented in Tables 4-5. The non-homogeneity functions of the material of the truncated conical shell are assumed to be linear and quadratic functions which $\varphi_0(\overline{\zeta})=\overline{\zeta}$ and $\overline{\zeta}^2$ (see, [9, 13]). Material properties of the isotropic truncated conical shell are given below [20]: E_0=2.11×10^{11} N/m^2, $\upsilon=0.3$.

The results given in all tables below for critical uniform lateral pressures corresponding to longitudinal wave number m=1, only the number of circumferential waves (n_{cr}) is presented in parentheses. In tables, H and NH show homogeneous and non-homogenous cases, respectively.

Table 4 shows the variation of the values of critical uniform lateral pressures and corresponding circumferential wave numbers of homogeneous and non-homogeneous truncated conical shells for different non-homogeneity functions, versus the Winkler foundation modulus K_w. As the foundation modulus K_w increases, the values of $P_{Lcr}^{K_w}$ and $n_{Lcr}^{K_w}$ increase continuously for homogeneous and non-homogeneous cases. As the foundation modulus K_w increases, the effect of the non-homogeneity on the values of critical uniform lateral pressure decreases and for high values of the foundation modulus K_w the effect of the non-homogeneity is insignificant. For example, when the truncated conical shells without a Winkler foundation, i.e. $K_w=0$ and the Young's modulus is changed, the effect on the critical uniform lateral pressure are (-5.48%) and 12.37% for $\varphi_0(\overline{\zeta})=\overline{\zeta}$ and $\varphi_0(\overline{\zeta})=\overline{\zeta}^2$, respectively. When truncated conical shells resting on a Winkler foundation (for $K_w=10^7$ N/m^3) and the Young's modulus is changed, the effect on the critical uniform lateral pressure are (-4.64%) and 8.54%, for $\varphi_0(\overline{\zeta})=\overline{\zeta}$ and $\varphi_0(\overline{\zeta})=\overline{\zeta}^2$, respectively. When the Young's modulus is kept constant and the Winkler foundation modulus is changed, the effect on the critical uniform lateral pressure are 0.001%, 0.45%, 4.77%, 46.39%, for $K_w=10^3, 10^5, 10^6, 10^7$ N/m^3, respectively with the homogeneous isotropic truncated conical shell.

Table 4
Variations of $P_{Lcr}^{K_w}$, P_{Lcr} , $n_{Lcr}^{K_w}$ and n_{Lcr} for the H and NH isotropic truncated conical shells versus the K_w (N/m^3) (R_1=1 m, h=0.01 m, L=2R_1, $\gamma = 30^o$, $\mu = 0.9$)

$\varphi_0(\bar{\zeta})$ / K_w (N/m^3)	P_{Lcr} (MPa), (n_{Lcr}) 0	$\bar{\zeta}$	$\bar{\zeta}^2$	$P_{Lcr}^{K_w}$ (MPa), ($n_{Lcr}^{K_w}$) 0	$\bar{\zeta}$	$\bar{\zeta}^2$
10^3	0.45786(8)	0.43278(8)	0.51449(8)	0.45789(8)	0.4328(8)	0.5145(8)
10^5				0.4600(8)	0.4349(8)	0.5166(8)
10^6				0.4785(8)	0.4534(8)	0.5352(8)
10^7				0.6644(9)	0.6336(9)	0.7211(8)

Table 5
Variations of $P_{Lcr}^{K_w}$, P_{Lcr} , $n_{Lcr}^{K_w}$ and n_{Lcr} for N and NH isotropic truncated conical shells versus the ratio R_1/h ($K_w = 10^6$ N/m^3, R_1=1 m, h=0.01 m, $\gamma = 30^o$, $\mu = 0.9$)

$\varphi_0(\bar{\zeta})$ / R_1/h	P_{Lcr} (MPa), (n_{Lcr}) 0	$\bar{\zeta}$	$\bar{\zeta}^2$	$P_{Lcr}^{K_w}$ (MPa), ($n_{Lcr}^{K_w}$) 0	$\bar{\zeta}$	$\bar{\zeta}^2$
100	0.4579(8)	0.4328(8)	0.5145(8)	0.4785(8)	0.4534(8)	0.5352(8)
200	0.0792(9)	0.0753(9)	0.0886(9)	0.1076(10)	0.1029(10)	0.1178(10)
300	0.0285(10)	0.0271(10)	0.0319(10)	0.0586(13)	0.0564(13)	0.0632(12)

As the values of the foundation modulus K_w is little the non-homogeneity has a considerable influence on the critical uniform lateral pressure.

In Table 5 variations of the values of critical uniform lateral pressures and corresponding circumferential wave numbers for homogeneous and non-homogeneous isotropic truncated conical shells with or without a Winkler foundation, versus the ratio R_1/h, as the $K_w = 10^6$ N/m^3, are presented. As the ratio R_1/h increases, the values of critical uniform lateral pressures decrease, whereas, corresponding wave numbers increase for both homogeneous and non-homogeneous truncated conical shells with or without a Winkler foundation. As the ratio R_1/h increases, the effect of the non-homogeneity on the values of the frequencies and corresponding wave numbers of the truncated conical shell resting on an elastic foundation decrease. The difference between $P_{Lcr}^{K_w}$ and P_{Lcr} are nearly 4.51%; 35.85 %, 105.67% as the ratio R_1/h are 100, 200, 300, respectively for homogeneous case. Consequently, the effect of a Winkler foundation on the values of $P_{Lcr}^{K_w}$ is increased rapidly, as the ratio R_1/h increase.

5 CONCLUSIONS

In this study, the buckling of non-homogeneous truncated conical shells under uniform lateral pressure and resting on a Winkler elastic foundation are presented. The basic relations have been obtained for non-homogeneous truncated conical shells, the elastic properties of which vary continuously in the thickness direction. The critical uniform lateral pressures of non-homogeneous truncated conical shells with or without a Winkler foundation are obtained. Finally, carrying out some computations, effects of the variation of the truncated conical shell characteristics, the non-homogeneity and a Winkler foundation on the values of the critical uniform lateral pressure have been studied.

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RFERENCES

[1] Pasternak P.L., 1954, *On a New Method of Analysis of an Elastic Foundation by Means of Two Foundation Constants*, Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvu i Arkhitekture, Moscow, USSR (in Russian).

[2] Kerr A.D., 1964, Elastic and visco-elastic foundation models, *ASME Journal of Applied Mechanics* **31**: 491-498.

[3] Bajenov V.A., 1975, *The Bending of the Cylindrical Shells in an Elastic Medium*, Kiev, Visha Shkola (in Russian).

[4] Sun B., Huang Y., 1988, The exact solution for the general bending problems of conical shells on the elastic foundation, *Applied Mathematics and Mechanics* **9**(5): 455-469.

[5] Eslami M.R., Ayatollahi M.R., 1993, Modal-analysis of shell of revolution on elastic-foundation, *International Journal of Pressure Vessels and Piping* **56**(3): 351-368.

[6] Paliwal D.N., Pandey R.K., Nath T., 1996, Free vibration of circular cylindrical shell on Winkler and Pasternak foundation, *International Journal of Pressure Vessels and Piping* **69**: 79-89.

[7] Ng T.Y., Lam K.Y., 2000, Free vibrations analysis of rotating circular cylindrical shells on an elastic foundation, *Journal of Vibration and Acoustics* **122**: 85-89.

[8] Tj H.G., Mikami T., Kanie S., Sato M., 2006, Free vibration characteristics of cylindrical shells partially buried in elastic foundations, *Journal of Sound and Vibration* **290**: 785-793.

[9] Lomakin V.A., 1976, *The Elasticity Theory of Non-homogeneous Materials*, Nauka, Moscow (in Russian).

[10] Awrejcewicz J., Krysko V.A., Kutsemako A.N., 1999, Free vibrations of doubly curved in-plane non-homogeneous shells, *Journal of Sound and Vibration* **225**(4): 701-722.

[11] Shen H.S., 2003, Post-buckling analysis of pressure loaded functionally graded cylindrical shells in thermal environments, *Engineering Structures* **25**: 487-497.

[12] Goldfeld Y., 2007, Elastic buckling and imperfection sensitivity of generally stiffened conical shells, *AIAA Journal* **45**(3): 721–729.

[13] Sofiyev A.H., Omurtag M., Schnack E., 2009, The vibration and stability of orthotropic conical shells with non-homogeneous material properties under a hydrostatic pressure, *Journal of Sound and Vibration* **319**(3-5): 963-983.

[14] Najafizadeh M.M., Hasani A., Khazaeinejad P., 2009, Mechanical stability of functionally graded stiffened cylindrical shells, *Applied Mathematical Modelling* **33**(2): 1151-1157.

[15] Tomar J., Gupta D., Kumar V., 1986, Natural frequencies of a linearly tapered non-homogeneous isotropic elastic circular plate resting on an elastic foundation, *Journal of Sound and Vibration* **111**: 1-8.

[16] Sofiyev A.H., 1987, The stability of non-homogeneous cylindrical shells under the effect of surroundings, *Soviet Scientific and Technical Research Institute (VINITI), Moscow* **3**(189): 1-9 (in Russian).

[17] Sofiyev A.H., Keskin S.N., Sofiyev A.L.H., 2004, Effects of elastic foundation on the vibration of laminated non-homogeneous orthotropic circular cylindrical shells, *Journal of Shock and Vibration* **11**: 89-101.

[18] Sheng G.G., Wang X., 2008, Thermal vibration, buckling and dynamic stability of functionally graded cylindrical shells embedded in an elastic medium, *Journal of Reinforced Plastics and Composites* **27**(2): 117-134.

[19] Singer J., 1962, The effect of axial constraint on the instability of thin conical shells under external pressure, *ASME Journal of Applied Mechanics*, 212-214.

[20] Agamirov V.L., 1990, *Dynamic Problems of Nonlinear Shells Theory*, Moscow, Nauka (in Russian).

[21] Tong L., 1996, Effect of axial load on free vibration of orthotropic conical shells, *Journal of Vibration and Acoustics* **118**: 164-168.

[22] Liew K.M., Ng T.Y., Zhao X., 2005, Free vibration analysis of conical shells via the element-free KP-Ritz method, *Journal of Sound and Vibration* **281**: 627-645.