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Research Paper

Arbitrary Angle Waveguide Bends Made of a New Heterostructure Phononic Crystal

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ABSTRACT

In this paper, an arbitrary angle waveguide bend made of a new has been studied. By creation of line heterostructure phononic crystal defects in the proposed heterostructure which is composed of square and rhombus phononic crystals, a simple structure waveguide bend with arbitrary angles is made. Analyzing the proposed bend showed that by creating line defects in the composition of the square and equilateral triangle lattices, 30˚ waveguide bend can be realized. Also the study showed that by creating line defects in the composition of the square and rhombus lattices, 40˚ (20˚) waveguide bend could be obtained if the angular constant of the rhombus lattice is 80˚ (40˚). The 40˚ and 20˚ waveguide bends have a narrow pass band which can be utilized as a filter to separate a specific frequency and guide it along a defined pass. Also the study shows that by incorporation of 90˚ bend within the presented heterostructure bends, waveguide bends can be realized that guide elastic waves in the arbitrary angles greater than 90˚.

Keywords: Phononic crystal; Arbitrary angle waveguide bends; Heterostructure.

1 INTRODUCTION

A GREAT deal of researches has been devoted to phononic crystals in recent years. These inhomogeneous structures are periodic arrangement of inclusions in an elastically different host material. The frequency range where the propagation of elastic waves is forbidden in any direction is called the complete band gap. By this feature, these structures can act as an acoustic filter, waveguide, liquid sensor [1-4], energy harvester [5] and etc. Vibration and noise reduction is another application of these structures. For example, a phononic crystal was used to meet the requirements of vibration and noise reduction for the wheel-driven electric bus [6], vibration reduction of car body [7], low-frequency noise and vibration reduction in ship and ocean engineering [8] and vibration reduction in piping systems [9]. Theoretical and empirical studies on phononic crystals have been done in several works [10-13]. Jia et al. demonstrated that by combining the band-gap formation mechanisms of Bragg scattering and local resonances, phononic crystal with wide and robust phononic band gaps can be established [14]. Aiming at solving the noise and vibration problem in vehicles, a phononic crystal structure was proposed by Shaobo et al. which could guide and

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filter the sound waves [15]. Also, the straight and bent waveguides have been analyzed by some researchers [16-23]. Yang et al. proposed a phononic crystal waveguide for enhancement of elastic wave energy harvesting [24]. The study of surface modes in phononic crystal waveguides in the hypersonic regime was done in Guo et al. ,s work [25]. In Korozlu et al. ,s research, a two-dimensional phononic crystal linear defect waveguide is utilized for size-based sorting of millimeter-sized solid particles in the air through acoustic radiation force [26]. In addition, heterostructure phononic crystal waveguides have been investigated in some studies [27]. Yao et al. studied heterostructure phononic crystal waveguides. They showed that when two or more than two different lattice constant phononic waveguides are combined together, there exist two or more than two perfect reflection frequency regions, and they appear independently without interfering with each other [28]. Photonic crystals as arbitrary angle waveguide bends have been studied in some researches. The arbitrary angle waveguide bends may have been fabricated by use of a circular photonic crystal [29-30] or by joining two truncated straight waveguides [31]. Also, arbitrary angle waveguide bends based on zero-index metamaterials have been proposed for electromagnetic waves [32]. Active elastic metamaterials [33] have been proposed and used for arbitrary angle guiding of elastic waves too. However according to the best of our knowledge, phononic crystals as arbitrary angle waveguide bends have not been studied yet.

By joining two-dimensional square and rhombus lattices, a new heterostructure phononic crystal is introduced in this paper. Then, by creation of line defects in the proposed heterostructure, an arbitrary angle waveguide bend is actualized. This bend was studied using displacement-based finite difference time domain method [34] and its ability as an arbitrary angle waveguide bend was examined afterwards.

2 PROPOSED ARBITRARY ANGLE WAVEGUIDE BEND

The proposed heterostructure phononic crystal which is constructed by joining square and rhombus phononic crystals has been illustrated in Fig. 1. The heterostructure has finite size in horizontal direction and extends infinitely in vertical direction. Heterostructure geometrical form forces the following relation between constants of square and rhombus lattices:

$$a' = a/(2\sin(\gamma/2)) \tag{1}$$

where a' and γ are constants of the rhombus lattice and a is constant of square lattice. Now by creating line defects in this heterostructure, the proposed arbitrary angle waveguide bend can be actualized as shown in Fig. 2. If in the heterostructure, the angular constant of rhombus lattice is γ°, then a $\gamma^\circ/2$ waveguide bend could be realized. To guide elastic waves through an arbitrary angle (α°) bend, the angular constant of the rhombus lattice of the heterostructure should be taken to be $2\alpha^\circ$. In Fig. 3 the proposed arbitrary angle waveguide bend has been shown which can guide elastic wave through over 90˚bend. Hereby it is supposed that the heterostructure has finite size in both horizontal and vertical directions.

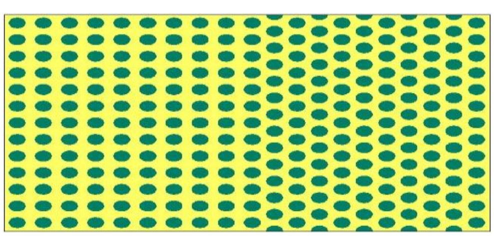

Fig.1
The proposed heterostructure phononic crystal that was created by joining square lattice (left) and rhombus lattice (right).

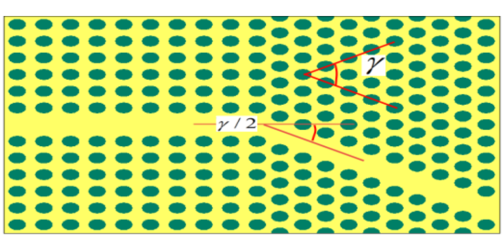

Fig.2
The proposed arbitrary angle waveguide bend that was constructed by created line defects in the heterostructure.

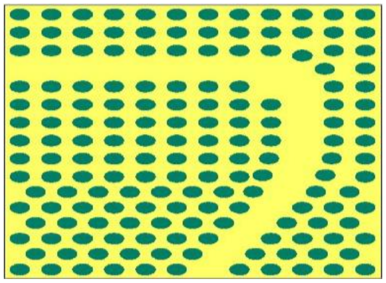

Fig.3
The proposed arbitrary angle waveguide bend that was constructed by created line defects in the heterostructure.

3 CALCULATION METHOD

Two-dimensional phononic crystals are considered here. The equations of elastic wave propagation can be described as:

$$\rho \dot{v}_i = \sigma_{ij,j} \tag{2}$$

$$v_i = \dot{u}_i \tag{3}$$

$$\sigma_{ij} = C_{ijmn} u_{m,n} \tag{4}$$

In the aforementioned equations, $\rho = \rho(x, y)$, $C_{ijkl}(x, y)$, v_i and u_i are respectively symbols of the density, elastic stiffness tensor, *ith* component of wave velocity and displacement in the structure. The summation agreement over dummy indices is considered, too. Since propagation of elastic waves in the *xy* plane is assumed, the displacement, velocity and stress tensor of the structure are independent of *z*, i.e., $u_i = u_i(x, y, t)$, $v_i = v_i(x, y, t)$ and $\sigma_{ij} = \sigma_{ij}(x, y, t)$. To discretize and solve the above equations, displacement-based finite difference time domain (DBFDTD) method was used which can be found in detail in [34].

4 NUMERICAL ANALYSIS

To prove the ability of the proposed bend as an arbitrary angle waveguide bend, several numerical examples are studied in this section. In the subsequent examples, the inclusion and the matrix are steel and epoxy, respectively. The density, longitudinal and transverse wave velocities of steel are presumed to be 7900 kg/m^3, 5950 *m/s* and 3240 *m/s* respectively, and those for epoxy are 1180 kg/m^3, 2540 *m/s* and 1160 *m/s*. The diameter of steel cylinders and the constant of the square lattice are *r*=5 and *a*=8 *mm* respectively.

To analyze the proposed bend when the bend angle was less than 90˚, left and right boundaries of the proposed arbitrary angle waveguide bend of Fig. 2 was surrounded by two homogenous regions and the perfectly matched layer (PML) [35] was adopted as the absorbing boundary condition. The periodic boundary condition was applied on the top and bottom boundaries.

To simulate the proposed bend when the bend angle was more than 90˚, all boundaries of the proposed arbitrary angle waveguide bend of Fig. 3 was surrounded by four homogenous regions. Because the heterostructure has finite size in all directions in this case, the perfectly matched layer (PML) was adopted as the absorbing boundary condition on all boundaries.

A Gaussian wave packet was launched along the *x* direction in the left homogenous region. The calculation model was discretized in both *x* and *y* directions with a grid interval of (*a*/60). The equations of motion were solved over 216 time steps with each time step lasting 7.91*ns*. By taking fast Fourier transform of the averaged *x* component of displacement over the guide width (right homogenous region) and normalizing it to the same quantity calculated for the wave packet in the absence of the phononic crystal, transmission coefficient can be calculated. The numerical analysis was done by a developed FORTRAN code based on parallel processing.

4.1 The 30˚ waveguide bend

To guide elastic waves through a 30˚ bend, the heterostructure was created by joining square and triangular lattices. The constant of the triangular lattice is 8 *mm*. The transmission spectra of the proposed bend in this case has been illustrated in Fig. 4. As this figure shows, the proposed bend can guide elastic waves with frequency range of 94-129 *kHz* through a 30˚ bend. In Fig. 5 the displacement field of the elastic wave with frequency of 102 *kHz* through the 30˚ bend has been presented. The figure demonstrates that the elastic wave is concentrated well within the waveguide and a few amounts of energy is leaked to surrounding medium, So the proposed bend can act as a 30˚ bend.

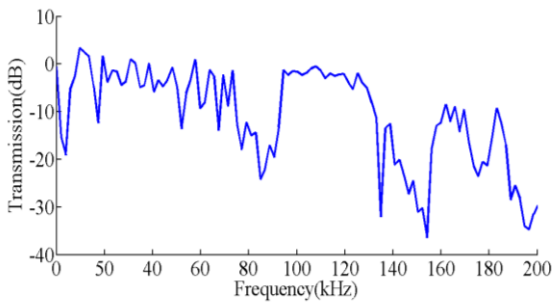

Fig.4
The transmission spectra of the proposed bend as a 30˚ waveguide bend.

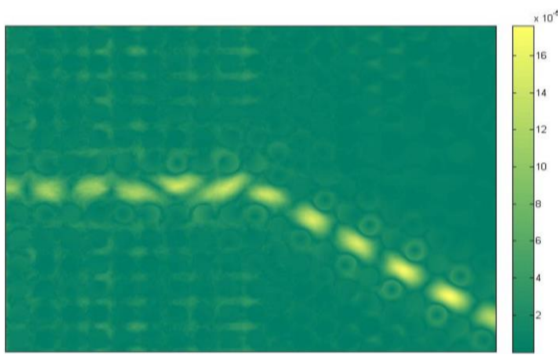

Fig.5
The displacement field of the elastic wave (in *m*) with frequency of 102 *kHz* through the 30˚ bend.

4.2 The 40˚ waveguide bend

In this case, the heterostructure was composed of square and rhombus lattices. The angular constant of the rhombus lattice is 80˚. Using Eq. (1), the constant of the rhombus lattice was calculated to be 6.22 *mm*. Fig. 6 shows the transmission spectra of the proposed bend as a 40˚ waveguide bend. As the figure shows, there is a narrow pass band extending from 121 *kHz* to 127 *kHz* where the proposed bend can guide elastic waves through a 40˚ bend.

In Fig. 7, the displacement field of the elastic wave with frequency of 124 *kHz* through the 40˚ bend has been illustrated. The result shows that the elastic wave propagation is restricted within the waveguide and hence the ability of the proposed bend as a 40˚ bend is proved.

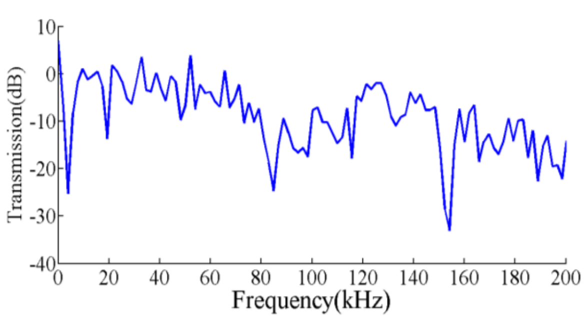

Fig.6
The transmission spectra of the proposed bend as a 40˚ waveguide bend.

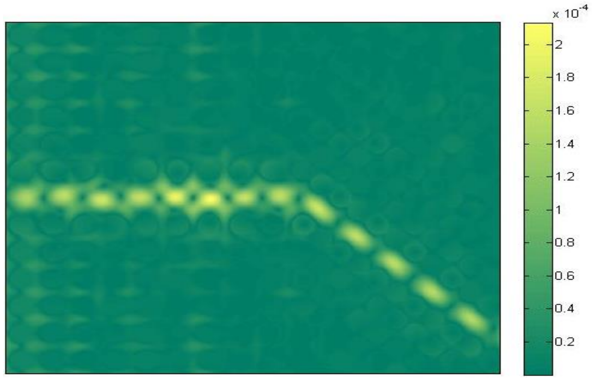

Fig.7
The displacement field of the elastic wave (in *m*) with frequency of 124 kHz through the 40˚ bend.

4.3 The 20˚ waveguide bend

To guide the elastic wave through a 20˚ bend, the heterostructure was composed of square and rhombus lattices. The angular constant of the rhombus lattice is 40˚. Using Eq. (1), the constant of the rhombus lattice was calculated to be 11.69 *mm*. Fig.8 represents the transmission spectra of the proposed bend as a 20˚ waveguide bend. As the figure indicates, the proposed bend can guide elastic waves at narrow frequency range of 100-106 *kHz* through a 20˚ bend. In Fig. 9 the displacement field of the elastic wave with frequency of 100 *kHz* through the 20˚ bend has been shown. This figure illustrates that the elastic wave is restricted within the guide. Although a few amounts of energy leaks to surrounding medium, the wave propagates along the bend. So the proposed bend can guide the elastic wave through a 20˚ bend.

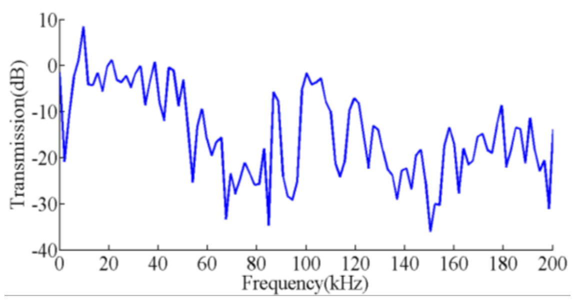

Fig.8
The transmission spectra of the proposed bend as a 20˚ waveguide bend.

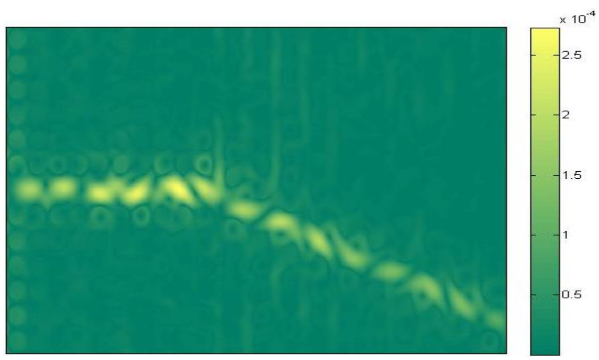

Fig.9
The displacement field of the elastic wave (in *m*) with frequency of 100 *kHz* through the 20˚ bend.

4.4 The over 90˚ *waveguide bend*

The ability of the proposed bend to guide elastic waves through over 90˚ bend is studied in this section. This feature is realized by incorporation of a 90˚ bend in the bends studied in previous sections.

Fig. 10 shows the displacement field of the elastic wave with frequency of 102 *kHz* through the 120˚ bend. Figs. 11and 12 shows the same field where the elastic wave with frequency of 124 (102) *kHz* propagates through the 130˚ (110˚) bend.

The 120˚ bend was created by joining the square and triangular lattices. The 130˚ (110˚) bend was created by joining the square and rhombus lattices with angular constant of 80˚ (40˚). As Figs. 10-12 demonstrate the proposed bend can confine elastic waves within the guide. The elastic wave propagation within the bend and its confinement proves that the proposed bend can guide elastic waves through over 90˚ bend.

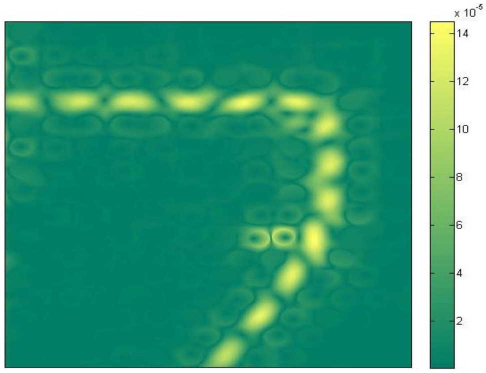

Fig.10
The displacement field of the elastic wave (in *m*) with frequency of 102 *kHz* through the 120˚ bend.

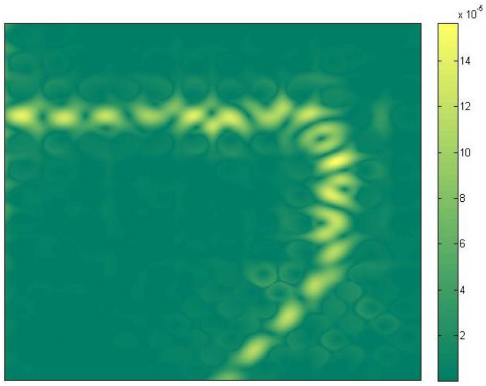

Fig.11
The displacement field of the elastic wave (in *m*) with frequency of 124 *kHz* through the 130˚ bend.

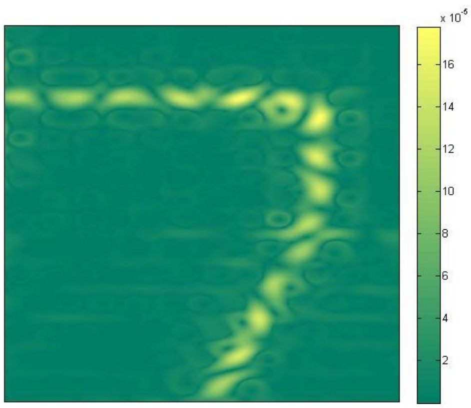

Fig.12
The displacement field of the elastic wave (in *m*) with frequency of 102 kHz through the 110˚ bend.

5 CONCLUSION

A new heterostructure phononic crystal was introduced and studied as an arbitrary angle waveguide bend in this paper. The heterostructure is created by joining square and rhombus phononic crystals. By creation of line defects in the proposed heterostructure, an arbitrary angle waveguide bend was introduced and studied. The numerical analysis showed the ability of the proposed bend as a 20˚, 30˚, 40˚, 110˚, 120˚ and 130˚ waveguide bend. Also the study proved that the proposed 40˚ and 20˚ waveguide bends have a narrow pass band, so the proposed bend can serve as a filter to separate a specific frequency.

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