Journal of Solid Mechanics Vol. 13, No. 4 (2021) pp. 448-459
DOI: 10.22034/jsm.2020.1905148.1625

A Comparison Between the Linear and Nonlinear Dynamic Vibration Absorber for a Timoshenko Beam

H. Kouhi[1,*], R. Ansari [1], E. Salahshoor [2], B. Miripour Fard [1]

[1]*Department of Mechanical Engineering, University of Guilan, Rasht, Iran*
[2]*Department of Mechanical Engineering, Izeh Branch, Islamic Azad University, Izeh, Iran*

Received 28 July 2021; accepted 1 October 2021

ABSTRACT

Dynamic vibration absorbers (DVAs) play an important role in the energy dissipation of a vibrating system. Undesirable vibrations of structures can be reduced by using the absorbers. This paper investigates the effect of an attached energy sink on the energy dissipation of a simply supported beam subjected to harmonic excitation. The aim is to design an optimal linear energy sink (LES) and a nonlinear energy sink (NES) and then compare them with each other. Each absorber includes a spring, a mass, and a damper. For each absorber, the optimum mass, stiffness, and damping coefficients are obtained in order to minimize the beam's maximum amplitude at the resonant frequencies. The optimization problem is minimizing the maximum amplitude of the beam subjected to an arbitrary harmonic force excitation. For consideration of the effects of rotary inertia and shear deformation, the Timoshenko beam theory is used. The mathematical model of beam with DVA is verified by using the ANSYS WORKBENCH software. Finally, by considering the uncertainty on the DVA parameters it was observed that the LES is more robust than the NES.

Keywords : Dynamic vibration absorber; Timoshenko beam; Optimization; Uncertainty; Robustness.

1 INTRODUCTION

AMONG the several techniques which are used for reducing the structural vibrations, DVAs are very useful and preferred. DVAs have received much attention for their good performance in suppressing of maximum deflections of continuous and discrete systems. They are widely used to reduce vibration especially when the system is under the harmonic excitations. A DVA includes a mass, a spring and a damper attached to the main structure. There are two types of DVAs; DVA with a linear spring and damper or LES and DVA with nonlinear spring and damper or NES. DVAs don't need an active control system with huge electrical power consumption (Samani et al. [12]). A DVA locally dissipates the vibrating energy. DVAs need tuning to have a good performance. A lot of research has been done to dissipate the vibration energy which is incited into the system by external excitation. In recent studies, linear and nonlinear vibration absorbers have been introduced to reduce the resonance peaks. In

[*]Corresponding author. Tel.: +98 1333690274.
E-mail address: hamed.koohi@guilan.ac.ir (H. Kouhi)

(Vakakis et al., [14]) it is shown that the vibrating energy of a linear discrete structure can be absorbed by a nonlinear oscillator. For a discrete system the evaluation of a nonlinear energy sink absorber based on the transmissibility has been addressed in (Zang et al., [16]). In (Chen et al., [3]) the vibration of a linear beam equipped with a NES is investigated. In addition to the continuous system, DVAs can be used for the vibration control of discrete systems (Starosvetsky and Gendelman, [13]). A nonlinear absorber tuning procedure is given in (Starosvetsky and Gendelman, [13]) by which the total system energy suppression is provided. In (Samani et al.,[13]) performances of dynamic vibration absorbers for beams subjected to moving loads have been addressed. The mentioned beam is equipped with a DVA to reduce the vibration. The optimization problem is minimization of maximum vibration amplitude of the beam. The robustness of a NES attached to a clamped-clamped beam with harmonic excitation is investigated in (Parseh et al., [11]). In (Younesian et al., [15]) a railway bridge is considered as a beam that is subjected to a moving load. The performance of a NES attached to the beam is evaluated. It is shown that NES can remarkably suppress the beam deflection. The robustness of the optimal NES is investigated by the sensitivity analysis. In (Ahmadabadi and Khadem, [1]) vibration control of a linear cantilever beam by two different NESs (grounded and ungrounded) has been investigated. The dynamics of a simply supported beam with an attached local NES have been studied in (Georgiades and Vakakis, [6]). It is shown that NES dissipates the shock energy without spreading it back to the beam. In (Avramov and Gendelman, [2]) they have shown that the nonlinear absorber can be useful for vibration dissipation of continuous harmonically forced system. Vibration control of a Timoshenko beam with optimized tuned mass damper (TMD) subjected to a moving load is investigated in (Moghaddas et al., [10]). The system of combined bridge-vehicle is studied in which the bridge is modeled as a Timoshenko beam. The objective is to determine the optimum values of the TMD parameters in order to minimize the maximum amplitude of beam. In (Esmailzadeh and Jalili, [5]) the maximum amplitude of beam has been minimized over a wide range of exciting frequency. The effect of TMDs attached to a simply-supported ends Timoshenko beam subjected to an exciting force has been addressed in (Chen and Huang, [4]). The effect of TMD number on the beam maximum amplitude is analyzed. In (Samani et al., [13], Parseh et al., [11], Younesian et al.,[15], Ahmadabadi and Khadem, [1]) and many other works for the beam modeling the Euler-Bernoulli model is employed which is not accurate modeling. In this work such as (Moghaddas et al., [10], Esmailzadeh and Jalili,[5], Chen and Huang, [4]) for consideration of the effects of rotary inertia and shear deformation, the Timoshenko beam theory is used. In many works such as (Samani et al., [13], Younesian et al., [15], Kani et al., [8], Avramov and Gendelman, [2], Moghaddas et al., [10], Chen and Huang, [4]) simply support ends beam is studied. In this paper we also use the mentioned boundary condition. Multiple DVAs is more effective than a single DVA, although there are many works that studied the effect of a single DVA attached to a beam, there are a few works such as (Moghaddas et al., [10], Chen and Huang, [4]) that have studied the performance of multi DVAs. In this paper beam with multi DVAs will be studied. In order to obtain suitable parameters for a DVA, an optimization problem should be defined. Many researches such as (Kani et al., [8], Moghaddas et al., [10], Chen and Huang, [4]) have used optimization methods to design absorber mass, spring and damper. In this paper the parameters are obtained from a min-max optimization problem. Although a great attention has been paid to use a NES rather than a LES (Ahmadabadi and Khadem, [1], Kani et al., [9], Georgiades and Vakakis, [6], Avramov and Gendelman, [2]) in the present paper these two types absorbers are compared with each other. In practice, the ratio of the total mass of DVAs to the beam mass should not be greater than 10% moreover it is not economic to design a heavy DVA (Chen and Huang, [4]). In our work, total mass of DVAs is assumed to be a constant value of 15*kg* (smaller than 10% of beam mass). In the other words we want to use all 15*kg* for the multi DVAs. This constraint must be considered in the optimization procedure. We use the total mass of the DVA equal to 15 *kg* which is far less than the beam mass of 235 *kg*.

In order to compare the robustness of LES and NES we use the uncertainty analysis (Parseh et al., [11], Younesian et al., [15]). In this method an uncertainty on exciting force amplitude and location as well as 20% uncertainty on stiffness and damping of each absorber will be considered and then the beam maximum amplitude for a wide range of exciting force frequency is studied. In the previous works the LES and NES have not been compared with each other.

2 BEAM TIME RESPONSE

In Fig.1 the simply supported beam which is equipped with three DVAs is shown. The structure is symmetric, At the $x=L/2$ the DVA mass is m_0 and for $x=L/4$ and $3L/4$ each DVA mass is m_1.

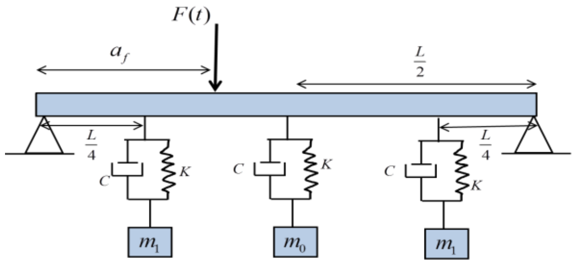


Fig.1
Simply supported Timoshenko beam with three DVAs.

In the following the governing equation of motion is derived. The Timoshenko beam theory is used to consider the effects of rotary inertia and shear deformation. To characterize the beam deformation, two quantities are used; $y(x,t)$ represent the transverse deflection and $\psi(x,t)$ is the rotation of the beam cross-section.

In order to derive equations by Lagrangian method (Esmailzadeh and Jalili, [5]) the assumed mode expansion is used in which $y(x,t)$ and $\psi(x,t)$ can be written as:

$$\begin{aligned} y(x,t) &= \sum_{i=1}^{n} Y_i(x) q_{bi}(t) \\ \psi(x,t) &= \sum_{i=1}^{n} \Psi_i(x) q_{bi}(t) \end{aligned} \tag{1}$$

where, $q_{bi}(t)$ is the time dependent generalized coordinate of the *i-th* mode of the beam, $\Psi_i(x)$ and $Y_i(x)$ are the rotational and transverse mode shape for the *i-th* mode of free vibration, respectively.

By applying the Lagrange's equation the equation of motion is obtained (Esmailzadeh and Jalili, [5]). For a beam with LES the spring is linear and we have

$$\begin{aligned} &\ddot{q}_{bi} N_i + \sum_{j=1}^{J} \left\{ k_j \left(\sum_{i=1}^{n} Y_i(a_j) q_{bi} - q_j \right) + c_j \left(\sum_{i=1}^{n} Y_i(a_j) \dot{q}_{bi} - \dot{q}_j \right) \right\} Y_i(a_j) + q_{bi} S_i = Y_i(a_f) F(t) \\ &M_j \ddot{q}_j - k_j \left(\sum_{i=1}^{n} Y_i(a_j) q_{bi} - q_j \right) - c_j \left(\sum_{i=1}^{n} Y_i(a_j) \dot{q}_{bi} - \dot{q}_j \right) = 0 \\ &,\ i = 1,2,...,n,\ j = 1,2,..,J \end{aligned} \tag{2}$$

and for a beam with NES the stiffness of spring is nonlinear (spring force $= kx^3$) and we have

$$\begin{aligned} &\ddot{q}_{bi} N_i + \sum_{j=1}^{J} \left\{ k_j \left(\sum_{i=1}^{n} Y_i(a_j) q_{bi} - q_j \right)^3 + c_j \left(\sum_{i=1}^{n} Y_i(a_j) \dot{q}_{bi} - \dot{q}_j \right) \right\} Y_i(a_j) + q_{bi} S_i = Y_i(a_f) F(t) \\ &M_j \ddot{q}_j - k_j \left(\sum_{i=1}^{n} Y_i(a_j) q_{bi} - q_j \right)^3 - c_j \left(\sum_{i=1}^{n} Y_i(a_j) \dot{q}_{bi} - \dot{q}_j \right) = 0 \\ &,\ i = 1,2,...,n,\ j = 1,2,..,J \end{aligned} \tag{3}$$

where, according to Fig.1, $J = 3$, $a_1 = L/4$, $a_2 = L/2$, $a_3 = 3L/4$, $M_1 = M_3 \equiv m_1$, $M_2 \equiv m_0$.

and $N_i = \rho \int_0^L \left[A Y_i^2(x) + I \psi_i^2(x) \right] dx$, $S_i = \int_0^L \left[EI \psi_i'^2(x) + \kappa AG \left(\psi_i(x) - Y_i'(x) \right)^2 \right] dx$. Derivations dot "." and prime " ' " represents partial derivative with respect to the time *t* and position *x*, respectively. ρ represents mass per unit volume of the beam, *A* is cross-sectional area of the beam, *I* is beam cross-sectional moment of inertia, *E* is the beam's Young's modulus of elasticity, κ is shear deformation coefficient in the Timoshenko beam theory, *G* is the beam's shear modulus.

From (Esmailzadeh and Jalili, [5]) the natural frequencies ω_i are obtaind by

$$S_i = N_i \omega_i^2 . \tag{4}$$

The details of determining of the Timoshenko-beam mode shape for $\Psi_i(x)$ and $Y_i(x)$ have been described in (Huang, [7]). In this paper, the type of beam is simply supported in which the boundary condition for each ends of the beam is $\Psi' = 0$ and $Y = 0$.

In the simply support ends, transverse deflections and bending moments are equal to zero at both ends. For mentioned boundary conditions the mode shapes of Timoshenko beam is (Huang, [7])

$$\begin{aligned} \Psi_i(x) &= \cos(\frac{i\pi x}{L}) \\ Y_i(x) &= C_4 \sin(\frac{i\pi x}{L}) \end{aligned} \tag{5}$$

where, $C_4 = \frac{L}{b\beta}\left[1 + b^2 s^2(\beta^2 - r^2)\right]$, $r = \left(\frac{I}{AL^2}\right)^{0.5}$, $s = \left(\frac{EI}{\kappa AGL^2}\right)^{0.5}$, $b = \left(\frac{1}{EI}\rho AL^4\omega_n^2\right)^{0.5}$,
$\beta = \frac{1}{\sqrt{2}}\left\{\left(r^2 + s^2\right) + \left[(r^2 - s^2)^2 + 4/b^2\right]^{0.5}\right\}^{0.5}$.

3 VERIFICATIONS OF EQUATIONS

In this section the mathematical equations of the Timoshenko beam is verified with ANSYS WORKBENCH software. Some of the beam parameters are given in
Table **1**. The parameters A and I are calculated with $A = bh$ and $I = bh^3/12$.

Table 1
beam parameters

$E\,(GPa)$	$G\,(GPa)$	L (m)	b(m)	h (m)	$\rho(kg/m^3)$	κ
200	80	6	0.1	0.05	7800	5/6

The beam with three linear DVA is modeled in ANSYS WORKBENCH as shown in Fig.2. In this verification $m_0 = 7kg$, $m_1 = 4kg$,$C = 27.5\ Ns/m$, $K = 1750 N/m$, exciting force amplitude is 1000N, exciting force location is $a_f = L/3$, exciting force frequency is $\omega = 5 rad/s$, step time is dt=0.0001s and finally simulation time is 30s. For the numerical solution of a continuous beam a discretization method is needed then the Runge-Kutta method is used for doing this purpose.

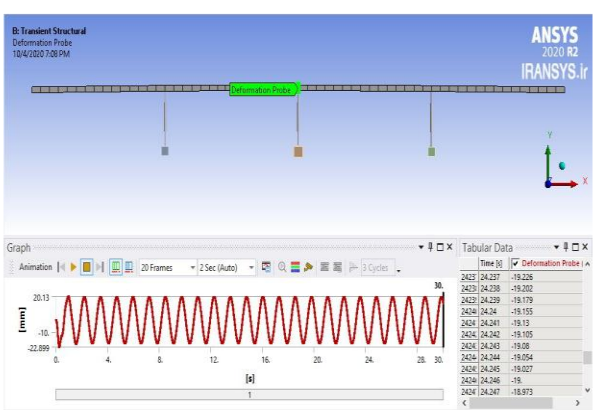


Fig.2
Beam modeling in the ANSYS WORKBENCH.

In this verification the first six natural frequencies of beam are used as $\omega_n = \{20.03, 80.11, 180.15, 320.01, 499.51, 718.41\}\ rad/s$ which are calculated from Eq.(4)). In the ANSYS WORKBENCH the time step is chosen to be 0.001s and the beam consists of 64 elements. In Table 2 some tuning in the ANSYS WORKBENCH is shown.

Table 2
Some parameters in the ANSYS WORKBENCH.

Number of Steps	Current Step Number	Step End Time	Auto Time Stepping	Define By	Time Step	Time Integration
1.	1.	30. *s*	Off	Time	1.e-003 *s*	On

In Fig.3 the response of the beam center Y_{cm} is shown, the small error between the ANSYS output and the mathematical model output shows the high accuracy of the mathematical modeling which is based on the Timoshenko beam theory.

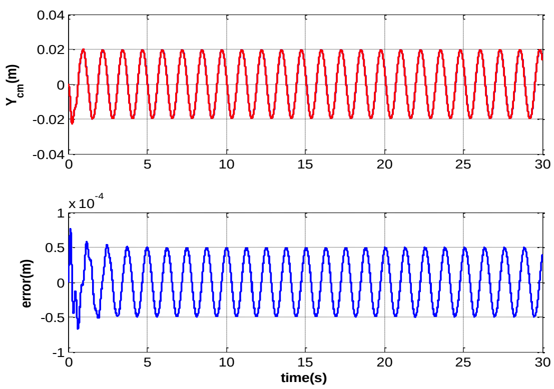


Fig.3
Deflection of beam center and the error between the ANSYS output and mathematical model output.

In Fig.4 the deflection of three DVAs is shown and in Fig.5 the error between the ANSYS output and the mathematical model output of three DVAs is shown. The results show the high accuracy of the mathematical modeling of the system of beam equipped with the DVAs.

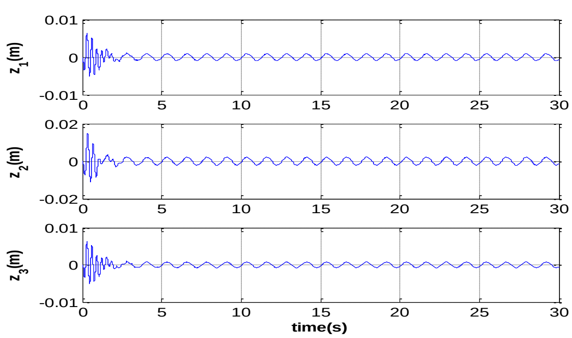


Fig.4
Deflection of the three DVAs.

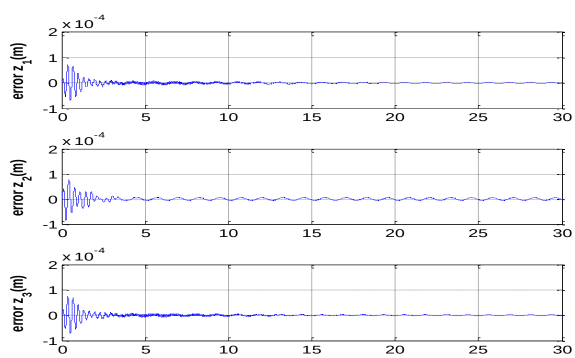


Fig.5
The error between the DVAs deflection from the ANSYS output and mathematical model output.

4 OPTIMIZATION

From the beam parameters in

Table 1, the beam mass is calculated as $m_{beam} = 234kg$. In this paper, the total mass of DVAs is considered $15kg$ which is far less than the beam mass (less than 10% of beam mass). For our problem, this relation is described as:

$$m_0 + 2m_1 = 15kg \tag{6}$$

That can be seen in Fig.6. This relation is used in the optimization as a constraint. Note that if we want to use a single DVA in Fig.1, by selecting $m_0 = 15kg$ we have $m_1 = 0$. Another reason for using the mentioned relation is that we want to use all $15kg$ for DVAs. This method has not been used in the previous works. In practice, in the optimization, the value of $m_0 = 15kg$ is not used because it results in $m_1 = 0$ that results in singularity in Eqs.(2) and (3).

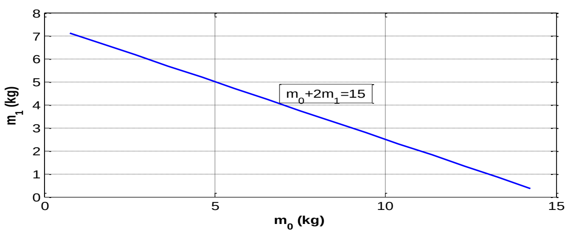


Fig.6
The relation between the mass of DVAs.

4.1 Problem formulation

In this paper, it is assumed that all absorbers have equal damping and stiffness or $c_i = C$, $k_i = K$. According to Eq.(6), only m_0 should be determined and m_1 is calculated as $m_1 = (15 - m_0)/2$. The point of acting of exciting harmonic force $F(t) = A_f \sin(\omega t)$ is considered $a_f = L/3$ in order to excite the beam's mode shapes (like many previous works such as (Avramov and Gendelman, [2], Esmailzadeh and Jalili, [5]) the place of exciting force and DVAs are fixed). Therefore, the vector of the optimization variables is $\boldsymbol{X} = [C \quad K \quad m_0]$. The rest of variables are fixed in optimization. The optimization purpose is to minimize the maximum deflection of the beam over a wide frequency range by using the DVAs. The absorber parameters should be tuned to suppress the resonance in the beam. The steady state amplitude of the beam's center deflection is $\hat{Y}$. The optimization problem can be expressed in the following condensed form.

$$\hat{Y}_{op} = \min_{\boldsymbol{X}} \left(\max_{\omega} \left| \hat{Y}(\omega, \boldsymbol{X}) \right| \right) \tag{7}$$

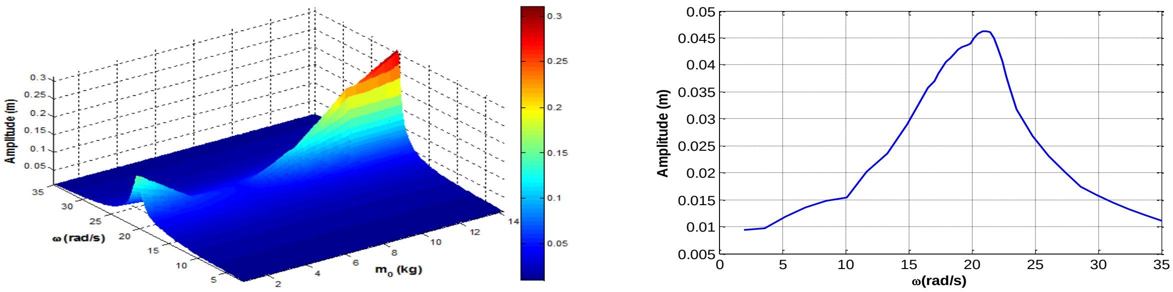
Thus, a min-max optimization problem should be solved. The traditional and simple optimization method has been the grid search method which is selected in this paper. In this method the whole domain of optimization variables is searched to find the optimal parameters. This method is a time consuming process but we are sure that the optimum parameters will be found. In the numerical optimization just three modes have been considered as enough. In this paper two types of DVAs is analyzed and compared with each other. First type is LES (by Eq.(2)) and second is NES (by Eq.(3)). In order to clarify the difference between the linear DVA and nonlinear DVA, optimization of the parameters will be done for the two force level $A_f = 500N$ and $A_f = 1000N$.

4.2 Optimization result for using linear DVA (LES)

In this case, the beam is equipped with three LESs. At first, the optimization result is given for $A_f = 500N$ and then the results related to $A_f = 1000N$ is given.

4.2.1 Optimization result for $A_f = 500N$

For the case of $A_f = 500N$ the optimal value for parameters of linear DVA is achieved as $m_0 = 5.65kg$, $C = 27.5\ Ns/m$ and $K = 1750N/m$ and m_1 is calculated as $m_1 = 4.425\text{kg}$. For the optimal values of $C = 27.5\ Ns/m$ and $K = 1750N/m$ the 3D surface of amplitude-ω-m_0 is shown in Fig.7(a) where the optimal amplitude is achieved for $m_0 = 5.65kg$. The amplitude-ω curve for the optimal value of $m_0 = 5.65kg$ is given in Fig.7 (b) where the maximum amplitude is $0.0462m$ for the frequency of about $21 rad/s$. This result shows that if we want an optimal linear DVA, the $15kg$ mass must be distributed as $m_0 = 5.65kg$ and $m_1 = 4.425\text{kg}$. From the 3D surface, it can be seen that this design is very robust against the changes in m_0.

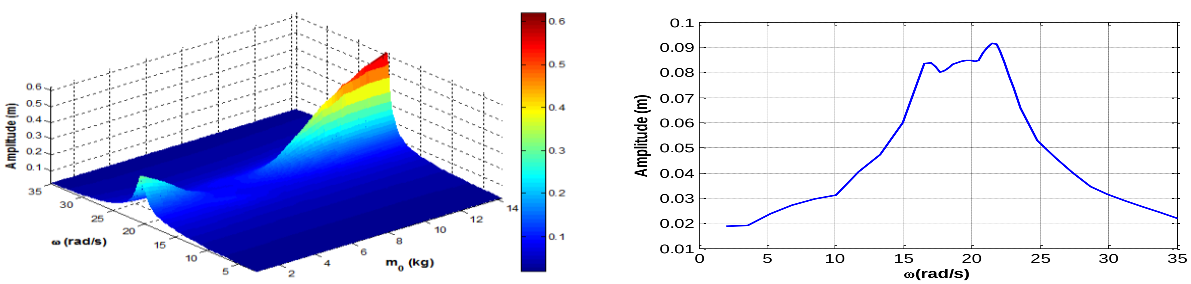


(a) (b)

Fig.7
Optimization result of beam with linear DVA for the force amplitude of $A_f = 500N$: a) amplitude as a function of ω and m_0 b) amplitude-ω curve for $m_0 = 5.65kg$.

4.2.2 Optimization result for $A_f = 1000N$

For a set of linear DVA the results of optimization with $A_f = 1000N$ and $A_f = 500N$ is similar, in other words, their optimum parameters are similar. Because the amplitude of $A_f = 1000N$ is two times bigger than the previous optimization value, in Fig.8(b) the maximum amplitude is $0.0916m$ which is about two times bigger than the previous optimization result. The aforementioned result is clear because the system is linear.



(a) (b)

Fig.8
Optimization result of beam with linear DVA for the force amplitude of $A_f = 1000N$: a) amplitude as a function of ω and m_0 b) amplitude-ω curve for the $m_0 = 5.65kg$.

4.3 Optimization result for using nonlinear DVA (NES)

In this case, a system of nonlinear energy sink is used for dissipating of the beam oscillatory energy. The beam is equipped with three NESs. At first the optimization result is given for $A_f = 500N$ and then the results related to $A_f = 1000N$ is presented.

4.3.1 Optimization result for $A_f = 500N$

For the case of $A_f = 500N$ the optimal value for parameters of nonlinear DVA is achieved as $m_0 = 14.25kg, C = 125\ Ns/m$ and $K = 75\times10^4\ N/m^3$. For the optimal values of $C = 125\ Ns/m$ and $K = 75\times10^4\ N/m^3$ the 3D surface of amplitude-ω-m_0 is shown in Fig.9(a) where the optimal amplitude is achieved for $m_0 = 14.25kg$. The amplitude-ω curve for the optimal value of $m_0 = 14.25kg$ is given in Fig.9(b) where the maximum amplitude is 0.063*m* that happens for the frequency of about 20 *rad/s*. Note that the linear DVA results in smaller amplitude of 0.0462*m*. This result shows that if we want an optimal nonlinear DVA, the 15*kg* mass must be approximately concentrated in the center of beam, in the other words, for a nonlinear stiffness, a single DVA should be used instead of three DVAs. From the 3D surface it can be seen that a little change in m_0 will result in a large amplitude for the beam, thus, this design is not robust against the changes in m_0.

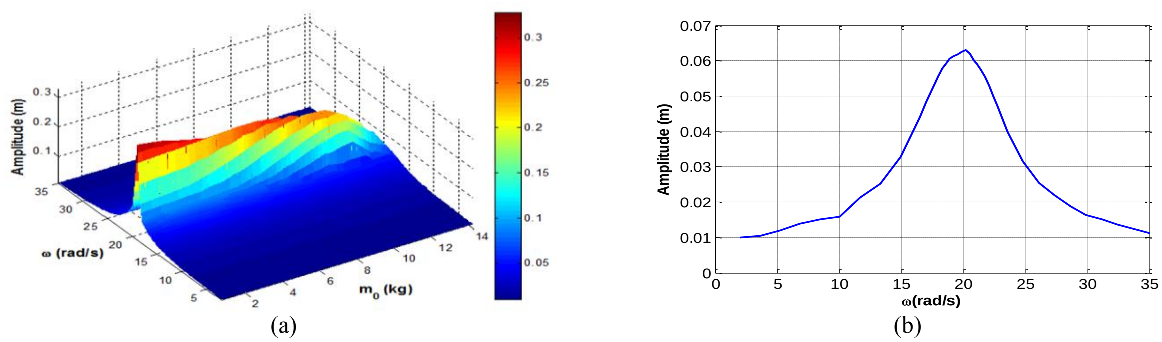


Fig.9
Optimization result of beam with nonlinear DVA for the force amplitude of $A_f = 500N$: a) amplitude as a function of ω and m_0 b) amplitude-ω curve for the $m_0 = 14.25kg$.

4.3.2 Optimization result for $A_f = 1000N$

Against the sysytem of beam with linear DVA, when the exciting force amplitude increase from $A_f = 500N$ to $A_f = 1000N$ the optimum parameters changes to $C = 80\ Ns/m$ and $K = 15\times10^4\ N/m^3$. This result show that when we use a system of nonlinear DVA, the optimum parameters depend on the amplitude of exciting force amplitude that is not suitable for a system of beam equipt with DVA. In this point of view, the linear DVA is better than the nonlinear one.

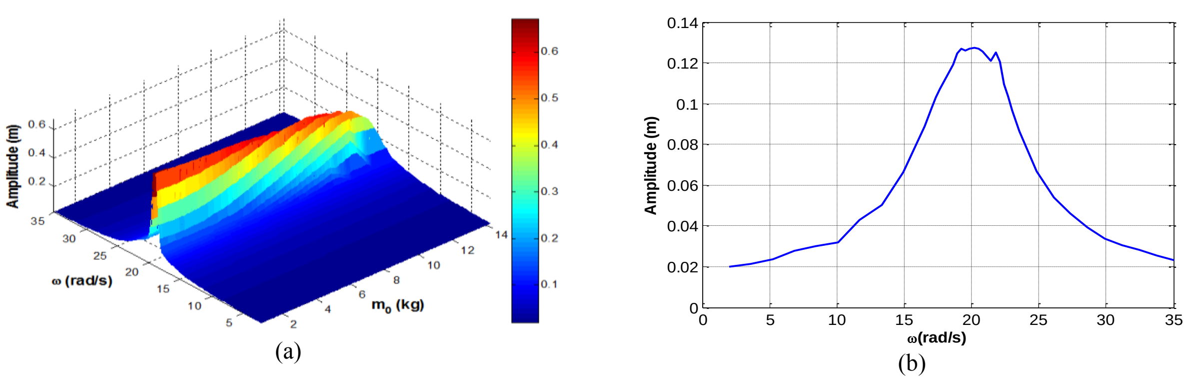


Fig.10
Optimization result of beam with nonlinear DVA for the force amplitude of $A_f = 1000N$: a) amplitude as afunction of ω and m_0 b) amplitude-ω curve for the $m_0 = 14.25kg$.

4.4 Robustness analysis

In this section four robustness analysis will be executed which are: robustness against force amplitude, exciting force location, damper and spring coefficient. The robustness of the optimal LES and NES will be investigated by putting the uncertainty on the four mentioned parameters.

4.4.1 Robustness analysis against the exciting force amplitude

In this section, the robustness of the optimal LES and NES will be investigated by putting the uncertainty on the exciting force amplitude. For this purpose, at first the optimal parameters of system for the force amplitude of $A_f = 500N$ are considered as the basic design. Then the force amplitude of $A_f = 800N$ is applied to the beam while the other parameters are fixed. In Fig.11(a) the amplitude-ω curve of beam with linear DVA is shown (with maximum amplitude 0.072 m) that is 1.56 times of the amplitude in Fig.7(b) (with the maximum amplitude 0.0462m) but when a nonlinear DVA is used in Fig.11(b) the amplitude will be 0.6m which is about 10 times of the amplitude in Fig.9(b) (with the maximum amplitude 0.063m). According to this analysis, the LES is very robust against the change in the exciting force amplitude.

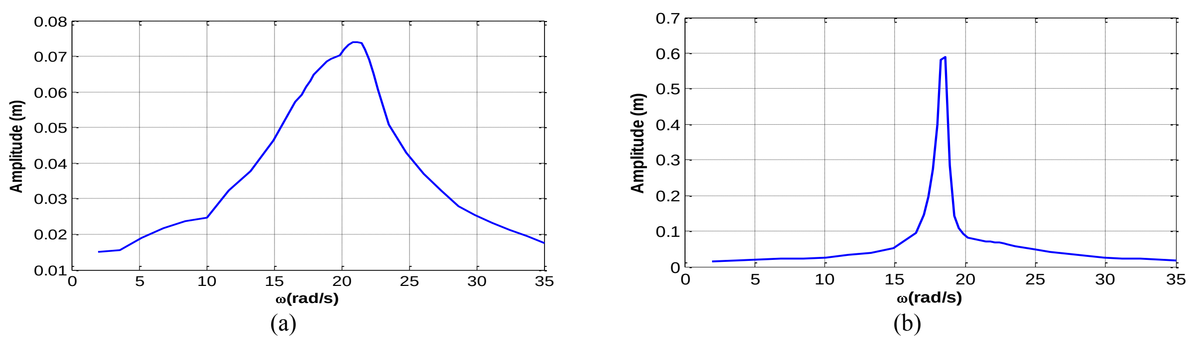


Fig.11
The amplitude of the beam with uncertainty on the amplitude of exciting force with $A_f = 800N$ a) linear DVA b) nonlinear DVA.

4.4.2 Robustness analysis against the exciting force location:

In this section, the robustness of the optimal LES and NES will be investigated by putting the uncertainty on the exciting force location. The previous design for the linear DVA was done for $a_f = L/3$ and $A_f = 1000N$ which is shown in Fig.8. In Fig.12 in order to investigate the robustness of linear DVA two values of $a_f = L/2$ and $a_f = L/4$ are considered for the location of exciting force. The amplitude of force is $A_f = 1000N$. Comparison of the result of Fig.12 and Fig.8 shows small changes in the beam amplitude.

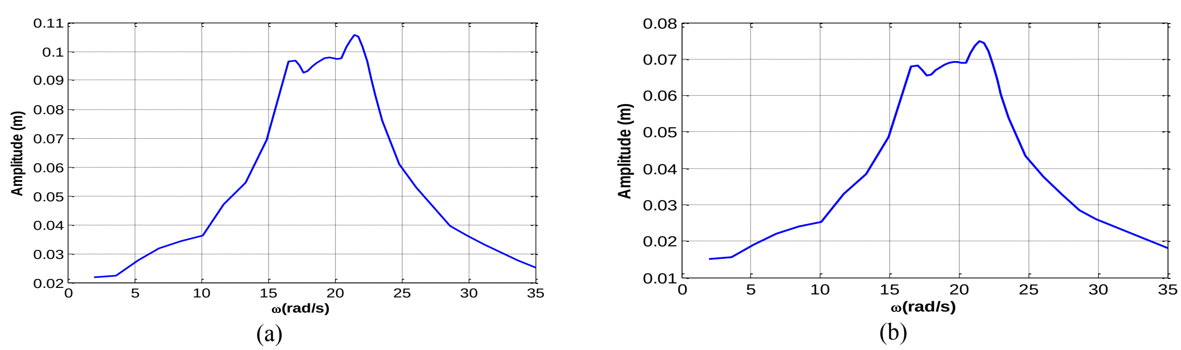


Fig.12
The amplitude of beam with linear DVA and with uncertainty on a_f a) $a_f = L/2$ b) $a_f = L/4$.

The maximum amplitude of the beam with nonlinear DVA and with $A_f = 500N$ and $a_f = L/3$ is $0.063m$ which is shown in Fig.9. In Fig.13 $a_f = L/2$ is considered as the new force location. Comparison of Fig.9 and Fig.13 shows that the beam with nonlinear DVA is not robust against the change in the exciting force location. In other words, when we use the nonlinear DVA, for each force location a special set of parameters should be employed. This conclusion shows that the linear DVA is more preferred than the nonlinear one.

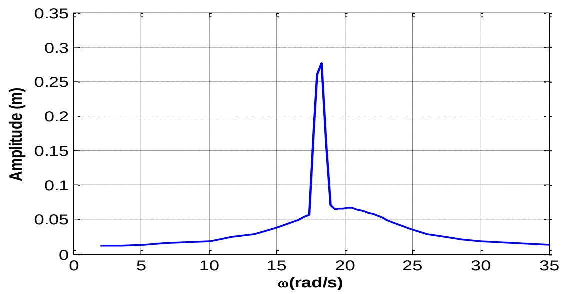


Fig.13
The amplitude of beam with nonlinear DVA and with $A_f = 500N$ and $a_f = L/2$.

4.4.3 Robustness analysis against the damper coefficient:

In this section, we consider 20% uncertainty on C for the both linear and nonlinear DVAs. The optimal parameters of linear and nonlinear DVAs with $A_f = 500N$ (Fig.7 and Fig.9) are considered as the basic design. In Fig.14(a) and Fig.14(b) the effect of +20% and -20% uncertainty on C is shown, respectively. In comparison with the result of Fig.7 for the linear DVA the amplitude changes is small. Thus the LES is very robust against the uncertainty on the damping coefficient.

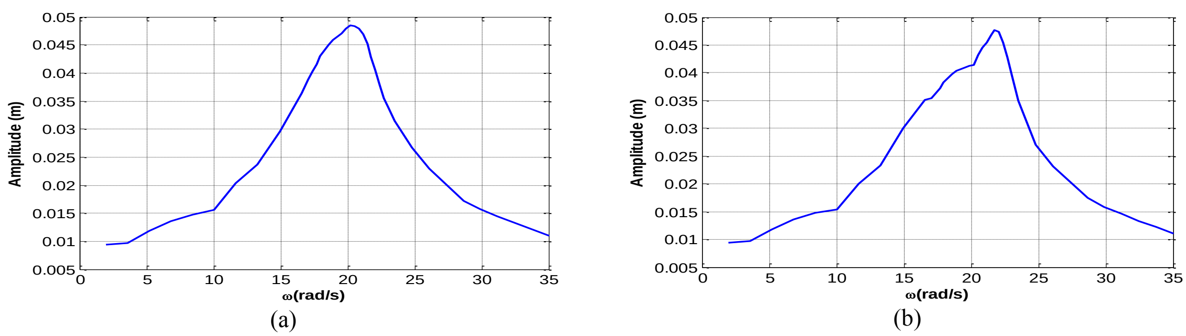


Fig.14
The amplitude of beam with linear DVA and with uncertainty on C a)+20% uncertainty b)-20% uncertainty.

The effect of 20% uncertainty on C for the nonlinear DVA is shown in Fig.15(a) and (b). In comparision with Fig.9, it can be seen that the nonlinear DVA is not robust against the -20% uncertainty on C. Therefore, from this point of view the LES is more robust than the NES.

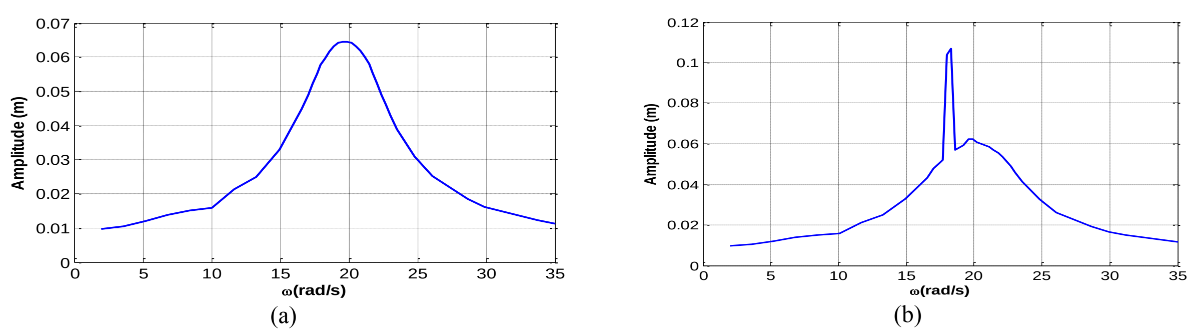
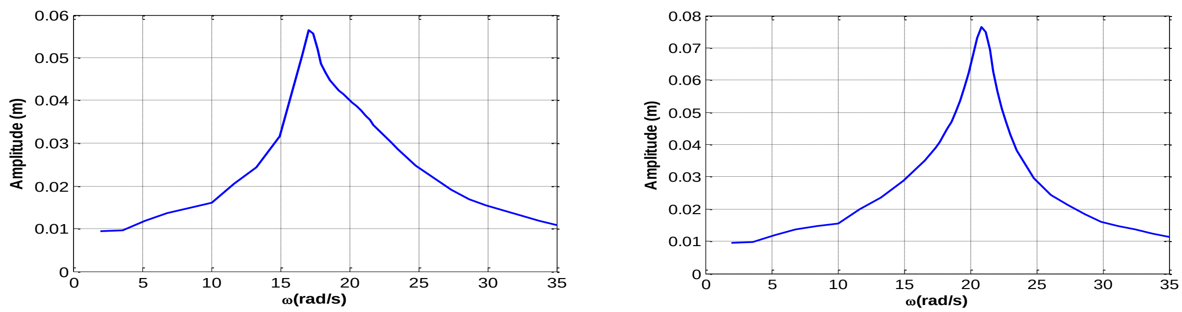


Fig.15
The amplitude of beam with nonlinear DVA and with uncertainty on C a)+20% uncertainty b)-20% uncertainty.

4.4.4 Robustness analysis against the spring coefficient

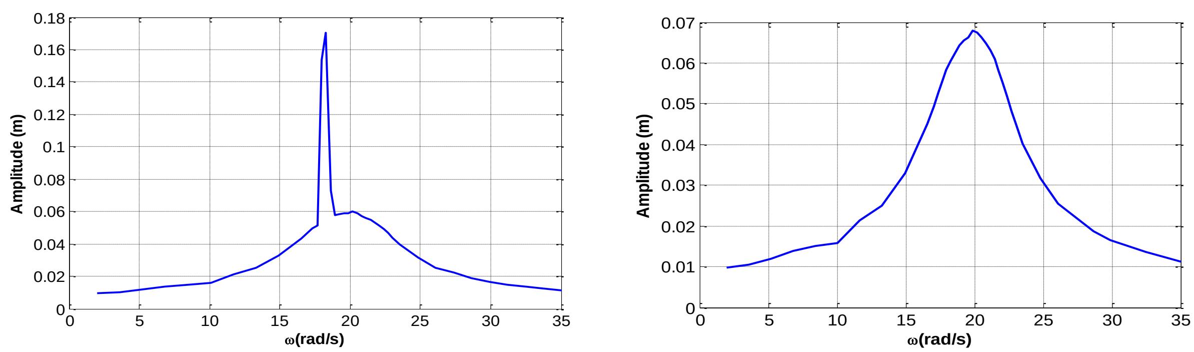
In this section we consider 20% uncertainty on *K* for the both linear and nonlinear DVAs. The optimal parameters of linear and nonlinear DVAs with $A_f = 500N$ (Fig.7 and Fig.9) are considered as the basic design. In Fig.16(a) and (b) the effect of +20% and -20% uncertainty of K is shown, respectively. In comparison with the result of Fig.7 for the linear DVA the amplitude changes is small. Thus the LES is very robust against the uncertainty on the spring coefficient.



(a) (b)

Fig.16
The amplitude of beam with linear DVA and with uncertainty on *K* a)+20% uncertainty b)-20% uncertainty.

The effect of 20% uncertainty of *K* for the nonlinear DVA is shown in Fig.17(a) and (b). In comparision with Fig.9 it can be seen that the nonlinear DVA is not robust against the +20% uncertainty on *K*. therefore, from this point of view the LES is more robust than the NES.



(a) (b)

Fig.17
The amplitude of beam with nonlinear DVA and with uncertainty on *K* a)+20% uncertainty b)-20% uncertainty.

5 CONCLUSION

In this paper, the effect of two types of DVAs (LES and NES) on the vibration dissipation is investigated. The Timoshenko beam theory is used to achieve accurate results. The mathematical model of beam with three DVAs is verified using the ANSYS WORKBENCH software. In order to design the LES and NES, a min-max optimization problem is formulated and the optimum mass, stiffness, and damping coefficients are obtained. For the first time the total mass constraint has been considered for the DVAs in the optimization procedure. In order to investigate the robustness of the DVA, the sensitivity analysis is done by applying the uncertainty on the exciting force amplitude, exciting force location, the damper coefficient and spring coefficient. The results show the linear DVA is more robust than the nonlinear DVA against the mentioned uncertainties. Moreover, the results show that LES is more robust than the NES against the absorber mass uncertainty. In the other hand, optimal parameters of the nonlinear DVA depend strongly on the external force amplitude and location and the damper and spring coefficient. The optimization results show that for the LES case, a special mass distribution must be used for the three DVAs but for the NES case, only single DVA (the whole of DVAs mass must be approximately concentrated in the middle of beam) is the optimal design.

REFERENCES

[1] Ahmadabadi Z.N., Khadem S., 2012, Nonlinear vibration control of a cantilever beam by a nonlinear energy sink, *Mechanism and Machine Theory* **50**: 134-149.

[2] Avramov K., Gendelman O., 2010, On interaction of vibrating beam with essentially nonlinear absorber, *Meccanica* **45**: 355-365.

[3] Chen J., Zhang W., Yao M., Liu J., Sun M., 2020, Thermal effect on dynamics of beam with variable-stiffness nonlinear energy sink, *International Journal of Nonlinear Sciences and Numerical Simulation* **21**: 1-10.

[4] Chen Y.-H., Hiang Y.-H., 2004, Timoshenko beam with tuned mass dampers and its design curves, *Journal of Sound and Vibration* **278**: 873-888.

[5] Esmailzadeh E., Jalili N., 1998, Optimum design of vibration absorbers for structurally damped Timoshenko beams, *Journal of Vibration and Acoustics* **120**(4): 833-841.

[6] Georgiades F., Vakakis A.F., 2007, Dynamics of a linear beam with an attached local nonlinear energy sink, *Communications in Nonlinear Science and Numerical Simulation* **12**: 643-651.

[7] Huang T., 1961, The effect of rotatory inertia and of shear deformation on the frequency and normal mode equations of uniform beams with simple end conditions, *Journal of Vibration and Acoustics* **28**(4): 579-584.

[8] Kani M., Khadem S., Pashaei M. Dardel M., 2016a, Vibration control of a nonlinear beam with a nonlinear energy sink, *Nonlinear Dynamics* **83**: 1-22.

[9] Kani M., Khadem S.E., Pashaei M. H., Dardel M., 2016b, Design and performance analysis of a nonlinear energy sink attached to a beam with different support conditions, *Proceedings of the Institution of Mechanical Engineers Part C, Journal of Mechanical Engineering Science* **230**: 527-542.

[10] Moghaddas M., Esmailzadeh E., Sedaghati R., Khasravi P., 2012, Vibration control of Timoshenko beam traversed by moving vehicle using optimized tuned mass damper, *Journal of Vibration and Control* **18**: 757-773.

[11] Parseh M., Dardel M., Ghasemi M. H., 2015, Investigating the robustness of nonlinear energy sink in steady state dynamics of linear beams with different boundary conditions, *Communications in Nonlinear Science and Numerical Simulation* **29**: 50-71.

[12] Samani F. S., Pellicano F. Masoumi A., 2013, Performances of dynamic vibration absorbers for beams subjected to moving loads, *Nonlinear Dynamics* **73**: 1065-1079.

[13] Starosvetsky Y., Gendelman O., 2008, Attractors of harmonically forced linear oscillator with attached nonlinear energy sink. II: Optimization of a nonlinear vibration absorber, *Nonlinear Dynamics* **51**: 47.

[14] Vakakis A. F., Manevitch L., Gendelman O., Bergman L., 2003, Dynamics of linear discrete systems connected to local, essentially non-linear attachments, *Journal of Sound and Vibration* **264**: 559-577.

[15] Younesian D., Nankali A., Motieyan E., 2011, Optimal nonlinear energy sinks in vibration mitigation of the beams traversed by successive moving loads, *Journal of Solid Mechanics* **3**:321-323.

[16] Zang J., Zhang Y.-W., Ding H., Yang T.-Z., Chen L.-Q., 2019, The evaluation of a nonlinear energy sink absorber based on the transmissibility, *Mechanical Systems and Signal Processing* **125**: 99-122.