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Research Paper

Free Vibration Analysis of Elastically Connected Beams with Step

H. Nayebi, M.M. Najafizadeh *

Department of Mechanical Engineering, Arak Branch, Islamic Azad University, Arak, Iran

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ABSTRACT

In this study, free vibration of stepped beam which is parallel to a uniform beam with same length and elastically connected to it, is considered. Euler-Bernoulli beam theory has been applied to drive equations of motion, abrupt change in height of beam considered as step and Winkler-type elastic layer model serve as connection between beams. The differential transform method (DTM) is applied to determine dimensionless frequencies and mode shapes. In the case of two uniform parallel beams accuracy of solution is verified by comparing with results reported by other methods. It is assumed all supports have one type and fully clamped and fully hinged supports considered for boundary conditions. The effects of different parameters such as: step location and ratio, connecting layer coefficient and boundary conditions on dimensionless frequencies and mode shapes investigated and discussed. This problem handled for first time in present study and results are completely new.

Keywords: Parallel beams; Stepped beam; Winkler-type elastic layer; Differential transform method.

1 INTRODUCTION

BEAMS are one of the most important and useful elements in structures and machines and have allocated a wide range of application to themselves in modern engineering and especially mechanical engineering, civil engineering and aerospace engineering. A lot of research has been conducted on the dynamical behavior of a single beam with homogeneous material. The behavior of this beam has been perfectly identified. Although, when there exists a discontinuity in the beam such as a step or there is a set of connected beams, still there are a lot of issues to be studied. Elastically connected beams have wide applications in tall buildings, railways and in nano technology such as multi-walled carbon nano tube. Oniszczuk [1] studied the free vibration of two parallel simply supported beams which were connected to a Winkler elastic layer. This researcher [2] studied the forced vibration of double-beams which had an elastic connection under the influence of harmonic load and moving force. Mao [3] concluded a general solution by using the Adomian modified decomposition method for the free vibration of elastically connected multiple-beams and also have elastic boundary conditions. Huang and Liu [4] investigated the free and

*Corresponding author. Tel.: +98 086 33412563.
E-mail address: m-najafizadeh@arak.iau.ir (M.M.Najafizadeh)

forced vibrations of double-parallel beams with different boundary conditions by the substructure method. Mirzabeigy and Madoliat [5] investigate the effect of cubic type nonlinearity in elastic inner layer on small amplitude vibration. The free and forced vibrations of the general form of the double-beam system with arbitrary intermediate supports and viscoelastic layer under the general boundary conditions are investigated by Zhao and Chang [6]. Hao et al [7] make use of modified Fourier-Ritz method for free vibration analysis of double-beam system with general boundary conditions. In none of the mentioned researches the effect of discontinuity such as step in the vibration of parallel beams has not been taken into consideration. The differential transform is a semi-analytic and reliable method for solving ordinary and partial differential equations. This method was first used in engineering problem by Chinese scientist for analyze electric circuits problems. Chen and Ho [8] applied this method for eigenvalue problems and Malik and Dang [9] applied method for vibration analysis of continuous systems, then, differential transform method (DTM) used for vibration analysis of different structures. Kaya and Ozgumus [10] study flexural–torsional-coupled vibration analysis of axially loaded closed-section composite Timoshenko beam by DTM. Shariyat and Alipour [11] applied DTM to vibration and modal stress analyses of circular plate made of functionally graded materials. Arikoglu and Ozkol [12] applied DTM for free vibration of composite beams with viscoelastic core. Mao [13] designed of piezoelectric modal sensors for cantilever beams with intermediate support by using differential transformation method. Shahba and Rajasekaran [14] proposed new method called differential transform element method for free vibration and stability analysis of tapered beams made of functionally graded material and many other problems is structural dynamics which handled by DTM [15-22]. Beside applications in structural analysis, DTM has wide range of applications in other fields of science and engineering [23-25]. João Fernandes da Silva et al. [26], focused on the free vibration analysis of Euler-Bernoulli beams under non-classical boundary conditions, and reached to this result that the behavior of these beams are similar to a free-free beam. Jingtao DU et al. [27] studied free vibration analysis of elastically connected multiple-beams with general boundary conditions using improved Fourier series method.

In this study, free vibration of step beam elastically connected by Winkler type spring to uniform beam is considered. DTM has been used for vibration analysis and step considered as abrupt change in beam’s height. Effect of different parameters on frequencies and mode shapes are investigated and some new and useful results reported for first time.

2 THE MATHEMATICAL MODEL

Consider a stepped beam which is parallel to a uniform beam with same length and elastically connected to it as shown in Fig. 1.

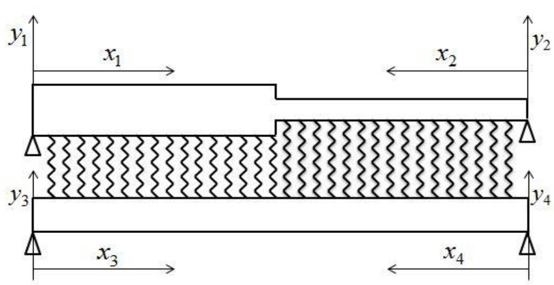


Fig.1
Stepped beam elastically connected to a uniform beam.

Four coordinates consider for derive mathematical model of system, although, in fact two in-dependent coordinates exist and others coordinates are dependent. Equations of motion with Euler-Bernoulli beam theory are as follow:

$$E_1 I_1 \frac{\partial^4 y_1(x_1,t)}{\partial x_1^4} + \rho_1 A_1 \frac{\partial^2 y_1(x_1,t)}{\partial t^2} + k\,(y_1(x_1,t) - y_3(x_3,t)) = 0, \quad 0 < x_1 < x_s \tag{1}$$

$$E_2 I_2 \frac{\partial^4 y_2(x_2,t)}{\partial x_2^4} + \rho_2 A_2 \frac{\partial^2 y_2(x_2,t)}{\partial t^2} + k\,(y_2(x_2,t) - y_4(x_4,t)) = 0, \quad 0 < x_2 < L - x_s \tag{2}$$

$$E_3I_3\frac{\partial^4 y_3(x_3,t)}{\partial x_3^4}+\rho_3A_3\frac{\partial^2 y_3(x_3,t)}{\partial t^2}+k\,(y_3(x_3,t)-y_1(x_1,t))=0,\quad 0<x_3<x_s \tag{3}$$

$$E_4I_4\frac{\partial^4 y_4(x_4,t)}{\partial x_4^4}+\rho_4A_4\frac{\partial^2 y_4(x_4,t)}{\partial t^2}+k\,(y_4(x_4,t)-y_2(x_2,t))=0,\quad 0<x_4<L-x_s \tag{4}$$

It is defined that for any $j\,(j=1..4)$, E_j is elasticity modulus, ρ_j is density, $I_j=\frac{b_jh_j^3}{12}$ is the cross-sectional moment of inertia, A_j is cross-section area. b_j and h_j are the width and thickness, respectively. k is the stiffness of the Winkler-type elastic layer between beams. Using separation of variables as:

$$y_j(x_j,t)=y_j(x_j)e^{i\omega t},\; j=1..4 \tag{5}$$

where ω is circular frequency and defining dimensionless coordinates as:

$$\bar{x}_j=\frac{x_j}{L},\bar{y}_j=\frac{y_j}{L},\; j=1..4 \tag{6}$$

where L is length of beams, we re-write Eqs. (1)-(4) as follow:

$$\frac{d^4\bar{y}_1}{d\bar{x}_1^4}-P_1\omega^2\bar{y}_1+K_1(\bar{y}_1-\bar{y}_3)=0, \tag{7}$$

$$\frac{d^4\bar{y}_2}{d\bar{x}_2^4}-P_2\omega^2\bar{y}_2+K_2(\bar{y}_2-\bar{y}_4)=0, \tag{8}$$

$$\frac{d^4\bar{y}_3}{d\bar{x}_3^4}-P_3\omega^2\bar{y}_3+K_3(\bar{y}_3-\bar{y}_1)=0, \tag{9}$$

$$\frac{d^4\bar{y}_4}{d\bar{x}_4^4}-P_4\omega^2\bar{y}_4+K_4(\bar{y}_4-\bar{y}_2)=0, \tag{10}$$

where

$$P_j=\frac{\rho_jA_jL^4}{E_jI_j},K_j=\frac{kL^4}{E_jI_j},\; j=1..4 \tag{11}$$

It is assumed supports have one type and two different types considered for supports: clamp and hinge. When supports type be clamp, then the relevant boundary conditions associated Eqs. (7)-(10) are:

$$\bar{y}_j(0)=\frac{\partial\bar{y}_j(0)}{\partial\bar{x}_j}=0,\; j=1..4 \tag{12}$$

And for hinge supports the relevant boundary conditions associated Eqs. (7)-(10) are:

$$\bar{y}_j(0)=\frac{\partial^2\bar{y}_j(0)}{\partial\bar{x}_j^2}=0,\; j=1..4 \tag{13}$$

For upper beam with discontinuous cross-section due to step, the continuity conditions in step location expressed as:

$$\bar{y}_1(R)=\bar{y}_2(1-R), \tag{14}$$

$$\frac{\partial \bar{y}_1(R)}{\partial \bar{x}_1}=-\frac{\partial \bar{y}_2(1-R)}{\partial \bar{x}_2}, \tag{15}$$

$$\frac{\partial^2 \bar{y}_1(R)}{\partial^2 \bar{x}_1}=\tau^3\frac{\partial^2 \bar{y}_2(1-R)}{\partial^2 \bar{x}_2}, \tag{16}$$

$$\frac{\partial^3 \bar{y}_1(R)}{\partial^3 \bar{x}_1}=-\tau^3\frac{\partial^3 \bar{y}_2(1-R)}{\partial^3 \bar{x}_2}, \tag{17}$$

where $R=\frac{x_s}{L}$ is dimensionless step location and $\tau=\frac{h_2}{h_1}$ is step ratio. We need other four equations for the continuity conditions in lower beam, because, although lower beam has uniform cross-section but employed coordinates for this beam forced us to use continuity conditions as follow:

$$\bar{y}_3(R)=\bar{y}_4(1-R), \tag{18}$$

$$\frac{\partial \bar{y}_3(R)}{\partial \bar{x}_3}=-\frac{\partial \bar{y}_4(1-R)}{\partial \bar{x}_4}, \tag{19}$$

$$\frac{\partial^2 \bar{y}_3(R)}{\partial \bar{x}_3^2}=\frac{\partial^2 \bar{y}_4(1-R)}{\partial \bar{x}_4^2}, \tag{20}$$

$$\frac{\partial^3 \bar{y}_3(R)}{\partial \bar{x}_3^3}=-\frac{\partial^3 \bar{y}_4(1-R)}{\partial \bar{x}_4^3}, \tag{21}$$

3 THE DIFFERENTIAL TRANSFORM METHOD

Differential transform method is an efficient semi-analytic approach for solving general differential equation that uses the form of polynomials as the approximations to the exact solutions that are sufficiently differentiable. The conceptual feature of the DTM is to transform the governing differential equations and boundary conditions as well as continuity conditions into a set of algebraic equations using a transformation role. Solving the algebraic equations in the usual way leads to accurate results with fast convergence rate and small computational effort.

A function $x(t)$, analytical in domain D, can be represented by a power series around any arbitrary point in this domain. Differential transform of a function $x(t)$ is defined as follows:

$$X(k)=\frac{1}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \tag{22}$$

In Eq. (22), $x(t)$ is the original function and $X(k)$ is the transformed function. Differential inverse transform of $X(k)$ is defined as:

$$x(t)=\sum_{k=0}^{\infty} X(k)t^k, \tag{23}$$

Combining Eq. (22) and Eq. (23), we obtain the following equation

$$x(t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \tag{24}$$

In principal applications, the function $x(t)$ is shown by a finite numbers of terms and Eq. (24) can be written as:

$$x(t)=\sum_{k=0}^{N}\frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \tag{25}$$

which implies that:

$$x(t)=\sum_{k=N+1}^{\infty}\frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}. \tag{26}$$

Is negligibly small. In this study, the convergence of the natural frequencies determines the value of N. Basic transformation rules depending on the DTM for differential equations and boundary conditions are tabulated in Tables 1 and 2, respectively.

4 APPLICATION OF THE DTM TO GOVERNING EQUATIONS

In this section, the DTM applied to Eqs. (7)-(10) by using the transformation rules given in Table 1 and the following recurrence equations is obtained:

$$\frac{(n+4)!}{n!}Y_1(n+4)-P_1\omega^2Y_1(n)+K_1(Y_1(n)-Y_3(n))=0, \tag{27}$$

$$\frac{(n+4)!}{n!}Y_2(n+4)-P_2\omega^2Y_2(n)+K_2(Y_2(n)-Y_4(n))=0, \tag{28}$$

$$\frac{(n+4)!}{n!}Y_3(n+4)-P_3\omega^2Y_3(n)+K_3(Y_3(n)-Y_1(n))=0, \tag{29}$$

$$\frac{(n+4)!}{n!}Y_4(n+4)-P_4\omega^2Y_4(n)+K_4(Y_4(n)-Y_2(n))=0, \tag{30}$$

Without loss of generality, assume all boundary conditions are clamp and solution procedure is explained for this type of boundary conditions. Using DTM rules given in Table 2 to Eq. (12), the boundary conditions at each ends transformed as:

$$Y_j(0)=Y_j(1)=0, j=1..4 \tag{31}$$

In clamp boundary condition the values of bending moment and shear force are unknown. Therefore, we assume the transformation of these values for each ends as follow:

$$Y_j(2)=a_j, Y_j(3)=b_j, j=1..4 \tag{32}$$

where a_j, b_j are unknown parameters.

From recurrence equations in Eqs. (27)-(30) and by using Eq. (31) and Eq. (32), $Y_j(n)(j=1..4)$. For all values of n, can be determined in terms of other parameters such as: $\omega,...$. For example for $Y_1(n)$ we have:

$$\begin{aligned}
&n=0: 4!Y_1(4)-P_1\omega^2 Y_1(0)+K_1(Y_1(0)-Y_3(0))=0 \xrightarrow{Eq.(31)} Y_1(4)=0,\\
&n=1: 5!Y_1(5)-P_1\omega^2 Y_1(1)+K_1(Y_1(1)-Y_3(1))=0 \xrightarrow{Eq.(31)} Y_1(5)=0,\\
&n=2: \frac{6!}{2!}Y_1(6)-P_1\omega^2 Y_1(2)+K_1(Y_1(2)-Y_3(2))=0 \xrightarrow{Eq.(31\&32)} Y_1(6)=\frac{2!}{6!}(P_1\omega^2 a_1-K_1(a_1-a_3)),\\
&...
\end{aligned} \tag{33}$$

Applying transformation rules in Table 2 to the continuity conditions of upper beam yields:

$$\sum_{n=0}^{N} R^n Y_1(n)=\sum_{n=0}^{N}(1-R)^n Y_2(n), \tag{34}$$

$$\sum_{n=0}^{N} nR^{n-1} Y_1(n)=-\sum_{n=0}^{N} n(1-R)^{n-1} Y_2(n), \tag{35}$$

$$\sum_{n=0}^{N} n(n-1)R^{n-2} Y_1(n)=\tau^3\sum_{n=0}^{N} n(n-1)(1-R)^{n-2} Y_2(n), \tag{36}$$

$$\sum_{n=0}^{N} n(n-1)(n-2)R^{n-3} Y_1(n)=-\tau^3\sum_{n=0}^{N} n(n-1)(n-2)(1-R)^{n-3} Y_2(n), \tag{37}$$

Also, continuity conditions of lower beam transformed as follows:

$$\sum_{n=0}^{N} R^n Y_3(n)=\sum_{n=0}^{N}(1-R)^n Y_4(n), \tag{38}$$

$$\sum_{n=0}^{N} nR^{n-1} Y_3(n)=-\sum_{n=0}^{N} n(1-R)^{n-1} Y_4(n), \tag{39}$$

$$\sum_{n=0}^{N} n(n-1)R^{n-2} Y_3(n)=\sum_{n=0}^{N} n(n-1)(1-R)^{n-2} Y_4(n), \tag{40}$$

$$\sum_{n=0}^{N} n(n-1)(n-2)R^{n-3} Y_3(n)=-\sum_{n=0}^{N} n(n-1)(n-2)(1-R)^{n-3} Y_4(n), \tag{41}$$

Substituting obtained $Y_j(n)(j=1..4)$ into transformed continuity conditions in Eqs. (34)-(41), yields eight algebraic equation which can be arranged and expressed in the matrix form as follow:

$$\begin{bmatrix} Q_1 & Q_2 & ... & Q_7 & Q_8\\ Q_9 & Q_{10} & ... & Q_{15} & Q_{16}\\ . & . & ... & . & .\\ . & . & ... & . & .\\ Q_{49} & Q_{50} & ... & Q_{55} & Q_{56}\\ Q_{57} & Q_{58} & ... & Q_{63} & Q_{64}\end{bmatrix}\begin{bmatrix} a_1\\ b_1\\ .\\ .\\ a_4\\ b_4\end{bmatrix}=0. \tag{42}$$

where $Q_r\,(r=1..64)$ are polynomials of ω. The coefficients of these polynomials are determined from values of $\tau, R, P_j, K_j\,(j=1..4)$. For the non-trivial solutions of Eq. (42), it is necessary that the determinant of the coefficient matrix is equal to zero. Mode shapes can be determined in similar approach as explained by other references [13,16]. The explained procedure can extended very easy for hinged beams. Using DTM rules given in Table 2 to Eq. (13), the boundary conditions at each ends with hinge support transformed as:

$$Y_j(0)=Y_j(2)=0, j=1..4 \tag{43}$$

In hinge boundary condition the values of slope and shear force are unknown. Therefore, we assume the transformation of these values for each ends as follow:

$$Y_j(1)=a_j, Y_j(3)=b_j, j=1..4 \tag{44}$$

Table1
Basic transformation rules of differential transform method.

Original function	Transformed function
$f(x)=\alpha.g(x)$	$F(n)=\alpha.G(n)$
$f(x)=g(x)\pm h(x)$	$F(n)=G(n)\pm H(n)$
$f(x)=\dfrac{d^m g(x)}{dx^m}$	$F(n)=\dfrac{(n+m)!}{n!}G(n+m)$
$f(x)=g(x).h(x)$	$F(n)=\sum_{r=0}^{n} G(n).H(n-r)$

Table 2
Basic rules of differential transform method for the boundary conditions.

Original B.C.	Transformed B.C.	Original B.C.	Transformed B.C.
$f(0)=0$	$F(0)=0$	$f(L)=0$	$\sum_{n=0}^{N} L^n F(n)=0$
$\dfrac{df(0)}{dx}=0$	$1!F(1)=0$	$\dfrac{df(L)}{dx}=0$	$\sum_{n=0}^{N} nL^{n-1}F(n)=0$
$\dfrac{d^2f(0)}{dx^2}=0$	$2!F(2)=0$	$\dfrac{d^2f(L)}{dx^2}=0$	$\sum_{n=0}^{N} n(n-1)L^{n-2}F(n)=0$
$\dfrac{d^3f(0)}{dx^3}=0$	$3!F(3)=0$	$\dfrac{d^3f(L)}{dx^3}=0$	$\sum_{n=0}^{N} n(n-1)(n-2)L^{n-3}F(n)=0$

5 NUMERICAL RESULTS

For the purpose of verification of the results, by setting $\tau=1$ we consider the state of two homogeneous beams and compare the results with the result due to the analysis method [1] and the substructure method [4] for the joint fulcrums. The considered physical parameters are as follows:

$$E_j=10^{10}Nm^{-2}, \rho_j=2000kgm^{-3}, A_j=0.05m^2, I_j=0.0004m^4, (j=1..4), L=10m \tag{45}$$

Comparison between the results due to suitable attention of the concluded solution has been presented in Table (3). In order to get new results, we have considered the below amounts and will apply them to all states studied:

$$E_j=10^{10}Nm^{-2}, \rho_j=2000kgm^{-3}, A_j=0.05m^2, I_j=0.0004m^4, (j=1,3,4), L=10m \tag{46}$$

It is obvious that with regard to the amount of parameter τ, we can determine the rest of the amounts as follows:

$$E_2 = E_1, \rho_2 = \rho_1, A_2 = \tau A_1, I_2 = \tau^3 I_1 \tag{47}$$

In Figs. 2 and 3 the influence of number of sentences used in the differential transform method has been conducted to determine the first six normal frequencies respectively for simply and clamp supported. It is obviously seen that differential transform has a suitable convergence speed and by increasing the number of sentences, we can find the frequency of higher modes. It is also observed that convergence speed in this method is dependent on the boundary conditions and convergence has a higher speed for simply supported.

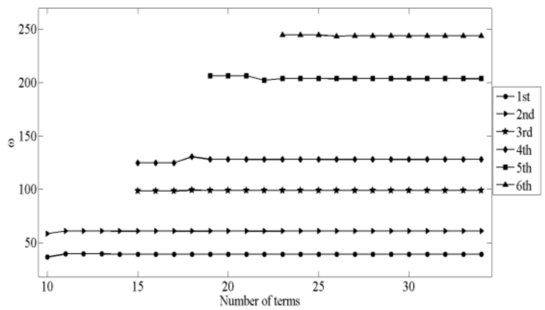


Fig.2
Convergence of frequencies with number of terms for clamped boundary conditions $(R = 0.75, \tau = 0.5, k = 10^5 Nm^{-2})$.

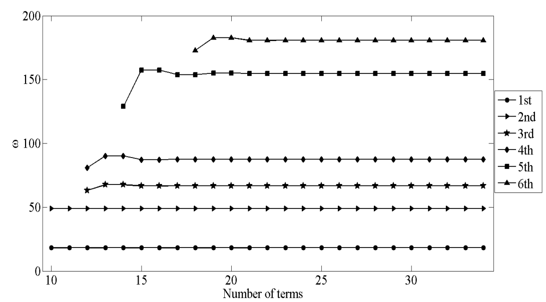


Fig.3
Convergence of frequencies with number of terms for hinged boundary conditions $(R = 0.75, \tau = 0.5, k = 10^5 Nm^{-2})$.

Table 3
Comparison between frequencies obtained via different approach for hinged boundary conditions ($\tau = 1$).

		1st mode	2nd mode	3rd mode	4th mode	5th mode
$k = 2\times10^5$	Present	19.7392	66.2543	78.9568	101.1641	177.6531
	[1]	19.7	66.3	79	101.2	177.7
	[4]	19.7	66.3	79	101.2	177.7
$k = 4\times10^5$	Present	19.7392	78.9568	91.5945	119.3071	177.6531
	[1]	19.7	79.9	91.6	119.3	177.7
	[4]	19.7	79.9	91.6	119.3	177.7

In Tables 4 and 5 in order, simply and clamp supported conditions, the effect of the step location and ratio on the first three normal frequencies has been studied. It is observed that in similar states, the frequency of clamp supported is higher than simply supported and a special model for frequency changes with the step location and ratio cannot be reached.

Table 4
Effects of step ratio & location on first three natural frequencies (Clamp boundary conditions) ($k = 2\times10^5 Nm^{-2}$).

	$R = 0.25$			$R = 0.5$		
τ	First mode	Second mode	Third mode	First mode	Second mode	Third mode
0.2	42.69323	96.64126	111.8353	38.76832	74.76845	102.564
0.5	40.93723	84.5356	93.60445	39.031	79.19804	101.8175
0.8	42.46136	79.05062	111.0572	42.31968	77.65222	115.9679
2	56.5971	80.93028	130.7973	52.32188	79.29161	130.5862
5	61.82164	91.47169	131.2663	61.82504	121.0564	131.4475
10	63.02413	130.203	133.1261	62.85439	130.7441	177.8633

Table 5
Effects of step ratio & location on first three natural frequencies (Hinge boundary conditions) ($k = 2\times10^5 Nm^{-2}$).

	$R = 0.25$			$R = 0.5$		
τ	First mode	Second mode	Third mode	First mode	Second mode	Third mode
0.2	17.54418	57.20208	84.30559	16.3901	59.78389	82.80892
0.5	16.88171	60.71742	77.00327	17.06414	66.70117	73.82652
0.8	18.12958	68.99217	71.93426	18.64353	67.62479	75.05734
2	27.96908	60.9605	89.08865	21.58367	62.16411	88.56852
5	29.06021	54.62537	90.73489	18.96197	56.33265	89.77614
10	23.10144	51.46593	90.77748	15.46677	52.81479	89.74423

In Table 6 and 7 the influence of the elastic layer coefficient between the two beams has been studied on the first five normal frequencies. It is observed that despite of the type of boundary conditions with increase in the elastic layer coefficient, the frequencies increase in any mode.

Table 6
Effect of connected layer coefficient on first five natural frequencies (Clamp boundary conditions) ($R = 0.5$ & $\tau = 0.5$).

$k\ (Nm^{-2})$	First mode	Second mode	Third mode	Fourth mode	Fifth mode
1×10^5	38.05052	62.63296	96.48185	128.108	168.0152
2×10^5	39.031	79.19804	101.8175	134.2529	172.1923
3×10^5	39.40934	92.53394	105.6076	141.3777	176.0512
4×10^5	39.62403	102.3921	109.8051	148.963	179.6053

Table 7
Effect of connected layer coefficient on first five natural frequencies (Hinge boundary conditions) ($R = 0.5$ & $\tau = 0.5$).

$k\ (Nm^{-2})$	First mode	Second mode	Third mode	Fourth mode	Fifth mode
1×10^5	16.83789	51.2115	67.19462	88.12379	124.5821
2×10^5	17.06414	66.70117	73.82652	100.2119	129.7141
3×10^5	17.17393	69.80956	86.5853	112.1904	132.0591
4×10^5	17.24407	70.77611	98.50526	123.4021	137.7016

In Tables 8 and 9 the effect of the step ratio and the elastic layer coefficient between the two beams on the main normal frequency has been studied respectively for simply and clamp supported condition. In these tables on the contrary to Tables 4 and 5, a smaller range of variation has been considered for the step ratio and the amounts are close to one. It can be said that in spite of the type of supported, by increasing the step ratio, the main frequency also increases. Also when the step ratio gets close to one, change in the elastic layer coefficient does not have much effect of the main frequency.

Table 8
Effects of step ratio & connected layer coefficient on fundamental natural frequency (Clamp boundary conditions) ($R = 0.3$).

$k\ (Nm^{-2})$	τ					
	0.5	0.7	0.9	1.1	1.3	1.5
15×10^4	40.6932	41.42416	43.45048	46.11192	48.75109	50.96466
25×10^4	41.04229	41.58558	43.47264	46.13726	48.99212	51.62307
35×10^4	41.20835	41.66264	43.48305	46.14903	49.10372	51.9308

Table 9
Effects of step ratio & connected layer coefficient on fundamental natural frequency (Hinge boundary conditions) ($R = 0.3$).

$k\ (Nm^{-2})$	τ					
	0.5	0.7	0.9	1.1	1.3	1.5
15×10^4	16.80372	17.54401	18.92045	20.58802	22.24276	23.6954
25×10^4	16.87507	17.58214	18.92562	20.59352	22.29097	23.81801
35×10^4	16.91101	17.60157	18.92831	20.59645	22.31727	23.88621

In Figs. 4 to 7, the effect of change in the location of step in the entire length of the beam on the first two normal frequencies has been studied for two different amounts of the step ratio and different fulcrum conditions. It is completely obvious that under the clamp supported conditions, changes in frequency with change in the step location is not predictable at all and when the step gets close to the supported, the slope of changes gets very steep. Although in the simply supported conditions the changes in the first mode are homogeneous and in the second mode depending on the step ratio a different behavior is observed. Also in this type of fulcrum, the frequency changes become really leveled with the step location and there are no steep slopes seen.

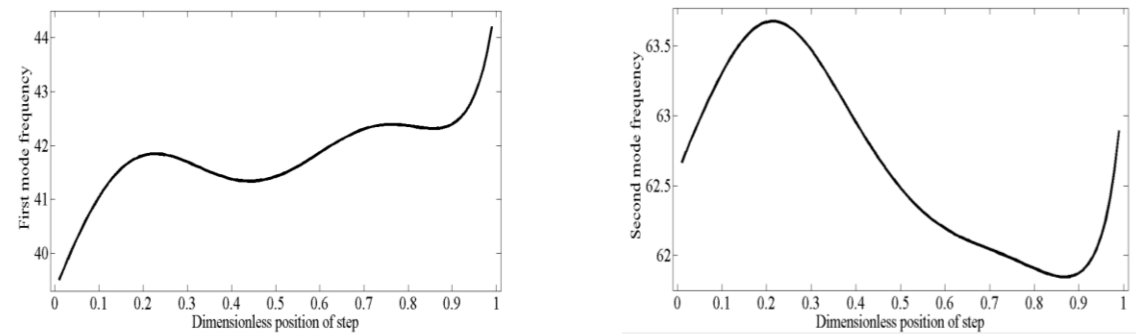


Fig.4
Effect of step location on first two natural frequencies (Clamp boundary conditions) ($\tau = 0.75, k = 10^5 Nm^{-2}$).

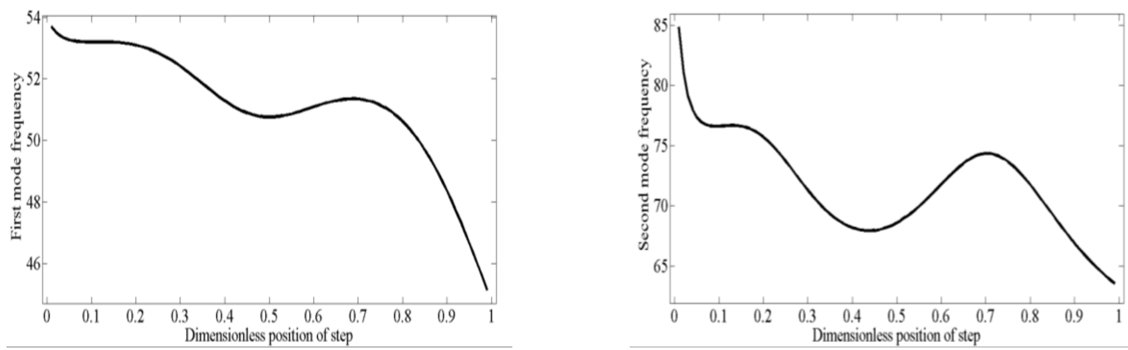


Fig.5
Effect of step location on first two natural frequencies (Clamp boundary conditions) ($\tau = 2, k = 10^5 Nm^{-2}$).

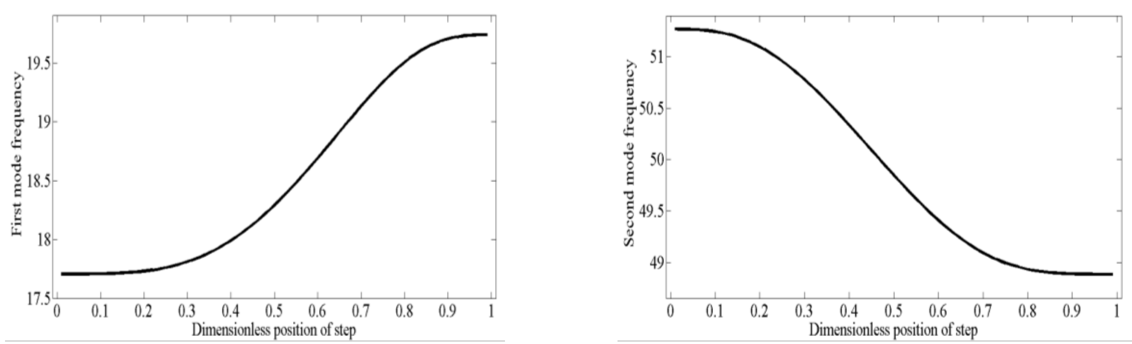


Fig.6
Effect of step location on first two natural frequencies (Hinge boundary conditions) ($\tau = 0.75, k = 10^5 Nm^{-2}$).

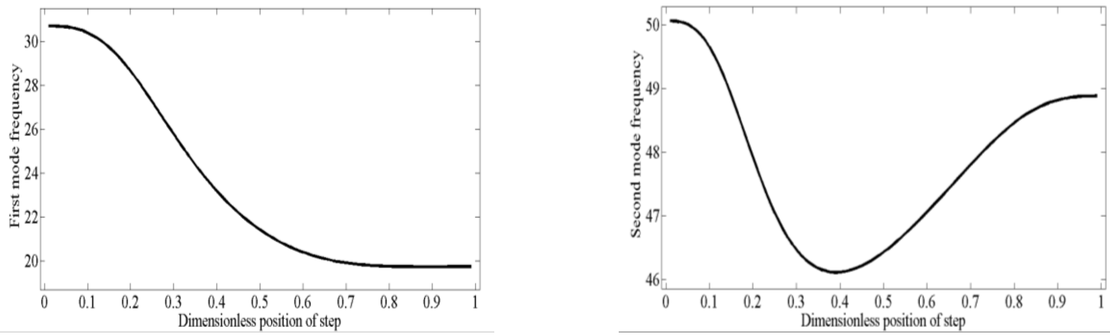


Fig.7
Effect of step location on first two natural frequencies (Hinge boundary conditions) ($\tau = 2, k = 10^5 Nm^{-2}$).

In Figs. 8 and 9, the effect of step location on the four shapes of the first normal mode of the beam with the clamp supported conditions is presented. As it is shown, alteration in the step location does not make much difference to the shape of modes. In Figs. (10) and (11), the effect of step location on the four shapes of the first normal mode of the beam with the simply supported conditions is presented. On the contrary to the clamp supported conditions, in this state alteration in the step location has a considerable influence on the shape of modes and changing the step location alters the phase and of-phase of the mode shapes.

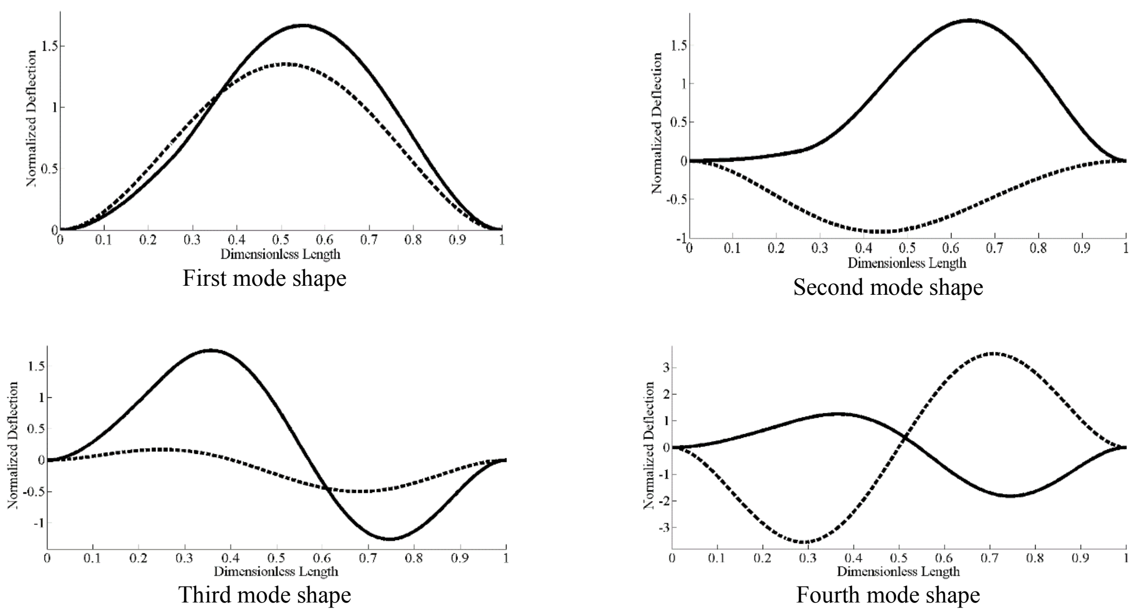
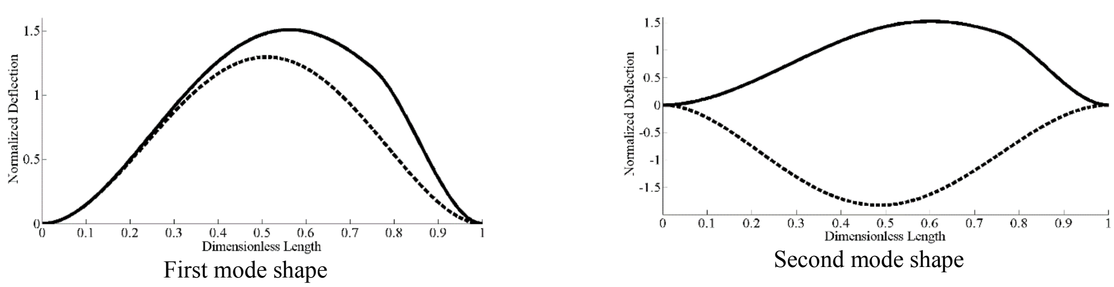
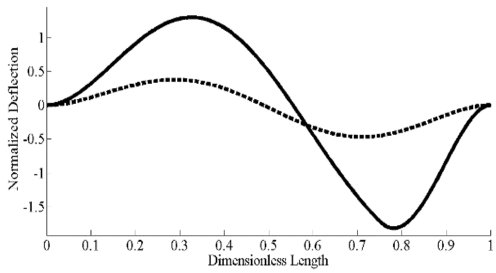
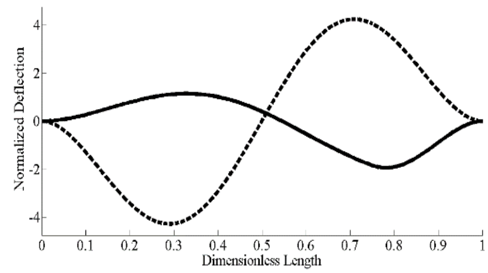


Fig.8
Normalized mode shapes in the case of clamp boundary conditions (Solid line: upper beam, Dashed line: lower beam), ($R = 0.25, \tau = 0.5, k = 2\times10^5 Nm^{-2}$).



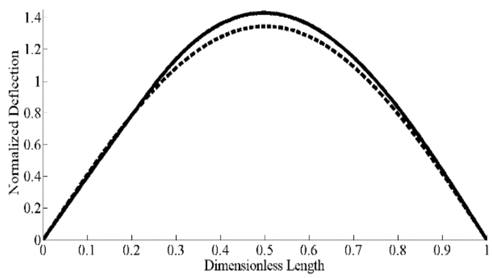


Third mode shape

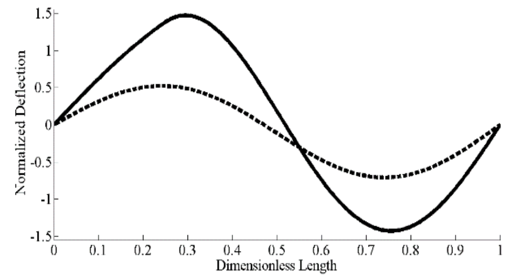


Fourth mode shape

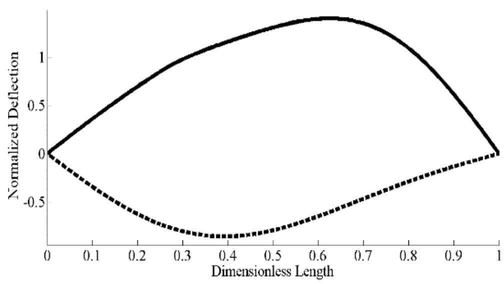
Fig.9
Normalized mode shapes in the case of clamp boundary conditions (Solid line: upper beam, Dashed line: lower beam), $(R = 0.75, \tau = 0.5, k = 2\times 10^5 Nm^{-2})$.



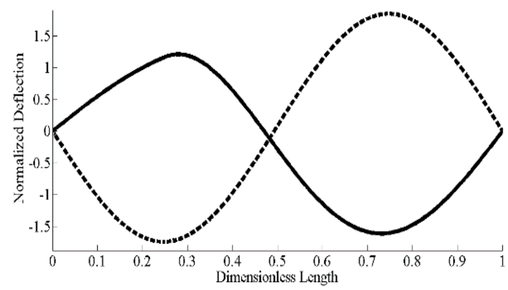
First mode shape



Second mode shape

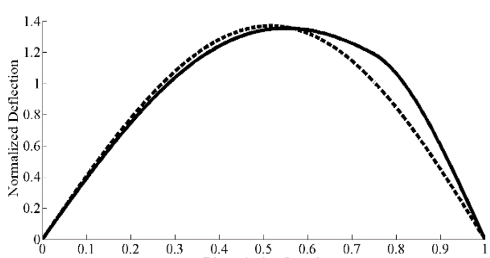


Third mode shape

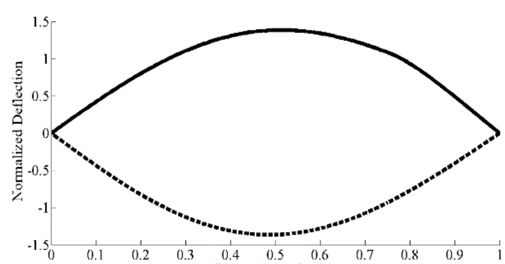


Fourth mode shape

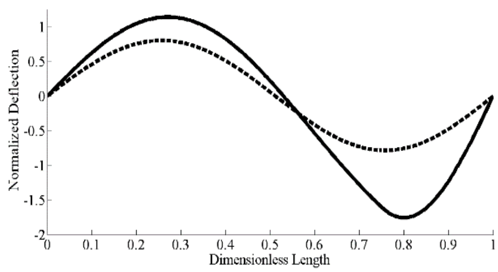
Fig.10
Normalized mode shapes in the case of hinge boundary conditions.(Solid line: upper beam, Dashed line: lower beam), $(R = 0.25, \tau = 0.5, k = 2\times 10^5 Nm^{-2})$.



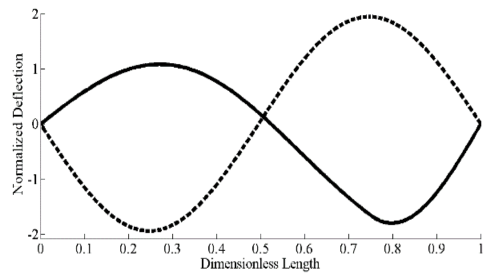
First mode shape



Second mode shape



Third mode shape



Fourth mode shape

Fig.11
Normalized mode shapes in the case of hinge boundary conditions (Solid line: upper beam, Dashed line: lower beam), $(R = 0.75, \tau = 0.5, k = 2\times10^5 Nm^{-2})$.

6 CONCLUSIONS

In present study, free vibration of elastically connected beams when one of them has step due to abrupt change in height investigated by means of differential transform method. Euler-Bernoulli beam theory has been applied to drive equations of motion. Applied method (DTM) yields high accuracy, rapid convergence and stability in computation. In special case of two parallel uniform beams, accuracy of results verified by comparing with other references. Effects of different parameters on dimensionless frequencies and normalized mode shapes investigated.

The main conclusions are as follows:

1) Convergence rate of the DTM depend on boundary conditions, although the DTM yields rapid convergence.
2) Despite the type of boundary condition, by increasing the elastic layer coefficient, frequencies increase in any mode.
3) When the step ratio is close to one, the elastic layer coefficient between the two beams does not have much influence on the main frequency.
4) Under the clamp supported conditions, the frequency changes in a completely irregular manner with other parameters.
5) Alteration in the step location does not influence the shape of modes much under the clamp supported conditions; although, in simply supported conditions, the step location is completely influential on the mode shapes.

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