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Three-Dimensional Stress Analysis for Semi-Elliptical Cracks in the Connection of Cylinder-Hemispherical Head for Thick-Walled Cylindrical Pressure Vessels

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ABSTRACT

These pressure vessels are made by different type of heads. One of them is hemi-spherical head. The area of geometrical discontinuity, like the connection of the cylinder to its hemi-spherical head, are the most susceptible areas for crack initiation along their welds. So it is worthwhile to consider cracks located at this connection. The purpose of this article is to investigate the effect of variation of stress field and geometry of problem on distribution of Stress Intensity Factor (SIF) for a semi-elliptical surface crack which is located at the connection of cylinder to its hemispherical head. The three dimensional finite element analysis is performed by employing singular elements along the crack front. The ratio of crack depth to crack length (*a/c*) ranged from 0.3 to 1.2; the ratio of crack depth to wall thickness (*a/t*) ranged from 0.2 to 0.8; and the cylinder geometry parameter of vessel $\gamma = R_0 / R_i$ ranged from 1.2 to 2. For better comparison the results are normalized and reported in non-dimensional formats. The results show that the crack configuration, vessel thickness and radius have significant influence on the stress intensity factor distribution along the crack front. Also For a fixed $R_0 / R_i = 1.5$ and $a / t = 0.4$ the maximum value of SIF occur in the cylindrical part and approximately near the deepest point of crack; not on the deepest point of crack depth and this may be due to changing stress field in this connection. The stress intensity factors are presented in suitable curves for various geometrical configurations providing useful tool for the fracture mechanics design of cracked pressure vessels.

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Keywords: Stress intensity factor; Cylinder-hemispherical head; Semi-elliptical crack; Cylindrical pressure vessel.

1 INTRODUCTION

CYLINDRICAL pressure vessels are widely used in many industries. Remarkable studies have been performed on the problem of semi-elliptical cracks contained in cylindrical and spherical pressure vessels. The

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analysis of surface cracks in pressure vessels is relatively more important as their catastrophic failure essentially leads to the loss of life and property. There are many studies, which carried out in considering semi-elliptical crack on cylindrical pressure vessel. In 1982 Raju and Newman [1] calculated SIFs for internal and external semi-elliptic surface cracks in cylindrical vessels under mechanical loading. In another study of Shahani and Nabavi [2], SIFs are obtained for internal longitudinal semi-elliptical cracks in finite length thick walled cylinder. Internal pressure is applied as a mechanical load. In this work especially cylinder length effect were observed. The results show that SIF increases as the cylinder length deacreases, especially at the corner point of the crack. A numerical analysis has been performed by Shiv Sahaya Shukla [3] in 2019 to study the Three-Dimensional (3-D) behavior of the stress field around the crack front of a semi-elliptical surface cracked thick plate under tension. It is found that the SIF for a semi-elliptical surface crack has a maximum at the center of the crack while the maximum of KI for semi-circular crack happens to be at the front free surface of the plate. In 2019, Numerical Analysis of the Effect of External Circumferential Elliptical Cracks in Transition Thickness Zone of Pressurized Pipes has been done by using Extended Finite Element Method (XFEM) [4].The results show that the XFEM is an effective tool for modeling crack in pipes. In another study by Perl in 2016[5] three dimensional, mode I SIF distributions for radial or coplanar crack arrays as well as ring cracks emanating from the inner surface of an autofrettaged spherical pressure vessel are evaluated. The 3-D analysis is performed via the finite element model. In 2016, Zareei [6] calculated the SIFs at the deepest point of an internal circumferential semi-elliptical crack in a pipe subjected to any arbitrary load. SIFs for internal semi-elliptical surface cracks in autofrettaged cylinders are obtained by Lin[7] and Three-Dimensional (3-D) finite element based displacement method with the crack tip square-root singularity of stresses and strains simulated is used to evaluate the SIFs along the crack front. As mentioned above many researchers have provided useful SIF solutions for surface cracks in cylindrical pressure vessels [8-14] and spherical pressure vessels [15-20].
These pressure vessels are made by different type of heads. One of them is hemi-spherical head. The connection of hemi-spherical head to the cylinder is susceptible to cracking along their welds due to several reasons such as cyclic pressurization-depressurization, the existence of heat-effected zone near the welds, the presence of corrosive agents, tensile residual stresses within this region. Furthermore, the area of geometrical discontinuity ,like the connection of the cylinder to its hemi-spherical head ,are the most susceptible areas for crack initiation ,where the stress field varies along the axial, moreover ,depending on the considered load case ,the circumferential direction. As a consequence, it is worthwhile to consider cracks located at this connection. The case of 3-D semi-elliptical crack in the connection of cylinder to its hemi-spherical head, in spite of its importance, has been analyzed by a few researchers. Diamantoudis [21] used finite element method in order to calculate the SIFs of a semi-elliptical crack located just close to the connection of the cylinder to its hemi-spherical head The difference of present study with Diamantoudis and Labeas work [21] is that the thickness of the present problem for cylinder and its hemispherical part are equal to "*t* ", and the other is that, in this study the half of crack is located in cylindrical part and next half in spherical part as can be seen in Fig. 1 , while Diamantoudis and Labeas [21] considered a semi-elliptical crack located just close to the connection of the cylinder to its hemi-spherical head and the thickness of the cylindrical part of pressure vessel is different from the hemi-spherical part. There isn't any stress intensity factor solutions for this kind of problem, neither analytically, nor numerically; Some solution that exist such as numerical results, Newman and Raju empirical equations, weight function solutions, etc., are only valid for especial ideal cases such as uniform stress distribution along the axial an circumferential direction and just for variation of stress through the thickness of the pressure vessels. Regarding to existence of non-continue stress in cylinder-head connection, In present study, the SIF for an internal semi-elliptical crack located in the connection of cylinder to its hemi-spherical head in cylindrical pressure vessels are calculated by finite element approach. The crack location considered is the connection of cylinder to its hemispherical head. The stress intensity factors are presented in suitable curves for various geometrical configurations, thus providing useful tool for the fracture mechanics design of cracked pressure vessels.
It is noteworthy that due to non- symmetry configuration full crack front is considered. Half of crack is located in cylindrical part and next half in spherical part. The radius of cylinder and hemi-spherical part assumed to be equal to t . The ratio of crack depth to wall thickness (*a/t*) ranged from 0.2 to 0.8; the ratio of crack depth to crack length (*a/c*) ranged from 0.3 to 1.2; and the outer to inner radii ratio of vessel $\gamma = R_0 / R_i$ ranged from 1.2 to 2.The purpose of this article is to investigate the effect of variation of stress field and geometry of problem on distribution of SIF for a semi-elliptical crack located in connection of cylindrical pressure vessel to its hemi-spherical head.

3-D Finite Element Analysis (FEA) is performed by using ANSYS 15.0 [22]. In order to facilitate modeling, APDL codes are developed for each crack case. After solving the problem by Finite Element Method by obtaining the displacement field around the crack front, stress intensity factors are calculated by using Displacement Correlation Technique (DCT).

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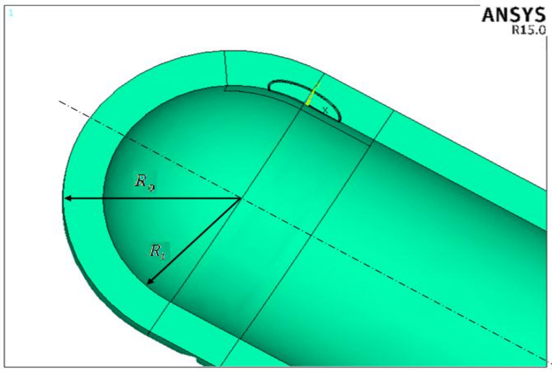

Fig.1
The geometry of the considered problem.

2 THREE-DIMENSIONAL ANALYSIS

The 3-D analysis is performed on an elastic cylindrical pressure vessel with its hemi-spherical head with inner radius R_i , outer radius R_0 and wall thickness *t*. the cylindrical pressure vessel contain an internal semi-elliptical crack located in the connection of cylinder to its hemi-spherical head with half-length of *c* and depth of *a* (see Fig. 2). The length of cylindrical part was always chosen large enough that the length would have a negligible effect on stress intensity factor.

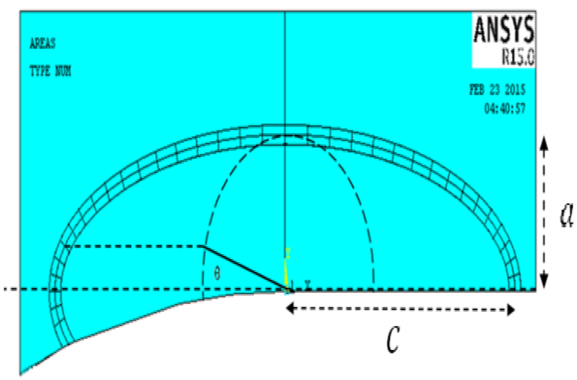

Fig.2
Configuration of crack located in the connection of cylinder to its hemi-spherical head.

2.1 Finite element modeling

In order to analyze the current problem, the 3D analysis is performed on an elastic cylinder with its hemi-spherical head of inner radius R_i , outer radius R_0 and wall thickness *t*, where $t = R_0 - R_i$. The finite element models of the problems studied in this paper are generated by the use of general purpose finite element analysis code ANSYS [22] During the creation of the model, ANSYS Parametric Design Language (APDL) codes are generated for modeling 3-D problems in ANSYS.

In this linear elastic problem, the displacements near the crack front vary as $\sqrt{r}$, where r is the distance from the crack tip, varying as $1/\sqrt{r}$. To produce this singularity in stresses and strains, the elements around the crack front should be quadratic, with the mid-side nodes placed at the quarter points. Such elements are called singular elements. The crack front should be surrounded by these elements, hence a small volume called the crack tunnel comprising the singular elements were created around the crack front (see Fig. 3). To mesh the others parts of the model 20-node solid-186 elements were employed in the finite element code ANSYS [22] (see Fig. 4 and 5).

The length of the cylindrical part of the model chosen in the way that there are no effects of the constraint parts on the obtained results.

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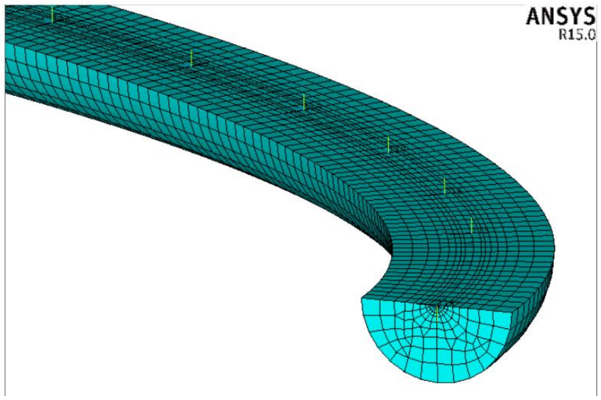

Fig.3
The elements around the crack-front of a semi-elliptic surface crack after sweeping.

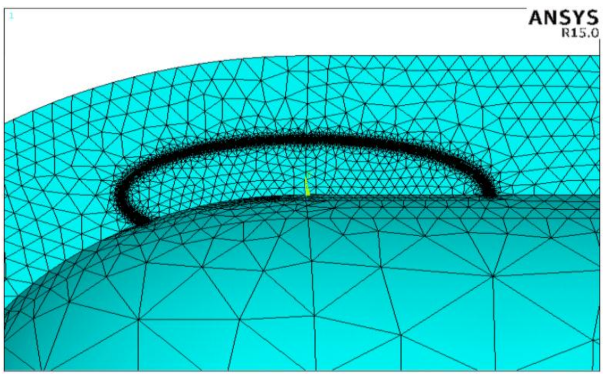

Fig.4
Closer view of the crack after meshing.

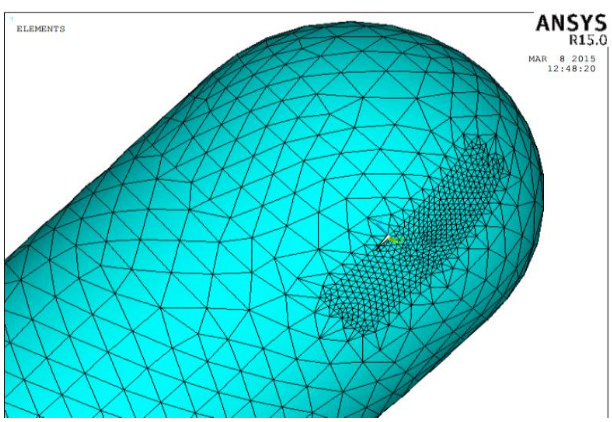

Fig.5
Full finite element model.

2.2 Validation of the fracture model

Two finite element modeling by using the commercial software ANSYS [22] were conducted to validate the results of current study and are presented in the following sections.

2.2.1 Semi-elliptical surface crack in finite plate

Among different solutions in the literature for computation of SIF for a semi-elliptical surface crack in a finite thickness plate under uniform tension, the comparison of results done with those reported by Raju and Newman [24] which is still most widely used for comparison and validation purposes. The first mode of stress intensity factors (K_I) obtained through finite element analysis are normalized by dividing it to K_0 which defined as:

$$K_0 = \sigma\sqrt{\pi a / Q} \tag{1}$$

where, Q =1.104 according to [24].

The comparison of present results with reference [24] along the whole crack front with *a/c* =0.2 as a function of the normalized coordinate ($2\phi/\pi$) and the crack depth to plate thickness ratio varying from 0.2 to 1 is done. The present fracture model gave accurate results and shown to be in relatively good agreement (see Fig. 6 and Table.1).

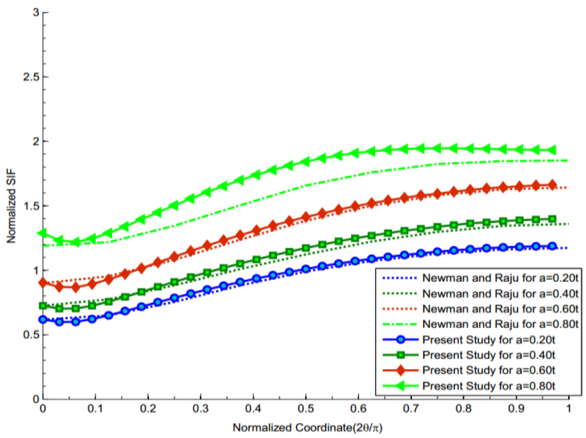

Fig.6
Validation of fracture model with Newman and Raju [24] for a semi-elliptical surface crack in a finite plate (*a*=0.2*c*).

Table 1
Comparison of validation data of fracture model with Newman and Raju [24] for a semi-elliptical surface crack in a finite plate (*a*=0.2*c*) and its percentage error.

Normalized Coordinate	*a*= 0.2 *c* and *a*=0.2 *t*			*a*= 0.2 *c* and *a*=0.2 *t*		
	Reference[24]	Present study	Percentage error	Reference[24]	Present study	Percentage error
0	0.62	0.58	6.56	0.72	0.68	7.17
0.125	0.65	0.64	2.13	0.78	0.74	4.40
0.25	0.75	0.78	-2.71	0.88	0.90	-2.32
0.375	0.88	0.89	-1.41	1.01	1.04	-3.30
0.5	0.99	1.00	-0.86	1.12	1.16	-3.66
0.625	1.07	1.07	0.25	1.22	1.25	-2.02
0.75	1.13	1.12	0.53	1.30	1.31	-0.90
0.875	1.16	1.14	1.43	1.34	1.34	0.67
1	1.17	1.16	0.85	1.36	1.36	0.18
Normalized Coordinate	*a*= 0.2 *c* and *a*=0.2 *t*			*a*= 0.2 *c* and *a*=0.2 *t*		
	Reference[24]	Present study	Percentage error	Reference[24]	Present study	Percentage error
0	0.90	0.83	8.64	1.19	1.02	16.66
0.125	0.95	0.91	4.81	1.22	1.10	10.76
0.25	1.08	1.11	-2.46	1.35	1.32	2.26
0.375	1.24	1.28	-3.22	1.50	1.54	-2.32
0.5	1.38	1.43	-2.98	1.66	1.71	-3.27
0.625	1.50	1.53	-1.74	1.76	1.85	-4.77
0.75	1.58	1.60	-1.37	1.82	1.93	-5.52
0.875	1.63	1.64	-0.50	1.85	1.97	-6.46
1	1.64	1.66	-1.18	1.85	2.00	-7.34

2.2.2 Semi-elliptical surface crack in spherical pressure vessel

The second finite element model to justify the reliability of the model was performed by comparing the distributions of K_I / K_0 along the front of a slender crescentic radial crack in a spherical vessel with, *a/c* =0.5, *a/t* =0.2 and spherical vessels ratio $\gamma = 4/3$; and compared with those reported by Perl and Bernstain [20] obtained by finite element implementation and using the standard-API 579-1[25]. The mode one stress intensity factors (K_I) are normalized by dividing it to K_0 as mentioned in section 2.2.1.

The results are presented in Fig. 7 and shows good agreement which gave confidence to use the current fracture model for complex cases like the semi-elliptical inner crack located in the connection of cylinder to its hemi-spherical head.

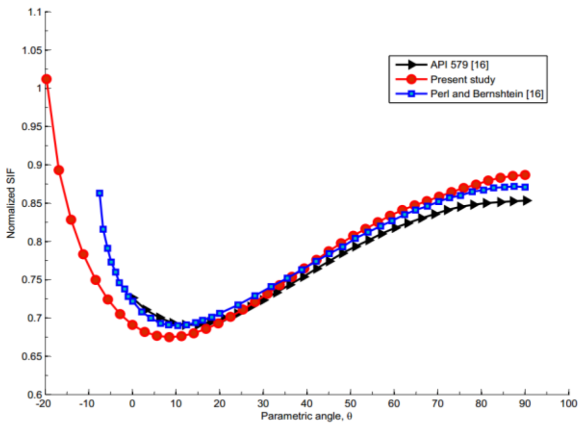

Fig.7
Comparison between the distributions of K_I / K_0 along the front of a slender crescentic radial crack, obtained herein, and a similar semi-elliptical crack calculated using the API 579-1[16] (*a/c*= 0.5, *a/t*= 0.2, and γ = 4/3).

2.3 Convergence study

In order to ensure the numerical accuracy of this simulation, a convergence study was conducted for $\gamma = 1.5$, *a/c* =0.4 and *a/t* =0.4 . Because of these trials, it is anticipated that for meshes of more than 164918 elements, the level of error will be less than 2% (see Fig. 8). Consequently, in order to maintain good accuracy while using reasonable computer resources all meshes contain almost more than this.

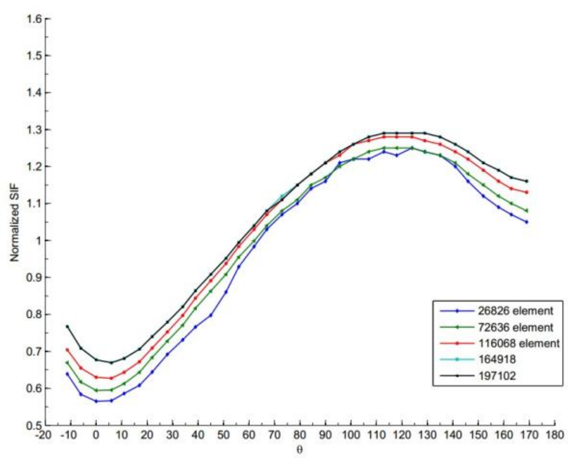

Fig.8
Convergence of normalized SIF for a semi-elliptical crack located in the connection of cylinder to its hemi-spherical head. (γ = 1.5, *a/c*=0.4 and *a/t*=0.4).

3 STRESS INTENSITY FACTORS CALCULATIONS

Due to importance of stress intensity parameter, Brittle fracture in cracked engineering components is usually examined by SIF. After the detecting crack in structure, SIF can be calculated by various experimental or theoretical methods such as finite element analysis. SIF can be calculated by Displacement Correlation Technique (DCT). Displacements calculated with finite element method are very accurate even near crack tip and are therefore the most obvious data for calculating stress intensity factors. In the current study, Displacement Correlation Technique (DCT) is used for this purpose. Displacement-based SIF calculation methods usually requires the following two conditions: 1) Quarter-point elements must be used in the first layer of elements around the crack tip; and 2) the mesh in the near-tip region has to be substantially refined for reasonable accuracy which considered during modeling and meshing of the current study.

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After the displacement values around the crack front are obtained with the help of the commercial analysis code by applying finite element method then SIF calculates according to Eq. (2) which reported by Köşker [26].

$$K_I = \frac{\sqrt{2\pi} \times E}{8(1-\upsilon^2)} \cdot \left[\frac{R_3^{\,3/2}(u_{b2u} - u_{b2d}) - R_2^{\,3/2}(u_{b3u} - u_{b3d})}{\sqrt{R_2}\sqrt{R_3}\,(R_3 - R_2)} \right] \tag{2}$$

where the local coordinate system located at point P (see Fig. 9) is composed of the tangential (t), normal (n) and binormal (b) directions and (r, θ) are the polar coordinates in the normal plane (n,b). The other terms in Eq. (2) can be found in Fig. 9.

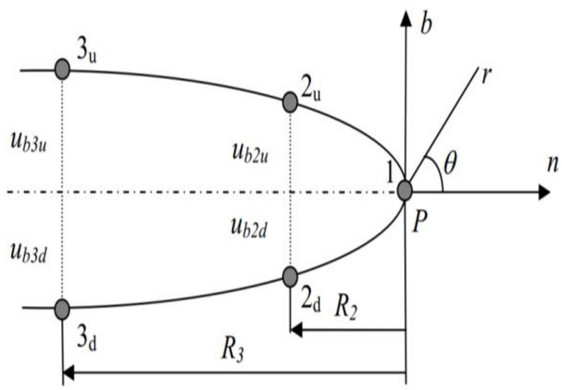

Fig.9
Deformed shape of the crack surface (non-symmetric).

4 RESULTS AND DISCUSSION

In present paper, a thick-walled cylinder with a semi-elliptical crack located in the connection of cylinder to its hemi-spherical head is considered. The pressure vessel subjected to the internal pressure of 100 *MPa* , the values of Young modulus (E) and Poisson's ratio (v) are respectively 200 *GPa* and 0.3.

The effects of non-dimensional parameters $\gamma = R_0 / R_i$, *a/t* and *a/c* on K_I / K_0 have been examined. In order to study the variation of SIFs along the crack front for this problem sixteen configurations of thick cylindrical pressure vessels with their hemi-spherical head are analyzed. All the stress intensity factors are normalized by dividing to K_0 given by:

$$K_0 = P\sqrt{R_i} \tag{3}$$

where P is internal pressure, and R_i inner radius of pressure vessel. It is not exactly clear that how stress varies in the connection of cylinder to its hemi-spherical head. Because of this fact, In order to study the effects of different configurations on the SIFs along the crack front of such problem K_I / K_0 considered. Actually, the values of K_I / K_0 represents the real value of SIFs. Hereafter the SIFs are considered in the non-dimensional form as K_I / K_0 . The dimensionless parameters such as *a/t*, *a/c* and $\gamma = R_0 / R_i$ varies to consider some essential geometries. The ratio of crack depth to wall thickness *a/t* = 0.2,0.4,0.6 and 0.8; the ratio of crack depth to crack length *a/c* = 0.3,0.4,0.8,1.0,1.10 and 1.20; and the outer to inner radii ratio of vessel $\gamma = R_0 / R_i$ =1.2,1.3,1.5,1.7 and 2.

In order to show the effect of depth of crack on SIFs four models are analyzed by fixing γ = 1.5 and *a/c* = 0.4 for different range of *a/t* = 0.2, 0.4, 0.6 and 0.8 as can be seen in Fig. 10.

As shown in Fig. 11, the variation of normalized SIFs are presented for $\gamma = R_0 / R_i$ = 1.5 and *a/t* = 0.4 with the ratio of crack depth to crack length *a/c* = 0.3, 0.4, 0.6, 0.8, 1.0, 1.10 and 1.20.

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Furthermore to study the effect of γ (cylinder geometry parameter) on the distribution of stress intensity factor on the crack front five model with *a/c* = 0.4, *a/t* = 0.4 and γ = 1.2, 1.3, 1.5, 1.7 and 2 are considered and presented in Fig. 12.

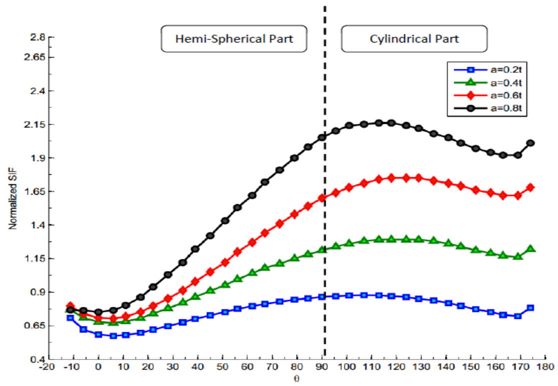

Fig.10
Variation of Normalized SIF along crack front for a semi-elliptical surface crack (R_0/R_i =1.5, *a/c*=0.4) in pressurized cylinder.

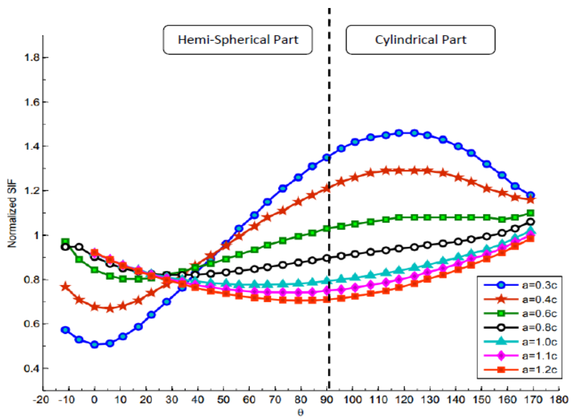

Fig.11
Variation of Normalized SIF along crack front for a semi-elliptical surface crack (R_0/R_i =1.5, *a/t*=0.4) in pressurized cylinder.

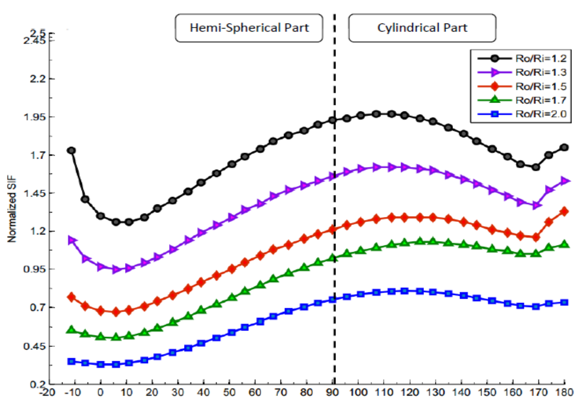

Fig.12
Variation of Normalized SIF along crack front for a semi-elliptical surface crack (*a/c*=0.4, *a/t*=0.4) in pressurized cylinder.

5 CONCLUSIONS

Three-Dimensional stress analysis of a semi-elliptical crack in the connection of cylinder-hemi spherical head for thick-walled cylindrical pressure vessels is investigated. The three dimensional finite element analysis is performed by employing singular elements along the crack front. The following results can be obtained from stress analysis of the results:

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- SIF results of cracks located in the connection of cylindrical pressure vessel to its hemi-spherical head are computed and presented in this article for different *a/c, a/t* and $\gamma = R_0 / R_i$ in a form suitable to study behavior of SIF along the cracks. the variation of presently computed SIF values differ significantly from SIF values of the same cracks located away from the connection of cylinder to its hemi-spherical head or which located just close to this area that reported by Diamantoudis and Labeas (2005).
- For a fixed $\gamma = R_0 / R_i = 1.5$ the stress intensity factors are higher for larger *a/t* ratio (see Fig. 10).
- For a fixed $R_0 / R_i = 1.5$ and *a/t* = 0.4 the maximum value of SIF occur in the cylindrical part and approximately near the deepest point of crack; not on the deepest point of crack depth and this may be due to changing stress field in this connection.
- As the *a/c* decreases, the distribution of SIF on the crack front tends to a parabola like shape and the critical point on the crack front lies on the cylindrical part; not on the deepest point of crack depth.
- The effect of aspect ratio on distribution of the stress intensity factor along the crack front with a constant value of relative depth of crack (*a/t* = 0.4) is shown in Fig. 11. The graphs demonstrate that the smaller aspect ratios produce the larger stress intensity factors. In other words, narrower the cracks, higher the stress intensity factors around crack fronts.

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