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Size-Dependent Higher Order Thermo-Mechanical Vibration Analysis of Two Directional Functionally Graded Material Nano beam

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ABSTRACT

This paper represented a numerical technique for discovering the vibrational behavior of a two-directional FGM (2-FGM) Nano beam exposed to thermal load for the first time. Mechanical attributes of two-directional FGM (2-FGM) Nano beam are changed along the thickness and length directions of nanobeam. The nonlocal Eringen parameter is taken into the nonlocal elasticity theory (NET). Uniform temperature rise (UTR), linear temperature rise (LTR), non-linear temperature rise (NLTR) and sinusoidal temperature rise (STR) during the thickness and length directions of Nano beam is analyzed. Third-order shear deformation theory (TSDT) is used to derive the governing equations of motion and associated boundary conditions of the two-directional FGM (2-FGM) Nano beam via Hamilton's principle. The differential quadrature method (DQM) is employed to achieve the natural frequency of two-directional FGM (2-FGM) Nano beam. A parametric study is led to assess the efficacy of coefficients of two-directional FGM (2-FGM), Nonlocal parameter, FG power index, temperature changes, thermal rises loading and temperature rises on the non-dimensional natural frequencies of two-directional FGM (2-FGM) Nano beam.

Keywords: Free vibration; Two directional FGM; Thermal load; Nonlocal theory; Nano beam.

1 INTRODUCTION

NANOTECHNOLOGY has made us able to develop many materials and innovate new composite materials with conflicting properties such as ductility a hardness, high strength and lightness and etc. simultaneously. Composite materials made up of two or more materials with distinct characteristics. one of the most frequent problems of this new materials is delamination that is called technically interlaminar cracking [1]. In order get better efficiency and solve the problem, functionally graded material was created. FG materials are a new type of

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nonhomogeneous composites that the volume fractions of the material components vary continuously along arbitrary directions as a function of position. Mainly constituent components of FG materials are metal and ceramics, because metals have high toughness and ceramics have great corrosive and heat resistance, the combination of metals and ceramics can make optimum composites. Continuous variation of volume fraction through FG materials prevents of discontinuous interfaces existence in composites which are where the fracture is initiated. The mechanical feature of FG materials provided a wide range of applications in medicine, biomechanics, aerospace, and etc. Various usages of FG materials in industrial sectors, attracted the interest of researchers to study mechanical behavior [2]–[4] of the structural elements that are manufactured from FG materials [5]. When the structure dimensions are in order of micro/nano, size effect gets importance on the mechanical properties. The elasticity theory or classical continuum unable to predict the size dependency and the scale effect of the nanostructure material properties. There are various continuum theories such as strain gradient, modified couple stress and nonlocal elasticity that consider size behavior. Studying mechanical behaviors of micro, nano and macro structures have been focused by many researchers like buckling [6]–[8] and vibration [9]–[14]. Nonlocal elasticity theory which was presented by Eringen assumes that the stress at a desired point relies on the strain at all around points of the body [15], [16]. Paddison et al [17] developed the nonlocal Euler/Bernoulli beam theory by applying Eringen's theory. Reddy [18] formulated nonlocal relations of Eringen's theory for various beam theories such as Timoshenko, Euler/Bernoulli, Levinson and Reddy beam theories. Wang an Liew [19] employed nonlocal continuum to analysis the static behavior of Euler/Bernoulli and Timoshenko beam theories. Aydogdu [20] utilized Eringen's theory for various beam theories in order to study buckling, bending and free vibration of nanoscale beams. J.Fernandez-Saez et al [21] formulated the problem of the static bending of Euler–Bernoulli beams using the Eringen integral constitutive equation. Pardhan and Padikor [22] presented finite element analysis for nonlocal elastic Kirchoff plate and Euler/Bernoulli beam. Pradhan and Murmu [23] reported flap wise vibration and bending properties of rotation nanoscale cantilevers via differential quadrature method. They considered that scale effects have a fundamental role in the vibration characteristics of the rotating nanoscale structures. Mahinzare et al [24] investigated the rotational and electrical load on the Nanoplate. Ebrahimi et al. [25]–[27]examined the applicability of differential transformation method in investigations on vibrational characteristics of FG size-dependent Nano beams. Ke and Wang [28] exploited the small-scale effects on the dynamic stability of FGM micro beams based upon Timoshenko beam model Thai [29] applied a nonlocal high-order theory in order to study mechanical behaviors of the Nano beams. Eltaher et al. [30]–[32] investigated a finite element analysis for free vibration of the FG Nano beams using nonlocal EBT. They also presented the stability and static responses of the FG Nano beams based on the nonlocal continuum theory. Mahinzare et al. [33] presented the vibrational effect of the Nano shell with conveying viscous fluid based on nonlocal strain gradient theory. Malekzadeh et al. [34] studied The surface and nonlocal effects on the nonlinear flexural free vibrations of elastically supported non-uniform cross section Nano beams simultaneously. Free vibration characteristics of functionally graded (FG) Nano beams based on third-order shear deformation beam theory by Navier-type solution are investigated by Ebrahimi et al. [35] Huang and Li [36] studied the free vibration of axially functionally graded beams with nonuniform cross-section. For different end supports, including clamped, simply-supported and free ends, the governing equation with varying coefficient was transformed into a Fredholm integral equation. Natural frequencies were determined by requiring that the resulting Fredholm integral equation has a non-trivial solution. Simsek et al. [37] presented a numerical approach to investigate the dynamic behavior of double-functionally graded beam systems with different boundary conditions subjected to a moving harmonic load. Two parallel functionally graded beams were connected with each other continuously by elastic springs. The material properties of the beam were assumed to vary continuously in the thickness direction according to power-law form. Rezaiee-Pajand and Hozhabrossadati [38] presented free vibration of double-axially functionally graded beams with elastic restraints Tounsi et al [39] investigated thermo-mechanical buckling behavior of Nano beams based on higher order beam theory. Al-Basyouni et al. [40] analyzed the bending and dynamic behaviors of FG micro beams based on new first and sinusoidal beam theories together with the classical beam theory and using the modified couple stress theory. Mahinzare et al. [41] studied the bi-dimensional FGM effect on the spinning micro plate with elastic foundations. Ansari et al. [42] investigated a numerical analysis is conducted to predict size-dependent nonlinear free vibration characteristics of third-order shear deformable micro beams made of functionally graded materials (FGMs) . they implemented the modified strain gradient elasticity theory and von Karman geometric nonlinearity into the classical third-order shear deformation beam theory to develop a nonclassical higher-order beam model. Rahmani and Jandaghian [43] presented buckling analysis of functionally graded Nano beams based on a nonlocal third-order shear deformation theory. FG materials for their excellent properties applied in micro/nanoscale structures and many researchers focused on studying the characteristics of this materials. Asghari et al [44], [45] have studied the vibration responses of the functionally graded TBT and EBT beams by considering the modified couple stress theory. Alshorbagy et al [46] examined the mechanical properties of FG beams with transversally and axially power law distribution. Ansari et al [47] analyzed the free vibration of FG material beam by using strain gradient TBT . They also presented the influence of the graduation values of material exponents in the vibration behavior of FG micro beams. Employing analytical method, the buckling, and bending of nonlocal EBT and TBT FG Nano beams were investigated by Simsek and Yurtcu [48] Huu-Tai Thai et al [49] developed various higher-order shear deformation beam theories for bending and free vibration of functionally graded beams. Niknam and Aghdam [50] investigated a semi analytical approach for large amplitude free vibration and buckling of FG Nano beams resting on an elastic foundation based on nonlocal elasticity theory. They showed that by increasing length of the beam, the effect of small scale parameter decreases. Ghiasian et al. [51], [52] studied the dynamic buckling behavior of FGM beams resting on elastic foundation and subjected to uniform temperature rise loading. Ebrahimi and Salari [53] presents the effect of various thermal loadings on buckling and vibrational characteristics of nonlocal temperature-dependent FG Nano beams. They have shown that various factors such as thermal environment and nonlocal parameter play important roles in dynamic behavior of FG Nano beams . Alibeigloo [54] presented thermal buckling analysis of piezoelectric FGM beams with simply supported ends . Zhu Su et al [55] presents a unified solution for free and transient vibration analyses of a functionally graded piezoelectric curved beam with general boundary conditions within the framework of Timoshenko beam theory . Li and Shi [56] studied the free vibration of functionally graded piezoelectric beams under different boundary conditions using the state-space based (DQM) . Xiang and Shi [57] presented static analysis for a functionally graded piezoelectric sandwich cantilever under a combined electro-thermal load, and the solution can be obtained by the Airy stress function method . Ebrahimi and Barati [58] investigates buckling response of higher-order shear deformable Nano beams made of functionally graded piezoelectric (FGP) materials embedded in an elastic foundation . Ebrahimi et al. [59] investigated thermal vibration behavior of (FG) Nano beams exposed to various kinds of thermomechanical loading including uniform, linear and nonlinear temperature rise embedded in a two-parameter elastic foundation based on third-order shear deformation beam theory which considers the influence of shear deformation.

As an extension of the authors' previous works [60]–[64], this paper will focus on, vibrational behaviors of two-directional FGM (2-FGM) Nano beam with variable thickness in frame work of third-order shear deformation beam theory are investigated based on nonlocal elastic theory. The assumptions of boundary conditions are considered in both case of simply supported and clamp supported ends. FGM properties are supposed to variation gradually based on power law through the whole body. The derivation of nonlocal equations is based on Hamilton's principle and the applied solution approach is the differential quadrature method (DQM). There are various numerical examples in order to show the resolution of results.

2 MATHEMATICAL RELATIONS

2.1 Kinematic equations

Assume a Nano beam with length *L*, as shown in Fig. 1(a). According to the third-order shear deformation beam theory, the displacements in the directions *x* and *z* can be approximated as:

$$u(x,z) = u_0(x) + z\,\psi(x) - \alpha z^3\left(\psi + \frac{\partial w}{\partial x}\right) \tag{1}$$

$$w(x,z) = w(x) \tag{2}$$

where *u* and *w* are the axial and transverse displacements and $\alpha = 4/(3h^2)$. The parameter ψ is the bending rotation of cross section at each point of the *y*-axis . Nonzero strain for the Reddy beam model are written as:

$$\varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z\,\varepsilon_{xx}^{(1)} + z^3\varepsilon_{xx}^{(3)} \tag{3}$$

$$\gamma_{xz} = \gamma_{xz}^{(0)} + z\,\gamma_{xz}^{(2)} \tag{4}$$

where

$$\varepsilon_{xx}^{(0)} = \frac{\partial u}{\partial x}\ ,\ \varepsilon_{xx}^{(1)} = \frac{\partial \psi}{\partial x}\ ,\ \varepsilon_{xx}^{(3)} = -\alpha\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) \tag{5}$$

$$\gamma_{xz}^{(0)}=\frac{\partial w}{\partial x}+\psi \;,\; \gamma_{xz}^{(2)}=-\beta(\frac{\partial w}{\partial x}+\psi) \tag{6}$$

and $\beta=4/h^2$. The motion of a structure with the elastic body in the arbitrary time interval $t_1 < t < t_2$ is the integral of total potential energy which by applying Hamilton's principle is extremum:

$$\int_0^t \delta(U_s-U_k-U_w)\, dt=0 \tag{7}$$

which U_s is strain energy, U_k is kinetic energy, U_w is work caused by the external load. The variation of strain energy can are expressed as:

$$\delta \mathrm{U}_s=\int_0^L\int_{-h/2}^{h/2}(\sigma_{xx}\,\delta\varepsilon_{xx}+\sigma_{xz}\,\delta\gamma_{xz})dz\; dx \tag{8}$$

By using Eqs. (3) and (4) into Eq.(8):

$$\delta \mathrm{U}_s=\int_0^L\int_{-h/2}^{h/2}(N\,\delta\varepsilon^{(0)}{}_{xx}+M\,\delta\varepsilon^{(1)}{}_{xx}+P\,\delta\varepsilon^{(3)}{}_{xx}+Q\,\delta\gamma^{(0)}{}_{xz}+R\,\delta\gamma^{(2)}{}_{xz})dz\; dx \tag{9}$$

where the introduced variables are defined as:

$$N=\int_A\sigma_{xx}dA \;,\; M=\int_A\sigma_{xx}z\, dA \;,\; P=\int_A\sigma_{xx}z^3 dA \;,\; Q=\int_A\sigma_{xz}dA \;,\; R=\int_A\sigma_{xz}z^2 dA \tag{10}$$

The variation of the work done by the applied load can be calculated as:

$$\delta \mathrm{U}_W=\int_0^L(N_T)\frac{\partial w}{\partial x}\frac{\delta\partial w}{\partial x}dx \tag{11}$$

where N_T is the compressive external axial load. The variation of the kinetic energy is defined as:

$$\delta \mathrm{U}_K=\frac{1}{2}\int_0^L\int_{-h/2}^{h/2}\rho((\frac{\partial u_x}{\partial t})^2+(\frac{\partial u_z}{\partial t})^2)dz\; dx \tag{12}$$

By substituting Eqs. (8) , (11) and (12) into Eq. (7) and making the coefficients of δw, δu and $\delta\psi$ to zero, the motion equations can be written as:

$$\frac{\partial N}{\partial x}=I_0\frac{\partial^2 u}{\partial t^2}+\hat{I}_1\frac{\partial^2\psi}{\partial t^2}-\alpha I_3\frac{\partial^3 w}{\partial x\,\partial t^2} \tag{13}$$

$$\frac{\partial\hat{M}}{\partial x}-\hat{Q}=\hat{I}_1\frac{\partial^2 u}{\partial t^2}+\hat{I}_2\frac{\partial^2\psi}{\partial t^2}-\alpha\hat{I}_4(\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^3 w}{\partial x\,\partial t^2}) \tag{14}$$

$$\frac{\partial\hat{Q}}{\partial x}-(N_T)\frac{\partial^2 w}{\partial x^2}+\alpha\frac{\partial^2 P}{\partial x^2}=I_0\frac{\partial^2 w}{\partial t^2}+\alpha I_3\frac{\partial^3 u}{\partial x\,\partial t^2}+\alpha I_4\frac{\partial^3\psi}{\partial x\,\partial t^2}-\alpha^2 I_6(\frac{\partial^3\psi}{\partial x\,\partial t^2}+\frac{\partial^4 w}{\partial x^2\partial t^2}) \tag{15}$$

where I_0, I_1, I_2, I_3, I_4 and I_6 are mass inertia and expressed as:

$$(I_0,I_1,I_2,I_3,I_4,I_6)=\int_A(1,z,z^2,z^3,z^4,z^6)\rho dA \tag{16}$$

In which $\hat{I}_1=I_1-\alpha I_3$, $\hat{I}_2=I_2-\alpha I_4$, $\hat{I}_4=I_4-\alpha I_6$ and in Eqs. (14) and (15) $\hat{M}=M-\alpha P$ and $\hat{Q}=Q-\beta R$.

3 THE NONLOCAL ELASTICITY THEORY FOR FG NANOBEAM

Based on nonlocal elasticity theory, the stress tensor at each point of the body is a function of strain tensors of all around points inside the body. For the elastic solids, the tensor components of the nonlocal stress σ_{ij} at each point can be obtained as:

$$\sigma_{ij}=\int_V\alpha(|x'-x|,\tau)[C_{ijkl}\varepsilon_{kl}(x')-C_{ijkl}\alpha_{kl}\Delta T]dV(x') \tag{17}$$

where C_{ijkl} is showing that the nonlocal stress at a desired point is the weighted average of the local stress of all around points and the term $\alpha\ (|x'-x|,\tau)$ is the nonlocal modules that accommodate the size effects into the constitutive equations. $|x'-x|$ represents the Euclidean distance and τ is constant as follow:

$$\tau=\frac{e_0a}{l}$$

where a and l are the internal and external characteristic lengths of Nano beam, respectively and e_0 is the material constant estimated experimentally. It is difficult to solve Eq. (17) for problems of nonlocal elasticity thus; the differential form of that equation is used as follow:

$$\sigma_{ij}-(e_0a)^2\nabla^2\sigma_{ij}=C_{ijkl}\varepsilon_{kl}-C_{ijkl}\alpha_{kl}\Delta T \tag{18}$$

In which σ_{ij} is the nonlocal stress and ε_{kl} is the nonlocal strain. Nonlocal theory in for a material can be expressed as:

$$\sigma_{xx}-(e_0a)^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=C_{11}\varepsilon_{xx}-C_{11}\alpha_1\Delta T \tag{19}$$

$$\sigma_{xz}-(e_0a)^2\frac{\partial^2\sigma_{xz}}{\partial x^2}=C_{55}\gamma_{xz} \tag{20}$$

where integrating Eqs. (19) and (20) over the cross-sectional area, one can define the moment strain and the force strain of the FG beams as:

$$N-\mu\frac{\partial^2N}{\partial x^2}=A_{xx}\frac{\partial u}{\partial x}+B_{xx}\frac{\partial\psi}{\partial x}-\alpha E_{xx}(\frac{\partial\psi}{\partial x}+\frac{\partial^2w}{\partial x^2})-N_T \tag{21}$$

$$M-\mu\frac{\partial^2M}{\partial x^2}=B_{xx}\frac{\partial u}{\partial x}+D_{xx}\frac{\partial\psi}{\partial x}-\alpha F_{xx}(\frac{\partial\psi}{\partial x}+\frac{\partial^2w}{\partial x^2}) \tag{22}$$

$$P-\mu\frac{\partial^2P}{\partial x^2}=E_{xx}\frac{\partial u}{\partial x}+F_{xx}\frac{\partial\psi}{\partial x}-\alpha H_{xx}(\frac{\partial\psi}{\partial x}+\frac{\partial^2w}{\partial x^2}) \tag{23}$$

$$Q-\mu\frac{\partial^2 Q}{\partial x^2}=-E_{15}^{e}\frac{\partial\phi}{\partial x}(A_{xz}-\beta D_{xz})(\psi+\frac{\partial w}{\partial x}) \tag{24}$$

$$R-\mu\frac{\partial^2 R}{\partial x^2}=-F_{15}^{e}\frac{\partial\phi}{\partial x}(D_{xz}-\beta F_{xz})(\psi+\frac{\partial w}{\partial x}) \tag{25}$$

where

$$\mu=(e_0a)^2$$
$$\{A_{xx},B_{xx},D_{xx},E_{xx},F_{xx},H_{xx}\}=\int_{-h/2}^{h/2}C_{11}\{1,z,z^2,z^3,z^4,z^6\}dz$$
$$\{A_{xz},\mathrm{D}_{xz},F_{xz}\}=\int_{-h/2}^{h/2}C_{55}\{1,z^2,z^4\}dz$$

By substation the second derivative of, that can be obtained from Eq.(13), into Eq.(21) the nonlocal normal force can be expressed as follows:

$$N_x=A_{xx}\frac{\partial u}{\partial x}+K_{xx}\frac{\partial\psi}{\partial x}-\alpha E_{xx}\frac{\partial^2 w}{\partial x^2}-N_T+\mu(I_0\frac{\partial^3 u}{\partial x\,\partial t^2}+\hat{I}_1\frac{\partial^2\psi}{\partial x\,\partial t^2}-\alpha I_3\frac{\partial^4 u}{\partial x^2\partial t^2}) \tag{26}$$

Second derivative of $\hat{M}$ by removing $\hat{Q}$ from Eqs. (14) and (15), can be obtained as following equation:

$$\frac{\partial^2\hat{M}}{\partial x^2}=-\alpha\frac{\partial^2 P}{\partial x^2}+(N_T)\frac{\partial^2 w}{\partial x^2}+I_0\frac{\partial^2 w}{\partial t^2}+I_1\frac{\partial^3 u}{\partial x\,\partial t^2}+I_2\frac{\partial^3\psi}{\partial x\,\partial t^2}-\alpha I_4(\frac{\partial^3\psi}{\partial x\,\partial t^2}+\frac{\partial^4\psi}{\partial x^2\partial t^2}) \tag{27}$$

In order to express the explicit equation of nonlocal bending moment we can put the second derivative of M into Eq.(22) and using Eq.(23) as follows:

$$\begin{aligned}\hat{M}&=K_{xx}\frac{\partial u}{\partial x}+I_{xx}\frac{\partial\psi}{\partial x}-\alpha J_{xx}(\frac{\partial\psi}{\partial x}+\frac{\partial^2 w}{\partial x^2})+\mu(-\alpha\frac{\partial^2 P}{\partial x^2}+\frac{\partial}{\partial x}(N_E+N_T)\frac{\partial w}{\partial x})+I_0\frac{\partial^2 w}{\partial t^2}+I_1\frac{\partial^3 u}{\partial x\,\partial t^2}+I_2\frac{\partial^3\psi}{\partial x\,\partial t^2}\\&-\alpha I_4(\frac{\partial^3\psi}{\partial x\,\partial t^2}+\frac{\partial^4\psi}{\partial x^2\partial t^2})\end{aligned} \tag{28}$$

In which $K_{xx}=B_{xx}-\alpha E_{xx}$, $I_{xx}=D_{xx}-\alpha F_{xx}$, $J_{xx}=F_{xx}-\alpha H_{xx}$. By using Eq.(15) to obtain the second derivative of Q and substitution it into Eq.(24) and using Eq.(25), the explicit equation of nonlocal shear force is obtained as:

$$\begin{aligned}\hat{Q}&=\bar{A}_{xz}(\frac{\partial w}{\partial x}+\psi)+\mu(-\alpha\frac{\partial^3 P}{\partial x^3}+(N_T)\frac{\partial^3 w}{\partial x^3})+\mu(I_0\frac{\partial^3 w}{\partial x\,\partial t^2}+\alpha I_3\frac{\partial^4 u}{\partial x^2\partial t^2}+\alpha I_4\frac{\partial^3\psi}{\partial x^2\partial t^2}\\&-\alpha^2 I_6(\frac{\partial^5\psi}{\partial x^3\partial t^2}+\frac{\partial^4\psi}{\partial x^2\partial t^2}))\end{aligned} \tag{29}$$

where $\bar{A}_{xz}=B^*_{xz}-\beta I^*_{xz}$, $A^*_{xz}=A_{xz}-\beta D_{xz}$, $I^*_{xz}=D_{xz}-\beta F_{xz}$. Now by applying $\hat{M}$ and $\hat{Q}$ from Eqs.(28) and (29) we calculate:

$$\alpha\frac{\partial^2}{\partial x^2}(P-\mu\frac{\partial^2 P}{\partial x^2})=\alpha(E_{xx}\frac{\partial^3 u}{\partial x^3}+F_{xx}\frac{\partial^3\psi}{\partial x^3}-\alpha H_{xx}(\frac{\partial^3\psi}{\partial x^3}+\frac{\partial^4 w}{\partial x^4}) \tag{30}$$

By substituting N, M and Q from Eqs.(26), (28) and (29), respectively, into Eqs.(13) , (14) and (15), the governing equations of nonlocal third-order shear deformation FG beam can be derived as follows:

$$A_{xx}\frac{\partial^2 u}{\partial x^2}+\frac{\partial A_{xx}}{\partial x}\frac{\partial u}{\partial x}+K_{xx}\frac{\partial^2\psi}{\partial x^2}+\frac{\partial K_{xx}}{\partial x}\frac{\partial\psi}{\partial x}-\alpha E_{xx}\frac{\partial^3 w}{\partial x^3}-\alpha\frac{\partial E_{xx}}{\partial x}\frac{\partial^2 w}{\partial x^2}+\mu(\frac{\partial^2 I_0}{\partial x^2}\frac{\partial^2 u}{\partial x^2}+2\frac{\partial I_0}{\partial x}\frac{\partial^3 u}{\partial x\partial t^2}$$
$$+I_0\frac{\partial^4 u}{\partial x^2\partial t^2}+\frac{\partial^2 I_1}{\partial x^2}\frac{\partial^2\psi}{\partial t^2}+2\frac{\partial I_1}{\partial x}\frac{\partial^3\psi}{\partial x\partial t^2}+I_1\frac{\partial^4\psi}{\partial x^2\partial t^2}-\alpha\frac{\partial^2 I_3}{\partial x^2}\frac{\partial^2\psi}{\partial t^2}-2\alpha\frac{\partial I_3}{\partial x}\frac{\partial^3\psi}{\partial x\partial t^2}-\alpha I_3\frac{\partial^4\psi}{\partial x^2\partial t^2} \tag{31}$$
$$-\alpha\frac{\partial^2 I_3}{\partial x^2}\frac{\partial^3 w}{\partial x\partial t^2}-2\alpha\frac{\partial I_3}{\partial x}\frac{\partial^4 w}{\partial x^2\partial t^2}-\alpha I_3\frac{\partial^5 w}{\partial x^3\partial t^2})-I_0\frac{\partial^2 u}{\partial t^2}-\hat{I}_1\frac{\partial^2\psi}{\partial t^2}+\alpha I_3\frac{\partial^2\psi}{\partial t^2}+\alpha I_3\frac{\partial^3 w}{\partial x\partial t^2}=0$$

$$K_{xx}\frac{\partial^2 u}{\partial x^2}+\frac{\partial K_{xx}}{\partial x}\frac{\partial u}{\partial x}+I_{xx}\frac{\partial^2\psi}{\partial x^2}+\frac{\partial I_{xx}}{\partial x}\frac{\partial\psi}{\partial x}-\alpha J_{xx}(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^3 w}{\partial x^3})-\alpha\frac{\partial J_{xx}}{\partial x}(\frac{\partial\psi}{\partial x}+\frac{\partial^2 w}{\partial x^2})$$
$$-\bar{A}_{xz}(\psi+\frac{\partial w}{\partial x})+(E^e_{31}-\alpha F^e_{31}+E^e_{15}-\beta F^e_{15})\frac{\partial\phi}{\partial x}-\hat{I}_1\frac{\partial^2 u}{\partial t^2}-\hat{I}_2\frac{\partial^2\psi}{\partial t^2}+\alpha\hat{I}_4(\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^3 w}{\partial x\partial t^2})$$
$$+\mu(\frac{\partial^2\hat{I}_1}{\partial x^2}\frac{\partial^2 u}{\partial t^2}+2\frac{\partial\hat{I}_1}{\partial x}\frac{\partial^3 u}{\partial x\partial t^2}+\hat{I}_1\frac{\partial^4 u}{\partial x^2\partial t^2}+\frac{\partial^2\hat{I}_2}{\partial x^2}\frac{\partial^2\psi}{\partial t^2}+2\frac{\partial\hat{I}_2}{\partial x}\frac{\partial^3\psi}{\partial x\partial t^2}+\hat{I}_2\frac{\partial^4\psi}{\partial x^2\partial t^2} \tag{32}$$
$$-\alpha\frac{\partial^2\hat{I}_4}{\partial x^2}(\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^3 w}{\partial x\partial t^2})-2\alpha\frac{\partial\hat{I}_4}{\partial x}(\frac{\partial^3\psi}{\partial x\partial t^2}+\frac{\partial^4 w}{\partial x^2\partial t^2})-\alpha\hat{I}_4(\frac{\partial^4\psi}{\partial x^2\partial t^2}+\frac{\partial^5 w}{\partial x^3\partial t^2}))=0$$

$$\bar{A}_{xz}(\frac{\partial\psi}{\partial x}+\frac{\partial^2 w}{\partial x^2})+\frac{\partial\bar{A}_{xz}}{\partial x}(\psi+\frac{\partial w}{\partial x})+\mu(\frac{\partial^2 N_T}{\partial x^2}\frac{\partial^2 w}{\partial x^2}+2\frac{\partial N_t}{\partial x}\frac{\partial^3 w}{\partial x^3}+N_T\frac{\partial^4 w}{\partial x^4})-N_T\frac{\partial^2 w}{\partial x^2}$$
$$-I_0\frac{\partial^2 w}{\partial t^2}+\alpha(E_{xx}\frac{\partial^3 u}{\partial x^3}+\frac{\partial^2 E_{xx}}{\partial x^2}\frac{\partial u}{\partial x}+J_{xx}\frac{\partial^3\psi}{\partial x^3}+\frac{\partial^2 J_{xx}}{\partial x^2}\frac{\partial\psi}{\partial x}-\alpha H_{xx}\frac{\partial^4 w}{\partial x^4}-\alpha\frac{\partial^2 H_{xx}}{\partial x^2}\frac{\partial^2 w}{\partial x^2})$$
$$-\alpha I_3\frac{\partial^3 u}{\partial x\partial t^2}-\alpha I_4\frac{\partial^3\psi}{\partial x\partial t^2}+\alpha^2 I_6(\frac{\partial^3\psi}{\partial x\partial t^2}+\frac{\partial^4 w}{\partial x^2\partial t^2})+\mu(\frac{\partial^2 I_0}{\partial x^2}\frac{\partial^2 w}{\partial^2 t}+2\frac{\partial I_0}{\partial x}\frac{\partial^3 w}{\partial x\partial t^2}$$
$$+I_0\frac{\partial^4 w}{\partial x^2\partial t^2}+\alpha\frac{\partial^2 I_3}{\partial x^2}\frac{\partial^3 u}{\partial x\partial t^2}+2\alpha\frac{\partial I_3}{\partial x}\frac{\partial^4 u}{\partial x^2\partial t^2}+\alpha I_3\frac{\partial^5\psi}{\partial x^3\partial t^2}+\alpha\frac{\partial^2 I_4}{\partial x^2}\frac{\partial^3\psi}{\partial x\partial t^2} \tag{33}$$
$$+2\alpha\frac{\partial I_4}{\partial x}\frac{\partial^4\psi}{\partial x^2\partial t^2}+\alpha I_4\frac{\partial^5\psi}{\partial x^3\partial t^2}-\alpha^2\frac{\partial^2 I_6}{\partial x^2}(\frac{\partial^3\psi}{\partial x\partial t^2}+\frac{\partial^4 w}{\partial x^2\partial t^2})-2\alpha^2\frac{\partial I_6}{\partial x}(\frac{\partial^4\psi}{\partial x^2\partial t^2}+\frac{\partial^5 w}{\partial x^3\partial t^2})$$
$$-\alpha^2 I_6(\frac{\partial^5\psi}{\partial x^3\partial t^2}+\frac{\partial^6 w}{\partial x^4\partial t^2}))=0$$

4 SOLUTION METHOD

Solution of the governing relations for simply supported boundary condition and clamped boundary condition of FG Nano beam is based on differential quadrature method (DQM). DQ method is a numerical method that is effective for the solution of ordinary and partial differential equation. This method is determined as the derivative of a function at a desired point that can be approximated as a linear sum of the weighted function at all sample points. According to DQ method, the partial derivatives of a displacement functions, w and φ as an example, are expressed as:

$$\left.\frac{\partial^{\boldsymbol{m}}\{u(x),w(x),\psi(x)\}}{\partial x^{\boldsymbol{m}}}\right|_{x=x_p}=\sum_{\boldsymbol{k}=1}^{\boldsymbol{n}}C_{\boldsymbol{ik}}^{(\boldsymbol{m})}\{u_{\boldsymbol{k}},v_{\boldsymbol{k}},w_{\boldsymbol{k}}\} \tag{34}$$

where

$$u_{\boldsymbol{k}}=u(x_{\boldsymbol{k}},\mathrm{t}) \tag{35}$$

$$w_k = w\left(x_k, t\right) \tag{36}$$

$$\psi_k = \psi\left(x_k, \mathrm{t}\right) \tag{37}$$

All number of nodes are expressed by n which are distributed during the r-axis and the weighting coefficients are defined by $C_{ik}^{(m)}$ that recessive formula is obtained as follows:

$$C_{ik}^{(1)} = \begin{cases} \dfrac{M\left(x_i\right)}{\left(x_i - x_k\right)M\left(x_k\right)} & i \neq k \\ -\sum\limits_{k=1, i\neq k}^{n} C_{ik}^{(1)} & i = k \end{cases} \tag{38}$$

where k=1, 2, 3,…, n . In order to achieve an optimizing distribution of mesh point, we applied cosine pattern to generate points of DQ method as follows:

$$x_i = \frac{1}{2}(1-\cos(\frac{(i-1)}{(n-1)}\pi)) \tag{39}$$

where i=1, 2, 3,…, n. The analysis shows that using this type of distribution makes the convergence of the solution faster. $M(x)$ is presented as follows:

$$M\left(\mathbf{x}\right) = \prod_{k=1,\ k\neq i}^{n} \left(x_i - x_k\right) \tag{40}$$

The related weighting factor for the x-th derivative of displacement functions is given as follows:

$$C_{ik}^{(m)} = \begin{cases} m\left[C_{ik}^{(m-1)} C_{ik}^{(1)} - \dfrac{C_{ik}^{(m-1)}}{\left(x_i - x_k\right)}\right] & i \neq k \text{ and } 2 \leq m \leq n-1 \\ -\sum\limits_{k=1, i\neq k}^{n} C_{ik}^{(m)} & i = k \text{ and } 1 \leq m \leq n-1 \end{cases} \tag{41}$$

By applying DQM into the governing equation, Eqs. (31) - (33), lead to Eqs. (45) - (47). Now, the matrix form of the motion equations presented as follows:

$$M \frac{\partial^2 d}{\partial t^2} + \left(K_e\right) d = 0 \tag{42}$$

In which K_e and M are the stiffness matrix and mass matrix, respectively. The dimensions of the matrixes are $3N\times 3N$ for thin shell love theory .in last theories, the displacement vector d, are given respectively as follows:

$$d = \left\{\left\{u_i\right\}^T, \left\{w_i\right\}^T, \left\{\psi_i\right\}^T\right\} \tag{43}$$

5 FREE VIBRATION ANALYSIS

Applying the dynamic displacement vector, d, in the form of $d^{*}=d\times e^{i\omega}$, would conduct to an eigenvalue system of the equation as follows:

$$\left(M\,\omega^2+K_e\right)d^*=0 \tag{44}$$

$$\begin{aligned}&A_{xx}\sum_{k=1}^{n_i}C_{ik}^{(2)}u_k+K_{xx}\sum_{k=1}^{n_i}C_{ik}^{(2)}\psi_k-\alpha E_{xx}\sum_{k=1}^{n_i}C_{ik}^{(3)}w_k+\mu(I_0\sum_{k=1}^{n_i}C_{ik}^{(2)}u_k\frac{\partial^2u}{\partial t^2}+I_1\sum_{k=1}^{n_i}C_{ik}^{(2)}\psi_k\frac{\partial^2\psi}{\partial t^2}\\&-\alpha I_3\sum_{k=1}^{n_i}C_{ik}^{(2)}\psi_k\frac{\partial^2\psi}{\partial t^2}-\alpha I_3\sum_{k=1}^{n_i}C_{ik}^{(3)}\psi_k\frac{\partial^3\psi}{\partial t^2})-I_0\frac{\partial^2u}{\partial t^2}-\hat{I}_1\frac{\partial^2\psi}{\partial t^2}+\alpha I_3\sum_{k=1}^{n_i}C_{ik}^{(1)}\psi_k\frac{\partial^2\psi}{\partial t^2}=0\end{aligned} \tag{45}$$

$$\begin{aligned}&K_{xx}\sum_{k=1}^{n_i}C_{ik}^{(2)}u_k+I_{xx}\sum_{k=1}^{n_i}C_{ik}^{(2)}\psi_k-\alpha J_{xx}(\sum_{k=1}^{n_i}C_{ik}^{(2)}\psi_k+\sum_{k=1}^{n_i}C_{ik}^{(3)}w_k)-\bar{A}_{xz}(\psi_i+\sum_{k=1}^{n_i}C_{ik}^{(1)}w_k)-\hat{I}_1\frac{\partial^2u}{\partial t^2}-\hat{I}_2\frac{\partial^2\psi}{\partial t^2}\\&+\alpha\hat{I}_4(\frac{\partial^2\psi}{\partial t^2}+\sum_{k=1}^{n_i}C_{ik}^{(1)}w_k\frac{\partial^2w}{\partial t^2})+\mu(\hat{I}_1\sum_{k=1}^{n_i}C_{ik}^{(2)}u_k\frac{\partial^2u}{\partial t^2}+\hat{I}_2\sum_{k=1}^{n_i}C_{ik}^{(2)}\psi_k\frac{\partial^2\psi}{\partial t^2}-\alpha\hat{I}_4(\sum_{k=1}^{n_i}C_{ik}^{(2)}\psi_k\frac{\partial^2\psi}{\partial t^2}+\sum_{k=1}^{n_i}C_{ik}^{(3)}\psi_k\frac{\partial^2\psi}{\partial t^2}))=0\end{aligned} \tag{46}$$

$$\begin{aligned}&\bar{A}_{xz}(\sum_{k=1}^{n_i}C_{ik}^{(1)}\psi_k+\sum_{k=1}^{n_i}C_{ik}^{(2)}w_k)+\mu((N_T)\sum_{k=1}^{n_i}C_{ik}^{(4)}w_k)-(N_T)\sum_{k=1}^{n_i}C_{ik}^{(2)}w_k-I_0\frac{\partial^2w}{\partial t^2}\\&+\alpha(E_{xx}\sum_{k=1}^{n_i}C_{ik}^{(3)}u_k+J_{xx}\sum_{k=1}^{n_i}C_{ik}^{(3)}\psi_k-\alpha H_{xx}\sum_{k=1}^{n_i}C_{ik}^{(4)}w_k)-\alpha I_3\sum_{k=1}^{n_i}C_{ik}^{(1)}u_k\frac{\partial^2u}{\partial t^2}-\alpha I_4\sum_{k=1}^{n_i}C_{ik}^{(1)}\psi_k\frac{\partial^2\psi}{\partial t^2}\\&+\alpha^2I_6(\sum_{k=1}^{n_i}C_{ik}^{(1)}\psi_k\frac{\partial^2\psi}{\partial t^2}+\sum_{k=1}^{n_i}C_{ik}^{(2)}w_k\frac{\partial^2w}{\partial t^2})+\mu(I_0\sum_{k=1}^{n_i}C_{ik}^{(2)}w_k\frac{\partial^2w}{\partial t^2}+\alpha I_3\sum_{k=1}^{n_i}C_{ik}^{(3)}\psi_k\frac{\partial^2\psi}{\partial t^2}\\&-\alpha^2I_6(\sum_{k=1}^{n_i}C_{ik}^{(3)}\psi_k\frac{\partial^3\psi}{\partial t^2}+\sum_{k=1}^{n_i}C_{ik}^{(4)}w_k\frac{\partial^2w}{\partial t^2}))=0\end{aligned} \tag{47}$$

6 TYPE OF THE THERMAL LOADING

6.1 Uniform temperature rise (UTR)

For a FG Nano beam at reference temperature T_0, the temperature is uniformly raised to a final value T which the temperature change is $\Delta T=T-T_0$.

6.2 Linear temperature rise (LTR)

For a FG Nano beam for which the beam thickness is thin enough, the temperature distribution is assumed to be varied linearly through the thickness as follows:

$$T=T_1+\Delta T\,(\frac{1}{2}+\frac{z}{h}) \tag{48}$$

where $\Delta T=T_2-T_1$ and T_2 and T_1 are the temperature of the top surface and the bottom surface, respectively.

6.3 Sinusoidal temperature rise (STR)

The temperature field when METE-FG Nano beam is exposed to sinusoidal temperature rise across the thickness can be defined as:

$$T=T_1+\Delta T\,(1-\cos\frac{\pi}{2}(\frac{1}{2}+\frac{z}{h})) \tag{49}$$

where $\Delta T=T_2-T_1$ is temperature change.

7 CONVERGENCE ANALYSIS

Firstly, a convergence test is performed to determine the sufficient numbers of grid points required to obtain accurate and stable results for DQM. As can be seen from Fig.1, twenty grid points are sufficient to obtain converged results for the frequency of the Nano beam in S-S and C-C boundary conditions.

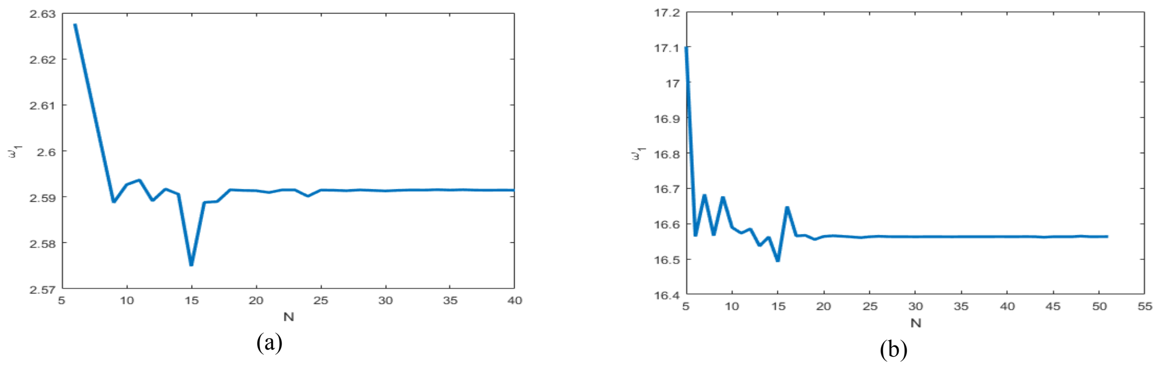


Fig.1
The effect of the number of grid points on evaluating convergence of the nondimensional natural frequency (a) S-S boundary condition (b) C-C boundary condition.

8 VALIDATION OF THE RESULTS WITH OTHER ARTICLES

In Table 1, the first natural frequencies of FG beam in macro size that obtained by the present study are compared with [30] and [65] which have been obtained based on the third order shear deformation theory (TBT) for hinged boundary condition. It is shown that there is an very good agreement with the present study between [30] and [65] . Employing the third order shear deformation theory (TBT), the first natural frequencies of a FGM nanobeam subjected to uniform thermal load with hinged boundary condition have excellent agreements with [66] as it has been observed in Table 2. (Results are closely coincident)

Table 1
Comparison of the first natural frequency of FG macro beam for different *n*.

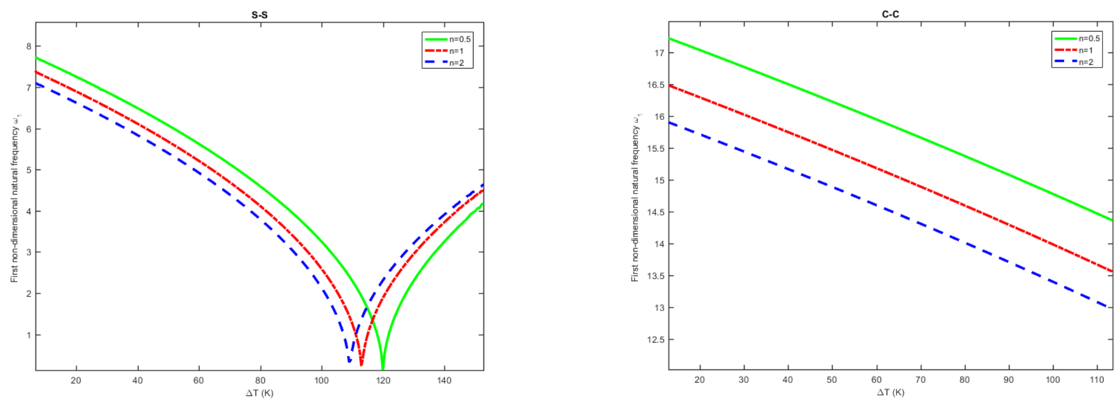
	Eltaher et al [30]	Rahmani and Pedram [65]	Present study
n=0	9.8797	9.8296	9.829567
n=0.5	7.8061	7.7149	7.778150
n=1	7.0904	6.9676	7.074459
n=5	6.0025	5.9172	5.983103

Table 2
Comparison of the first natural frequency of FG Nano beam for different ΔT, n and μ

		n=1		*n*=5	
μ	ΔT	Ebrahimi and Barati [66]	Present study	Ebrahimi and Barati [66]	Present study
0	20	7.89464	7.89659639	7.32331	7.30344017
	50	6.98874	6.83028555	6.41992	6.40063911
1	20	7.47792	7.45481028	6.93323	6.88847202
	50	6.51432	6.47910419	5.97112	5.94238816
2	20	7.1112	7.08269715	6.58978	6.56470677
	50	6.08985	6.05734908	5.56865	5.54084313

9 RESULTS AND DISCUSSION

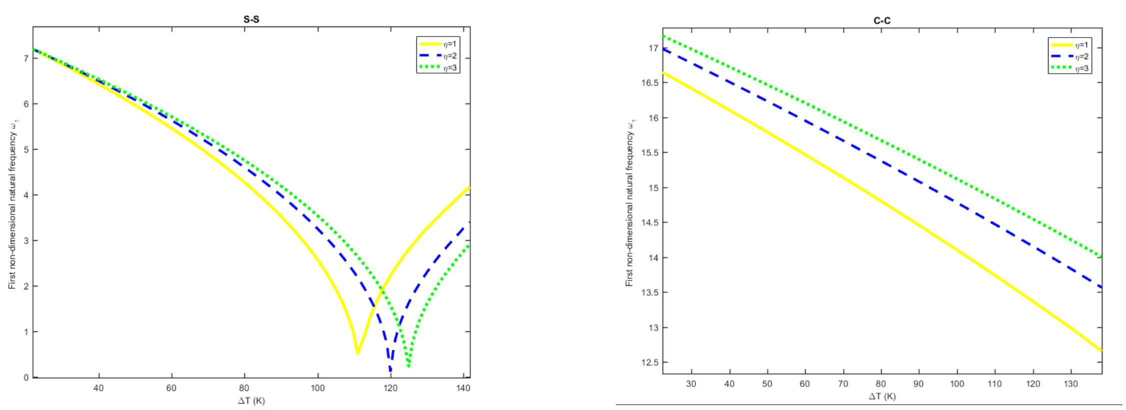
The Fig. 2 demonstrates the variation of the first non-dimensional natural frequency with respect to rises of temperature for different power index. It is observed that increasing the temperature leads to decreasing the non-dimensional natural frequency until critical temperature. It is shown in both boundary conditions before the critical temperature by increasing the power index natural frequency decrease. It can be observed after the critical temperature, that the natural frequencies for the *S-S* case increase with temperature.



(a) (b)

Fig.2
The variation of the first non-dimensional natural frequency with respect rises of temperature for different power index.

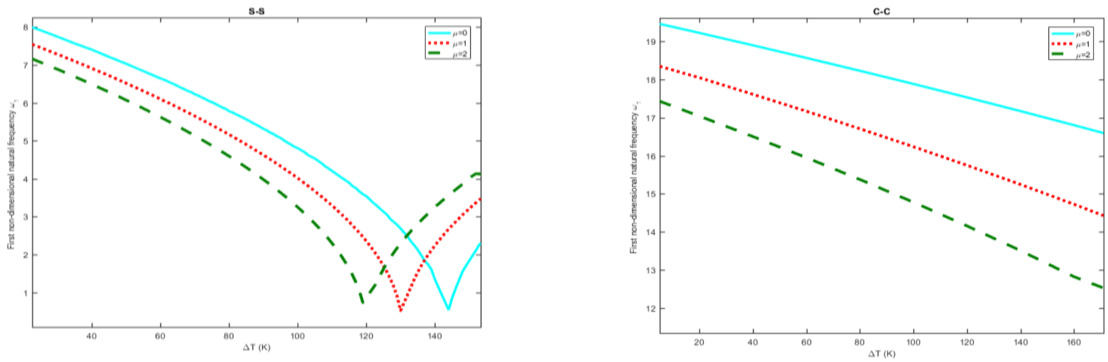
In Fig. 3, It can be shown by rising the temperature, natural frequency tends to decrease until the critical temperature. It can be conclude that there is a direct relation between the η parameter and natural frequency before critical temperature. It is shown in the *S-S* case for lower η parameter the critical temperature is lower. It can observe in the *C-C* case and for the *S-S* case before critical temperature, by increasing η parameter the diagram slope decreases. This figure shows as the temperature increases, the dependence of the frequency and parameter η increases.



(a) (b)

Fig.3
Variation of the first non-dimensional natural frequency with rises of temperature for different η parameter.

The Fig. 4 shows us the changes of the first non-dimensional natural frequency for the different μ parameter versus rises of temperature. As is shown by increasing the temperature natural frequency tends to decrease and we can see in the *C-C* case and in the *S-S* case before the critical temperature in the same temperature the higher μ parameter has a lower frequency. In *c-c* case, it can be observed the diagram slope has a direct relation with μ parameter.

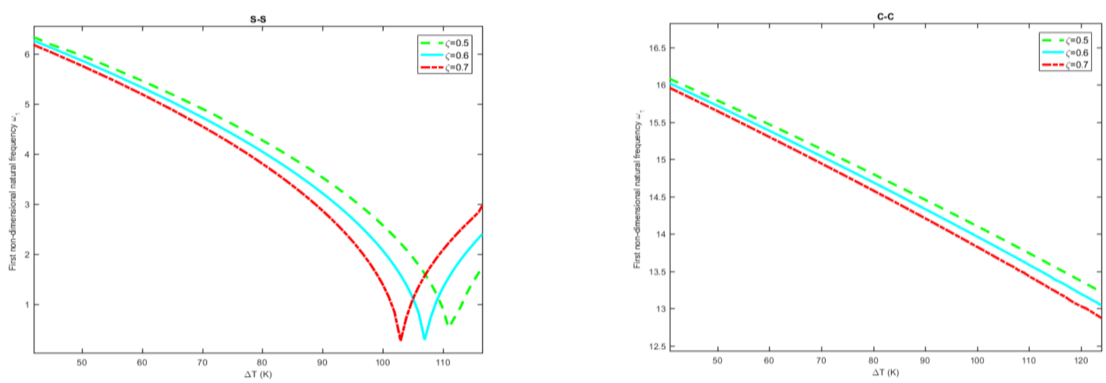


(a) (b)

Fig.4
The changes of the first non-dimensional natural frequency for different μ parameter versus rises of temperature.

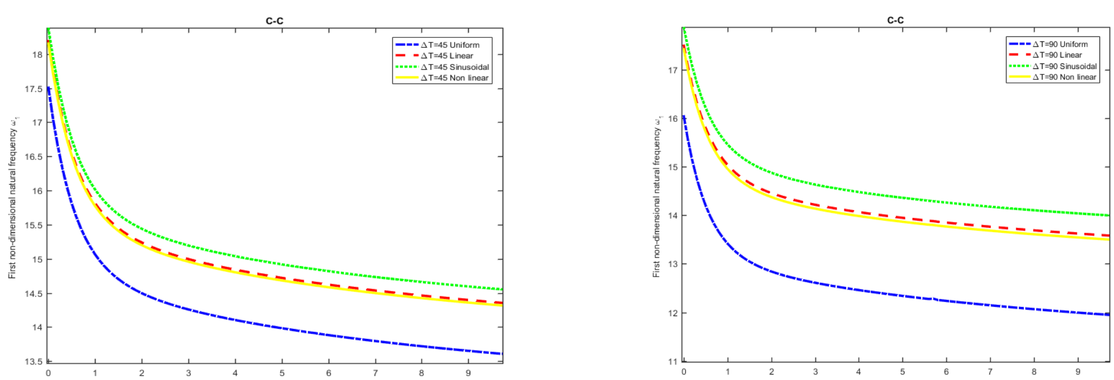
The Fig. 5 demonstrates the relationship between temperature and natural frequency for a different ξ parameter. As is shown by rising the temperature, natural frequency tends to zero before the critical temperature and by decreasing the ξ parameter natural frequency increase .it can be observed increasing the temperature would increase the difference between the values of the natural frequency for the different ξ parameter.

The Fig. 6 presents the variation of natural frequency with power index for the different functions of variation temperature. It can be concluded there is an inverse relationship between the variation of power index and natural frequency for different temperature changes and it can be observed for the all temperature distribution types with the same variation of the power index, variation of the frequency will be the same. The lowest frequencies value is related to the uniform temperature rise (UTR) and in this case, the critical condition is happened when the amount of the power index is about 7.8.

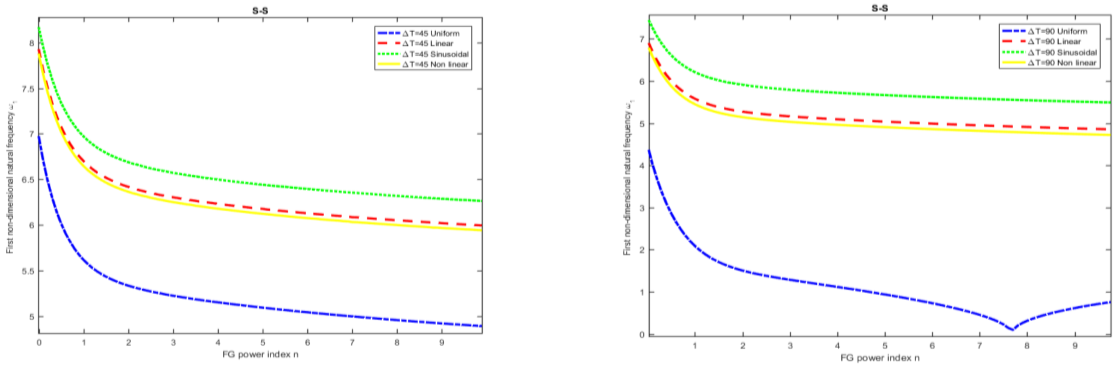


(a) (b)

Fig.5
Effect of the variation of temperature in natural frequency for different ξ parameter.



(a) (b)



(c) (d)

Fig.6
Variation of natural frequency with power index for different function of variation temperature rise.

10 CONCLUSION

This paper investigated the thermo-mechanical behavior of two-directional FGM (2-FGM) Nano beam based on nonlocal elasticity theory (NET). Adopting nonlocal elasticity theory to capture the nonlocal Eringen parameter of Nano beam, the governing equations of the motion are derived via Hamilton's principle and solved with generalized differential quadrature method (DQM). Mechanical attributes of the two-directional FGM (2-FGM) Nano beam are dependent based on power-law along the thickness and length directions of Nano beam. Provided results display the effect of the coefficients of two-directional FGM (2-FGM), FG power index, differential temperature, nonlocal Eringen parameter, coefficients (ξ,η) of two-directional FGM on the first non-dimensional natural frequencies of two-directional FGM (2-FGM) Nano beam. It is seen that the one-directional FGM power index 'n', differential temperature, two-directional FGM parameter (ξ) and nonlocal Eringen parameter μ yields in a reduction in rigidity of the nanobeam and first non-dimensional natural frequency. While, the rigidity of the two-directional FGM (2-FGM) Nano beam and non-dimensional the natural frequency results increase with the rise of the two-directional FGM parameter η. Also the value of the non-dimensional frequency of Nano beam is highest for sinusoidal temperature rise (STR) and it is lowest for Uniform temperature rise (UTR).

APPENDIX

Fig 1(a) shows a schematic of the bi-directional FGM (2-FGM) Nano beam subjected to the thermal load. L and h represent the length and the thickness of Nano beam respectively.

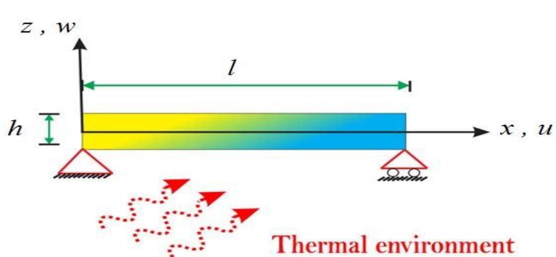


Fig.1(a)
The schematic of the two-directional FGM (2-FGM) Nano beam.

Characteristics of a two-dimensional FGM Nano beam such as , mass density ρ Young's modulus E, thermal expansion η and Poisson ratio v alter through the thickness and also in radius direction with respect to the volume fraction of the constituent, V_i, as below [67]:

$$P(r,z) = (\mathrm{P}_t \mathrm{V}_c(\mathrm{z}) + \mathrm{P}_b \mathrm{V}_m(z))(1+\xi(\frac{r}{R})^{\eta}) \tag{A.1}$$

where the V_m and Vc are the volume fraction of the metal and ceramic constituent respectively, the subscripts t and b denote the top and bottom surfaces of the plate which are made of metal and ceramic respectively. η and ξ are the power and the coefficients of the changes in material character, in radial-direction. It is worth mentioning that:

$$V_c(z)+V_m(z)=1 \tag{A.2}$$

Hence, Eq. (2) can be rewritten as below:

$$P(r,z)=((\mathrm{P}_t-\mathrm{P}_b)\,\mathrm{V}_c(z)+P_b)(1+\xi(\frac{x}{L})^{\eta}) \tag{A.3}$$

The ceramic volume fraction, V_c, would be defined by the following power law formula by setting the center of the coordinate system in the center of the plate (as shown in Fig. 1(a)):

$$V\,(r,\mathrm{z})=(\frac{1}{2}+\frac{z}{h})^{n} \tag{A.4}$$

Consequently, it can be shown that Young's modulus, Poisson ratio, mass density, and thermal expansion of the FGM are as below:

$$E(r,z)=((E_c-E_m)(\frac{1}{2}+\frac{z}{h})^{n}+E_m)(1+\xi(\frac{x}{L})^{\eta}) \tag{A.5}$$

$$\nu(r,z)=((\nu_c-\nu_m)(\frac{1}{2}+\frac{z}{h})^{n}+\nu_m)(1+\xi(\frac{x}{L})^{\eta}) \tag{A.6}$$

$$\rho(r,z)=((\rho_c-\rho_m)(\frac{1}{2}+\frac{z}{h})^{n}+\rho_m)(1+\xi(\frac{x}{L})^{\eta}) \tag{A.7}$$

$$\beta(r,z)=((\beta_c-\beta_m)(\frac{1}{2}+\frac{z}{h})^{n}+\beta_m)(1+\xi(\frac{x}{L})^{\eta}) \tag{A.8}$$

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