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Research Paper

# Characterization of the Viscoelastic *Q*-Factor of Electrically Actuated Rectangular Micro-Plates

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**ABSTRACT**

Regarding the necessity of designing high *Q*-resonators in micro electromechanical systems, this paper investigates the viscoelastic behavior of a rectangular micro-plate subjected to electrical actuation. Equations governing the vibrations of the homogeneous plate were obtained on the basis of classical plate theory (Kirchhoff's model). The Kelvin–Voigt model was also employed to consider the viscoelastic properties. The Galerkin decomposition method was used for decomposition of the governing differential equations. Additionally, the effects of various parameters were investigated on the *Q*-factor. Furthermore, a Finite Element simulation is carried out using COMSOL Multiphysics. The verification of the proposed model was conducted by comparing the obtained results with those from previous studies which revealed the validity of the proposed approach and the accuracy of the assumptions made The suggested design approach proposed in this this study is expected to design high *Q*-factor micro resonators and may be used to improve the performance of many MEMS devices. 

**Keywords :** Electrical actuation; Micro-plate; MEMS; *Q*-factor; Viscoelasticity.

## 1 INTRODUCTION

MICRO Electro Mechanical Systems (MEMS) have experienced rapid growth and received considerable attention since the early 1980s. Light weight, small dimensions, low energy consumption, and durability of MEMS make them even more attractive. In contrast to electronic components that are produced by IC fabrication procedures, MEMS components are made by micromachining. The micro-machine technology has enabled the production of sub-millimeter devices such as accelerometers, microscopic motors, and gears. Several applications could be named for these systems, including micro-pumps, vehicle airbag accelerometers, and inkjet printer heads [1-3]. Regarding to its excellent physical and mechanical properties, silicon is often used in MEMS in the form of micro-beams and micro-plates [4]. Vibrating micro-plates are used as resonators in MEMS devices [5,6]. It is of utmost significance in these applications that the microstructure features a high *Q*-factor (low damping) [7]. Several mechanisms are at play in the damping of structural vibrations, which most of them can be canceled through proper

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design. For example, the thin film damping effect can be minimized by increasing the gap length and reducing the applied pressure on the microstructure. However, intrinsic mechanisms are more difficult to control because they depend primarily on the material and geometric properties of structures [8,9]. Thermoelastic and viscoelastic damping are the most notable types of intrinsic damping. Using a Galerkin approach Vogl and Nayfeh [10] developed a reduced-order model for resonant frequency the circular micro-plates. Zhao et al [11] studied the linear vibration characteristics of clamped rectangular micro-plates using a reduced-order model with the stretching of middle surface taken into account. Based on a non-classical continuum mechanics approach, Murmu and Pradhan [12] studied the small-scale effects on the free vibrations of nano-plates. To investigate the temperature effects, Talebian et al [13] studied small-amplitude vibrations of a rectangular micro-plate under an applied electrostatic force using the Kirchhoff theory. The results were suggestive of the significant effects of temperature variations and residual stress on the pull-in voltage and resonant frequencies. Saghir and Younes [14] investigated both the static and dynamic behaviors of electrically actuated rectangular micro-plates using a reduced-order model. Li et al [15] developed a closed-form approach for the resonant frequency analysis of electrically actuated rectangular micro-plates. Grover and Seth [16] studied the visco-thermoelastic behavior of circular micro-plate resonators. They obtained an analytical expression for thermoelastic damping based on dual phase-lagging heat conduction and Kelvin-Voigt model. Reviewing the open literature reveals that studies conducted on the linear or the nonlinear vibrations of electrically actuated micro-plates have employed elastic models and to the best of authors' knowledge, the viscoelastic *Q*-factor of the electro-actuated rectangular micro-plates has not been investigated yet. Considering the extensive researches dedicated to thermoelastic damping [17-21].

The present study aims to investigate the effect of pure viscoelastic damping on the vibrational behavior of rectangular micro-plates. The remainder of this paper is organized as follows. First, the governing equations are derived based on the Kirchhoff thin plate theory. Then the derived governing differential equations are discretized by the Galerkin method. Finally in the results section, following a validation, the effects of various design parameters are discussed.

## 2 MODELING

Fig. 1 shows a fully clamped, thin, viscoelastic, micro-plate with length $2a$, width $2b$, uniform thickness $h$ and gap $g$ which is subjected to the voltage $V(t) = V_{dc} + V_{ac}(t)$.

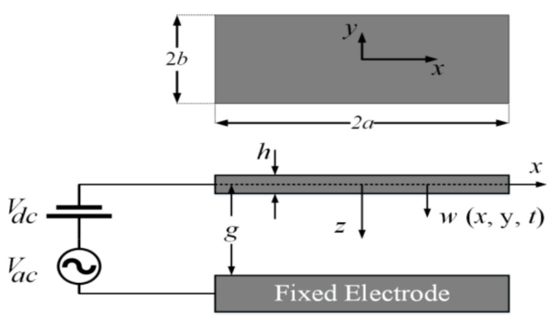



**Fig.1**
Schematic model of the micro-plate.

### *2.1 Governing equations*

The stress-strain relations for the elastic structure are presented in Eq. (1) [22,23].

$$\begin{aligned}\sigma_{xx} &= \frac{E}{1-\upsilon^2}\varepsilon_{xx} + \frac{\nu E}{1-\upsilon^2}\varepsilon_{yy}\\ \sigma_{yy} &= \frac{E}{1-\upsilon^2}\varepsilon_{yy} + \frac{\nu E}{1-\upsilon^2}\varepsilon_{xx}\\ \sigma_{xy} &= G\,\varepsilon_{xy}\end{aligned} \tag{1}$$

where $E$ is Young's modulus, $G$ is shear modulus and $\upsilon$ is the Poisson's ratio. The strain components are related to transverse displacement ($z$ direction) as:

$$\varepsilon_{xx} = -z\,\frac{\partial^2 w}{\partial x^2}$$
$$\varepsilon_{yy} = -z\,\frac{\partial^2 w}{\partial y^2} \tag{2}$$
$$\varepsilon_{xy} = -2z\,\frac{\partial^2 w}{\partial x\,\partial y}$$

Eq. (3) presents the viscoelastic stress-strain relations in the form of the Kelvin–Voigt model [24,25].

$$\sigma_{xx} = \frac{E}{1-\upsilon^2}\left(\varepsilon_{xx} + \upsilon\varepsilon_{yy}\right) + \tau_c\,\frac{E}{1-\upsilon^2}\frac{\partial}{\partial t}\left(\varepsilon_{xx} + \upsilon\varepsilon_{yy}\right)$$
$$\sigma_{yy} = \frac{E}{1-\upsilon^2}\left(\varepsilon_{yy} + \upsilon\varepsilon_{xx}\right) + \tau_c\,\frac{E}{1-\upsilon^2}\frac{\partial}{\partial t}\left(\varepsilon_{yy} + \upsilon\varepsilon_{xx}\right) \tag{3}$$
$$\sigma_{xy} = G\,\varepsilon_{xy} + \tau_c G\,\frac{\partial \varepsilon_{xy}}{\partial t}$$

Here $t$ is time and $\tau_c$ is the mechanical relaxation time in the Kelvin–Voigt model. The bending moments are obtained by integrating the moments of the in-plane stresses over the plate thickness. That is,

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}\right) - \tau_c D\,\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}\right)$$
$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \upsilon\frac{\partial^2 w}{\partial x^2}\right) - \tau_c D\,\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial y^2} + \upsilon\frac{\partial^2 w}{\partial x^2}\right) \tag{4}$$
$$M_{xy} = -D\left(1-\upsilon\right)\frac{\partial^2 w}{\partial x\,\partial y} - \tau_c D\left(1-\upsilon\right)\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)$$

where $D$ represents the flexural rigidity of the plate obtained as follows:

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \tag{5}$$

The transverse shearing forces are as follows:

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}$$
$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \tag{6}$$

Therefore, the vibration equation of the micro-plate could be obtained as:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + f^e\left(x, y, t\right) = \rho h\,\frac{\partial^2 w}{\partial t^2} \tag{7}$$

where $f^e$ is the electrical actuation force per unit area. Substituting Eq. (4) into Eq. (6) and then the resulting into Eq. (7), yields

$$D\left(\nabla^4 w+\tau_c \frac{\partial}{\partial t}\left(\nabla^4 w\right)\right)+\rho h \frac{\partial^2 w}{\partial t^2}=\frac{\varepsilon V^2(t)}{2(g-w)^2} \tag{8}$$

where

$$\nabla^4 w=\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4} \tag{9}$$

As shown in Fig. 1, the applied electric excitation, $V(t)$, applied to the micro-plate can be written as:

$$V(t)=V_{\mathrm{dc}}+V_{\mathrm{ac}}(t) \tag{10}$$

The deflection of the micro-plate is correspondingly composed of the static deflection induced by the DC voltage ($V_{\mathrm{dc}}$) and the small vibrations around the deflected state induced by the AC voltage ($V_{\mathrm{ac}}$). Therefore the deflection under the excitation voltage could be written as [15]

$$w(x,y,t)=w_{\mathrm{dc}}(x,y)+w_{\mathrm{ac}}(x,y,t) \tag{11}$$

where $w_{\mathrm{dc}}$ and $w_{\mathrm{ac}}$ are the static and dynamic deflections respectively. Substituting Eqs. (10) and (11) into Eq. (8) and expanding the electrostatic force term by Taylor-series at the static equilibrium, Eq. (8) could be decomposed into a static deflection governing equation and another dynamic governing equation as:

$$D\nabla^4 w_{\mathrm{dc}}=\frac{\varepsilon V_{\mathrm{dc}}^2}{2(g-w_{\mathrm{dc}})^2} \tag{12}$$

and

$$\begin{aligned} D\left(\nabla^4 w_{ac}+\tau_c \frac{\partial}{\partial t}\left(\nabla^4 w_{ac}\right)\right)+\rho h \frac{\partial^2 w_{ac}}{\partial t^2}&=\frac{\varepsilon V_{\mathrm{dc}}^2}{(g-w_{\mathrm{dc}})^3} w_{ac}+\frac{\varepsilon V_{dc}}{(g-w_{\mathrm{dc}})^2} V_{ac} \\ &+\frac{3\varepsilon V_{\mathrm{dc}}^2}{2(g-w_{\mathrm{dc}})^4} w_{ac}^2+\frac{2\varepsilon V_{\mathrm{dc}}^2}{(g-w_{\mathrm{dc}})^3} V_{ac} w_{ac}+\frac{\varepsilon}{2(g-w_{\mathrm{dc}})^2} V_{ac}^2+\cdots \end{aligned} \tag{13}$$

For small vibrations, the amplitude of the applied AC voltage is much smaller than that of the DC voltage. This indicates that $V_{\mathrm{ac}} \ll V_{\mathrm{dc}}$ and $w_{\mathrm{ac}} \ll w_{\mathrm{dc}}$. Based on this the higher order terms could be eliminated and the eigenvalue equation of the electrostatically actuated rectangular micro-plate is derived as:

$$D\left(\nabla^4 w_{ac}+\tau_c \frac{\partial}{\partial t}\left(\nabla^4 w_{ac}\right)\right)+\rho h \frac{\partial^2 w_{ac}}{\partial t^2}=\frac{\varepsilon V_{\mathrm{dc}}^2}{(g-w_{\mathrm{dc}})^3} w_{ac} \tag{14}$$

*2.2 Reduced order model*

The well-known Galerkin method is employed to decompose the deflection governing equation into a coupled set of algebraic equations. For the linear damped eigenvalue problem, we let

$$w_{dc}(x,y)=\sum_{n,m=0}^{N} C_{nm}\varphi_{nm}(x,y) \tag{15}$$

$$w_{ac}(x,y,t)=\sum_{n,m=0}^{N} C_{nm}\varphi_{nm}(x,y)e^{i\omega t} \tag{16}$$

where $C_{nm}$ is the unknown coefficient to be determined and $\varphi_{nm}$ is the undamped mode shape of fully clamped rectangular micro-plate. The expression for the mode shape is given by [26]

$$\varphi_{nm}=\cos^2\left(\frac{(2n+1)\pi x}{2a}\right)\cos^2\left(\frac{(2m+1)\pi y}{2b}\right) \tag{17}$$

Substituting Eq. (15) into (12) and Eq. (16) into (14), and then multiplying both sides of the resulting equation with $\varphi_{lr}$ and integrating the outcome over the area of the micro-plate, yields the following system of algebraic equations.

$$D\sum_{n,m=0}^{N} C_{nm}A_{nm}=\frac{\varepsilon V_{dc}^2}{\left(g-\sum_{n,m=0}^{N} C_{nm}A_{nm}\right)^2} \tag{18}$$

$$D(1+i\omega\tau_c)\sum_{n,m=0}^{N} C_{nm}A_{nm}-\rho h\omega^2\sum_{n,m=0}^{N} C_{nm}B_{nm}=\frac{\varepsilon V_{dc}^2}{(g-w_{dc})^3}\sum_{n,m=0}^{N} C_{nm}B_{nm} \tag{19}$$

where

$$\begin{aligned} A_{nm}&=\iint\left(\frac{\partial^4\varphi_{nm}}{\partial x^4}\varphi_{lr}+2\frac{\partial^4\varphi_{nm}}{\partial x^2\partial y^2}\varphi_{lr}+\frac{\partial^4\varphi_{nm}}{\partial y^4}\varphi_{lr}\right)dxdy \\ B_{nm}&=\iint\varphi_{nm}\varphi_{lr}\,dxdy \qquad n,m,l,r=0,1,2,3,\dots \end{aligned} \tag{20}$$

Once all the unknown coefficients $C_{nm}$ are determined by solving Eq. (18), the value of the static deflection $w_{dc}$ could be obtained by Eq. (15) and finally Eq. (19) could be solved. By considering the viscoelastic damping, resonant frequency will be a complex number. The real part of the complex frequency gives the eigen frequency in the presence of viscoelastic coupling, and the imaginary part gives the attenuation of the vibration. The amount of viscoelastic damping is characterized in terms of the *Q*-factor. This factor is defined as the ratio of the real and imaginary parts of the complex resonant frequency [27,17].

$$Q_{nm}=\frac{1}{2}\left|\frac{\mathrm{Re}(\omega_{nm})}{\mathrm{Im}(\omega_{nm})}\right|=\frac{1}{2\zeta_{nm}} \tag{21}$$

where $\zeta$ is damping ratio. Higher $Q$ indicates a lower rate of energy loss and the oscillations die out more slowly.

## 3 RESULTS

In this section as a case study, a fully clamped square micro-plate which is made of silicon is considered with the characteristics tabulated in Table 1 [13,28]. The results were calculated by writing program in MATLAB. In order to validate the proposed model, the obtained results were compared with those from previous literature.

As well as the presented theoretical solution, a non-linear Finite Element analysis has also been studied. The Finite Element Method (FEM) is a powerful numerical method in the analysis of MEMS. In this paper the COMSOL Multiphysics 5.5 package was used for the Finite Element simulation. The adopted model was "three-dimensional space dimension". Moreover, the physics was selected to be "Electromechanics," which helps COMSOL to identify the governing coupled electromechanical equations.

**Table 1**
Geometrical and mechanical properties [13,28]

| | |
|---|---|
| $2a$ ($\mu m$) | 250 |
| $2b$ ($\mu m$) | 250 |
| $h$ ($\mu m$) | 3 |
| $g$ ($\mu m$) | 1 |
| $E$ ($GPa$) | 250 |
| $\rho$ ($kg/m^3$) | 2331 |
| $\tau_c$ ($s$) | Order of $1\times10^{-10}$ |
| $\varepsilon$ ($F/m$) | $8.8541878\times10^{-12}$ |
| $\upsilon$ | 0.06 |

The variation of the un-damped resonant frequency corresponding to the first mode shape versus the applied voltage is depicted in Fig. 2. As shown in Fig. 2, the natural frequency is reduced when the applied voltage is raised which means the stiffness is decreasing. Fig. 3 shows a comparison of the static mid-point deflection under different voltages. Based on the Figs. 2 and 3, it is obvious that the calculated values agree well with the previously published results [13] which demonstrates the validity of the proposed approach and the accuracy of the assumptions made. The three-dimensional illustrations of the first mode shape of un-damped and damped (considering viscoelastic behavior) vibrations are shown in Fig. 4. For this Figure the applied voltage is zero.

The time histories and phase portraits of the considered micro-plate for two different step DC voltages are plotted in Fig. 5. As illustrated in this figure, the system becomes unstable for some step DC voltages ($V_{dc}$ = 53.6 $V$) smaller than the value of the static pull-in voltage ($V_{dc}$ = 62 $V$). This phenomenon is called dynamic pull-in and the critical (minimum) step DC voltage, in which the dynamic pull-in phenomenon happens, is called dynamic pull-in voltage. The dynamic pull-in voltages are about 90% of the static pull-in voltages [29], which is consistent with the obtained results. Fig. 6 illustrates the long time oscillation of the micro-plate due the suddenly applied step DC voltage $V$ = 50 $V$. Since the vibration of the micro-plate weakens so slowly that it is hardly distinguished from the curve in Fig. 6, the difference between the deflections with and without consideration of viscoelastic damping, $\Delta w_{max}$, is shown in Fig. 7.

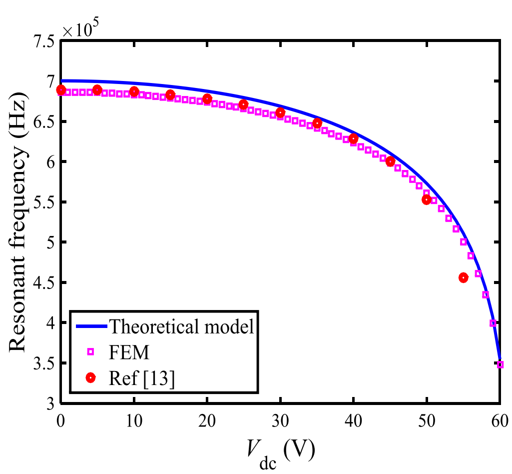



**Fig.2**
The variation of the first un-damped resonant frequency $\omega_{11}$ versus applied voltage $V_{dc}$.

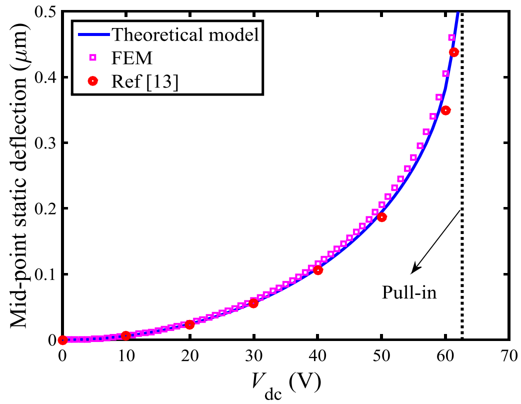
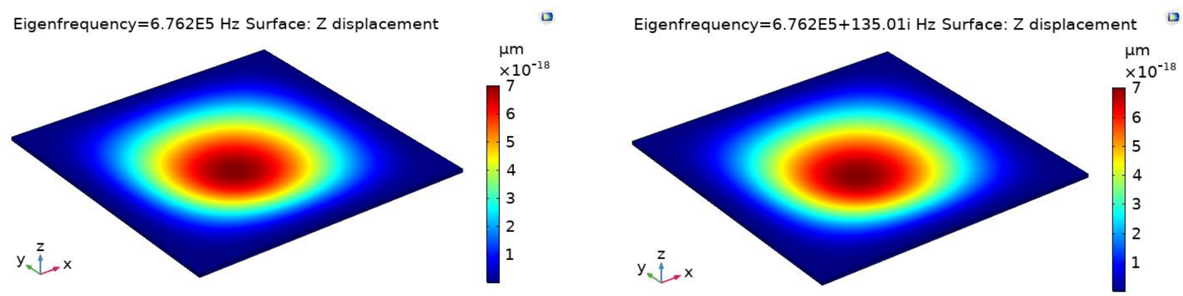
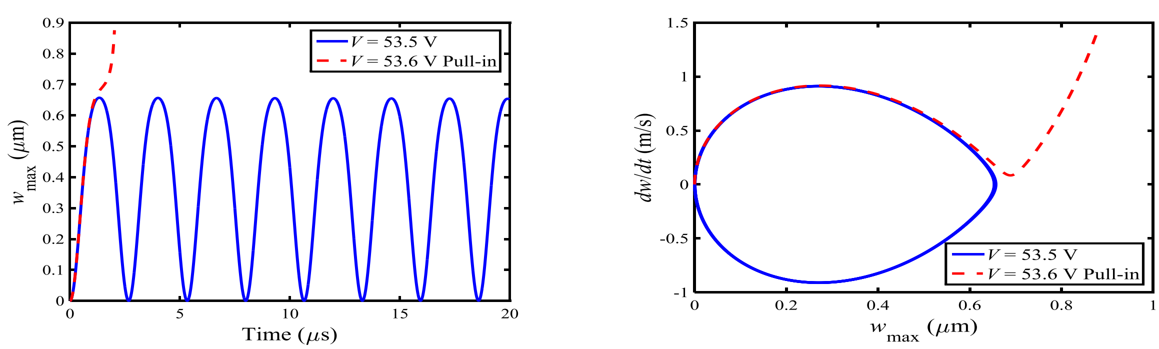



**Fig.3**
The variation of the static deflection $w_{dc}$ versus applied voltage $V_{dc}$.




(a) (b)

**Fig.4**
First mode shape of the micro-plate without electric actuation, (a) un-damped vibration, (b) damped vibration.




(a) (b)

**Fig.5**
Dynamic response of the micro-plate, (a) time history, (b) phase portrait.

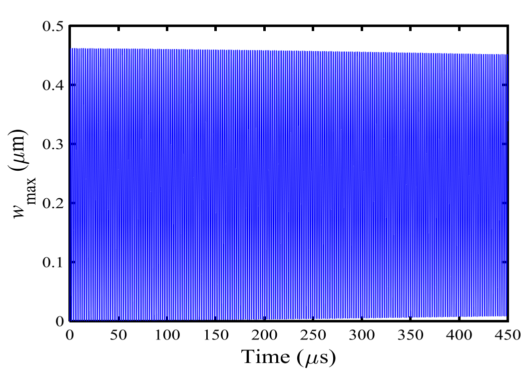



**Fig.6**
Dynamic response of the micro-plate due to the suddenly applied voltage $V = 50$ $V$ for long period of time.

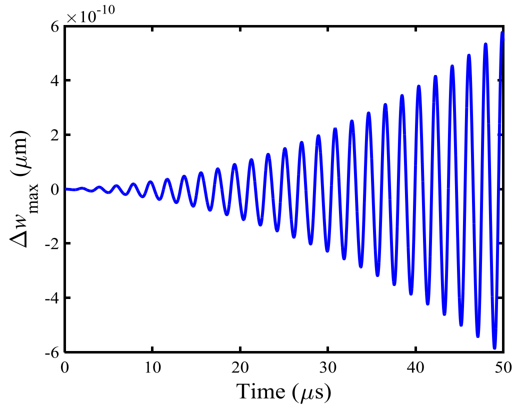



**Fig.7**
The displacement difference with and without consideration of viscoelastic damping.

In order to investigate the effect of the viscoelastic damping, Figures 8-11 show the variation of the first quality factor as functions of the various design parameters. For each case the remaining parameters are the same as Table 1.

Fig. 8 shows the $Q$-factor with various values of the thickness $h$. It is observed that the $Q$-factor decrease when the thickness increases and this is due to the increasing of flexural rigidity $D$. As the applied voltages for each thickness are not close to the pull-in point, therefore their effect are not considerable on the $Q$-factor.

Fig. 9 shows the effect of the gap size on the $Q$-factor. The value of the pull-in voltage increases when the gap size increases. Therefore in Fig. 9 at first the $Q$-factor rapidly increases. Then its changes become very slowly as the pull-in voltage becomes much greater than the considered voltage.

Figs. 10 and 11 respectively show the $Q$-factor variation with applied voltages for micro-plates with the length to width ratios, $a/b$, and gap size to plate thickness ratios, $g/h$. For each case the applied voltage varies from 0 to the pull-in point. It is evident that as the applied voltage gets closer to the pull-in point the $Q$-factor drops more rapidly. It means that the stiffness of the micro-plate is softened.

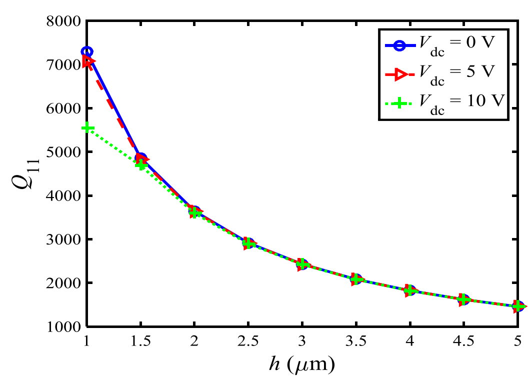



**Fig.8**
The variation of the $Q$-factor versus thickness of the micro-plate $h$.

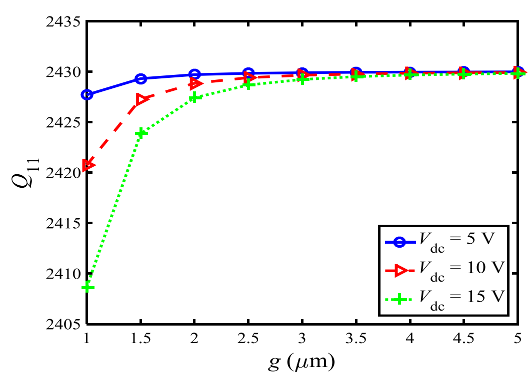



**Fig.9**
The variation of the $Q$-factor versus gap $g$.

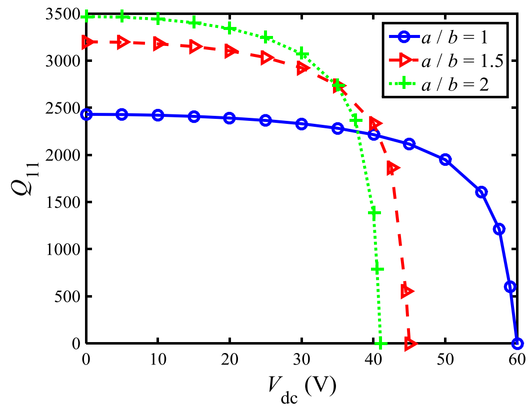



**Fig.10**
The variation of the $Q$-factor versus applied voltage for different length to width ratios $a/b$.

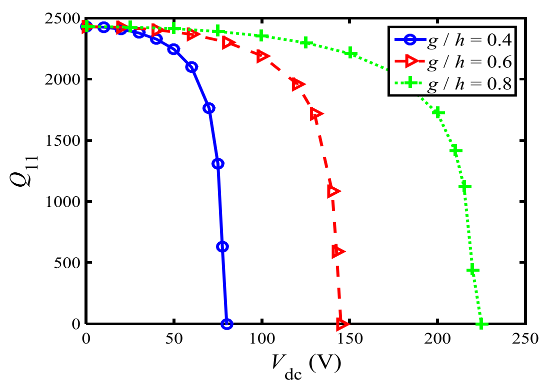



**Fig.11**
The variation of the $Q$-factor versus applied voltage for different gap to thickness ratios $g/h$.

## 4 CONCLUSION

In this study, a reduced-order model was established for the pure analysis of viscoelastic $Q$-factor of electrically actuated micro-plates. The Kirchhoff plate theory and the Kelvin–Voigt damping model were employed to derive the governing differential equations. The reduced order model was obtained with the aid of the Galerkin technique. The obtained results from the proposed semi-analytical solution were well validated by FEM simulations and literature. Further the effects of various design parameters were investigated.

For the considered case study it was found out that, the viscoelastic damping is a very weak damping mechanism and the decay of displacement takes place slowly. Moreover, the obtained results depicted that the $Q$-factor significantly depends on the applied voltage especially when the voltage becomes closer to the pull-in point, while its effect is not significant for voltages far from the pull-in limit.

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