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Investigation of Strain Gradient Theory for the Analysis of Free Linear Vibration of Nano Truncated Conical Shell

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ABSTRACT

In this paper the nano conical shell model is developed based on modified strain gradient theory. The governing equations of the nano truncated conical shell are derived using the FSDT, and the size parameters through modified strain gradient theory have been taken into account. Hamilton's principle is used to obtain the governing equations, and the shell's equations of motion are derived with partial differentials along with the classical and non-classical boundary conditions. Galerkin's method and the Generalized Differential Quadrature (GDQ) approach are applied to obtain the linear free vibrations of the carbon nano cone (CNC). The CNC is studied with simply supported boundary condition. The results of the new model are compared with those of the classical and couple stress theories, which point to the conclusion that the classical and couple stress models are special cases of modified strain gradient theory. Results also reveal that rigidity of the nano truncated conical shell in the strain gradient theory is greater than that in the modified couple stress and classical theories respectively, which leads to an increase in dimensionless natural frequency ratio. Moreover, the study investigates the effect of the size parameters on nano shell vibration for different lengths and vertex angles.

Keywords: GDQ method; Galerkin's method; Strain gradient theory; Carbon nano cone (CNC).

1 INTRODUCTION

THE use of thin walled conical shells is of much importance in a number of different branches of engineering. In aerospace engineering, such structures are used for aircraft and satellites. In ocean engineering, they are used for submarines, torpedoes, ballistic missiles, and offshore drilling rigs, while in civil engineering conical shells are used in containment vessels in tanks. Therefore, the vibration characteristics of conical shells must be studied for

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safety and stability reasons. Intense dependency of material elastic constants on structural dimension at the micro/nano-scale has been indicated by experimental research. The stiffness and resistance of the material increases by size effect, which is size reduction as a result of a scale down in structure dimensions. Although the focus of the study of nano shells is on nano dimensions, a precise anticipation of structural behavior is not feasible by classical theory. Since considering the small-scale size effects are not possible by the classical theory of continuum, we should apply higher order continuum theories. These theories include nonlocal elasticity theories, the modified couple stress theory (MCST) and the strain gradient theory (SGT). Lam *et al*. [24] developed the modified strain gradient theory. This theory provides a more comprehensive reflection of the size effects on microstructures. It encompasses three material length scale parameters corresponding to the dilatation gradient vector, the deviatory stretch gradient tensor and the curvature tensor. Field of non-classical shell has witnessed the accomplishment of numerous studies. In this paper, the first-order shear deformable model for the nano-conical shell is developed for the first time using the strain gradient theory. Firouz-Abadi *et al*. [1] have studied the free vibration of the cone shell investigated on the basis of non-local theory. Stability of the nano scale cone under external pressure and compressive axial force have been investigated by Firouz-Abadi *et al.* [2] based on non-local theory. Fotouhi *et al* .[3] also investigated the free vibration of the nano-cone shell in an elastic substrate based on non-local theory. Elastic foundation was simulated using Winkler and Pasternak models and motion equations based on the modal analysis technique and using the Galerkin' method is obtained and solved. The results of this study emphasize the effects of geometry and scale parameter on the natural frequency of the nano-shell cone. Tadi Beni *et al*.[4] examined the torsional vibrations of the nano-cone shell based on the modified couple-stress theory. In this study, the results of the dimensionless frequency of the cone shell are compared with the couple-stress theory and the classical theory. It is clear that the rigidity of nano-shells in couple-stress model is more than classical theory. Zeighampour *et al.*[5] investigated shear deformable conical shell formulation in the framework of couple stress theory using the first-order shear deformable shell model, in order to obtain the governing equations, Hamilton's principle is used and the equations of shell motion with partial differentials are derived along with classical and non-classical boundary conditions. Finally, the free vibration of the single-walled carbon nano cone (SWCNC) is scrutinized through examples. The SWCNC is modeled as simply supported, and the Galerkin' method is used to solve the vibration problem. Analysis of the thin conical shell was investigated by Zeighampour and Y. Tadi Beni [6] based on the modified couple stress theory using the classic shear deformable shell model, the equations of motion with partial differentials and classical and non-classical boundary conditions are derived using Hamilton's principle. This non-classical formulation can incorporate size effects in nano/micro scales. The free vibrations of the single-walled carbon nano cone (SWCNC) are examined as a special case. The SWCNC is modeled as simply supported, and the Galerkin' method is used to solve the vibrational problem. Results of the new formulation are compared to the classical theory. Sofiyev [7] investigated the FGM truncated conical shells in a nonlinear dynamics study, taking into account the theory of large deformations and the nonlinear kinetic model of van Karman-Donald, using the principle of super positions and Galerkin and Hamilton's method and Harmonic balance method, the problem of non-linear vibration of the FG truncated conical shell surrounded by an elastic medium is solved.Finally, the effect of shell structure on the nonlinear frequency parameter and the ratio of nonlinear frequency variations to linear frequency was studied.

In another study, Sofiyev and Kuruoglu [8] examined the natural frequencies of orthotropic multilayer shells under different boundary conditions on the elastic substrate. Sofiyev and Kuruoglu [9] also investigated the nonlinear buckling of cone shells from elastic foundation. Sofiyev [10] in a study investigated the free vibration with a large domain of cone shells from composite orthotropic materials. The bifurcation and vibration responses of a composite truncated conical shell with embedded single-walled carbon nano tubes (SWCNTs) subjected to an external pressure and axial compression simultaneously was investigated by M. Mehri *et al.*[11]. The equations of motion are established using Green-Lagrange type nonlinear kinematics within the framework of Novozhilov nonlinear shell theory. Linear membrane pre buckling analysis is conducted to extract the pre buckling deformations. A semi-analytical solution on the basis of the trigonometric expansion through the circumferential direction along with the harmonic differential quadrature (HDQ) discretization in the meridional direction is developed. A series of comparison studies are carried out to assure the accuracy and the convergence of the HDQ method. The research indicates that the superb accuracy and efficiency of solutions with few grid points are attributed to the higher-order harmonic approximation function in the HDQ method. Ansari *et al.*[12] investigated the free vibration of carbon nano cones (CNCs) under different types of boundary conditions. The Donnell shell theory and nonlocal elasticity are used to derive the governing equations of motion. The analytical Galerkin' method together with beam mode shapes as weighting functions are employed to solve the problem. Making use of the beam modal functions enables us to examine the role of boundary condition in the vibrational behavior of CNCs. Kamarian *et al*.[13] investigated the free vibration analysis of Carbon Nano tube-Reinforced Composite (CNTRC) conical shells is performed considering the agglomeration effect of Carbon Nano tubes (CNTs). The material properties of the nano composite conical shell are estimated employing the Eshelby-Mori-Tanaka approach based on an equivalent fiber assumption. The equations of motion are derived based on the First-order Shear Deformation Theory (FSDT). The Generalized Differential Quadrature (GDQ) technique is originally implemented to solve the governing equations of the problem and to obtain the natural frequencies of the structures, since it has proven to be an efficient and accurate numerical tool. A parametric study is herein developed to investigate the influence of some characteristic parameters on the vibrational behavior of the CNTRC conical shell, *e.g*. The CNTs volume fraction and agglomeration, or the boundary conditions and geometrical parameters like the thickness to radius ratio. The non-linear vibration of truncated conical shells made of functionally graded materials (FGMs) was investigated by Sofiyev *et al*. [14] by using the large deformation theory and von Karman–Donnell-type of kinematic non-linearity. The material properties of FGMs are assumed to vary continuously through the thickness of the shell. The fundamental relations, the non-linear motions and compatibility equations of the FGM truncated conical shell were derived, by using Superposition' method, Galerkin' method and Harmonic balance method. Tohidi *et al*. [15] investigated the forced vibration and nonlinear buckling of cylindrical shell nano shells under different underlying conditions. The vibratory behavior of cylindrical shells was investigated by Zeighampour *et al*.[16] based on the modified strain gradient theory. The governing equations are based on the vibration of Navier's procedure. Tadi Beni *et al*.[17] investigated the free vibrations of the shear modulator cylindrical shell have performed the shear deformation based on coupling-stress theory. Golami *et al*.[18] studied the vibration and buckling of the deformable circular cylindrical nano and micro circular shells based on the Mindelin strain theory with first-order theory. B. Zhang *et al*.[19] studied the free vibration analysis of four -unknown shear deformable functionally graded cylindrical micro shells based on the strain gradient elasticity theory . Bakhtiari *et al*.[20] investigated the formulation of nonlinear kinematics of shells in three different shell theories namely Donnell, Sanders and Nemeth including shear deformation for anisotropic materials.

A finite element solution for the equilibrium equation of Sander's improved first-approximation theory is developed and has been used to develop the nonlinear finite element amplitude equation of vibration of conical shells of Donnell, Sanders and Nemeth theories using generalized coordinates methods and Lagrange equations of motions. The amplitude equation of nonlinear vibration of conical shell has been solved for multiple cases of isotropic materials with neglecting the shear deformation. Tohidi *et al*.[[21] studied the dynamic stability of FG-CNT-reinforced viscoelastic micro cylindrical shells resting on nonhomogeneous orthotropic viscoelastic medium subjected to harmonic temperature distribution and 2D magnetic field. Ghadiri *et al*.[22], studied the nonlinear bending vibration of a rotating Nano beam based on nonlocal Eringen's theory using differential quadrature method. Malekzadeh *et al*.[23] investigated the nonlinear free flexural vibration of skew nano plates by considering the influences of free surface energy and size effect (small scale) simultaneously based on classical plate theory (CPT) using Hamilton's principle and Green's strain tensor together with von Kármán. The solution algorithm is based on the transformation of the governing differential equation from the physical domain to a rectangular computational one, and discretization of the spatial derivatives by employing the differential quadrature method (DQM).

2 PROBLEM DESCRIPTION

Fig.1 shows the nano conical shell, where *R(a), h* and *2α* are minimum radius, thickness and vertex angle, respectively.

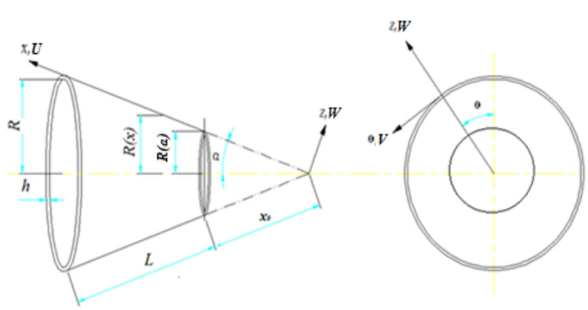

Fig.1
Coordinates and displacements of nano cone shell.

2.1 Displacement field in the nano cone shell

Based on the first-order shear deformable shell model, displacement components of an arbitrary point can be written as follows (Reddy [25]):

$$U\left(x,\theta,z,t\right)=u_{0}\left(x,\theta,t\right)+z\phi\left(x,\theta,t\right), \tag{1}$$

$$V\left(x,\theta,z,t\right)=v_{0}\left(x,\theta,t\right)+z\psi\left(x,\theta,t\right), \tag{2}$$

$$W\left(x,\theta,z,t\right)=w_{0}\left(x,\theta,t\right), \tag{3}$$

In the above equations, u_0 , v_0 and w_0 stand for the displacement vector in the middle surface of the nano conical shell, and t represents time. In addition, ϕ and ψ are rotations around the x and θ axes so, the strain displacements for nano truncated conical shell are (Zeighampour [5]):

$$\varepsilon_{xx}=\frac{\partial u_0}{\partial x}+z\frac{\partial \phi}{\partial x}, \tag{4a}$$

$$\varepsilon_{\theta\theta}=\frac{\frac{\partial v_0}{\partial \theta}}{x sin\alpha}+z\frac{\frac{\partial \psi}{\partial \theta}}{x sin\alpha}+\frac{u_0}{x}+z\frac{\phi}{x}+\frac{w_0 cos\alpha}{x sin\alpha}, \tag{4b}$$

$$\varepsilon_{\theta z}=\frac{1}{2}\left(\frac{\frac{\partial w_0}{\partial \theta}}{x sin\alpha}+\psi-\frac{v_0 cos\alpha}{x sin\alpha}\right), \tag{4c}$$

$$\varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial w_0}{\partial x}+\phi\right), \tag{4d}$$

$$\varepsilon_{x\theta}=\frac{1}{2}\left(\frac{\partial v_0}{\partial x}+z\frac{\partial \psi}{\partial x}-\frac{v_0}{x}+z\frac{\psi}{x}+\frac{\frac{\partial u_0}{\partial \theta}}{x sin\alpha}+z\frac{\frac{\partial \phi}{\partial \theta}}{x sin\alpha}\right) \tag{4e}$$

2.2 Governing motion equations

In this section, the equations of motion are structured using the energy method and based on the Hamilton' principle, which can be described as follows.

$$\delta\int_{t_1}^{t_2}(T-U+W)dt=0 \tag{5}$$

In above equation T represents the kinetic energy; U and V are the total potential energy and W is the external work. In this paper W is zero. In addition, δ specifies the variation operator. In accordance with strain gradient theory, potential strain energy is stored in structure; it can be obtained as follows (Lam *et al*. [24]; Zhang *et al*. [19]):

$$U = \frac{1}{2}\int_{\Omega} (\sigma_{ij}\varepsilon_{ij} + P_i\gamma_i + \tau^{(1)}_{ijk}\eta^{(1)}_{ijk} + m^{s}_{ij}\chi^{s}_{ij})d\Omega \tag{6}$$

In the above relation ε_{ij} ، γ_i ، $\eta_{ijk}^{(1)}$ and χ_{ij} indicate the strain tensor, the dilatation gradient vector, the deviatory stretch gradient tensor and the symmetric rotation gradient tensor, respectively and are defined by the following relations

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{7}$$

$$\gamma_i = \varepsilon_{mm,i} \tag{8}$$

$$\eta^{(1)}_{ijk} = \frac{1}{3}\left(\varepsilon_{ij,k} + \varepsilon_{jk,i} + \varepsilon_{ki,j}\right) - \frac{1}{15}\left(\delta_{ij}\varepsilon_{mm,k} + 2\varepsilon_{mk,m}\right) - \frac{1}{15}\left(\delta_{jk}\varepsilon_{mm,i} + 2\varepsilon_{mi,m}\right) - \frac{1}{15}\left(\delta_{ki}\varepsilon_{mm,j} + 2\varepsilon_{mj,m}\right) \tag{9}$$

$$\chi^{s}_{ij} = \frac{1}{4}(\varepsilon_{ipq}\eta_{jpq} + \varepsilon_{jpq}\eta_{ipq}) \tag{10}$$

That ‘‘,’’, u_i and δ_{ij} specifies the partial derivative, displacement vector and the Kronecker delta respectively. The nonzero components of the dilatation gradient vector, the deviatory stretch gradient tensor and the symmetric rotation gradient tensor can be calculated. Besides, the classical stress tensor σ_{ij} and the higher-order stresses containing P_i, $\eta^{(1)}_{ijk}$ and m^{s}_{ij} can be given by the following relations (Zhang *et al.* [19])

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} \tag{11}$$

$$P_i = 2\mu l_0^2\gamma_i \tag{12}$$

$$\tau^{(1)}_{ijk} = 2\mu l_1^2\eta^{(1)}_{ijk} \tag{13}$$

$$m^{s}_{ij} = 2\mu l_2^2\chi^{s}_{ij} \tag{14}$$

In which l_0, l_1 and l_2 are the material length scale parameters; λ and μ define the bulk and shear modulus which can be denoted in terms of Young's modulus (E) and Poisson's ratio (υ) as:

$$\lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)} \tag{15}$$

$$\mu = \frac{E}{2(1+\upsilon)} \tag{16}$$

The kinetic energy of the nano cone can be calculated using the following equation:

$$T = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{0}^{2\pi}\int_{x_0}^{x_0+L}\left[\left(\frac{\partial U}{\partial t}\right)^2 + \left(\frac{\partial V}{\partial t}\right)^2 + \left(\frac{\partial W}{\partial t}\right)^2\right] x \sin\alpha dx d\theta dz \tag{17}$$

By substituting Eqs. (7) to (14) into Eq. (6), and substituting Eqs. (6) and (17) into Eq (5), the equations of motion of the nano truncated cone can be expressed as:

$$
\begin{aligned}
& N_{xx}\sin\alpha + x\sin\alpha\frac{\partial N_{xx}}{\partial x} - N_{\theta\theta}\sin\alpha + \frac{\partial N_{x\theta}}{\partial\theta} + \frac{6}{5x}\frac{\partial Y_{\theta\theta\theta}}{\partial\theta} + \frac{1}{5}\sin\alpha\frac{\partial Y_{zxz}}{\partial x} - \frac{1}{5}\frac{Y_{xxz}}{x}\cos\alpha \\
& + \frac{Y_{x}\sin\alpha}{x} - \frac{3}{5}\frac{Y_{xxx}\sin\alpha}{x} - \frac{1}{3x\tan\alpha}\frac{\partial Y_{xz\theta}}{\partial\theta} + \frac{1}{5}x\sin\alpha\frac{\partial^2 Y_{\theta\theta x}}{\partial x^2} + \frac{2}{15x\sin\alpha}\frac{\partial^2 Y_{zzx}}{\partial\theta^2} \\
& + \frac{2}{5}x\sin\alpha\frac{\partial^2 Y_{zzx}}{\partial x^2} + \frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{xxx}}{\partial\theta^2} - \frac{4}{15x\sin\alpha}\frac{\partial^2 Y_{x\theta\theta}}{\partial\theta^2} + \frac{1}{15x\sin\alpha}\frac{\partial^2 Y_{xzz}}{\partial\theta^2}\frac{1}{2x\sin\alpha}\frac{\partial^2 Y_{\theta z}}{\partial\theta^2} \\
& + \frac{1}{5}x\sin\alpha\frac{\partial^2 Y_{\theta x\theta}}{\partial x^2} + \frac{1}{2}\frac{\partial^2 Y_{xz}}{\partial\theta^2} + \frac{2}{15}\frac{\partial^2 Y_{z\theta z}}{\partial x\partial\theta} - \frac{4}{15x}\frac{\partial Y_{z\theta z}}{\partial\theta} - \frac{8}{15x}\frac{\partial Y_{\theta zz}}{\partial\theta} + \frac{1}{2x}\frac{\partial Y_{xz}}{\partial\theta} - \frac{28}{15x}\frac{\partial Y_{\theta xx}}{\partial\theta} - \frac{14}{15x}\frac{\partial Y_{x\theta x}}{\partial\theta} \\
& - \sin\alpha\frac{\partial Y_{x}}{\partial x} + \frac{1}{5}x\sin\alpha\frac{\partial Y_{xzz}}{\partial x} - \frac{1}{2x\tan\alpha}\frac{\partial Y_{\theta\theta}}{\partial\theta} - \frac{4}{3x\tan\alpha}\frac{\partial Y_{x\theta z}}{\partial\theta} - \frac{Y_{zzx}\sin\alpha}{5x} + \frac{4Y_{\theta x\theta}\sin\alpha}{5x} \\
& - \frac{2\sin\alpha Y_{xzz}}{5x} + \frac{4\cos\alpha Y_{\theta z\theta}}{5x} + \frac{8\sin\alpha Y_{x\theta\theta}}{5x} - \frac{Y_{xzx}\cos\alpha}{5x} + \frac{12}{5}\sin\alpha\frac{\partial Y_{x\theta\theta}}{\partial x} - \frac{3}{5}\frac{Y_{zzz}\cos\alpha}{x} \\
& + \frac{1}{2x\tan\alpha}\frac{\partial Y_{zz}}{\partial\theta} - \frac{Y_{zxx}\cos\alpha}{5x} + \frac{1}{5}\sin\alpha\frac{\partial Y_{zzx}}{\partial x} - \frac{7}{5}\sin\alpha\frac{\partial Y_{xxx}}{\partial x} - \frac{4}{15x\sin\alpha}\frac{\partial^2 Y_{\theta\theta x}}{\partial\theta^2} + \frac{1}{5}x\sin\alpha\frac{\partial^2 Y_{xzz}}{\partial x^2} \\
& - \frac{4}{15x\sin\alpha}\frac{\partial^2 Y_{\theta x\theta}}{\partial\theta^2} - x\sin\alpha\frac{\partial^2 Y_{x}}{\partial x^2} - x\sin\alpha\frac{\partial^2 Y_{\theta}}{\partial x\partial\theta} + \frac{1}{5}x\sin\alpha\frac{\partial^2 Y_{x\theta\theta}}{\partial x^2} - \frac{2}{5}x\sin\alpha\frac{\partial^2 Y_{xxx}}{\partial x^2} - \frac{8}{15}\frac{\partial^2 Y_{x\theta x}}{\partial x\partial\theta} \\
& + \frac{6}{5}x\sin\alpha\frac{\partial Y_{\theta x\theta}}{\partial x} - \frac{16}{15}\frac{\partial^2 Y_{\theta xx}}{\partial x\partial\theta} + \frac{2}{5}\frac{\partial^2 Y_{\theta\theta\theta}}{\partial x\partial\theta} + \frac{4}{5}\frac{\partial^2 Y_{\theta zz}}{\partial x\partial\theta} + \frac{8}{5}\frac{\cos\alpha Y_{\theta\theta z}}{x} = I_0 x\sin\alpha\frac{\partial^2 u_0}{\partial t^2} + I_1 x\sin\alpha\frac{\partial^2\phi}{\partial t^2}
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
& \frac{\partial N_{\theta\theta}}{\partial\theta} + 2x\sin\alpha N_{x\theta} + x\sin\alpha\frac{\partial N_{x\theta}}{\partial x} + N_{\theta z}\cos\alpha - \frac{8}{15}\frac{Y_{\theta XX}\sin\alpha}{x} + \frac{2}{3}\frac{Y_{x\theta z}\cos\alpha}{x} + \frac{2}{5}\frac{Y_{\theta\theta\theta}}{x\sin\alpha} \\
& - \frac{2}{5}\frac{Y_{zz\theta}}{x\sin\alpha} - \frac{1}{5}\frac{Y_{z\theta z}}{x\sin\alpha} - \frac{1}{2x\tan\alpha}\frac{\partial Y_{x\theta}}{\partial\theta} + \frac{2}{15}\frac{Y_{zz\theta}\sin\alpha}{x} + \frac{2}{5}\frac{Y_{\theta\theta\theta}\sin\alpha}{x} - \frac{16}{15x\tan\alpha}\frac{\partial Y_{z\theta\theta}}{\partial\theta} \\
& + \frac{1}{2x}\frac{\partial Y_{\theta z}}{\partial\theta} - \frac{16}{15x}\frac{\partial Y_{\theta x\theta}}{\partial\theta} - \frac{\partial^2 Y_{\theta}}{\partial\theta^2} - \frac{1}{2}\frac{\partial^2 Y_{\theta z}}{\partial x\partial\theta} - \frac{\partial Y_{x\theta z}}{\partial x}\cos\alpha - \frac{10}{3}\frac{Y_{x\theta z}}{x}\cos\alpha - \frac{Y_{xx}}{2x}\cos\alpha \\
& + \frac{2}{15}\frac{Y_{z\theta z}\sin\alpha}{x} - \frac{1}{5}\frac{Y_{\theta xx}}{\sin\alpha} + \frac{2}{15x\tan\alpha}\frac{\partial Y_{zxx}}{\partial\theta} - \frac{8}{15}\frac{Y_{x\theta x}\sin\alpha}{x} + \frac{2}{5x\tan\alpha}\frac{\partial Y_{zzz}}{\partial\theta} + \frac{1}{15}\sin\alpha\frac{\partial Y_{zz\theta}}{\partial x} \\
& + \frac{Y_{zz}}{2x}\cos\alpha + \frac{1}{15}\sin\alpha\frac{\partial Y_{z\theta z}}{\partial x} - \frac{1}{2}\sin\alpha\frac{\partial Y_{xz}}{\partial x} - \frac{4}{15}x\sin\alpha\frac{\partial^2 Y_{x\theta x}}{\partial x^2} + \frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{x\theta x}}{\partial\theta^2} + \frac{1}{5}x\sin\alpha\frac{\partial^2 Y_{\theta\theta\theta}}{\partial x^2} \\
& - \frac{1}{2}x\sin\alpha\frac{\partial^2 Y_{xz}}{\partial x^2} - \frac{8}{15}\frac{\partial^2 Y_{\theta\theta x}}{\partial x\partial\theta} + \frac{2}{15}\frac{\partial^2 Y_{zzx}}{\partial x\partial\theta} + \frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{z\theta z}}{\partial\theta^2} + \frac{1}{15}x\sin\alpha\frac{\partial^2 Y_{zz\theta}}{\partial x^2} - \frac{4}{15}x\sin\alpha\frac{\partial^2 Y_{xx\theta}}{\partial x^2} + \frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{\theta zz}}{\partial\theta^2} \\
& + \frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{\theta xx}}{\partial\theta^2} + \frac{1}{15}\sin\alpha\frac{\partial Y_{\theta zz}}{\partial x} - \frac{1}{x}\frac{\partial Y_{x}}{\partial\theta} - \frac{1}{2}\frac{\partial Y_{xx}}{\partial x}\cos\alpha + \frac{6}{5}\sin\alpha\frac{\partial Y_{\theta\theta\theta}}{\partial x} + \frac{4}{5x}\frac{\partial Y_{xxx}}{\partial\theta} - \frac{32}{15x}\frac{\partial Y_{\theta\theta x}}{\partial\theta} - \frac{8}{15}\frac{\sin\alpha}{x}Y_{xx\theta} \\
& - \frac{Y_{xx\theta}}{5x\sin\alpha} + \frac{2}{15x\tan\alpha}\frac{\partial Y_{xzx}}{\partial\theta} - \frac{Y_{x\theta x}}{15x\sin\alpha} + \frac{1}{2}\frac{\sin\alpha}{x}Y_{xz} - \frac{8}{15x\tan\alpha}\frac{\partial Y_{\theta z\theta}}{\partial\theta} + \frac{2}{15}\frac{\sin\alpha}{x}Y_{\theta zz} + \frac{2}{15x\tan\alpha}\frac{\partial Y_{xxz}}{\partial\theta} \\
& - \frac{19}{15}\sin\alpha\frac{\partial Y_{x\theta x}}{\partial x} + \frac{1}{15}x\sin\alpha\frac{\partial^2 Y_{z\theta z}}{\partial x^2} + \frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{xx\theta}}{\partial\theta^2} + \frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{zz\theta}}{\partial\theta^2} - \frac{2}{5x\sin\alpha}\frac{\partial^2 Y_{\theta\theta\theta}}{\partial\theta^2} + \frac{1}{15}x\sin\alpha\frac{\partial^2 Y_{\theta\theta z}}{\partial x^2} \\
& - \frac{19}{15}\sin\alpha\frac{\partial Y_{xx\theta}}{\partial x} + \frac{2}{15}\frac{\partial^2 Y_{xzz}}{\partial x\partial\theta} + \frac{8}{15x}\frac{\partial Y_{xzz}}{\partial\theta} + \frac{4}{15x}\frac{\partial Y_{zzx}}{\partial\theta} - \frac{\partial^2 Y_{x}}{\partial x\partial\theta} - \frac{8}{15}\frac{\partial^2 Y_{\theta x\theta}}{\partial x\partial\theta} + \frac{2}{5}\frac{\partial^2 Y_{xxx}}{\partial x\partial\theta} - \frac{16}{15}\frac{Y_{\theta zz}\cos\alpha}{x\tan\alpha} + \frac{2}{15}\frac{Y_{x\theta x}\cos\alpha}{x\tan\alpha}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&+\frac{2}{15}\frac{\cos\alpha}{x\sin\alpha}\frac{\partial Y_{xzx}}{\partial\theta}+\frac{2}{15}\frac{Y_{xx\theta}\cos\alpha}{x\tan\alpha}+\frac{2}{15}\frac{\cos\alpha}{x\sin\alpha}\frac{\partial Y_{zxx}}{\partial\theta}-\frac{8}{15}\frac{\cos\alpha}{x\sin\alpha}\frac{\partial Y_{\theta z\theta}}{\partial\theta}-\frac{8}{15}\frac{Y_{\theta z\theta}\cos\alpha}{x\tan\alpha}+\frac{2}{5}\frac{Y_{\theta\theta\theta}\cos\alpha}{x\tan\alpha}\\
&+\frac{2}{5}\frac{\cos\alpha}{x\sin\alpha}\frac{\partial Y_{zzz}}{\partial\theta}-\frac{16}{15}\frac{\cos\alpha}{x\sin\alpha}\frac{\partial Y_{z\theta\theta}}{\partial\theta}+\frac{2}{15}\frac{Y_{\theta xx}\cos\alpha}{x\tan\alpha}-\frac{1}{2}\frac{Y_{xz}\cos\alpha}{x\tan\alpha}+\frac{2}{15}\frac{\cos\alpha}{x\sin\alpha}\frac{\partial Y_{xxz}}{\partial\theta}-\frac{\partial Y_{x\theta z}}{\partial x}\cos\alpha-\frac{8}{15}\frac{\partial^2 Y_{x\theta\theta}}{\partial x\,\partial\theta}\\
&-\frac{1}{2}\frac{\partial Y_{\theta\theta}}{\partial x}\cos\alpha-\frac{19}{15}\sin\alpha\frac{\partial Y_{\theta xx}}{\partial x}-\frac{4}{15}x\sin\alpha\frac{\partial^2 Y_{\theta xx}}{\partial x^2}+\frac{2}{15}\frac{\partial^2 Y_{xzz}}{\partial x\,\partial\theta}=I_0\,x\sin\alpha\frac{\partial^2 u_0}{\partial t^2}+I_1\,x\sin\alpha\frac{\partial^2\psi}{\partial t^2}
\end{aligned}
$$

$$
\begin{aligned}
&-N_{\theta\theta}\cos\alpha+N_{xz}\sin\alpha+\frac{\partial N_{\theta z}}{\partial\theta}+x\sin\alpha\frac{\partial N_{xz}}{\partial x}+\frac{Y_x}{x}\cos\alpha-Y_x\sin\alpha\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial Y_x}{\partial x}\cos\alpha-\frac{1}{2}\frac{\partial^2 Y_{xx}}{\partial x\,\partial\theta}\\
&+\frac{\partial Y_\theta}{\partial\theta}\cos\alpha-\frac{1}{2x}\frac{\partial Y_{xx}}{\partial\theta}+\frac{1}{2x}\frac{\partial Y_{\theta\theta}}{\partial\theta}+\frac{1}{2}\frac{\partial^2 Y_{\theta\theta}}{\partial x\,\partial\theta}-\frac{1}{2x\tan\alpha}\frac{\partial Y_{xz}}{\partial\theta}-\frac{2}{5}\frac{\partial Y_{xxx}}{\partial x}\cos\alpha+\frac{1}{2}\frac{\partial Y_{\theta z}}{\partial x}\cos\alpha\\
&+\frac{3}{2}\sin\alpha\frac{\partial Y_{x\theta}}{\partial x}+\frac{1}{2}x\sin\alpha\frac{\partial^2 Y_{x\theta}}{\partial x^2}-\frac{1}{2x\sin\alpha}\frac{\partial^2 Y_{x\theta}}{\partial\theta^2}-\frac{3}{5}\frac{Y_{xxx}}{x}\cos\alpha+\frac{6}{5x\tan\alpha}\frac{\partial Y_{\theta\theta\theta}}{\partial\theta}+\frac{1}{5}x\sin\alpha\frac{\partial^2 Y_{zzz}}{\partial x^2}+\frac{1}{5x\sin\alpha}\frac{\partial^2 Y_{zzz}}{\partial\theta^2}\\
&+\frac{1}{5}\sin\alpha\frac{\partial Y_{zzz}}{\partial x}-\frac{3}{5x\tan\alpha}Y_{zzz}\cos\alpha-\frac{8}{15x\tan\alpha}\frac{\partial Y_{xx\theta}}{\partial\theta}-\frac{4}{15x\tan\alpha}\frac{\partial Y_{x\theta x}}{\partial\theta}+\frac{8}{5x}Y_{\theta\theta x}\cos\alpha\\
&+\frac{4}{5x}Y_{\theta x\theta}\cos\alpha+\frac{16}{15}\frac{\partial Y_{\theta\theta x}}{\partial x}\cos\alpha+\frac{8}{15}\frac{\partial Y_{\theta x\theta}}{\partial x}\cos\alpha+\frac{2}{15x\sin\alpha}\frac{\partial^2 Y_{xxz}}{\partial\theta^2}+\frac{1}{15x\sin\alpha}\frac{\partial^2 Y_{xzx}}{\partial\theta^2}-\frac{8}{15}x\sin\alpha\frac{\partial^2 Y_{xxz}}{\partial x^2}\\
&-\frac{4}{15}x\sin\alpha\frac{\partial^2 Y_{xzx}}{\partial x^2}-\frac{6}{5}\sin\alpha\frac{\partial Y_{xxz}}{\partial x}-\frac{3}{5}\sin\alpha\frac{\partial Y_{xzx}}{\partial x}-\frac{6}{15x\tan\alpha}Y_{xxz}\cos(\alpha)-\frac{3}{15x\tan\alpha}Y_{xzx}\cos\alpha\\
&+\frac{4}{5}\sin\alpha\frac{\partial Y_{\theta\theta z}}{\partial x}+\frac{2}{5}\sin\alpha\frac{\partial Y_{\theta z\theta}}{\partial x}-\frac{8}{15x\sin\alpha}\frac{\partial^2 Y_{\theta\theta z}}{\partial\theta^2}-\frac{4}{15x\sin\alpha}\frac{\partial^2 Y_{\theta z\theta}}{\partial\theta^2}+\frac{2}{15}x\sin\alpha\frac{\partial^2 Y_{\theta\theta z}}{\partial x^2}+\frac{1}{15}x\sin\alpha\frac{\partial^2 Y_{\theta z\theta}}{\partial x^2}\\
&+\frac{24}{15x\tan\alpha}Y_{\theta\theta z}\cos\alpha+\frac{12}{15x\tan\alpha}Y_{\theta z\theta}\cos\alpha-\frac{2}{5x}Y_{xzz}\cos\alpha-\frac{1}{5x}Y_{zzx}\cos\alpha-\frac{4}{15}\frac{\partial Y_{xzz}}{\partial x}\cos\alpha\\
&-\frac{2}{15}\frac{\partial Y_{zzx}}{\partial x}\cos\alpha-\frac{28}{15x\tan\alpha}\frac{\partial Y_{\theta zz}}{\partial\theta}-\frac{14}{15x\tan\alpha}\frac{\partial Y_{z\theta z}}{\partial\theta}-\frac{2}{x}\frac{\partial Y_{x\theta z}}{\partial\theta}-2\frac{\partial^2 Y_{x\theta z}}{\partial x\,\partial\theta}=I_0 x\sin\alpha\frac{\partial^2 w_0}{\partial t^2}
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&M_{xx}\sin\alpha+x\sin\alpha\frac{\partial M_{xx}}{\partial x}-M_{\theta\theta}\sin\alpha+\frac{\partial M_{x\theta}}{\partial\theta}-N_{xz}x\sin\alpha+x\sin\alpha\frac{\partial Y_z}{\partial x}+\frac{1}{2}x\sin\alpha\frac{\partial Y_{x\theta}}{\partial x}\\
&+Y_{x\theta}\sin\alpha+\frac{1}{2}\frac{\partial Y_{\theta\theta}}{\partial\theta}+\frac{1}{2}Y_{\theta z}\cos\alpha-\frac{1}{2}\frac{\partial Y_{zz}}{\partial\theta}+\frac{\partial Y_{x\theta z}}{\partial\theta}+\frac{1}{5}Y_{xxx}\cos\alpha-\frac{8}{15}Y_{x\theta\theta}\cos\alpha-\frac{4}{15}Y_{\theta x\theta}\cos\alpha\\
&+\frac{16}{15}x\sin\alpha\frac{\partial Y_{xzz}}{\partial x}+\frac{8}{15}x\sin\alpha\frac{\partial Y_{zxz}}{\partial x}+\frac{4}{3}Y_{xxz}\sin\alpha+\frac{2}{3}Y_{xzx}\sin\alpha-\frac{4}{3}Y_{\theta\theta z}\sin\alpha-\frac{2}{3}Y_{\theta z\theta}\sin\alpha+\frac{\partial Y_{x\theta z}}{\partial\theta}\\
&+\frac{2}{15}Y_{xzz}\cos\alpha-\frac{4}{15}x\sin\alpha\frac{\partial Y_{\theta\theta z}}{\partial x}-\frac{2}{15}x\sin\alpha\frac{\partial Y_{\theta z\theta}}{\partial x}+\frac{1}{15}Y_{zzx}\cos\alpha-\frac{2}{5}x\sin\alpha\frac{\partial Y_{zzz}}{\partial x}-x\sin\alpha\frac{\partial^2 T_x}{\partial x^2}\\
&+\frac{\sin\alpha}{x}T_x-\sin\alpha\frac{\partial T_x}{\partial x}-x\sin\alpha\frac{\partial^2 T_\theta}{\partial x\,\partial\theta}+\frac{1}{2}\frac{\partial^2 T_{xz}}{\partial x\,\partial\theta}+\frac{1}{2x}\frac{\partial T_{xz}}{\partial\theta}+\frac{1}{2x\sin\alpha}\frac{\partial^2 Y_{\theta z}}{\partial\theta^2}-\frac{2}{5}x\sin\alpha\frac{\partial^2 T_{xxx}}{\partial x^2}\\
&-\frac{7}{5}\sin\alpha\frac{\partial T_{xxx}}{\partial x}-\frac{3}{5}\frac{\sin\alpha}{x}T_{xxx}+\frac{1}{5x\sin\alpha}\frac{\partial^2 T_{xxx}}{\partial\theta^2}+\frac{2}{5}\frac{\partial^2 T_{\theta\theta\theta}}{\partial x\,\partial\theta}+\frac{6}{5x}\frac{\partial T_{\theta\theta\theta}}{\partial x}-\frac{24}{15}\frac{\partial^2 T_{\theta xx}}{\partial x\,\partial\theta}+\frac{12}{5}\frac{\sin\alpha}{x}T_{x\theta\theta}\\
&+\frac{12}{5}\frac{\cos\alpha}{x}T_{\theta\theta z}-\frac{42}{15x}\frac{\partial T_{\theta xx}}{\partial\theta}+\frac{18}{5}\sin\alpha\frac{\partial T_{x\theta\theta}}{\partial x}-\frac{12}{15x\sin\alpha}\frac{\partial^2 T_{x\theta\theta}}{\partial\theta^2}+\frac{3}{5}x\sin\alpha\frac{\partial^2 T_{x\theta\theta}}{\partial x^2}-\frac{2}{x\tan\alpha}\frac{\partial T_{x\theta z}}{\partial\theta}\\
&-\frac{3}{5}\frac{T_{xxz}}{x}\cos\alpha-\frac{3}{5}\frac{T_{zzx}}{x}\sin\alpha+\frac{3}{5}x\sin\alpha\frac{\partial^2 T_{xzz}}{\partial x^2}+\frac{3}{5}\sin\alpha\frac{\partial T_{xzz}}{\partial x}+\frac{3}{15}\frac{1}{x\sin\alpha}\frac{\partial^2 T_{xzz}}{\partial\theta^2}-\frac{12}{15x}\frac{\partial T_{\theta zz}}{\partial\theta}\\
&+\frac{6}{5}\frac{\partial^2 T_{\theta zz}}{\partial x\,\partial\theta}-\frac{3}{5}\frac{T_{zzz}}{x}\cos\alpha=x\sin\alpha\left(I_1\frac{\partial^2 u_0}{\partial t^2}+I_2\frac{\partial^2\phi}{\partial t^2}\right)
\end{aligned}
\tag{21}
$$

$$\begin{aligned}
&\frac{\partial M_{\theta\theta}}{\partial\theta}+x\sin\alpha\frac{\partial M_{x\theta}}{\partial x}+2M_{x\theta}\sin\alpha-N_{\theta z}x\sin\alpha+\frac{\partial Y_z}{\partial\theta}-\frac{1}{2}Y_{xx}\sin\alpha-\frac{1}{2}x\sin\alpha\frac{\partial Y_{xx}}{\partial x}+\frac{1}{2}Y_{\theta\theta}\sin\alpha\\
&-\frac{1}{2}\frac{\partial Y_{x\theta}}{\partial\theta}+\frac{Y_{xz}\cos\alpha}{2}+\frac{x\sin\alpha}{2}\frac{\partial Y_{zz}}{\partial x}+\frac{2}{15}Y_{xx\theta}\cos\alpha+\frac{1}{15}Y_{x\theta x}\cos\alpha-\frac{4}{5}Y_{\theta\theta\theta}\cos\alpha-\frac{4}{15}\frac{\partial Y_{zxx}}{\partial\theta}-\frac{2}{15}\frac{\partial Y_{xzx}}{\partial\theta}\\
&+\frac{16}{15}\frac{\partial Y_{\theta\theta z}}{\partial\theta}+\frac{8}{15}\frac{\partial Y_{\theta z\theta}}{\partial\theta}+\frac{22}{15}Y_{\theta zz}\cos\alpha+\frac{11}{15}Y_{z\theta z}\cos\alpha-\frac{2}{5}\frac{\partial Y_{zzz}}{\partial\theta}+4Y_{x\theta z}\sin\alpha+2x\sin\alpha\frac{\partial Y_{x\theta z}}{\partial x}-\frac{1}{x}\frac{\partial T_x}{\partial\theta}\\
&-\frac{\partial^2 T_x}{\partial x\partial\theta}-\frac{\partial^2 T_\theta}{\partial\theta^2}-\frac{T_x}{2x}\cos\alpha+\frac{T_{xz}}{2x}\sin\alpha-\frac{1}{2}x\sin\alpha\frac{\partial^2 T_{xz}}{\partial x^2}-\frac{1}{2}\sin\alpha\frac{\partial T_{xz}}{\partial x}+\frac{1}{2}x\cos\alpha\frac{\partial T_{\theta\theta}}{\partial x}\\
&+\frac{1}{2x}\frac{\partial T_{\theta z}}{\partial\theta}-\frac{1}{2}\frac{\partial^2 T_{\theta z}}{\partial x\partial\theta}+\frac{4}{5x}\frac{\partial T_{xxx}}{\partial\theta}+\frac{2}{5}\frac{\partial^2 T_{xxx}}{\partial x\partial\theta}+\frac{2}{5}\frac{T_{\theta\theta\theta}}{x\sin\alpha}+\frac{2}{5}\frac{T_{\theta\theta\theta}\cos\alpha}{x\tan\alpha}-\frac{2}{5x\sin\alpha}\frac{\partial^2 T_{\theta\theta\theta}}{\partial\theta^2}+\frac{6}{5}\sin\alpha\frac{\partial T_{\theta\theta\theta}}{\partial x}\\
&+\frac{1}{5}x\sin\alpha\frac{\partial^2 T_{\theta\theta\theta}}{\partial x^2}+\frac{2}{5}\frac{\sin\alpha}{x}T_{\theta\theta\theta}-\frac{57}{15}\sin\alpha\frac{\partial T_{xx\theta}}{\partial x}+\frac{2}{5x\tan\alpha}\frac{\partial T_{zzz}}{\partial\theta}-\frac{48}{15x}\frac{\partial T_{\theta\theta x}}{\partial\theta}-\frac{24}{15}\frac{\partial^2 T_{\theta\theta x}}{\partial x\partial\theta}-\frac{24}{15x\tan\alpha}\frac{\partial T_{z\theta\theta}}{\partial\theta}\\
&+\frac{6T_{xx\theta}\cos\alpha}{15x\tan\alpha}+\frac{3}{5x\sin\alpha}\frac{\partial^2 T_{xx\theta}}{\partial\theta^2}-\frac{12}{15}x\sin\alpha\frac{\partial^2 T_{xx\theta}}{\partial x^2}-\frac{24}{15}\frac{\sin\alpha}{x}T_{xx\theta}-\frac{3T_{xx\theta}}{5x\sin\alpha}-\frac{3T_{\theta zz}}{5x\sin\alpha}+\frac{3}{15}\sin\alpha\frac{\partial T_{\theta zz}}{\partial x}\\
&+\frac{3}{5x\sin\alpha}\frac{\partial^2 T_{\theta zz}}{\partial\theta^2}+\frac{3}{15}x\sin\alpha\frac{\partial^2 T_{\theta zz}}{\partial x^2}+\frac{6}{15}\frac{\sin\alpha}{x}T_{\theta zz}-\frac{24}{15}\frac{T_{\theta zz}\cos\alpha}{x\tan\alpha}+\frac{6}{15x\tan\alpha}\frac{\partial T_{xxz}}{\partial\theta}+\frac{6}{15}\frac{\partial^2 T_{xzz}}{\partial x\partial\theta}+\frac{12}{15x}\frac{\partial T_{xzz}}{\partial\theta}\\
&+\frac{2}{5x\tan\alpha}\frac{\partial T_{zzz}}{\partial\theta}-\frac{24}{15x\tan\alpha}\frac{\partial T_{\theta zz}}{\partial\theta}-2\cos\alpha\frac{\partial T_{x\theta z}}{\partial x}-\frac{4}{x}\cos\alpha T_{x\theta z}=x\sin\alpha\left(I_1\frac{\partial^2 v_0}{\partial t^2}+I_2\frac{\partial^2\psi}{\partial t^2}\right)
\end{aligned}\tag{22}$$

In above equations, I_0; I_1 and I_2 are the moment inertia and are defined as:

$$\left(I_0,I_1,I_2\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho\left(1,z,z^2\right)dz\tag{23}$$

According to the strain gradient theory, the classical and high-tension derivations are as follows:

$$\begin{aligned}
&N_{ij}=\int\sigma_{ij}dz\quad,\quad M_{ij}=\int\sigma_{ij}.zdz\quad,\quad Y_i=\int P_i dz\quad,\quad T_i=\int P_i.zdz\\
&Y_{ijk}=\int\tau_{ijk}dz\quad,\quad T_{ijk}=\int\tau_{ijk}.zdz\quad,\quad Y_{ij}=\int m_{ij}dz\quad,\quad T_{ij}=\int m_{ij}.zdz
\end{aligned}\tag{24}$$

The following relation also expresses the classical and non-classical boundary conditions:

$$\left[N_x+\frac{\partial Y_x}{\partial x}-\frac{Y_{xxx}}{x}-\frac{2}{5}\frac{\partial Y_{xxx}}{\partial x}+\frac{Y_{x\theta\theta}}{x}+\frac{2}{5}\frac{\partial Y_{x\theta\theta}}{\partial x}+\frac{2}{5}\frac{\partial Y_{xzz}}{\partial x}+\frac{Y_{\theta x\theta}}{x}+\frac{1}{5}\frac{\partial Y_{\theta x\theta}}{\partial x}+\frac{Y_{\theta\theta x}}{x}+\frac{1}{5}\frac{\partial Y_{zzx}}{\partial x}\right]\Bigg|_{x=x_0,x_0+L}=0\tag{25a}$$

or

$$\delta u_0\Big|_{x=x_0,x_0+L}=0\tag{25b}$$

$$\left[Y_x x\sin\alpha+\frac{2}{5}Y_{xxx}x\sin\alpha-\frac{2}{5}Y_{x\theta\theta}x\sin\alpha-\frac{2}{5}Y_{xzz}x\sin\alpha-\frac{1}{5}Y_{\theta x\theta}x\sin\alpha-\frac{1}{5}Y_{zzx}x\sin\alpha\right]\Bigg|_{x=x_0,x_0+L}=0\tag{26a}$$

or

$$\delta\left(\frac{\partial u_0}{\partial x}\right)\Big|_{x=x_0,x_0+L}=0 \tag{26b}$$

$$\begin{aligned}&\left[N_{x\theta}-\frac{Y_{xx}\cos\alpha}{2x\sin\alpha}+\frac{Y_{\theta\theta}\cos\alpha}{2x\sin\alpha}-\frac{1}{2}\frac{\partial Y_{xz}}{\partial x}-2\frac{Y_{xx\theta}}{x}-\frac{8}{15}\frac{\partial Y_{xx\theta}}{\partial x}-\frac{Y_{x\theta x}}{x}-\frac{4}{15}\frac{\partial Y_{x\theta x}}{\partial x}-2\frac{Y_{x\theta z}\cos\alpha}{x\sin\alpha}+\frac{Y_{\theta\theta\theta}}{x}\right.\\&\left.+\frac{1}{5}\frac{\partial Y_{\theta\theta\theta}}{\partial x}+\frac{1}{15}\frac{\partial Y_{z\theta z}}{\partial x}+\frac{2}{15}\frac{\partial Y_{zz\theta}}{\partial x}\right]\Bigg|_{x=x_0,x_0+L}=0\end{aligned} \tag{27a}$$

or

$$\delta v_0\Big|_{x=x_0,x_0+L}=0 \tag{27b}$$

$$\left[\frac{1}{2}Y_{xz}x\sin\alpha+\frac{8}{15}Y_{xx\theta}x\sin\alpha+\frac{4}{15}Y_{x\theta x}x\sin\alpha-\frac{Y_{\theta\theta\theta}}{5}x\sin\alpha-\frac{2Y_{\theta zz}}{15}x\sin\alpha-\frac{Y_{z\theta z}}{15}x\sin\alpha\right]\Big|_{x=x_0,x_0+L}=0 \tag{28a}$$

or

$$\delta\frac{\partial v_0}{\partial x}\Big|_{x=x_0,x_0+L}=0 \tag{28b}$$

$$\begin{aligned}&\left[N_{xz}+\frac{1}{5}\frac{\partial Y_{zzz}}{\partial x}+\frac{1}{15}\frac{\partial Y_{\theta z\theta}}{\partial x}+\frac{8}{15}\frac{Y_{\theta x\theta}\cos\alpha}{x\sin\alpha}+\frac{16}{15}\frac{Y_{x\theta\theta}\cos\alpha}{x\sin\alpha}+\frac{Y_{x}\cos\alpha}{x\sin\alpha}-\frac{4}{15}\frac{Y_{xzz}\cos\alpha}{x\sin\alpha}-\frac{2}{15}\frac{Y_{zxz}\cos\alpha}{x\sin\alpha}-\frac{2}{3}\frac{Y_{xxz}}{x}\right.\\&-\frac{2}{5}\frac{Y_{xxx}\cos\alpha}{x\sin\alpha}-\frac{Y_{x\theta}\cos\alpha^2}{x\sin\alpha^2}-\frac{1}{2}\frac{\partial Y_{x\theta}}{\partial x}\frac{\cos\alpha^2}{\sin\alpha^2}+\frac{Y_{\theta z}\cos\alpha}{2x\sin\alpha}-\frac{1}{3}\frac{Y_{xzx}}{x}+\frac{1}{3}\frac{Y_{\theta z\theta}}{x}+\frac{2}{3}\frac{Y_{\theta\theta z}}{x}+\frac{2}{15}\frac{\partial Y_{\theta\theta z}}{\partial x}\\&\left.+\frac{1}{2\sin(\alpha)^2}\frac{\partial Y_{x\theta}}{\partial x}+\frac{Y_{x\theta}}{x\sin(\alpha)^2}-\frac{4}{15}\frac{\partial Y_{xzx}}{\partial x}+\frac{8}{15}\frac{\partial Y_{xxz}}{\partial x}\right]\Bigg|_{x=x_0,x_0+L}=0\end{aligned} \tag{29a}$$

or

$$\delta w_0\Big|_{x=x_0,x_0+L}=0 \tag{29b}$$

$$\begin{aligned}&\left[-\frac{1}{2}\frac{x}{\sin\alpha}Y_{x\theta}+-\frac{1}{2}\frac{x\cos\alpha^2}{\sin\alpha}Y_{x\theta}+\frac{8}{15}Y_{xxz}x\sin\alpha+\frac{4}{15}Y_{xzx}x\sin\alpha-\frac{1}{2}\frac{x}{\sin\alpha}Y_{zzz}+\frac{x\sin\alpha}{2}Y_{xx\theta}\right.\\&\left.-\frac{2}{15}x\sin(\alpha)Y_{\theta\theta z}-\frac{1}{15}x\sin(\alpha)Y_{\theta z\theta}-\frac{1}{5}x\sin(\alpha)Y_{zzz}\right]\Big|_{x=x_0,x_0+L}=0\end{aligned} \tag{30a}$$

or

$$\delta\frac{\partial w_0}{\partial x}\Big|_{x=x_0,x_0+L}=0 \tag{30b}$$

$$\begin{aligned}&\left[M_{xx}+\frac{Y_{x\theta}}{2\sin(\alpha)^2}-\frac{Y_{x\theta}\cos(\alpha)^2}{2\sin(\alpha)^2}-\frac{\partial T_x}{\partial x}+Y_z-\frac{T_{xxx}}{x}-\frac{2}{5}\frac{\partial T_{xxx}}{\partial x}+\frac{16}{15}Y_{xxz}+2\frac{T_{x\theta\theta}}{x}+\frac{2}{5}\frac{\partial T_{x\theta\theta}}{\partial x}+\frac{8}{15}Y_{xzx}\right.\\&\left.+\frac{1}{5}\frac{\partial T_{xzz}}{\partial x}+\frac{T_{\theta x\theta}}{x}+\frac{1}{5}\frac{\partial T_{\theta x\theta}}{\partial x}-\frac{4}{15}Y_{\theta\theta z}-\frac{2}{15}Y_{\theta z\theta}+\frac{1}{5}\frac{\partial T_{zxz}}{\partial x}+\frac{1}{5}\frac{\partial T_{zzx}}{\partial x}-\frac{2}{15}Y_{z\theta\theta}-\frac{2}{5}Y_{zzz}\right]\Bigg|_{x=x_0,x_0+L}=0\end{aligned} \tag{31a}$$

or

$$\delta\phi\Big|_{x=x_0,x_0+L}=0 \tag{31b}$$

$$\left[x\sin(\alpha)T_x+\frac{2}{5}x\sin(\alpha)T_{xxx}-\frac{2}{5}x\sin(\alpha)T_{xzz}-\frac{1}{5}x\sin\alpha T_{\theta x\theta}-\frac{1}{5}x\sin\alpha T_{\theta\theta x}-\frac{1}{5}x\sin\alpha T_{zzx}\right]\Big|_{x=x_0,x_0+L}=0 \tag{32a}$$

or

$$\delta\frac{\partial\phi}{\partial x}\Big|_{x=x_0,x_0+L}=0 \tag{32b}$$

$$\begin{aligned}&\Big[M_{x\theta}-\frac{4}{15}\frac{\partial T_{x\theta x}}{\partial x}+\frac{1}{15}\frac{\partial T_{z\theta z}}{\partial x}-\frac{4}{15}\frac{\partial T_{\theta xx}}{\partial x}-2\frac{T_{x\theta z}\cos\alpha}{x\sin\alpha}-\frac{T_{x\theta x}}{x}-2\frac{T_{xx\theta}}{x}-\frac{1}{2}\frac{\partial T_{xz}}{\partial x}-\frac{1}{2}Y_{xx}+\frac{1}{2}Y_{zz}-\frac{4}{15}\frac{\partial T_{xx\theta}}{\partial x}\\&+2Y_{x\theta z}+\frac{1}{2}\frac{T_{\theta\theta}\cos\alpha}{x\sin\alpha}+\frac{2}{15}\frac{\partial T_{\theta zz}}{\partial x}+\frac{1}{15}\frac{\partial T_{\theta\theta\theta}}{\partial x}+\frac{T_{\theta\theta\theta}}{x}\Big]\Big|_{x=x_0,x_0+L}=0\end{aligned} \tag{33a}$$

or

$$\delta\psi\Big|_{x=x_0,x_0+L}=0 \tag{33b}$$

$$\left[\frac{1}{2}T_{xz}x\sin\alpha+\frac{8}{15}T_{xx\theta}x\sin\alpha+\frac{4}{15}T_{x\theta x}x\sin\alpha-\frac{1}{5}T_{\theta\theta\theta}x\sin\alpha-\frac{2}{15}T_{\theta zz}x\sin\alpha-\frac{1}{15}T_{z\theta z}x\sin\alpha\right]\Big|_{x=x_0,x_0+L}=0 \tag{34a}$$

or

$$\delta\frac{\partial\psi}{\partial x}\Big|_{x_0,x_0+L}=0 \tag{34b}$$

In this section, linear free vibrations of the simply supported conical shell are considered as special cases for the evaluation of formulation. The essential boundary conditions are as follows (Zeighampour [5]):

$$v_0=0,\quad w_0=0,\quad \psi=0,\ \text{at}\ \ x_0,\ x_0+L \tag{35}$$

It could also be argued that, considering freedom from shear force and bending moment in the classic case and from stresses in the non-classical cases, the natural boundary conditions are expressed as (Zeighampour [5]):

$$(25a)=0\ ,\ (28a)=0\ ,\ (30a)=0\ ,\ (31a)=0\ \text{and}\ (34a)=0\ \ \text{at}\ \ x_0,\ x_0+L \tag{36}$$

3 SOLUTION METHOD

3.1 GDQ method

The GDQ method defines that the derivatives of a sufficiently smooth function with respect to a co-ordinate direction at a discrete grid point can be approximated by a weighted linear sum of functional values of all the discrete mesh points in that co-ordinate direction. It is based on the analyses of a higher order polynomial approximation in linear vector space to reach at the weighting coefficient required by the method. The mathematical expression of this basic theorem is expressed as follows:

$$f^{(m)}(x)\Big|_{x=x_i}=\sum_{k=1}^{N}A_{ik}^{m}f(x_k) \quad , \tag{37}$$

where A_{ik}^{m} are the weighting coefficients, and the non-diagonal matrix of the first-order derivative weighted coefficient is calculated as follows:

$$\begin{aligned}&A_{ik}^{(1)}=\frac{\prod(x_i)}{(x_i-x_k)\prod(x_k)} \quad i,k=1,2,....,N_k \ \mathit{and} \quad i\neq k\\ &\prod(x_i)=\prod_{\upsilon=1,\upsilon\neq i}^{N_x}(x_i-x_\upsilon) \qquad ,\prod(x_k)=\prod_{\upsilon=1,\upsilon\neq k}^{N_x}(x_k-x_\upsilon)\end{aligned} \tag{38}$$

The second-order and higher-order of the non-diagonal matrices weighted coefficients are obtained as follows:

$$\begin{aligned}&A_{ik}^{(r)}=r\left[A_{ii}^{(r-1)}.A_{ik}^{(1)}-\frac{A_{ik}^{(r-1)}}{x_i-x_k}\right] \ \text{, where } 2\leq r\leq N_x-1\\ &\mathit{for} \quad i,k=1,2,...,N_x \qquad \mathit{and} \quad k\neq i\end{aligned} \tag{39}$$

The diagonal terms of the weighting matrix will be obtained as follows:

$$A_{ii}^{(r)}=-\sum_{\upsilon=1,\upsilon\neq i}^{N}A_{i\upsilon}^{(r)} \qquad ,1\leq m=N_x-1 \tag{40}$$

The choice of precision points in convergence and the accuracy of this method is very effective. We choose to use the cosine grid point distribution in our computations:

$$x_i=\left[1-\cos\left((i-1)\pi/(N-1)\right)\right]L/2 \tag{41}$$

3.2 Displacement

For free vibration of nano conical shell the displacements are assumed to be in the following forms: (Ghadiri and Shafiei [22])

$$\begin{aligned}&u_o(x,\theta,t)=u(x)\cos(n\theta)\cos(\omega t),\\ &v_0(x,\theta,t)=v(x)\cos(n\theta)\cos(\omega t),\\ &w_0(x,\theta,t)=w(x)\cos(n\theta)\cos(\omega t),\\ &\phi(x,\theta,t)=\phi(x)\cos(n\theta)\cos(\omega t),\\ &\psi(x,\theta,t)=\psi(x)\cos(n\theta)\cos(\omega t),\end{aligned} \tag{42}$$

After replacing the displacements (42) in the equations of motion (18) to (22) and boundary conditions, the equations of motion can be expressed as matrices:

$$\left([K]-\Omega^2[M]\right)\{d\}=0 \tag{43}$$

That:

$$\{d\} = \left[u\ v\ w\ \ \phi\ \psi\right]^T \tag{44}$$

In the above equation, Ω represents the natural frequency of the nano cone. The natural frequency of the system is obtained through the solution of the eigenvalue problem in Eq. (43).

3.3 Implementation of boundary condition

By applying the substitution boundary condition in governing equations (*SBCGE*), divide the matrix equation into two parts: Internal points $\{U_i\}$ and boundary points $\{U_b\}$. In the governing equations, we present the weighted coefficients of the interior points with $[A_{ii}]$ and the weighted coefficients of the boundary points with $[A_{ib}]$.

$$[A_{ii}]\{U_i\} + [A_{ib}]\{U_b\} = (\ )\omega^2\{U_i\} \tag{45}$$

We also divide the other weighting coefficients into two parts $[A_{bi}]$, which include the weighting coefficients of the interior points in the boundary condition, and $[A_{bb}]$, which relate to the weighting coefficients of the boundary points in the boundary condition.

$$\begin{gathered}
[A_{bb}]\{U_b\} + [A_{bi}]\{U_i\} = 0 \rightarrow \{U_b\} = -[A_{bb}]^{-1}[A_{bi}]\{U_i\} \\
\left[A^*\right] = [A_{ii}] - [A_{ib}][A_{bb}]^{-1}[A_{bi}] \\
\left[A^*\right]\{U_i\} = (\ )\omega^2\{U_i\} \rightarrow \left[A^* - \omega^2 I\right]\{U\} = 0
\end{gathered} \tag{46}$$

4 NUMERICAL SOLUTION

In this section, the geometrical and mechanical properties of the structure are expressed to obtain the numerical results and examine the effect of various parameters on the behavior of the system. The thickness and R_a/h of nano cone are considered 0.34 *nm* and 25 respectively. The structure is composed of isotropic aluminum with the Young's modulus of E=70 *GPa*, Poisson's ratio of υ=0.3 and ρ=2710 $Kg/m^{3.}$

4.1 Validation

To validate the results of present study, the results of this study are compared with the other available literature. For this purpose, in the table one frequency parameter λ of truncated macro conical shell with different the semi vertex angles is compared. In this Comparison, The thickness and R_a/h of cone are considered 4 *mm* and 100 respectively. The structure is composed of isotropic aluminum with the Young's modulus of E=70 *GPa,* Poisson's ratio of υ=0.3 and ρ=2710 Kg/m^3.

In the second table Compared of frequency parameter λ of nano truncated conical shell with different the semi vertex angles. In this Comparison The thickness and R_a/h of nano cone are considered 0.34 *nm* and 25 respectively. The structure is composed of isotropic aluminum with the Young's modulus of E=70 *GPa,* Poisson's ratio of υ=0.3 and ρ=2710 Kg/m^3. The results of this study have a good agreement.

4.2 Parametric study

In this paper, the free vibration of conical nano-shell with simply support using GDQ method is investigated for the first time by using first order shear theory and modified strain gradient theory which includes the main equations of motion and classical and non-classical boundary conditions. The results of the analysis are compared with the classical and coupled modified stress theories. Also, the effect of size on length and radius in dimensionless frequency is investigated.

Table 1

A comparison on the non-dimensional natural frequencies ($\Omega = \omega R_a \sqrt{\frac{\rho(1-\nu^2)}{E}}$) of isotropic truncated conical shell (*Lsinα* /*R*= 0.25, *R/h* = 100, *h* = 4 *mm*, υ= 0.3, *E* = 70 *GPa*, *ρ*= 2710 *Kg*/*m*^{3}).

$\alpha(^\circ)$		Modes			
		2	3	4	5
$\alpha = 30$	Mehri *et al* [11]	0.7909	0.7285	0.6356	0.5537
	Present study	0.7906	0.7279	0.6346	0.5520
$\alpha = 45$	Mehri *et al*[11]	0.6878	0.6974	0.6668	0.6312
	Present study	0.6873	0.6965	0.6651	0.6285
$\alpha = 60$	Mehri *et al* [11]	0.5721	0.6004	0.6060	0.6088
	Present study	0.5711	0.5985	0.6031	0.6045

In Fig. 2 it can be seen that the effect of variation of the dimensionless coefficient R_a/L on the ratio Ω in the different vertex angles. This comparison is done *for* m=1, n=2, l/h=1, L/R_a=5. The material length scale parameter for strain gradient theory is assumed $l_0=l_1=l_2=l$. By increasing the ratio R_a/h in the gradient strain theory, decrease the dimensionless frequency Ω. The lowest values are for the 60 degree angle.

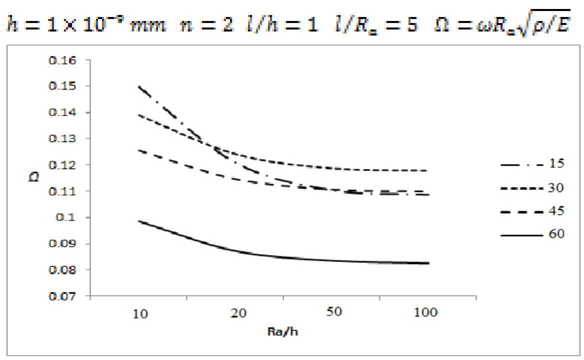

Fig.2
Variations of dimensionless frequency Ω for dimensionless ratio *Ra/h* in different vertex angles.

In Fig. 3 the comparison ratio of material length scale parameter per minimum radius on dimensionless frequency in the different vertex angles is investigated. This comparison is done for m=1,n=3,R_a/h=20,L/h=1.It can be seen that by increasing the ratio of $l\,/\,R_a$ decrease dimensionless frequency ratio results. The lowest values are for the 60 degree angle.

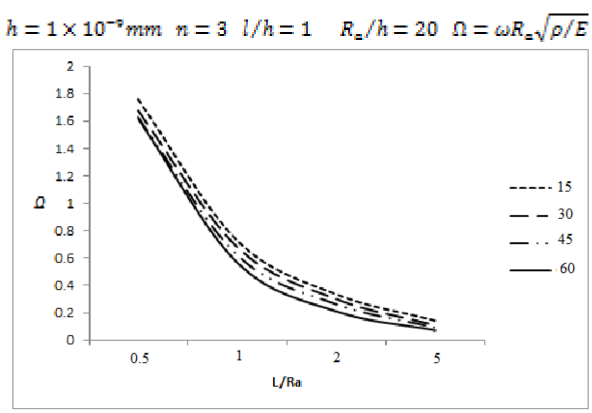

Fig.3
Effect of different L/R_a on dimensionless frequency Ω

In Fig. 4 it can be seen that the effect of variation of the dimensionless coefficient l/h on the ratio Ω in the different vertex angles. This comparison is done for m=1, n=1, R_a/h=25, L/R_a=1. The material length scale parameter for strain gradient theory is assume $l_0=l_1=l_2=l$. By increasing ratio l/h in various vertex angles, the dimensionless Ω Increases almost linearly. The lowest values are for the 60 degree angle.

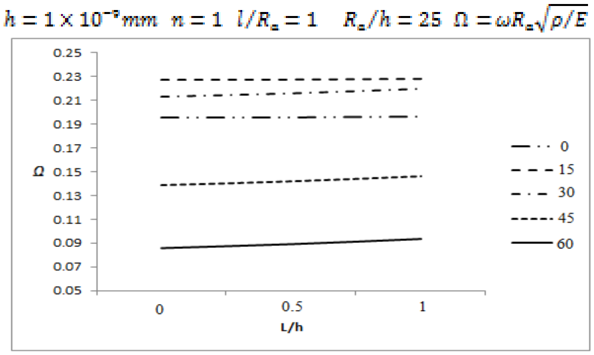

Fig.4
Effect of different L/h on dimensionless frequency Ω

In Figs. 5, 6 and 7 it can be seen that the effect of variation of the dimensionless coefficient L/R_a on the dimensionless frequency Ω in the three theories of the classic, couple stress and strain gradient is compared. This comparison is done for m=1, n=5, R_a/h=25. The material length scale parameter for strain gradient theory is assume $l_0=l_1=l_2=l$ and for couple stress $l_2=l$. For both above non classical theory h/l=1. By increasing the rate L/R_a decrease the ratio of Ω These changes in the strain gradient theory occur at a higher dimensionless frequency Ω.

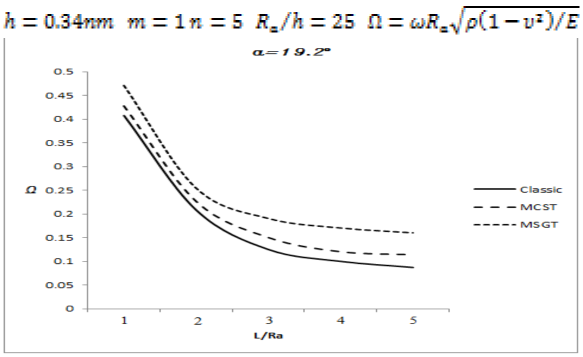

Fig.5
Effect of different L/R_a on dimensionless frequency Ω in three theory.

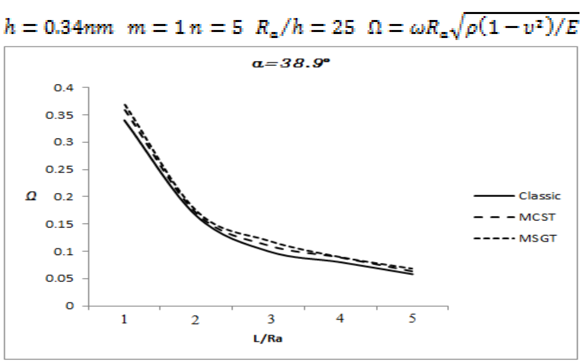

Fig.6
Effect of different L/R_a on dimensionless frequency Ω in three theory.

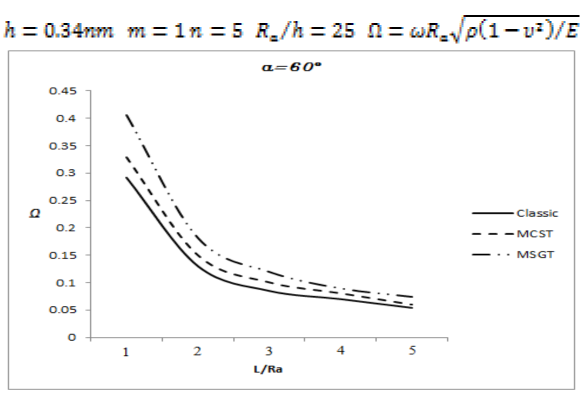

Fig.7
Effect of different L/R_a on dimensionless frequency Ω in three theory.

Below is the shape of the conical nano shell modes for the states n =1, m =1 and n =1, m =2 and n =1, m =3 n is the half circumfrential wave number and m is the longitudinal wave number.

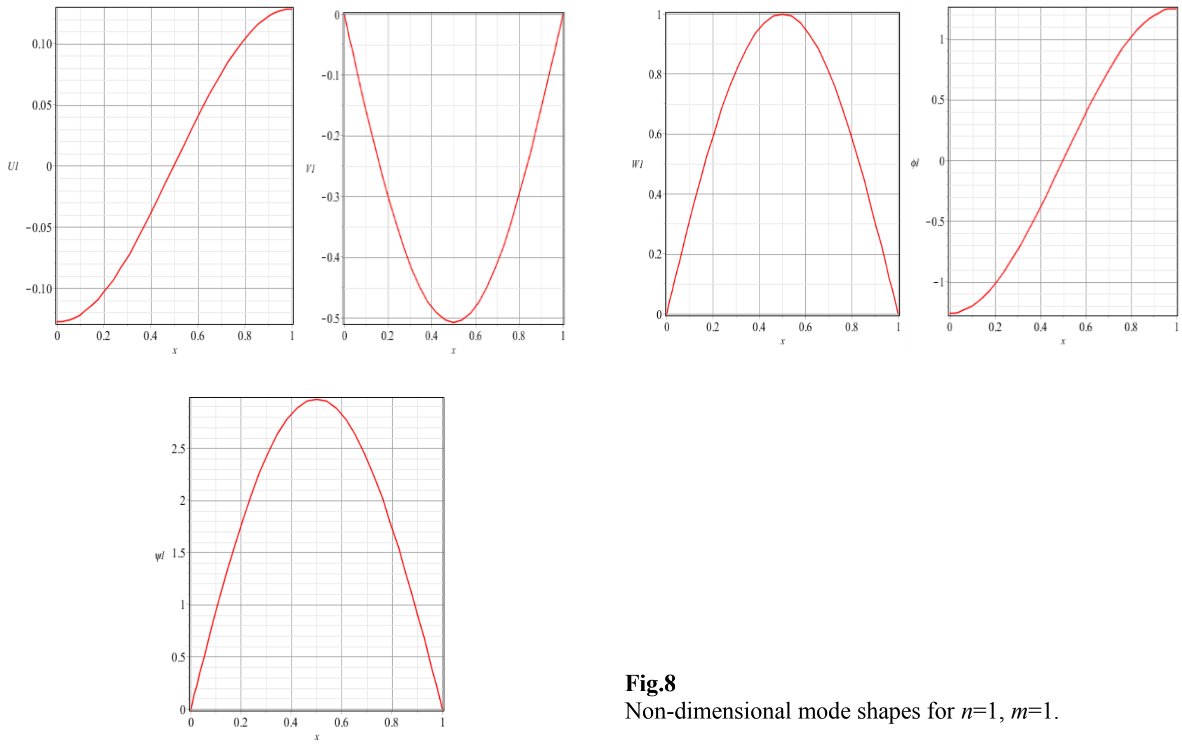

Fig.8
Non-dimensional mode shapes for n=1, m=1.

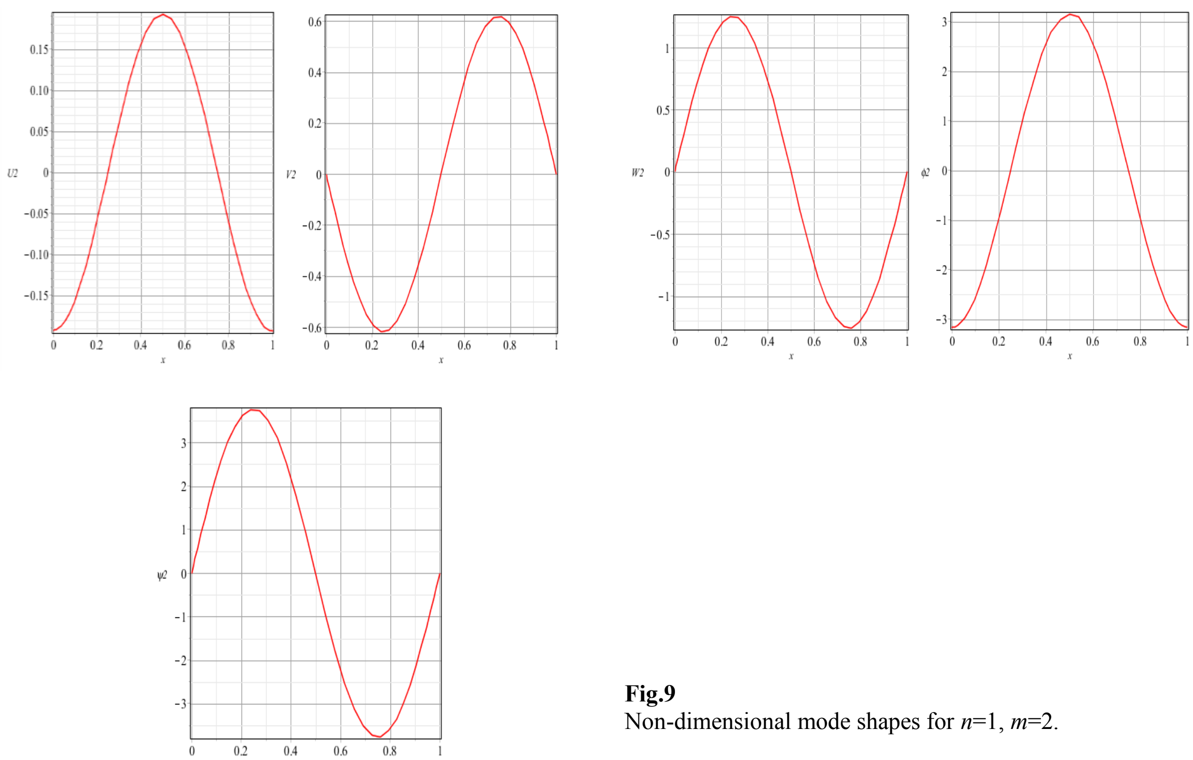

Fig.9
Non-dimensional mode shapes for n=1, m=2.

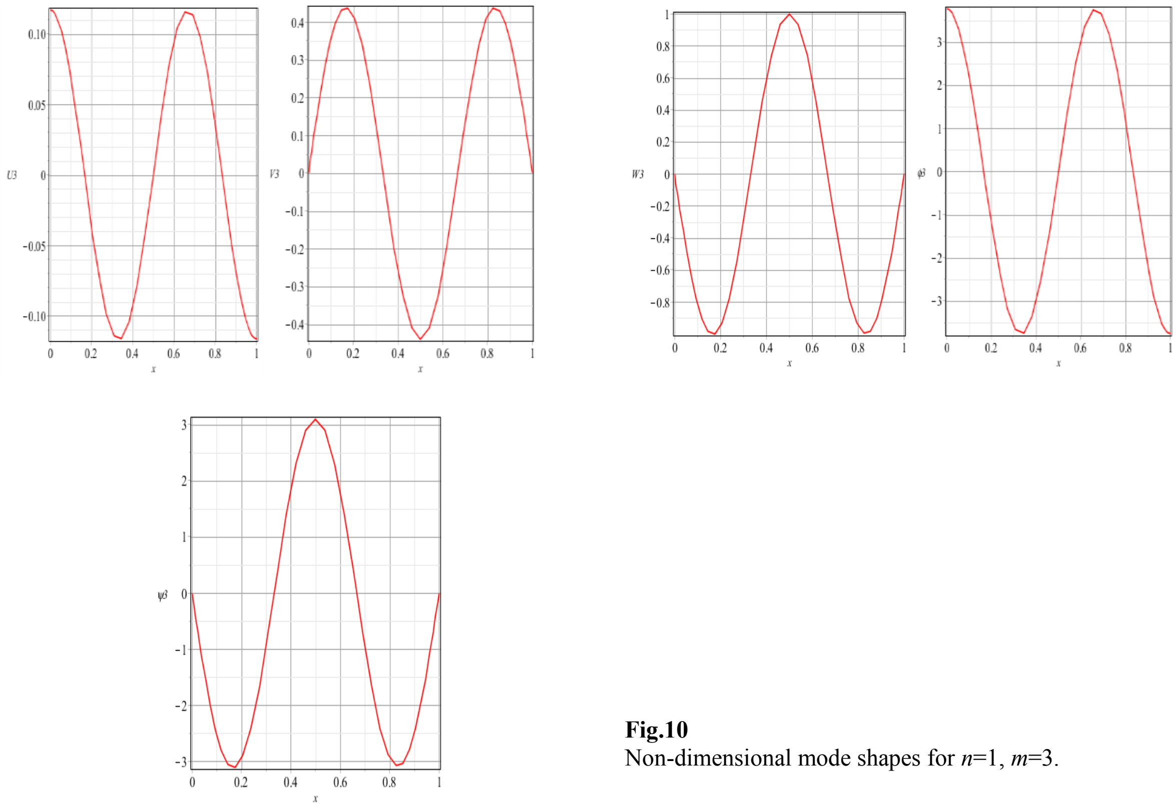

Fig.10
Non-dimensional mode shapes for n=1, m=3.

5 CONCLUSIONS

Free linear vibration analysis of nano truncated conical shell using the strain gradient theory was carried out. The simply support was considered .Using first-order shear shell and MSGT theories and Hamilton's principle, the motion equations were derived. The DQM and Galerkin' methods were employed to solve the problem. The influence of three theories on frequency was investigated. Also the Effect of different ratios l / h, l/R_a and R_a / h of different vertexes on dimensionless frequency ratio were investigated and finally the variations of dimensionless frequency on different rate l/R_a for strain gradient theory and modified couple stress and classical theories were investigated and compared together. Results also reveal that rigidity of the nano truncated conical shell in the strain gradient theory is greater than that in the modified couple stress and classical theories respectively, which leads to an increase in dimensionless natural frequency ratio.Following Results were obtained:

- As can be seen by increasing dimensionless rate L/ R_a , decrease the dimensionless frequency Ω and rigidity in the strain gradient theory is greater than MCST and classic theories. Thats means the rigidity of the nano truncated conical shell in the strain gradient theory is greater than that in the modified couple stress and classical theories respectively.
- Increasing the l/h ratio *in* each vertex angle, increase dimensionless frequency linearly for m=1 and n=1.
- By increasing the R_a/h *in* each vertex angle , decreasing the dimensionless frequency rapidly.

As can be seen by increasing L/ R_a, dimensionless frequency ratio decrease rapidly. The lowest values are for the 60 degree angle.

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