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Vibration of Timoshenko Beam-Soil Foundation Interaction by Using the Spectral Element Method

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ABSTRACT

This article presents an analysis of free vibration of elastically supported Timoshenko beams by using the spectral element method. The governing partial differential equation is elaborated to formulate the spectral stiffness matrix. Effectively, the non-classical end boundary conditions of the beam are the primordial task to calibrate the phenomenon of the Timoshenko beam-soil foundation interaction. Non-dimensional natural frequencies and shape modes are obtained by solving the partial differential equations, numerically. Upon solving the eigenvalue problem, non-dimensional frequencies are computed for the first three modes of vibration. Obtained results of this study are intended to describe multiple objects, such as: (1) the establishment of the modal analysis with and without elastic springs, (2) the quantification of the influence of the beam soil foundation interaction, (3) the influence of soil foundation stiffness' on free vibration characteristics of Timoshenko beam. For this propose, the first three eigenvalues of Timoshenko beam are calculated and plotted for various stiffness of translational and rotational springs.

Keywords : Free-vibration; Non classical boundary conditions; Timoshenko beam; Spectral element method; Finite element method; Beam-soil foundation interaction; Mechanical properties of soil.

1 INTRODUCTION

STRUCTURES like buildings, bridges, pipelines, scaffoldings…etc, are directly in contact with the soil foundation that generates a transfer of loading between them. For its importance, it must be taken into account in the analysis and design under static or dynamic loadings [1]. Early, classical models do not consider the effect of the interaction between the structure and the soil foundation. Really, the interaction effect is neglected in simplify mathematical models in computing phase. In this concept, Mohod and dhadse [2] studied the importance and the role of the soil structure interaction task in computational modeling of ground-structure interaction. In the civil

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engineering domain, many numerical methods are used to obtain the solutions for static or dynamic responses of structures. It underlined that the finite element method is one of most popular and powerful of them [3]. This method can converge to the solution of a problem if many assumptions must be verified, such as: the robustness of the algorithm, mesh refinement, incompatible element forms, the structural form and boundary conditions. On the opposite hand, the spectral finite element method provide as a very important tool for structural dynamics. Its performance and rapidity have been highly observed comparing it with the finite element method [4-5]. Instead, many studies have already been devoted to free vibration analysis of beam with elastically supports using Euler-Bernoulli theory with the finite element method and spectral element method (SEM) [5-6]. Many approaches have already presented assuming that beams are sufficiently slender to be considered as an Euler-Bernoulli beam neglecting the shear deformations and do not give accurate solutions for thick beams. These studies have been elaborated related to the problem of free vibration of beams with elastically supports based using Euler-Bernoulli theory or Timoshenko theory of beams. The common factor between these theories is that the Timoshenko assumption takes into consideration the shear deformations. The solutions of partial differential equations governing the Timoshenko beam depend on the time and the spatial parameter. Using the separate method of variable, it is possible to eliminate the temporal variable [7]. The transformation reduces the governing partial differential equations to a set of ordinary differential equation [8]. Firstly, the task problem of free vibration of Timoshenko beams using finite element procedure is solved by Abbas [9], in which the influence of translational and rotational support flexibilities on the natural frequencies of free vibrations of Timoshenko beams with non-idealized end conditions are investigated. Adding, Hernandez et al [10] analyzed a mixed finite element method for computing the vibration modes of a Timoshenko curved rods with arbitrary geometry. Free vibrations of elastically supported beams are investigated using Euler-Lagrange equation [11]. The two main matrices, which are the stiffness and mass matrices for a two-node beam element with two degree of freedom for each node, are computed based upon Hamilton's principle. Lee and Schultz [12] applied the pseudo-spectral method to the eigenvalue analysis of Timoshenko beams. Zhou [13] used the Rayleigh-Ritz method for the free vibration of multi-span Timoshenko beams. Farghaly [14] has investigated the natural frequencies and the critical buckling load coefficients for a multi-span Timoshenko beam. Banerjee [15] investigated the free vibration analysis of axially loaded Timoshenko beams by using the dynamic stiffness method. The free vibration of Timoshenko beams with internal hinge and subjected to axial tensile load is carried out by Lee et al. [16]. A dynamic investigation method for the analysis of Timoshenko beams which takes into account shear deformation is proposed by Auciello and Ercolano [17]. In [17], the solution of the problem is obtained through the iterative variational Rayleigh-Ritz method. The free vibration of Timoshenko beams having classical boundary conditions, which was satisfied by Lagrange multipliers, was investigated for different thickness-to-length ratios by Kocatürt and Şimşek [6]. In this study, free vibrations of elastically supported Timoshenko beam for the first three eigenvalues of the Timoshenko beam are analyzed using Lagrange equations. The higher-order Timoshenko beam element is developed and employed in studying free vibration of 2-D functionally graded materials Timoshenko beams. The material properties of the beams are considered to vary in both the thickness and longitudinal directions by a power-law distribution. Based on Timoshenko beam theory, equations of motion are derived from Hamilton's principle and they are solved by a finite element procedure based on the developed beam element. The beam element, using hierarchical functions to interpolate the displacement field, is formulated by constraining the shear strain constant for improving its efficiency [18]. More, the geometrical characteristics; length to depth ratio, and different end conditions of a Timoshenko beam are examined by using the differential transform method [19]. In this work, the end conditions, such as: hinge-hinge, fix-hinge, fix-fix and fix-free beams are studied. In this case, the vibration frequencies are computed for the beam with various values of the length to depth ratio. Finally, Magdalena [20] has presented a review on spectral methods for modeling of wave propagation in structures. Therefore, he has exploited it as a tool for diagnostic and damage detection method using the phenomenon of mechanical wave propagation. More, nonlinear vibrations of a slightly curved beam with arbitrary rising function are handled [21]. In this study, the Method of multiple scales is used to solve the equations of motion dealing that the primary resonance is resulted in the steady-state vibrations and thus, the natural frequencies can be obtained for different supports' types, locations of the masses and linear coefficient of the foundation.

In this contribution, the vibrations of the soil foundation-Timoshenko beam are studied by using the spectral element method. The partial differential equations governing the dynamic behavior of the beam with non-classical boundary conditions are illustrated. This topic is of a primary interest by the authors who have already developed it to analyze the riddle of the soil structure interaction [4-5, 22-23]. Upon solving the eigenvalue problem, non-dimensional frequency of the beam are plotted in function of the soil intensity. The sensitivity analysis is also carried out based on the mechanical parameters of compounds of the system. Obtained results are intended to many objects, such as: (1) the establishment of the modal analysis with and without elastic springs, (2) the quantification of the influence of the interaction between the beam and the soil foundation, (3) the influence of soil foundation stiffness' on free vibration characteristics of Timoshenko beam. For this propose, the first three eigenvalues of Timoshenko beam are calculated for various rigidity of translational and rotational springs, and obtained results are presented in 2D plots.

2 MATHEMATICAL FORMULATION

Consider a straight uniform Timoshenko beam of length *L*, flexional rigidity *EI* and cross-section Ω. The Cartesian axis (*xx*) is associated to the central one of the beam. The beam studied is restricted on both ends with elastic supports, which are made of translational and rotational stiffness's, k_0, k_1, K_0 and K_1 (Fig. 1).

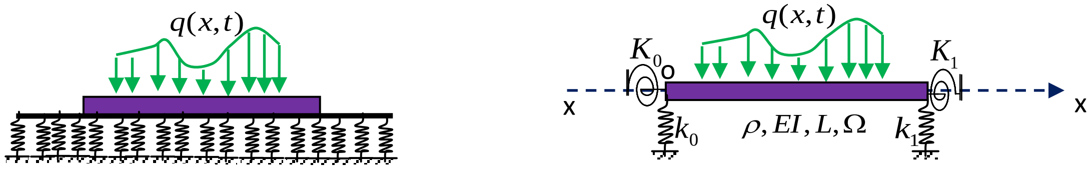


Fig.1
Mechanical and geometrical data of the beam.

The governing coupled differential equations for transverse vibrations of Timoshenko beams are

$$\Omega^* G\left[\frac{\partial^2 v(x,t)}{\partial x^2}-\frac{\partial \varphi(x,t)}{\partial x}\right]-\rho\Omega\frac{\partial^2 v(x,t)}{\partial t^2}-q(x,t)=0 \tag{1a}$$

$$EI\,\frac{\partial^2 \varphi(x,t)}{\partial x^2}+\Omega^* G\left(\frac{\partial v(x,t)}{\partial x}-\varphi(x,t)\right)-\rho I\,\frac{\partial^2 \varphi(x,t)}{\partial t^2}=0 \tag{1b}$$

In which $\Omega^* = R_s\Omega$ and R_s is the reduced shear factor, *G*, the modulus of rigidity, ρ, the mass per unite the volume, $v(x,t)$, the transverse deflection, $\varphi(x,t)$ the bending slope and $q(x,t)$. Particularly for free-vibration of the beam, the equation system (1) becomes

$$\Omega^* G\left[\frac{\partial^2 v(x,t)}{\partial x^2}-\frac{\partial \varphi(x,t)}{\partial x}\right]-\rho\Omega\frac{\partial^2 v(x,t)}{\partial t^2}=0 \tag{2a}$$

$$EI\,\frac{\partial^2 \varphi(x,t)}{\partial x^2}+\Omega^* G\left(\frac{\partial v(x,t)}{\partial x}-\varphi(x,t)\right)-\rho I\,\frac{\partial^2 \varphi(x,t)}{\partial t^2}=0 \tag{2b}$$

The slope of an *x*-abscise section can be coupled of the flexion and shear effect (Fig. 2). The corresponding expression can be expressed by

$$\frac{\partial v(x,t)}{\partial x}=\gamma(x,t)+\varphi(x,t) \tag{3}$$

where $\gamma(x, t)$ is the slope of the beam due to shear strains.

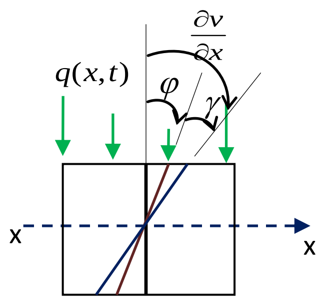


Fig.2
Deformations due to flexion and shear forces.

The system of Eqs. (2) becomes after integration the Eq. (3)

$$\Omega^* G(v''(x,t) - \varphi'(x,t)) - \rho\Omega \ddot{v} = 0 \tag{4a}$$

$$EI\varphi''(x,t) + \Omega^* G(v'(x,t) - \varphi(x,t)) - \rho I \ddot{\varphi} = 0 \tag{4b}$$

The partial differential Eq. (4) can be solved using Fourier decomposition of the displacement field into the sum of harmonic vibration with absence of external loading as:

$$v(x,t) = \frac{1}{N}\sum_{N=0}^{N-1} W_n(x) e^{i\omega t} \tag{5a}$$

$$\varphi(x,t) = \frac{1}{N}\sum_{N=0}^{N-1} \Phi_n(x) e^{i\omega t} \tag{5b}$$

The substitute the Eq. (5) into the Eq. (4), we obtain

$$\Omega^* G(W''(x) - \Phi'(x)) + \rho\Omega\omega^2 W(x) = 0 \tag{6a}$$

$$EI\Phi''(x) + \Omega^* G(W'(x) - \Phi(x)) + \rho I \omega^2 \Phi(x) = 0 \tag{6b}$$

So, the relation (6) can be written as matrix form

$$\begin{bmatrix} \Omega^* G & 0 \\ 0 & EI \end{bmatrix} \begin{Bmatrix} W''(x) \\ \Phi''(x) \end{Bmatrix} + \begin{bmatrix} 0 & -\Omega^* G \\ \Omega^* G & 0 \end{bmatrix} \begin{Bmatrix} W'(x) \\ \Phi'(x) \end{Bmatrix} + \begin{bmatrix} \omega^2 \rho\Omega & 0 \\ 0 & \omega^2 \rho I - \Omega^* G \end{bmatrix} \begin{Bmatrix} W(x) \\ \Phi(x) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{7}$$

The solutions of the Eq. (6) can be written

$$W(x) = Ae^{i\alpha(\omega)x} \tag{8a}$$

$$\Phi(x,t) = \beta A e^{i\alpha(\omega)x} \tag{8b}$$

Subtitling the Eq. (8) into the Eq. (7), we obtain

$$\begin{bmatrix} \rho\Omega\omega^2 - \Omega^* G\alpha^2 & -\Omega^* G\alpha i \\ \Omega^* G\alpha i & \rho I\omega^2 - \Omega^* G - EI\alpha^2 \end{bmatrix} \begin{Bmatrix} 1 \\ \beta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{9}$$

In order to obtain non-trivial solutions, the determinant of the above matrix must be zero.

$$\alpha^4 - \alpha_F^4 \eta \alpha^2 + \alpha_F^4 \left(\alpha_G^4 \eta_1 - 1\right) = 0 \tag{10}$$

with $\alpha_F = \sqrt{\omega}\left(\dfrac{\rho\Omega}{EI}\right)^{1/4}$, $\alpha_G = \sqrt{\omega}\left(\dfrac{\rho}{R_s G}\right)^{1/4}$, $\eta_1 = \dfrac{I}{\Omega}$, $\eta_2 = \dfrac{EI}{R_s G \Omega}$ and $\eta = \eta_1 + \eta_2$

The roots of the Eq. (10) are

$$\alpha_1 = \frac{\alpha_F}{\sqrt{2}} \sqrt{\alpha_F^2 \eta - \sqrt{\left(\alpha_F^4 \eta^2 - 4(\alpha_G^4 \eta_1 - 1)\right)}} \tag{11a}$$

$$\alpha_2 = \frac{\alpha_F}{\sqrt{2}} \sqrt{\alpha_F^2 \eta + \sqrt{\left(\alpha_F^4 \eta^2 - 4(\alpha_G^4 \eta_1 - 1)\right)}} \tag{11b}$$

with $\beta_1 = \dfrac{i}{\alpha_1}(\alpha_1^2 - \alpha_G^4)$, $\beta_2 = \dfrac{i}{\alpha_2}(\alpha_2^2 - \alpha_G^4)$

Thus, the displacement and rotation expressions can be obtained with

$$W(x) = (A_1 e^{i\alpha_1 x} + A_2 e^{-i\alpha_1 x} + A_3 e^{i\alpha_2 x} + A_4 e^{-i\alpha_2 x}) \tag{12a}$$

$$\Phi(x) = (\beta_1 A_1 e^{i\alpha_1 x} - \beta_1 A_2 e^{-i\alpha_1 x} + \beta_2 A_3 e^{i\alpha_2 x} - \beta_2 A_4 e^{-i\alpha_2 x}) \tag{12b}$$

The nodal degree of freedom vector can be deduced as:

$$\{q_e\} = [D(\omega)]\{A\} \tag{13}$$

with

$$[D(\omega)] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \beta_1 & -\beta_1 & \beta_2 & -\beta_2 \\ e_1 & e_1^{-1} & e_2 & e_2^{-1} \\ \beta_1 e_1 & -\beta_1 e_1^{-1} & \beta_2 e_2 & -\beta_2 e_2^{-1} \end{bmatrix}, \; e_1 = e^{i\alpha_1 L} \text{ and } e_2 = e^{i\alpha_2 L}.$$

In the same manner, the nodal force vector is

$$[F] = \begin{bmatrix} \left(-\Omega^* G\left(i\alpha_1 - \beta_1\right) + k_0\right) & \left(\Omega^* G\left(i\alpha_1 - \beta_1\right) + k_0\right) & \left(-\Omega^* G\left(i\alpha_2 - \beta_2\right) + k_0\right) & \left(\Omega^* G\left(i\alpha_2 - \beta_2\right) + k_0\right) \\ i\alpha_1\left(EI\beta_1 - K_0\right) & i\alpha_1\left(EI\beta_1 + K_0\right) & i\alpha_2\left(EI\beta_2 - K_0\right) & i\alpha_2\left(EI\beta_2 + K_0\right) \\ \left(-k_1 - \Omega^* G\left(i\alpha_1 - \beta_1\right)\right)e_1 & \left(-k_1 + \Omega^* G\left(i\alpha_1 - \beta_1\right)\right).e_1^{-1} & \left(-k_1 - \Omega^* G\left(i\alpha_2 - \beta_2\right)\right)e_2 & \left(-k_1 + \Omega^* G\left(-i\alpha_2 - \beta_2\right)\right)e_2^{-1} \\ i\alpha_1\left(EI\beta_1 + K_1\right)e_1 & i\alpha_1\left(EI\beta_1 - K_1\right).e_1^{-1} & i\alpha_2\left(EI\beta_2 + K_1\right)e_2 & i\alpha_2\left(EI\beta_2 - K_1\right)e_2^{-1} \end{bmatrix} \tag{14}$$

The relationship between nodal force and degree of freedom vectors is expressed by

$$\{F_e\} = [C(\omega)][D(\omega)]^{-1}\{q_e\} \tag{15}$$

where $[C(\omega)][D(\omega)]^{-1}$ is the spectral stiffness matrix of the Timoshenko beam on non-classical conditions.

3 NUMERIC EXAMPLES

3.1 validation of the approach

3.1.1 Beam without soil foundation

The free vibration of the Timoshenko beam without considering the soil foundation is investigated to show the validity of this approach for particularly cases (Fig. 3). Different boundary conditions were considered and circular frequencies were compared to references [24-25]. The mechanical properties and geometrical dimensions of the beam are given by the Table 1.

Table 1
Mechanical and geometrical properties of the beam.

$L(m)$	$b(m)$	$h(m)$	$E\,(\text{GP}a)$	$G\,(Pa)$	$\rho(\text{Kg}/m^3)$	R_s
0.4	0.02	0.08	2.1×10^2	$3E/8$	7850	5/6

The first three circular frequencies of Timoshenko beam free vibrations using various classical boundary conditions (Fig.3) are compared to reference [24-25] (Table 2).

Table 2
Convergence study for first three mode circular frequencies $\omega(rad/s)$.

Boundary conditions	Method used	ω_1	ω_2	ω_3
Pinned-pinned	SEM	6838.833559	23190.827069	43443.493061
	Reference [24]	6838.8336	23190.827	43443.493
Clumped-free	SEM	2529.492708	13279.905185	31044.790965
	Reference [24]	2529.4927	13279.905	31044.791
Clamped-pinned	SEM	9741.946896	26150.250677	45545.510089
	Reference [25]	9741.9469	26150.251	45545.510

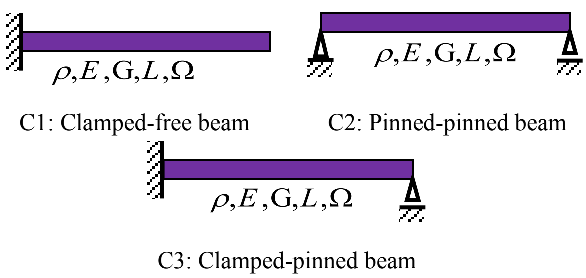


Fig.3
Beams studied.

This section shows not only the convergence of this approach but also the influence of supported classical conditions on vibration modes of the Timoshenko beam. The Fig. 4 regroups the relative non-dimensional frequency ratio for above beam configurations showing the influence of supported end conditions on the modes of Timoshenko beam vibration. This diminution of the influence is remarkable for the first mode and decreases corresponding to higher modes. As example, it varies from 4 times for the first mode to 1.5 times for the third mode of vibration.

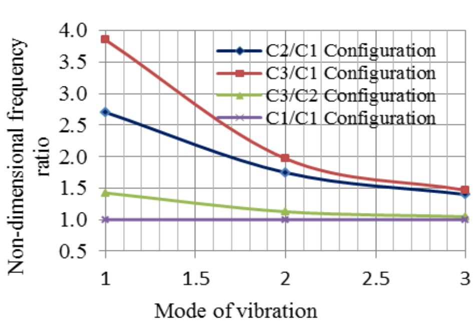


Fig.4
Influence of Supported classical conditions.

3.1.2 Beam with translational spring

The mechanical properties and geometrical dimensions of a stainless beam are regrouped in Table 3.

Table 3
Mechanical and geometrical properties of the beam.

Young's modulus, E=210 *GPa* Density, ρ = 7850 kg/m^3 Poisson's ratio, ν =0.3 Spring stiffness, k_0=$10^4 N/m$ Shear correction factor, R_s=5/6	Length, L=450 *mm* Width, b=20 *mm* Thickness, t=3 *mm*

Beam with translational spring (Fig. 5)

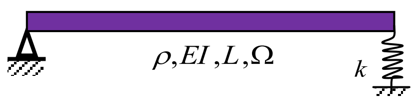


Fig.5
Beam on transversal spring.

The comparison of the first three circular frequencies is elaborated by using the finite element method and the spectral element method (Table 4). The convergence is largely observed between the spectral element method and the finite element method meshing the beam to 200 finite elements.

Table 4
First three frequencies of vibration of the beam.

Method	$\omega_1(rad/s)$	$\omega_2(rad/s)$	$\omega_3(rad/s)$
FEM (200 elements)	196.839179	580.351268	1202.704901
SEM (1 element)	196.839191	580.351264	1202.704858

3.1.3 Beam with rotational spring

Beam with rotational spring (Fig. 6)

Fig.6
Beam on rotational spring**.**

The rotational stiffness of the spring is estimated at $K = 10^4 N.m/rad$ and the first three frequencies of the beam vibration are regrouped in the Table 5.

Table 5
First three frequencies of vibration of the beam.

Method	$\omega_1(rad/s)$	$\omega_2(rad/s)$	$\omega_3(rad/s)$
FEM (200 elements)	54.463076	490.098502	1360.991677
SEM (1 element)	54.463053	490.098494	1360.991615

3.1.4 Beam with translational and rotational spring

Beam with translational and rotational spring (Fig. 7)

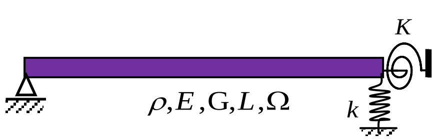


Fig.7
Beam on translational and rotational springs.

In the same manner, the first three frequencies of the beam on translational and rotational springs are regrouped in the Table 6.

Table 6
First three frequencies of vibration by finite element method.

Method	$\omega_1(rad/s)$	$\omega_2(rad/s)$	$\omega_3(rad/s)$
FEM (200 elements)	54.463076	490.098502	1360.991677
SEM (1 element)	54.463053	490.098494	1360.991615

Many free vibrations of beams have been studied in this section and the comparison made with and without translational and rotational springs. The robustness of the spectral element method via the finite element method is observed when the beam meshed to about 200 finite elements.

3.2 Analysis using this approach

In order to calibrate the effect of the interaction between the Timoshenko beam and the soil foundation on free vibrations, the first three eigenvalues of Timoshenko beam with various translational and rotational spring configurations are computed. The primordial objective of this section is to illustrate how the non-dimensional frequency parameters change for different configurations (Fig. 8). In all cases, the stiffness parameters, $k_i\ (N/m)$ and $K_i\ (N.m/rad)$, are taken into account with the same values for all support stiffness'. The geometrical and mechanical characteristics of a concrete beam are regrouped in the Table 7.

Table 7
Geometrical and mechanical characteristics of beams.

Concrete beam	$E\,(GPa)$	$\rho(Kg/m^3)$	$L(m)$	$b(m)$	$h(m)$	$K_S\left(KN/m^3\right)$	Ω^*/Ω	ν
	30	2000	10	1.10	0.50	5×10^4	5/6	0.20

3.2.1 Beam on translational and rotational springs

The study is devoted to analyze free vibration of clamped-clamped beam (Fig. 8).

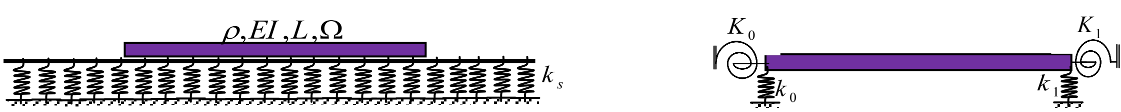
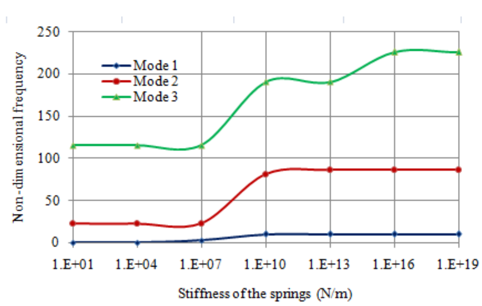
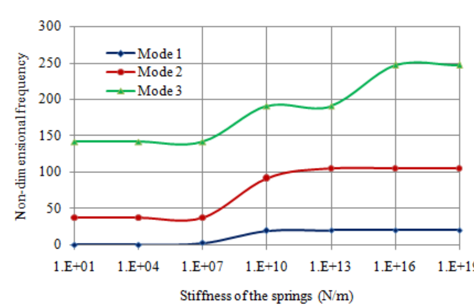


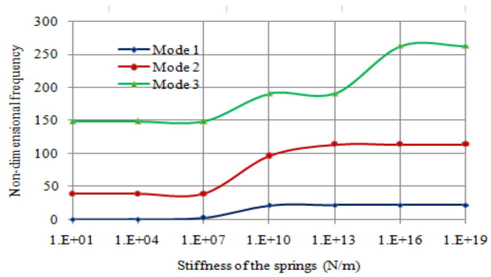
Fig.8
Case 1.



(1)

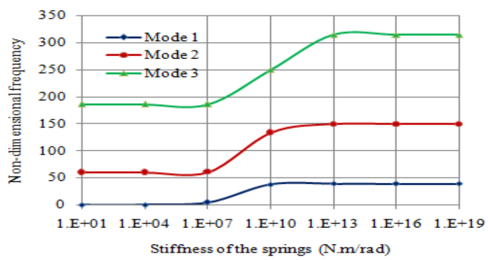


(2)

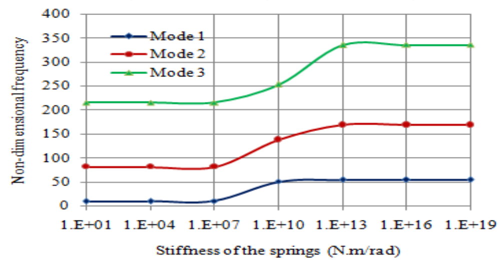


(3)

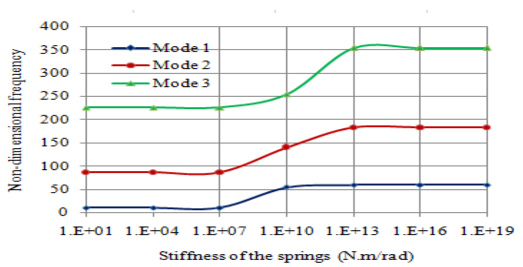
(a) First three modes of translational vibration.



(1)



(2)



(3)

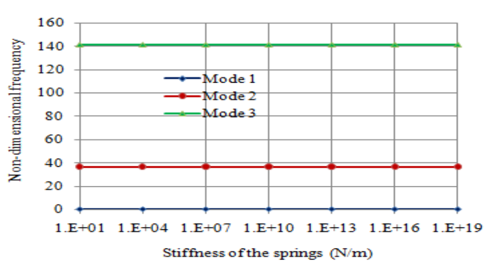
(b)First three modes of rotational vibration.

Fig.9
Effect of boundary spring stiffness on the frequency parameter: (a) Translational springs (b) Rotational springs: (1) k_i=10^0 *N/m*(*N.m/rad*), (2) k_i=10^9 *N/m*(*N.m/rad*), (3) k_i=10^{18} *N/m*(*N.m/rad*).

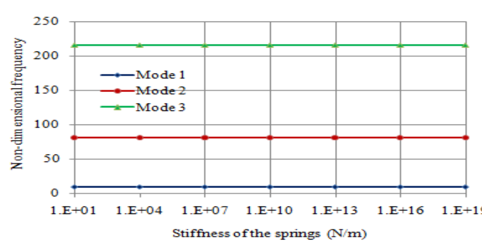
3.2.2 Beam on rotational springs

Beam on rotational springs (Fig. 10)

Fig.10
Case 2.



(1)



(2)

Fig.11
First three modes of (1) translational vibration (2) rotational vibration.

Translational springs do not have an influence on the non-dimensional frequency parameter. Independently, the beam vibrates with a constant non-dimensional frequency parameter.

3.2.3 Beam on translational springs

Beam on translational springs (Fig. 12)

Fig.12
Case 3.

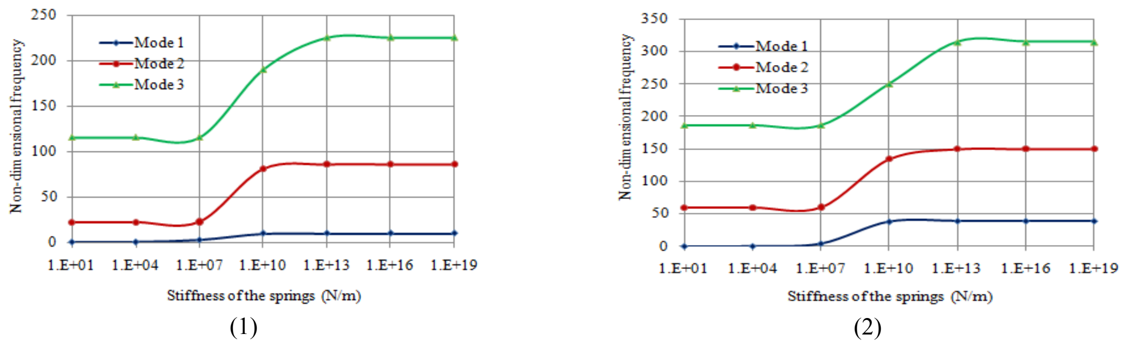


Fig.13
First three modes of (1) translational vibration (2) rotational vibration.

3.2.4 Beam on translational and (translational and rotational) springs

Beam on translational and (translational and rotational) springs (Fig. 14)

Fig.14
Case 4.

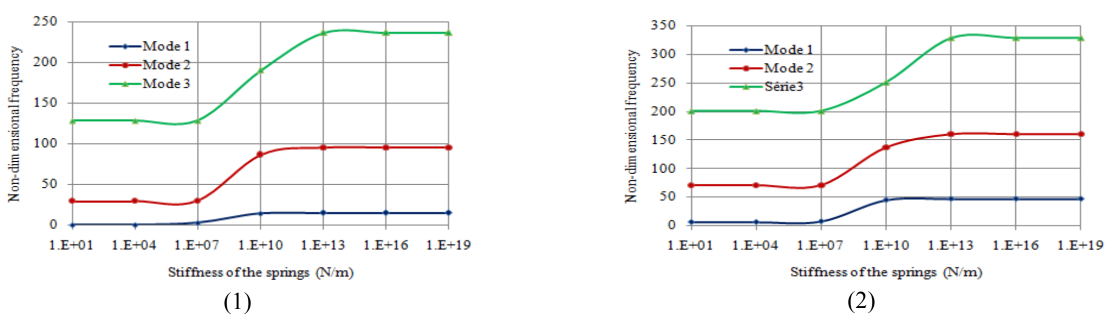


Fig.15
First three modes of (1) translational vibration (2) rotational vibration for $k_1=10^9$(*N.m/rad*).

3.2.5 Beam on (translational and rotational) and rotational springs

Beam on (translational and rotational) and rotational springs (Fig. 16)

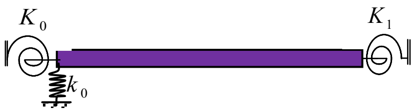
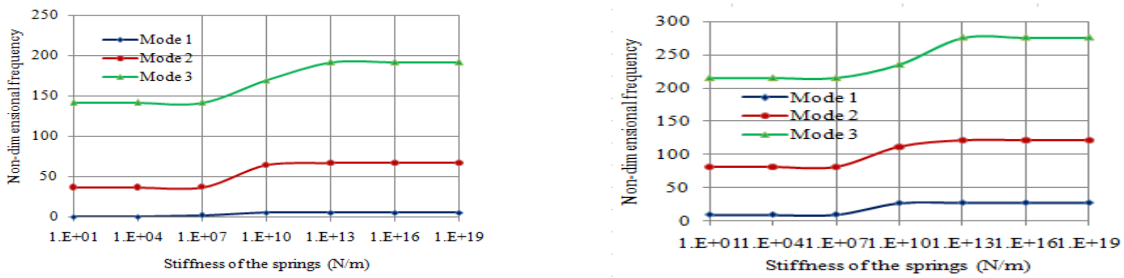


Fig.16
Case 5.



(1) (2)

Fig.17
First three modes of (1) translational vibration (2) rotational vibration for $k_0=10^9(N/m)$.

In three-dimensional plot of the first three mode of vibration is shown in Fig. 18.

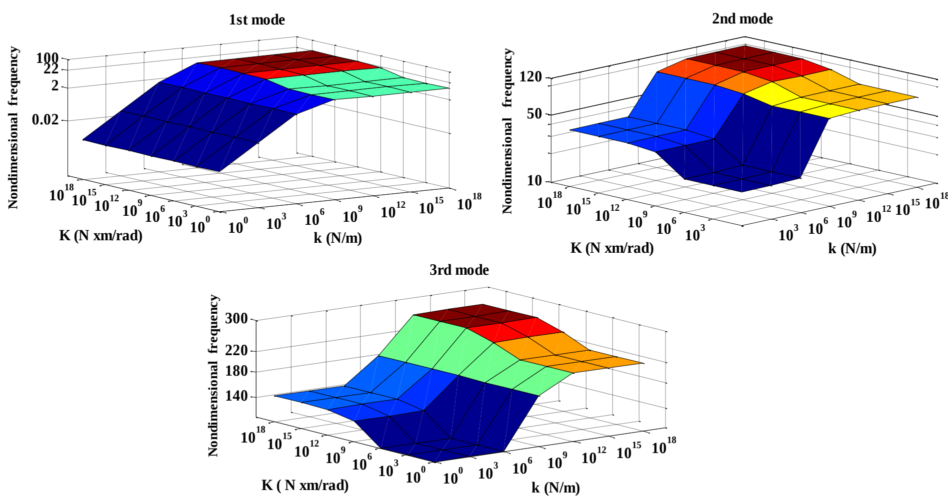


Fig.18
First three modes of vibration for case 1.

In this section, five cases are considered (Figs. 8, 10, 12, 14 and 16) to study the effect of non-classical boundary conditions or of soil-structure interaction on the free vibration of Timoshenko beam. The effect of translational and rotational springs is quantified and commented. It is observed that from Figs. 9, 11, 13, 15 and 17 that, translational springs are much effective on the frequency parameters than rotational springs. The analysis statement shows that a stable of the non-dimensional parameter can be observed when the stiffness of beams is either less than $10^5 N/m$ or more than $10^{10} N/m$. More, it seems that the configuration 2 is more appropriate to transversal free vibration of the Timoshenko beam.

4 CONCLUSIONS

In this paper, the spectral element method was used to analyze the vibrations of Timoshenko beam on non-classical supported end conditions. Obtained results are compared to those of the finite element method and analytical data. Non-dimensional frequencies of beam vibration were plotted and corresponding conclusions that can be inspired are

- The approach is validated using a beam with and without translational and rotational springs. This verification is done through the comparison of obtained results with finite element data and analytical method.
- A single spectral element was used to modeling the Timoshenko beam while a large number of finite elements are required for achieving results accuracy.
- Many numerical programs are developed for these concerns using the spectral element method and the finite element method. These numerical tools can be developed to analyze various problems.
- Numerical applications using the spectral element method show lower computation cost compared to the finite element method.
- Translational springs are much effective on the frequency parameters than rotational springs.
- The analysis shows that a stable of the non-dimensional parameter can be observed for feeble and higher values of spring stiffness's.

REFERENCES

[1] Tabatabaiefar H.R., Clifton T., 2016, Significance of considering soil-structure interaction effects on seismic design of unbraced building frames resting on soft soils, *Australian Geomechanics* **5**(1): 55-66.

[2] Mohod M.V., Dhadse G.D., 2014, Importance of soil structure interaction for framed structure*, International Conference on Advances in Civil and Mechanical Engineering Systems*, Surat, India.

[3] Lee U., 2009, Spectral element analysis method, *In Spectral Element Method in Structural Dynamics*, Chichester, UK.

[4] Hamioud S., Khalfallah S., 2016, Free-vibration of Bernoulli-Euler Beam by the spectral element method, *Technical Journal* **10**(3-4): 106-112.

[5] Hamioud S., Khalfallah S., 2018, Free-vibration of Timoshenko Beam using the spectral element method, *International Journal for Engineering Modelling* **31**(1-2): 61-76.

[6] Kocaturk T., Şimşek M., 2005, Free vibration analysis of Timoshenko beams under various boundary conditions, *Sigma Journal of Engineering and Natural Sciences* **3**: 79-93.

[7] Lee U., Cho J., 2008, FFT-based spectral element analysis for the linear continuum dynamic systems subjected to arbitrary initial conditions by using the pseudo-force method, *International Journal for Numerical Methods in Engineering* **74**: 159-174.

[8] Gopalakrishnan S., Chakraborty A., Roy Mahapatra D., 2008, Spectral Finite Element Method: Wave Propagation, Diagnostics and Control in Anisotropic and Inhomogenous Structures, Springer, London.

[9] Abbas B.A.H., 1984, Vibrations of beams with elastically restrained end, *Journal of Sound and Vibration* **97**: 541-548.

[10] Hernandez E., Otrola E., Rodriguez R., Sahueza F., 2008, Finite element approximation of the vibration problem for a Timoshenko curved rod, *Revista de La Union Matemtica* **49**: 15-28.

[11] Azevedo A.S.D.C, Vasconcelos A.C.A., Hoefel S.D.S., 2016, Dynamic analysis of elastically supported Timoshenko beam, *XXXVII Iberian Latin American Congress on Computational Methods in Engineering*, Brazilia, Brazil.

[12] Lee J., Schultz W.W., 2004, Eigenvalue analysis of Timoshenko beams and axi-symmetric Mindlin plates by the pseudospectral method, *Journal of Sound and Vibration* **269**: 609-621.

[13] Zhou D., 2001, Free vibration of multi-span Timoshenko beams using static Timoshenko beam functions, *Journal of Sound and Vibration* **241**: 725-734.

[14] Farghaly S.H., 1994, Vibration and stability analysis Timoshenko beams with discontinuities in cross-section, *Journal of Sound and Vibration* **174**: 591-605.

[15] Banerjee J.R., 1998, Free vibration of axially loaded composite Timoshenko beams using the dynamic stiffness matrix method, *Computers & Structures* **69**: 197-208.

[16] Lee Y.Y., Wang C.M., Kitipornchai S., 2003, Vibration of Timoshenko beams with internal hinge, *Journal of Engineering Mechanics* **129**(3): 293-301.

[17] Auciello N.M., Ercolano A., 2004, A general solution for dynamic response of axially loaded non-uniform Timoshenko beams, *Journal of Solids and Structures* **41**(18-19): 4861-4874.

[18] Thom T.T., Kien N.D., 2018, Free vibration of two-directional FGM beams using a higher-order Timoshenko beam element, *Vietnam Journal of Science and Technology* **56**(3): 380-396.

[19] Shali S., Jafarali P., Nagaraja S.R., 2018, Identification of second spectrum of a Timoshenko beam using differential transform method**,** *Journal of Engineering Science and Technology* **13**(4): 893-908.

[20] Magdalena P., 2018, Spectral methods for modeling of wave propagation in structure in terms of damage detection- A review, *Applied Sciences Journal* **8**(7): 1-25.

[21] Sarigul M., 2018, Effect of elastically supports on nonlinear vibrations of a slightly curved beam, *Uludag University Journal of the Faculty of Engineering* **23**(2): 255-274.

[22] Hamioud S., Khalfallah S., 2017, Dynamic analysis of rods using the spectral element method, *Algerian & Equipment Journal* **57**: 49-55.

[23] Boudaa S., Khalfallah S., Hamioud S., 2019, Dynamic analysis of soil-structure interaction by the spectral element method, *Innovative Infrastructure Solutions* **4**(1): 40.

[24] Ruta P., 1999, Application of Chebyshev series to solution of non-prismatic beam vibration problems, *Journal of Sound and Vibration* **227**(2): 449-467.

[25] Chen G., Qian L., Yin Q., 2014, Dynamic analysis of a Timoshenko beam subjected to an accelerating mass using spectral element method, *Shock and Vibration* **2014**: 12.