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Impact of Initial Stress on Reflection and Transmission of SV-Wave between Two Orthotropic Thermoelastic Half-Spaces

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ABSTRACT

Reflection and transmission of plane waves between two initially stressed thermoelastic half-spaces with orthotropic type of anisotropy is studied. Incidence of a SV-type wave from the lower half-space is considered and the amplitude ratios of the reflected and transmitted SV-wave, P-wave and thermal wave are obtained by using appropriate boundary conditions. Numerical computation for a particular model is performed and graphs are plotted to study the effect of angle of incidence of the wave and the initial stress parameters of the half-spaces. From the graphical results, it is found that the modulus of reflection and transmission coefficients of the thermal wave is very less in comparison to reflection and transmission coefficients of P- and SV-waves. It is also observed that for vertical incidence of SV-wave we have only reflected and refracted SV-waves and there is no reflected or refracted P and thermal waves, whereas for horizontal incidence of SV-wave, there exists only reflected SV-wave and no other reflected or transmitted wave exists. Moreover, it is found that all the reflection and transmission coefficients are strongly affected by the initial stress parameters of the both half-spaces.

Keywords: Thermal wave; Anisotropic; Initial stress; Reflection coefficient; Transmission coefficient.

1 INTRODUCTION

THE study of reflection and refraction of seismic waves at the interface between two different elastic media is of great importance to seismologist and geophysicist. The seismic waves propagating through the Earth travel through different layers and interfaces. The velocities of these waves are influenced by the properties of the layers through which it travel, and whenever these waves comes across the discontinuities between different layers, the phenomena of reflection and refraction take place. The signals of reflected waves are not only helpful in providing information about the internal structures of the Earth but are also helpful in exploration of valuable materials such as

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minerals, crystals, metals, oil, etc. The technique of wave propagation is one of the most suitable in terms of time saving and economy. Therefore, the problems of seismic wave propagation and their reflection and refraction are of great help to geophysicists, engineers in mineral companies and future researchers in the pertinent area. Moreover, these studies are useful in earthquake engineering, non-destructive evaluation, signal processing, sound system and wireless communication.

The presence of initial stresses in solid materials can have a substantial effect on their subsequent reaction to applied loads that is very different from the corresponding reaction in the initial stresses free case. The stresses which exist in an elastic body even though external forces are absent are named as initial stresses and the body is said to be in the state of initial stress. Rock mechanics, mechanics of materials and structural elements, geophysics, seismology, mechanics of composites and similar fields are fundamental scientific areas in which it is necessary to study the effect of initial stresses or strains as applied to elastic waves. The fact that the Earth is in a state of high initial stress was first predicted by Love [1]. Due to atmospheric pressure, gravity variation, creep, difference in temperature, large initial stresses may exist inside the Earth. The high stress developed below the Earth's surface due to gravity has a strong influence on the propagation of elastic waves generated due to earthquake, explorations and impacts. Thus, it is necessary to study the properties of wave propagation in the presence of initial stress. Biot [2] showed that the elastic wave propagation in the presence of initial stress was different from the case of absence of initial stress and could not be described with the classical linear theory of elasticity and stress-dependent elastic coefficients. Biot [3] described the theory of incremental deformation in his well-known book "Mechanics of Incremental Deformations" and later many researchers applied this theory to study the propagation of elastic waves in pre-stressed elastic bodies. Dey and Addy [4] studied reflection of P- and SV-waves from free surface of an elastic half-space under initial stress. Chattopadhyay et al. [5] discussed the reflection of P- and SV-waves at free surface of an initially stressed sandy medium. Dey et al. [6] investigated reflection and refraction of P waves under initial stress. However, in none of the above work effect of temperature field was considered. Sinha and Sinha [7] and Sinha and Elsibai [8] studied reflection of thermoelastic waves from the free surface of a solid half-space considering one and two relaxation times, respectively. Sinha and Elsibai [9] discussed reflection and refraction of thermoelastic waves at an interface of two semi-infinite media with two relaxation times. Abd-Alla and Al-Dawy [10] studied reflection of SV-waves in a generalized thermoelastic medium. Kumar and Singh [11] investigated reflection and transmission of plane waves at an imperfectly bounded interface of two generalized thermoelastic half-spaces. Singh et al. [12] studied reflection and transmission of P- and SV-waves at an interface between two dissimilar thermoelastic solids with diffusion. However, none of the above researchers considered the effect of initial stress. Montanaro [13] employed Biot's theory to investigate isotropic linear thermoelasticity with hydrostatic initial stress. Singh et al. [14] applied Lord-Shulman theory to study reflection of generalized thermoelastic waves from free surface of a half-space under hydrostatic initial stress. Othman and Song [15] used Green and Naghdi theory to investigate reflection of plane waves from a free surface of a generalized thermoelastic half-space under hydrostatic initial stress. Singh et al. [16] studied reflection and refraction of thermoelastic waves at an interface between two solid half-spaces under hydrostatic initial stress. A good amount of information regarding reflection and refraction of waves in initially stressed thermoelastic medium can be gained by the works of Chakraborty and Singh [17], Singh and Chakraborty [18, 19], however the above investigations were done for isotropic medium. Kumar and Kumar [20] investigated wave propagation in orthotropic generalized thermoelastic half-space with voids under initial stress. Abd-Alla et al. [21] studied propagation of Rayleigh waves in generalized magneto-thermoelastic orthotropic material under the effect of initial stress and gravity field. Ahmed and Abo-Dahab [22] discussed the effect of gravity and initial stress on the propagation of Rayleigh and Stoneley waves in a thermoelastic orthotropic granular layer over a thermoelastic orthotropic granular half-space. Pal et al. [23] investigated plane wave propagation in inhomogeneous anisotropic generalized thermoelastic medium with two relaxation times. Works of Sharma [24, 25], Gupta and Gupta [26], Sharma and Kaur [27], Kakar and Kakar [28], Kumar et al. [29], Sur and Kanoria [30-33], Karmakar et al. [34], Prasad et al. [35], Kumar and Kaur [36], Sur et al. [37, 38], Purkait et al. [39] may also be cited who have studied propagation of waves and related phenomena in anisotropic, thermoelastic and initially stressed medium with different geometry.

In this problem, an attempt is made to investigate reflection and transmission phenomena due to incidence of a plane SV- wave at a plane interface between two homogeneous orthotropic thermoelastic solid half-spaces under the effect of initial stress. Due to incidence of a SV-wave three reflected waves, namely P-wave, SV-wave and thermal wave, and same type of three refracted waves are generated. Applying appropriate boundary conditions, amplitude ratios corresponding to the three reflected and three refracted components are calculated and graphs of these ratios are plotted against the angle of incidence for different set of values of the initial stress parameters of the half-spaces. It is found that the amplitude ratios corresponding to all the reflected and refracted waves are strongly affected by the angle of incidence of the wave and the initial stress present in the medium.

2 FORMULATION OF THE PROBLEM

Two orthotropic solid half-spaces both of which are homogeneous and at uniform absolute temperature in the undisturbed state, separated by a plane interface $z = 0$ are considered. Both the half-spaces are in the state of initial stress. A plane SV-wave travelling through the lower half-space (medium *M*) is incident at the interface and is partially reflected as one SV-wave (rotational wave), one P-wave (dilatational wave) and one thermal wave (dilatational wave). Rest of the wave continues to travel in the upper half-space (medium M') after refraction, as one SV-wave, one P-wave and one thermal wave (Fig. 1). We shall calculate the amplitude ratios corresponding to the three reflected and three refracted components.

Since we are dealing a two-dimensional problem, we restrict our analysis to plane strain parallel to oxz - plane. Hence, all the field variables depend only on space coordinates *x*, *z* and time *t*, and are independent of coordinate *y*. For convenience, we shall use notation of prime in all quantities corresponding to the medium M' .

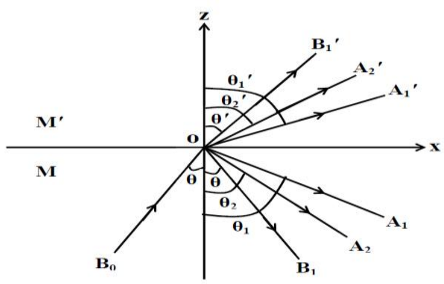


Fig.1
Geometry of the problem.

3 BASIC EQUATIONS AND SOLUTIONS

The dynamical equations of motion for a plane strain under initial compressive stress P in *x*- direction in absence of heat source and body forces are given by Biot [3]:

$$\begin{aligned} &\frac{\partial \tau_{11}}{\partial x}+\frac{\partial \tau_{21}}{\partial y}+\frac{\partial \tau_{31}}{\partial z}+\frac{P}{2}\left(\frac{\partial w_{21}}{\partial y}-\frac{\partial w_{31}}{\partial z}\right)=\rho\frac{\partial^2 u}{\partial t^2}\\ &\frac{\partial \tau_{12}}{\partial x}+\frac{\partial \tau_{22}}{\partial y}+\frac{\partial \tau_{32}}{\partial z}+\frac{P}{2}\frac{\partial w_{12}}{\partial x}=\rho\frac{\partial^2 v}{\partial t^2}\\ &\frac{\partial \tau_{13}}{\partial x}+\frac{\partial \tau_{23}}{\partial y}+\frac{\partial \tau_{33}}{\partial z}-\frac{P}{2}\frac{\partial w_{31}}{\partial x}=\rho\frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{1}$$

where

$$w_{ij}=u_{ij}-u_{ji} \tag{2}$$

(u,v,w) are displacement components, τ_{ij} are incremental stress components, ρ is the density of the medium, P is the compressive initial stress.

Eq. (1) in two dimensions (x,z) reduces to

$$\begin{aligned} &\frac{\partial \tau_{11}}{\partial x}+\frac{\partial \tau_{31}}{\partial z}-\frac{P}{2}\frac{\partial w_{31}}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2}\\ &\frac{\partial \tau_{13}}{\partial x}+\frac{\partial \tau_{33}}{\partial z}-\frac{P}{2}\frac{\partial w_{31}}{\partial x}=\rho\frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{3}$$

The modified heat conduction equation is given by

$$\nabla^2 T = \frac{2\rho s}{\delta_1+\delta_2}\frac{\partial}{\partial t}\left[1+t_2\frac{\partial}{\partial t}\right]T + \frac{T_0(\upsilon_1+\upsilon_3)}{\delta_1+\delta_2}\frac{\partial}{\partial t}\left[1+t_2\delta_{ij}\frac{\partial}{\partial t}\right]\vec{\nabla}\cdot\vec{u} \tag{4}$$

where s is the specific heat per unit mass, T_0 is the initial temperature, T is the absolute temperature, $\vec{u}=(u,v,w)$ is the displacement vector, α_1 and α_2 are thermal expansion coefficients, δ_{ij} is Kronecker delta, δ_1 and δ_2 are thermal conductivities, t_1 and t_2 are mechanical relaxation times.

The stress-strain relations are given by

$$\begin{aligned}\tau_{11} &= (c_{11}+P)\frac{\partial u}{\partial x}+(c_{13}+P)\frac{\partial w}{\partial z}-\upsilon_1\left(1+t_1\frac{\partial}{\partial t}\right)T\\ \tau_{33} &= c_{13}\frac{\partial u}{\partial x}+c_{33}\frac{\partial w}{\partial z}-\upsilon_3\left(1+t_1\frac{\partial}{\partial t}\right)T\\ \tau_{13} &= c_{44}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\end{aligned} \tag{5}$$

where

$$\upsilon_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_2 \qquad \upsilon_3=2c_{13}\alpha_1+c_{33}\alpha_2 \tag{6}$$

Moreover, c_{ij} are the stiffness constants.

Using Eqs. (5) and (2) in Eq. (3), we get

$$(c_{11}+P)\frac{\partial^2 u}{\partial x^2}+\left(c_{13}+c_{44}+\frac{P}{2}\right)\frac{\partial^2 w}{\partial x\,\partial z}+\left(c_{44}+\frac{P}{2}\right)\frac{\partial^2 u}{\partial z^2}-\upsilon_1\frac{\partial}{\partial x}\left[1+t_1\frac{\partial}{\partial t}\right]T=\rho\frac{\partial^2 u}{\partial t^2} \tag{7a}$$

$$\left(c_{44}-\frac{P}{2}\right)\frac{\partial^2 w}{\partial x^2}+\left(c_{13}+c_{44}+\frac{P}{2}\right)\frac{\partial^2 u}{\partial x\,\partial z}+c_{33}\frac{\partial^2 w}{\partial z^2}-\upsilon_3\frac{\partial}{\partial z}\left[1+t_1\frac{\partial}{\partial t}\right]T=\rho\frac{\partial^2 w}{\partial t^2} \tag{7b}$$

In the present problem, we shall use Classical-dynamical theory in which $t_1=t_2=0$, $\delta_{ij}=0$, so Eqs. (4),(7a) and (7b) reduce to

$$\nabla^2 T = \frac{2\rho s}{\delta_1+\delta_2}\frac{\partial T}{\partial t}+\frac{T_0(\upsilon_1+\upsilon_3)}{\delta_1+\delta_2}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) \tag{8}$$

$$(c_{11}+P)\frac{\partial^2 u}{\partial x^2}+\left(c_{13}+c_{44}+\frac{P}{2}\right)\frac{\partial^2 w}{\partial x\,\partial z}+\left(c_{44}+\frac{P}{2}\right)\frac{\partial^2 u}{\partial z^2}-\upsilon_1\frac{\partial T}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2} \tag{9a}$$

$$\left(c_{44}-\frac{P}{2}\right)\frac{\partial^2 w}{\partial x^2}+\left(c_{13}+c_{44}+\frac{P}{2}\right)\frac{\partial^2 u}{\partial x\,\partial z}+c_{33}\frac{\partial^2 w}{\partial z^2}-\upsilon_3\frac{\partial T}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2} \tag{9b}$$

We introduce displacement potentials ϕ and ψ as:

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z}\ ,\ \ v=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x} \tag{10}$$

Using Eq. (10) in Eq. (9a), we get

$$(c_{11}+P)\frac{\partial^2\phi}{\partial x^2}+(c_{13}+2c_{44}+P)\frac{\partial^2\phi}{\partial z^2}-\upsilon_1 T=\rho\frac{\partial^2\phi}{\partial t^2} \tag{11a}$$

$$\left(c_{11}-c_{13}-c_{44}+\frac{P}{2}\right)\frac{\partial^2\psi}{\partial x^2}+\left(c_{44}+\frac{P}{2}\right)\frac{\partial^2\psi}{\partial z^2}=\rho\frac{\partial^2\psi}{\partial t^2} \tag{11b}$$

Using Eq. (10) in Eq. (9b), we get

$$(c_{13}+2c_{44})\frac{\partial^2\phi}{\partial x^2}+c_{33}\frac{\partial^2\phi}{\partial z^2}-\upsilon_3 T=\rho\frac{\partial^2\phi}{\partial t^2} \tag{12a}$$

$$\left(c_{44}-\frac{P}{2}\right)\frac{\partial^2\psi}{\partial x^2}+\left(c_{33}-c_{13}-c_{44}-\frac{P}{2}\right)\frac{\partial^2\psi}{\partial z^2}=\rho\frac{\partial^2\psi}{\partial t^2} \tag{12b}$$

Using Eq. (10) in Eq. (8), we get

$$\nabla^2 T=\frac{1}{\chi}\frac{\partial T}{\partial t}+\eta\frac{\partial}{\partial t}\left(\nabla^2\phi\right) \tag{13}$$

where

$$\chi=\frac{\delta_1+\delta_2}{2\rho s}\quad \text{and} \quad \eta=\frac{T_0(\upsilon_1+\upsilon_3)}{\delta_1+\delta_2} \tag{14}$$

when we put Eq. (10) into Eq. (9a) and simplify, Eqs. (11a) and (11b) are obtained by integrating partially with respect to x and z , respectively. Similarly, using Eq. (10) in Eq. (9b), Eqs. (12a) and (12b) are obtained by integrating partially with respect to z and x, respectively. Thus Eqs. (11a) and (12a) involving scalar potential ϕ represent dilatational waves along x- axis and z- axis, respectively and Eqs. (11b) and (12b) involving vector potential ψ represent rotational waves along z- axis and x- axis, respectively.

We shall consider Eqs. (11a) and (12b) which represents dilatational and rotational waves propagating along x-axis. So, the equations for dilatational and rotational waves propagating along x axis in terms of potential functions ϕ and ψ are given by

$$(c_{11}+P)\frac{\partial^2\phi}{\partial x^2}+(c_{13}+2c_{44}+P)\frac{\partial^2\phi}{\partial z^2}=\rho\frac{\partial^2\phi}{\partial t^2}+\upsilon_1 T \tag{15a}$$

$$\left(c_{44}-\frac{P}{2}\right)\frac{\partial^2\psi}{\partial x^2}+\left(c_{33}-c_{13}-c_{44}-\frac{P}{2}\right)\frac{\partial^2\psi}{\partial z^2}=\rho\frac{\partial^2\psi}{\partial t^2} \tag{15b}$$

Observing Eqs. (13), (15a) and (15b), we find that dilatational waves are affected by the presence of the temperature field but the rotational wave is independent of the temperature field.

For a harmonic wave propagating along the x- direction, with the wave normal in xz- plane, making an angle θ with the z- axis, we can assume the solution of Eqs. (13), (15a) and (15b) as follows:

$$T=\bar{T}\exp[i\{k(x\sin\theta+z\cos\theta)-\omega t\}] \tag{16a}$$

$$\phi=\bar{\phi}\exp[i\{k(x\sin\theta+z\cos\theta)-\omega t\}] \tag{16b}$$

$$\psi=\bar{\psi}\exp[i\{l(x\sin\theta+z\cos\theta)-\omega t\}] \tag{16c}$$

where k and l are the wave numbers of the dilatational and rotational waves, respectively and ω is the angular frequency.

Using Eqs. (16a) and (16b) in Eq. (15a), we get

$$\left[k^2 c_1^2 \left\{ (1+2\zeta)\sin^2\theta + \left(\frac{c_{13}+2c_{44}}{c_{11}} + 2\zeta \right)\cos^2\theta \right\} - \omega^2 \right]\bar{\phi} + \frac{\upsilon_1}{\rho}\bar{T} = 0 \tag{17a}$$

and using Eqs. (16a) and (16b) in Eq. (13) gives

$$i\,\eta\omega k^2\bar{\phi} + \left(k^2 - i\,\frac{\omega}{\chi} \right)\bar{T} = 0 \tag{17b}$$

where

$$c_1^2 = \frac{c_{11}}{\rho} \quad \text{and} \quad \zeta = \frac{P}{2c_{11}} \tag{18}$$

Eqs. (17a) and (17b) will be simultaneously satisfied only if the determinant of their coefficients vanishes. Equating to zero the determinant of coefficients of Eqs. (17a) and (17b) yields

$$V^4 - (\xi + \beta - i\,\gamma)V^2 - i\,\xi\gamma = 0 \tag{19}$$

where

$$\xi = (1+2\zeta)\sin^2\theta + \left(\frac{c_{13}+2c_{44}}{c_{11}} + 2\zeta \right)\cos^2\theta \quad , \beta = \frac{\upsilon_1\eta\chi}{\rho c_1^2} \;,\; \gamma = \frac{\omega\chi}{c_1^2}\,,\; V = \frac{\omega}{kc_1} \tag{20}$$

Eq. (19) is quadratic in V^2, which indicates that two types of dilatational waves would travel in the medium M, namely P-wave and thermal wave, travelling with two different velocities. We take the roots of Eq. (19) as $V_1 = \frac{\omega}{k_1 c_1}$ and$V_2 = \frac{\omega}{k_2 c_1}$, where V_1 and V_2 represents velocities of the P-wave and thermal wave, respectively in dimensionless form. Substituting Eq. (16c) in Eq. (15b) gives $\left[\left(\frac{c_{44}}{c_{11}} - \zeta \right)\sin^2\theta + \left(\frac{c_{33}-c_{13}-c_{44}}{c_{11}} - \zeta \right)\cos^2\theta \right]^{1/2} = \frac{\omega}{lc_1}$. $\omega/lc_1 \;\; (=V_0 \text{ say})$ gives the velocity of SV-wave in dimensionless form. k_1, k_2 and l are the wave vectors of the P-wave, thermal wave and SV-wave, respectively.

when a SV-wave travelling through the lower half-space is incident at the interface $z = 0$ at angle θ with the negative z- axis, we get reflected P-wave, thermal wave and SV-wave making angles θ_1, θ_2 and θ, respectively with the negative z- axis in the medium M and refracted P-wave, thermal wave and SV-wave making angles θ_1', θ_2' and θ', respectively with the positive z- axis in the medium M'. Using the symbol of prime for the quantities corresponding to the medium M', we have

$$\left.\begin{array}{c} \chi' = \dfrac{\delta_1' + \delta_2'}{2\rho' s'}, \quad \eta' = \dfrac{T_0'(\upsilon_1' + \upsilon_3')}{\delta_1' + \delta_2'}, \\ c_1'^2 = \dfrac{c_{11}'}{\rho'}, \\ \zeta' = \dfrac{P'}{2c_{11}'}, \\ \xi' = (1+2\zeta')\sin^2\theta' + \left(\dfrac{c_{13}' + 2c_{44}'}{c_{11}'} + 2\zeta' \right)\cos^2\theta', \\ \beta' = \dfrac{\upsilon_1'\eta'\chi'}{\rho' c_1'^2}, \quad \gamma' = \dfrac{\omega\chi'}{c_1'^2}, \\ V_1' = \dfrac{\omega}{k_1' c_1'}, \quad V_2' = \dfrac{\omega}{k_2' c_1'}, \quad V_0' = \dfrac{\omega}{l' c_1'}. \end{array}\right\} \tag{21}$$

The angles θ, θ_1, θ_2, θ', θ_1', θ_2' and the corresponding wave vectors l, k_1, k_2, l', k_1', k_2' are related by

$$l\sin\theta = k_1\sin\theta_1 = k_2\sin\theta_2 = l'\sin\theta' = k_1'\sin\theta_1' = k_2'\sin\theta_2' \tag{22}$$

Eq. (22) in terms of dimensionless velocities V 's can be written as:

$$\frac{\sin\theta}{c_1V_0} = \frac{\sin\theta_1}{c_1V_1} = \frac{\sin\theta_2}{c_1V_2} = \frac{\sin\theta'}{c_1V_0'} = \frac{\sin\theta_1'}{c_1V_1'} = \frac{\sin\theta_2'}{c_1V_2'} \tag{23}$$

In medium M, the displacement potentials and temperature may be taken in the following form:

$$\phi = A_1\exp[i\{k_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\}] + A_2\exp[i\{k_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\}] \tag{24a}$$

$$\psi = B_0\exp[i\{l(x\sin\theta + z\cos\theta) - \omega t\}] + B_1\exp[i\{l(x\sin\theta - z\cos\theta) - \omega t\}] \tag{24b}$$

$$T = D_1\exp[i\{k_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\}] + D_2\exp[i\{k_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\}] \tag{24c}$$

In medium M', the displacement potentials and temperature may be taken in the following form:

$$\phi' = A_1'\exp[i\{k_1'(x\sin\theta_1' + z\cos\theta_1') - \omega t\}] + A_2'\exp[i\{k_2'(x\sin\theta_2' + z\cos\theta_2') - \omega t\}] \tag{25a}$$

$$\psi' = B_1'\exp[i\{l'(x\sin\theta' + z\cos\theta') - \omega t\}] \tag{25b}$$

$$T' = D_1'\exp[i\{k_1'(x\sin\theta_1' + z\cos\theta_1') - \omega t\}] + D_2'\exp[i\{k_2'(x\sin\theta_2' + z\cos\theta_2') - \omega t\}] \tag{25c}$$

Using Eqs. (24a) and (24c) in Eq. (13), the values of the coefficients D_1 and D_2 in terms of A_1 and A_2 can be written as:

$$D_1 = \frac{\rho\omega^2}{\upsilon_1}\frac{\beta}{V_1^2 + i\gamma}A_1 \quad \text{and} \quad D_2 = \frac{\rho\omega^2}{\upsilon_1}\frac{\beta}{V_2^2 + i\gamma}A_2 \tag{26}$$

Similarly, we can obtain

$$D_1' = \frac{\rho'\omega^2}{\upsilon_1'}\frac{\beta'}{V_1'^2 + i\gamma'}A_1' \quad \text{and} \quad D_2' = \frac{\rho'\omega^2}{\upsilon_1'}\frac{\beta'}{V_2'^2 + i\gamma'}A_2' \tag{27}$$

Using Eqs. (26) and (27) in Eqs. (24c) and (25c) respectively, we get

$$T = \frac{\rho\omega^2}{\upsilon_1}\frac{\beta}{V_1^2 + i\gamma}A_1\exp[i\{k_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\}] + \frac{\rho\omega^2}{\upsilon_1}\frac{\beta}{V_2^2 + i\gamma}A_2\exp[i\{k_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\}] \tag{28}$$

and

$$T' = \frac{\rho'\omega^2}{\upsilon_1'}\frac{\beta'}{V_1'^2 + i\gamma'}A_1'\exp[i\{k_1'(x\sin\theta_1' + z\cos\theta_1') - \omega t\}] + \frac{\rho'\omega^2}{\upsilon_1'}\frac{\beta'}{V_2'^2 + i\gamma'}A_2'\exp[i\{k_2'(x\sin\theta_2' + z\cos\theta_2') - \omega t\}] \tag{29}$$

4 BOUNDARY CONDITIONS AND REFLECTION AND TRANSMISSION COEFFICIENTS

Following boundary conditions will hold at the interface $z = 0$:

1. Continuity of tangential displacement i.e. $u = u'$ at $z = 0$ which leads to

$$\frac{\partial \phi}{\partial x}-\frac{\partial \psi}{\partial z}=\frac{\partial \phi'}{\partial x}-\frac{\partial \psi'}{\partial z} \quad \text{at} \quad z=0$$

2. Continuity of normal displacement i.e. $w=w'$ at $z=0$ which leads to

$$\frac{\partial \phi}{\partial z}+\frac{\partial \psi}{\partial x}=\frac{\partial \phi'}{\partial z}+\frac{\partial \psi'}{\partial x} \quad \text{at} \quad z=0$$

3. Continuity of tangential force per unit area i.e. $\tau_{13}+Pe_{xz}=\tau'_{13}+Pe'_{xz}$ at $z=0$ which leads to

$$(2c_{44}+P)\left[\frac{\partial^2 \psi}{\partial x^2}-\frac{\partial^2 \psi}{\partial z^2}+2\frac{\partial^2 \phi}{\partial x\,\partial z}\right]=(2c'_{44}+P')\left[\frac{\partial^2 \psi'}{\partial x^2}-\frac{\partial^2 \psi'}{\partial z^2}+2\frac{\partial^2 \phi'}{\partial x\,\partial z}\right] \text{at} \quad z=0$$

4. Continuity of normal force per unit area i.e. $\tau_{33}=\tau'_{33}$ at $z=0$ which leads to

$$c_{13}\frac{\partial^2 \phi}{\partial x^2}+c_{33}\frac{\partial^2 \phi}{\partial z^2}+(c_{13}-c_{33})\frac{\partial^2 \psi}{\partial x\,\partial z}-\upsilon_3 T=c'_{13}\frac{\partial^2 \phi'}{\partial x^2}+c'_{33}\frac{\partial^2 \phi'}{\partial z^2}+(c'_{13}-c'_{33})\frac{\partial^2 \psi'}{\partial x\,\partial z}-\upsilon'_3 T'$$

5. Continuity of temperature gradient i.e. $\frac{\partial T}{\partial z}=\frac{\partial T'}{\partial z}$ at $z=0$
6. Continuity of temperature i.e. $T=T'$ at $z=0$

Using Eqs. (5), (22), (23), (24a), (24b), (25a), (25b), (28) and (29) in the above boundary conditions, we get the following system of six equations:

$$\sin\theta\frac{A_1}{B_0}+\sin\theta\frac{A_2}{B_0}-\sin\theta\frac{A'_1}{B_0}-\sin\theta\frac{A'_2}{B_0}+\cos\theta\frac{B_1}{B_0}+\frac{c_1V_0}{c'_1V'_0}\cos\theta'\frac{B'_1}{B_0}=\cos\theta \tag{30a}$$

$$\frac{V_0}{V_1}\cos\theta_1\frac{A_1}{B_0}+\frac{V_0}{V_2}\cos\theta_2\frac{A_2}{B_0}+\frac{c_1V_0}{c'_1V'_1}\cos\theta'_1\frac{A'_1}{B_0}+\frac{c_1V_0}{c'_1V'_2}\cos\theta'_2\frac{A'_2}{B_0}-\sin\theta\frac{B_1}{B_0}+\sin\theta\frac{B'_1}{B_0}=\sin\theta \tag{30b}$$

$$\left[\begin{array}{l}\left(\frac{V_0}{V_1}\right)^2\sin 2\theta_1\frac{A_1}{B_0}+\left(\frac{V_0}{V_2}\right)^2\sin 2\theta_2\frac{A_2}{B_0}+\left(\frac{2c'_{44}+P'}{2c_{44}+P}\right)\left(\frac{c_1V_0}{c'_1V'_1}\right)^2\sin 2\theta'_1\frac{A'_1}{B_0}\\ +\left(\frac{2c'_{44}+P'}{2c_{44}+P}\right)\left(\frac{c_1V_0}{c'_1V'_2}\right)^2\sin 2\theta'_2\frac{A'_2}{B_0}+\cos 2\theta\frac{B_1}{B_0}-\left(\frac{2c'_{44}+P'}{2c_{44}+P}\right)\left(\frac{c_1V_0}{c'_1V'_0}\right)^2\cos 2\theta'\frac{B'_1}{B_0}\end{array}\right]=-\cos 2\theta, \tag{30c}$$

$$\left[\begin{array}{l}\left[(c_{13}\sin^2\theta_1+c_{33}\cos^2\theta_1)\left(\frac{V_0}{V_1}\right)^2+\frac{\upsilon_3\rho c_1^2V_0^2}{\upsilon_1}\left(\frac{\beta}{V_1^2+i\gamma}\right)\right]\frac{A_1}{B_0}\\ +\left[(c_{13}\sin^2\theta_2+c_{33}\cos^2\theta_2)\left(\frac{V_0}{V_2}\right)^2+\frac{\upsilon_3\rho c_1^2V_0^2}{\upsilon_1}\left(\frac{\beta}{V_2^2+i\gamma}\right)\right]\frac{A_2}{B_0}\\ -\left[(c'_{13}\sin^2\theta'_1+c'_{33}\cos^2\theta'_1)\left(\frac{c_1V_0}{c'_1V'_1}\right)^2+\frac{\upsilon'_3\rho' c_1^2V_0^2}{\upsilon'_1}\left(\frac{\beta'}{V_1'^2+i\gamma'}\right)\right]\frac{A'_1}{B_0}\\ -\left[(c'_{13}\sin^2\theta'_2+c'_{33}\cos^2\theta'_2)\left(\frac{c_1V_0}{c'_1V'_2}\right)^2+\frac{\upsilon'_3\rho' c_1^2V_0^2}{\upsilon'_1}\left(\frac{\beta'}{V_2'^2+i\gamma'}\right)\right]\frac{A'_2}{B_0}\\ +\left(\frac{c_{33}-c_{13}}{2}\right)\sin 2\theta\frac{B_1}{B_0}+\left(\frac{c'_{33}-c'_{13}}{2}\right)\sin 2\theta'\frac{B'_1}{B_0}\end{array}\right]=\left(\frac{c_{33}-c_{13}}{2}\right)\sin 2\theta, \tag{30d}$$

$$\left[\begin{array}{l}\frac{\rho}{\upsilon_1}\frac{V_0}{V_1}\left(\frac{\beta}{V_1^2+i\gamma}\right)\cos\theta_1\frac{A_1}{B_0}+\frac{\rho}{\upsilon_1}\frac{V_0}{V_2}\left(\frac{\beta}{V_2^2+i\gamma}\right)\cos\theta_2\frac{A_2}{B_0}\\ +\frac{\rho'}{\upsilon'_1}\frac{c_1V_0}{c'_1V'_1}\left(\frac{\beta'}{V_1'^2+i\gamma'}\right)\cos\theta'_1\frac{A'_1}{B_0}+\frac{\rho'}{\upsilon'_1}\frac{c_1V_0}{c'_1V'_2}\left(\frac{\beta'}{V_2'^2+i\gamma'}\right)\cos\theta'_2\frac{A'_2}{B_0}\end{array}\right]=0, \tag{30e}$$

$$\frac{\rho}{\upsilon_1}\left(\frac{\beta}{V_1^{\,2}+i\,\gamma}\right)\frac{A_1}{B_0}+\frac{\rho}{\upsilon_1}\left(\frac{\beta}{V_2^{\,2}+i\,\gamma}\right)\frac{A_2}{B_0}-\frac{\rho'}{\upsilon_1'}\left(\frac{\beta'}{V_1'^2+i\,\gamma'}\right)\frac{A_1'}{B_0}-\frac{\rho'}{\upsilon_1'}\left(\frac{\beta'}{V_2'^2+i\,\gamma'}\right)\frac{A_2'}{B_0}=0 \tag{30f}$$

Solving the above system of equations we can find the values of the amplitude ratios $\frac{A_1}{B_0}$, $\frac{A_2}{B_0}$, $\frac{A_1'}{B_0}$, $\frac{A_2'}{B_0}$, $\frac{B_1}{B_0}$ and $\frac{B_1'}{B_0}$. $\frac{A_1}{B_0}$, $\frac{A_2}{B_0}$ and $\frac{B_1}{B_0}$ are the amplitude ratio of the reflected P-wave, reflected thermal wave and reflected SV-wave, respectively, whereas $\frac{A_1'}{B_0}$, $\frac{A_2'}{B_0}$ and $\frac{B_1'}{B_0}$ are the amplitude ratio of the refracted P-wave, refracted thermal wave and refracted SV-wave, respectively. For numerical purpose, we shall use the approximate solution of Eq. (19). Solution of Eq. (19) can be written as:

$$V^{\,2}=\frac{1}{2}\left[(\xi+\beta-i\,\gamma)\pm\sqrt{(\xi+\beta-i\,\gamma)^2+4i\,\xi\gamma}\,\right]$$

For most of the elastic materials, $\beta<<1\left(\beta=\frac{\upsilon_1\eta\chi}{\rho c_1^2}\right)$ and $\gamma<<1\left(\gamma=\frac{\omega\chi}{c_1^2}\right)$, so by using approximation, the roots of above equation can be obtained as:

$$V_1=\xi^{1/2}\left[1+\frac{\beta}{2\xi}\left(1-i\,\frac{\gamma}{\xi}\right)\right] \quad \text{and} \quad V_2=i^{\,1/2}\gamma^{1/2}\left(\frac{\beta}{\xi}-1\right)^{1/2}$$

Similarly, for the medium M', we can write

$$V_1'=\xi'^{1/2}\left[1+\frac{\beta'}{2\xi'}\left(1-i\,\frac{\gamma'}{\xi'}\right)\right] \quad \text{and} \quad V_2'=i^{\,1/2}\gamma'^{1/2}\left(\frac{\beta'}{\xi'}-1\right)^{1/2}$$

5 NUMERICAL RESULTS AND DISCUSSION

In order to examine the effect of initial stress on the behaviour of reflection and transmission of P-wave, thermal wave and SV-wave, we have plotted graphs for the modulus of reflection and transmission coefficients versus angle of incidence for different set of values of initial stress parameters ζ and ζ' $\left(\zeta=P/2c_{11}\right.$ and $\left.\zeta'=P'/2c_{11}'\right)$. For numerical purpose, materials chosen for the lower and upper half-spaces are magnesium and cobalt, respectively, physical data for which are given by:

For the lower half-space M (Magnesium):

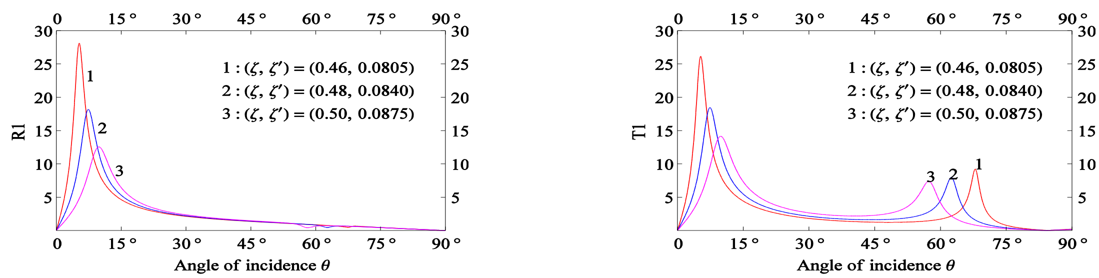
$c_{11}=5.974\times10^{10}\,N/m^2$, $c_{12}=2.624\times10^{10}\,N/m^2$, $c_{13}=2.170\times10^{10}\,N/m^2$, $c_{33}=6.170\times10^{10}\,N/m^2$,
$c_{44}=1.639\times10^{10}\,N/m^2$, $\rho=1740\,kg/m^3$, $s=1040\,JKg^{-1}K^{-1}$, $\delta_1=\delta_2=170\,Wm^{-1}k^{-1}$,
$\alpha_1=\alpha_2=24.80\times10^{-6}K^{-1}$, $T_0=298K$

For the upper half-space M' (cobalt):

$c_{11}'=3.071\times10^{11}\,N/m^2$, $c_{12}'=1.650\times10^{11}\,N/m^2$, $c_{13}'=1.027\times10^{11}\,N/m^2$, $c_{33}'=3.581\times10^{11}\,N/m^2$,
$c_{44}'=0.755\times10^{11}\,N/m^2$, $\rho'=8836\,kg/m^3$, $s'=427\,JKg^{-1}K^{-1}$, $\delta_1'=\delta_2'=69\,Wm^{-1}k^{-1}$,
$\alpha_1'=\alpha_2'=13.00\times10^{-6}K^{-1}$, $T_0'=298K$

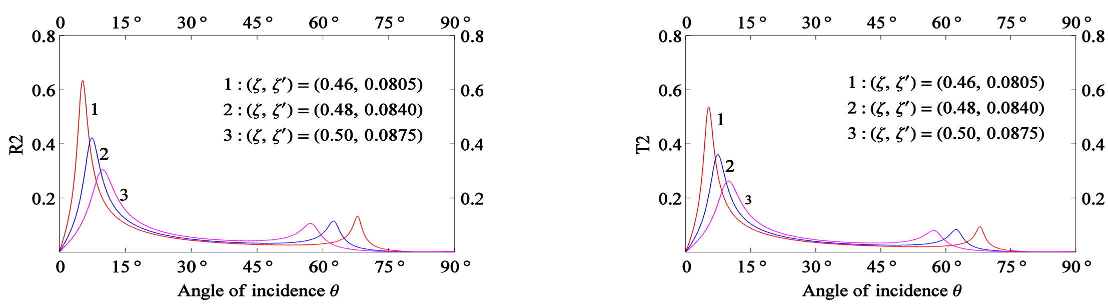
Moreover $\omega=2\pi\times100\ MHz$.

Figs. 2-4 show the variation of modulus of reflection and transmission coefficients with angle of incidence for different set of values of initial stress parameters ζ and ζ'. Figs. 2(a), 3(a) and 4(a) show the variation of modulus of reflection coefficients of P-wave, thermal wave and SV-wave, respectively with the angle of incidence, whereas Figs. 2(b), 3(b) and 4(b) show the variation of modulus of transmission coefficients of P wave, thermal wave and SV-wave, respectively. In the figures, we have used the symbols $R1$, $R2$ and $R3$ to denote the modulus of reflection coefficients of P- wave, thermal wave and SV-wave, whereas symbols $T1$, $T2$ and $T3$ have been used to denote the modulus of transmission coefficients of the corresponding waves. Curve labeled as 1, 2 and 3 corresponds to $(\zeta,\zeta') = (0.46, 0.0805)$, $(\zeta,\zeta') = (0.48, 0.0840)$ and $(\zeta,\zeta') = (0.50, 0.0875)$. The above values are taken in such a manner that $P' = 0.9P$. From Figs. 2, 3 and 4 we observe that the modulus of reflection and transmission coefficients of the thermal wave is very less in comparison to reflection and transmission coefficients of P- and SV-waves. Observing all the figures, we find that for vertical incidence of SV-wave (i.e. $\theta = 0^\circ$) we have only reflected SV-wave and transmitted SV-wave and there is no reflected and transmitted P and thermal waves. For horizontal incidence of SV-wave (i.e. $\theta = 90^\circ$), we have only reflected SV-wave and no other reflected or transmitted waves exist. From all the figures, we observe that reflection and transmission coefficients are strongly affected by the initial stress parameter. There are two critical angles of incidence for all the reflection and transmission coefficients and these angles vary with the initial stress parameter.



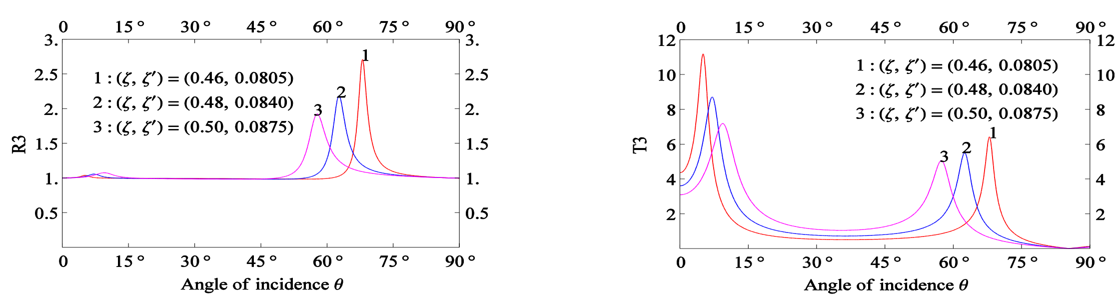
(a) (b)

Fig.2
Variation of modulus of reflection and transmission coefficients of P-wave.



(a) (b)

Fig.3
Variation of modulus of reflection and transmission coefficients ($R2\times10^3$, $T2\times10^4$) of thermal wave.



(a) (b)

Fig.4
Variation of modulus of reflection and transmission coefficients of SV-wave.

6 CONCLUSIONS

Reflection and transmission phenomena due to incidence of a plane SV-wave at a plane interface between two homogeneous orthotropic thermoelastic solid half-spaces under the effect of initial stress is studied in this problem. Due to incidence of a SV wave three reflected waves, namely P-wave, SV-wave and thermal wave, and same type of three refracted waves are generated. Applying appropriate boundary conditions, amplitude ratios corresponding to the three reflected, three refracted components are calculated, and graphs of these ratios are plotted against the angle of incidence for different set of values of initial stress parameters of the half-spaces. It is found that:

1. The modulus of reflection and transmission coefficients of the thermal wave is very less in comparison to reflection and transmission coefficients of the P- and SV-waves.
2. For vertical incidence of SV-wave, we have only reflected and refracted SV-waves and there does not exist reflected or refracted P- and thermal waves.
3. For horizontal incidence of SV-wave, we have only reflected SV-wave and no other reflected or transmitted wave exists.
4. All the reflection and transmission coefficients are strongly affected by the initial stress parameters of the both half-spaces.

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