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Influence of the Imperfect Interface on Love-Type Mechanical Wave in a FGPM Layer

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ABSTRACT

In this study, we consider the propagation of the Love-type wave in piezoelectric gradient covering layer on an elastic half-space having an imperfect interface between them. Dispersion relation has been obtained in the form of determinant for both electrically open and short cases. The effects of different material gradient coefficients of functionally graded piezoelectric material (FGPM) and imperfect boundary on the phase velocity of Love-type waves are discussed. Also, the influence of mechanically and electrically imperfect interface on the surface wave phase velocity is obtained and shown graphically. The dispersion curves are plotted and the effects of material properties of both FGPM and orthotropic material are studied. Moreover, dispersion relation of the considered microstructure depends substantially on the material gradient coefficients and width of the guiding plate. Numerical results are highlighted graphically and are validated with existing literature. The present study is the prior attempt to show the interfacial imperfection influence with the considered structure on wave phase velocity. The outcomes are widely applicable and useful for the development and characterization of Love-type mechanical waves in FGPM-layered media, SAW devices and other piezoelectric devices.

Keywords: FGPM, Love-type mechanical wave, Imperfect, Dispersion relation, Analytical analysis.

1 INTRODUCTION

MATERIALS play a significant role in the modernization of society by introducing advance devices which result in ease in the livings. Advanced materials like Functionally Graded Piezoelectric Materials (FGPM) are one of the most useful engineering composites which are developed by mixing two or more distinct constituent's phases with smooth continuous variation. Using such materials and surface wave propagation phenomenon through them creates valuable gadgets like SAW devices, transducers, etc. The articles carrying the wave propagation phenomenon in which the material coefficients of the guiding plate play a substantial role in describing the dispersive nature of the considered seismic wave are critical. In the theory of elastodynamics, Love waves are horizontally polarized surface waves and these waves is an outcome of the interfaces of many shear waves in

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layered structure, i.e. elastic layer welded on a elastic substrate on one side and vacuum medium on the other side. Therefore, such study of surface guided waves has collected much attention in the area of material crack or defects estimating capabilities of seismic waves. These surface waves are also beneficial for investigating the surface mechanical properties of underlying solids by non-destructive testing techniques (NDT) and in electronics industry even. During the manufacturing time of some material cracks or defects may occur at the interface due to thermal mismatch or some faults which result into an imperfect interface. Displacements fields' components are not continuous at the surface of common boundary of two distinct media in case of an imperfect interface, and this interfacial imperfection at the joint interface affects the surface wave propagation. Such types of mechanical problems are very helpful which explored the influence of material gradients, width of the plate and elastic constants on phase velocity of the surface wave. Also provide the better prediction of imperfectness effect in electrically short and open case. Love-type waves in layered piezoelectric structures/FGPM structure are interesting because of their many device applications in the acoustic and microwave fields. Following Love [1], many researchers have presented comprehensive results of seismic wave propagation in the isotropic and anisotropic medium. It is a well-known fact that Love waves propagating in the piezoelectric materials are extensively used in sensors, transducers, and surface acoustic wave (SAW) devices because of their better performances. So the study of Love-type waves in piezoelectric materials is of great importance. Du et al. [2] have studied the Love wave propagation in the functionally gradient piezoelectric material layer and analytically derived some special cases. The problem of transverse surface waves in a piezoelectric material carrying a functionally graded layer of finite thickness was studied by Qian et al. [3]. Eskandari and Shodja [4] have discussed the problem of Love waves in FGPM with elastic properties having quadratic variation. Cao et al. [5] and Singhal et al. [6-7] have investigated the propagation of Love waves in the FGPM-layered composite system. Fan et al. [8] studied the antiplane piezoelectric surface wave over a ceramic half-space with an imperfectly bonded layer. Recently, Rayleigh wave on the half-space with a piezoelectric gradient layer and imperfect interface investigated by Li et al. [9]. Chaudhary et al. [10] has examined the surface wave propagation in piezoelectric layer lying over an orthotropic substrate and deduced some particular cases.

As for FGPM framework, especially those who considered smart material layered structure, no endeavor has been taken to explore the concept of mechanical wave propagation phenonmenon in the FGPM layer deposited on elastic substrate with interfacial imperfection. The main aim of the present study is to explore the remarkable influence of material gradients parameters in relation with the imperfect interfacial parameter grahically. In this present article, we analytically derive dispersion relation of Love-type wave propagation in smart material structure following the elastic wave theory. Distinct parametric graphs are drawn (numerically) to exhibit the influence of parameters, like material gradients, mechanical imperfect and electrical imperfect on phase velocities. Moreover, the effects of these considered parameters with wave propagation analysis over imperfectness of the boundary on dispersion relation are observed and discussed in details. The results obtained show that possible imperfections of interface bonding must be considered in the design and fabrication of composites materials and also a benchmark for further investigation of FGPM coupled structures and design of SAW devices.

2 FORMULATION OF THE PROBLEM

We have considered an FGPM layer of finite thickness over a vertically elastic substrate with imperfect interface as shown in Fig. 1. We choose a Cartesian coordinate system in such a way that y axis is in the direction of wave propagation and x axis pointing vertically downward. The variations in parameters of FGPM layer are considered in the form

$$c_{44}(x)=c_{44}^0 e^{\sigma x},\ e_{15}(x)=e_{15}^0 e^{\sigma x},\ \varepsilon_{11}(x)=\varepsilon_{11}^0 e^{\sigma x},\ \rho(x)=\rho^0 e^{\sigma x}.$$

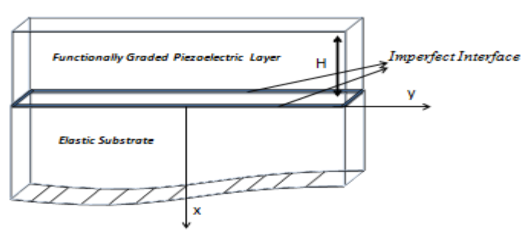


Fig.1
Geometry of the problem.

For the piezoelectric layer with initial stress, the equilibrium equations of elasticity without body forces and the Gauss' law of electrostatics without free charge are given as follows [11, 12 and 13]

$$\begin{aligned} &\sigma_{ij,j} + (u_{i,k}\sigma^0_{kj})_{,j} = \rho_i \ddot{u}, \\ &D_{i,j} + (u_{i,j}D^0_j)_i = 0 \end{aligned} \tag{1}$$

where $i, j, k = 1,2,3, \rho$ is the mass density, u_i and D_i denote the mechanical and electrical displacements in the i *th* direction, respectively, σ_{ij} is the stress tensor, σ^0_{kj} is the initial stress tensor and D^0_j is the initial electric displacement. The dots denote time differentiation and the comma denotes spatial differentiation.

The well-known piezoelectric constitutive equations are

$$\begin{aligned} &\sigma_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \\ &D_j = e_{jkl}S_{kl} + \varepsilon_{jk}E_k \end{aligned} \tag{2}$$

where σ_{ij} and S_{kl} are the stress and strain tensors, D_j and E_k are the electrical displacement and electric field intensity and $c_{ijkl}, e_{kij}, \varepsilon_{jk}$ are the elastic, piezoelectric and dielectric coefficients respectively. For FGPM layer the material properties are only function of *x*-axis.

Mechanical displacement and strain components are related as:

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

According to the quasi-static Maxwell's equation, relation between the electrical intensity and the electrical potential is

$$E_i = \frac{\partial \phi}{\partial x_i} \tag{4}$$

where ϕ is the electrical potential function [Cao et al.14] On the assumption that the Love wave propagates in the y direction and with the initial stress component σ^0_y, the mechanical displacement components and the scalar electric potential function can be given as:

$$u = v = 0, \qquad w = w(x, y, t), \qquad \phi = \phi(x, y, t). \tag{5}$$

Hence, the motion equation and the electrical displacement equilibrium equation for FGPM buffer layer are given by

$$\begin{aligned} &c_{44}(x)\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) + e_{15}(x)\left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2}\right) + \frac{\partial c_{44}(x)}{\partial x}\frac{\partial w_1}{\partial x} + \frac{\partial e_{15}(x)}{\partial x}\frac{\partial \phi_1}{\partial x} = \frac{\partial^2 w_1}{\partial t^2}, \\ &e_{15}(x)\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) - \varepsilon_{11}(x)\left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2}\right) + \frac{\partial e_{15}(x)}{\partial x}\frac{\partial w_1}{\partial x} - \frac{\partial \varepsilon_{11}(x)}{\partial x}\frac{\partial \phi_1}{\partial x} = 0 \end{aligned} \tag{6}$$

where w_1 and ϕ_1 denote the mechanical displacement and the electrical potential in the FGPM buffer layer, respectively.

The field equations for elastic substrate can written as:

$$\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} = \frac{1}{c^{s\,2}_{sh}}\frac{\partial^2 w_2}{\partial t^2}, \tag{7}$$

$$\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial y^2} = 0 \tag{8}$$

where $c_{sh}^s = \sqrt{\frac{c_{44}^s}{\rho^s}}$ is the shear wave velocity in the elastic substrate with ρ^s , c_{44}^s and ε_{11}^m being the density, shear modulus and dielectric constant in the substrate half-space.

3 ANALYTICAL SOLUTIONS

3.1 Solution for FGPM layer

By assuming

$$\psi_2 = \phi_1 - \frac{e_{15}}{\varepsilon_{11}} w_1 \tag{9}$$

Eq. (12) and (13) can be rewritten as:

$$\left(c_{44}^0 + \frac{e_{15}^{0\,2}}{\varepsilon_{11}^0} \right) \left(\sigma \frac{\partial w_2}{\partial x} + \nabla^2 w_1 \right) = \rho^0 \frac{\partial^2 w_1}{\partial t^2} \tag{10}$$

$$\sigma \frac{\partial \psi_2}{\partial x} + \nabla^2 \psi_2 = 0. \tag{11}$$

The solutions of Eqs. (10) and (11) are assumed as

$$w_1(x,y,t) = W_1(x) \exp\left[ik\left(y - ct \right) \right], \tag{12}$$

$$\psi_2(x,y,t) = \xi_2(x) \exp\left[ik\left(y - ct \right) \right]. \tag{13}$$

The above assumed solution reduces Eqs. (10) and(11) to

$$\xi_2''(x) + \sigma \xi_2'(x) - k^2 \xi_2(x) = 0, \tag{14}$$

$$W_1''(x) + \sigma W_1'(x) + k^2 \left(\frac{c^2}{c_{sh2}^2} - 1 \right) W_1(x) = 0 \tag{15}$$

where $c_{sh2}^2 = \frac{1}{\rho_0} \left(c_{44}^0 + \frac{e_{15}^{0\,2}}{\varepsilon_{11}^0} \right)$.

The solution of Eq. (14) is

$$\xi_2(x) = c_5 e^{m_1 x} + c_6 e^{m_2 x} \tag{16}$$

where $m_{1,2} = \frac{-\sigma/2 \pm \sqrt{\sigma^2 + 4k^2}}{2}$. When $m_{1,2} = c^2 > c_{sh2}^2 \left(1 + \frac{\sigma^2}{4k^2} \right)$, the solution of Eq. (16) is obtained as:

$$W_1(x) = c_1 e^{sx} \cos(\mu x) + c_2 e^{sx} \sin(\mu x), \tag{17}$$

where $s = -\sigma/2, \quad \mu = \frac{1}{2}\sqrt{4k^2\left(\frac{c^2}{c_{sh2}^2}-1\right)-\sigma^2}$. Substituting Eqs. (16) and (17) into Eqs. (12) and (13), we get

$$w_1(x,y,t) = \left(c_1 e^{sx}\cos(\mu x) + c_2 e^{sx}\sin(\mu x)\right)\exp\left[ik(y-ct)\right], \tag{18}$$

$$\phi_1(x,y,t) = \left\{c_3 e^{m_1 x} + c_4 e^{m_2 x} + \frac{e_{15}^0}{\varepsilon_{11}^0}\left[c_1 e^{sx}\cos(\mu x) + c_2 e^{sx}\sin(\mu x)\right]\right\}\exp\left[ik(y-ct)\right]. \tag{19}$$

3.2 Solution for elastic substrate

Since we have the condition $w_2 \to 0$ and $\phi_2 \to 0$ when $x \to \infty$.

Therefore the solution of (14) and (15), satisfying the above condition may be written as:

$$w_2(x,y,t) = c_5 e^{-kd_2 x}\exp\left[ik(y-ct)\right] \tag{20}$$

$$\phi_2(x,y,t) = c_6 e^{-kx}\exp\left[ik(y-ct)\right] \tag{21}$$

where $d_2 = \sqrt{1-\frac{c^2}{c_{sh}^{s\,2}}}$.

4 BOUNDARY CONDITIONS

For Love waves propagating in the considered structure, mechanical displacement and electrical potential satisfy the following boundary conditions and interface continuity conditions. It should be pointed out that two kinds of electrical boundary conditions, i.e. electrical open and short conditions, would be considered in this study.

The mechanical and electrically open conditions at the free surface can be given as:

$$\tau_{xz1}(-h,y) = 0, \; D_{x1}(-h,y) = 0. \tag{22}$$

The mechanical and electrically short conditions at the free surface are expressed as follows:

$$\tau_{xz1}(-h,y) = 0, \quad \phi_1(-h,y) = 0. \tag{23}$$

Here we introduce the imperfection interface factors K_T and K_N to indicate the imperfections exist along the interfaces between the upper piezoelectric layer and the FGPM layer.

$$\begin{aligned} &D_{x1}(0,y) = D_{x2}(0,y), && \tau_{xz1}(0,y) = \tau_{xz2}(0,y), \\ &\left(w_2(0,y) - w_1(0,y)\right)K_T = \tau_{xz2}(0,y), && \left(\phi_2(0,y) - \phi_1(0,y)\right)K_N = D_{x2}(0,y), \end{aligned} \tag{24}$$

5 DISPERSION RELATION

5.1 Dispersion relation for electrically open case

Using boundary conditions i.e. (Eqs. (22) and (24)) and eliminating the arbitrary constants, we get

$$\left|A_{ij}\right|_{6\times 6}=0 \tag{25}$$

where

$$A_{11}=\left(c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}\right)\left(e^{sx}s\cos\mu x-e^{sx}\mu\sin\mu x\right),\ A_{12}=\left(c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}\right)\left(e^{sx}s\sin\mu x+e^{sx}\mu\cos\mu x\right)$$

$$A_{13}=e_{15}m_1e^{-m_1H},\ A_{14}=e_{15}m_2e^{-m_2H},\ A_{15}=A_{16}=A_{21}=A_{22}=0$$

$$A_{23}=\varepsilon_{11}m_1e^{-m_1H},A_{24}=\varepsilon_{11}m_2e^{-m_2H},\ A_{25}=A_{26}=0$$

$$A_{31}=\left(c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}\right)s,\ A_{32}=\left(c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}\right)\mu, A_{33}=e_{15}m_1, A_{34}=e_{15}m_2$$

$$A_{35}=-c_{44}'kd_2, A_{36}=-e_{15}'k, A_{41}=A_{42}=0$$

$$A_{43}=\varepsilon_{11}m_1, A_{44}=\varepsilon_{11}m_2, A_{45}=-e_{15}'kd_2, A_{46}=-\varepsilon_{11}'k$$

$$A_{51}=K_T, A_{52}=A_{53}=A_{54}=0, A_{55}=\left(c_{44}'kd_2+K_T\right), A_{56}=-e_{15}'k$$

$$A_{61}=-\frac{e_{15}}{\varepsilon_{11}}K_N, A_{62}=0, A_{63}=A_{64}=-K_N, A_{65}=e_{15}'kd_2, A_{66}=K_N-k\,\varepsilon_{11}'.$$

Eq. (25) is the dispersion relation for Love type wave in the considered structure.

5.2 Dispersion relation for electrically short case

Using boundary conditions i.e. (Eqs. (23) and (24)) and eliminating the arbitrary constants, we get

$$\left|B_{ij}\right|_{6\times 6}=0 \tag{26}$$

where all the terms are similar to those given in Eq. (25) except

$$B_{21}=\frac{e_{15}}{\varepsilon_{11}}e^{-sH}\cos\mu H, B_{22}=\frac{e_{15}}{\varepsilon_{11}}e^{-sH}\sin\mu H, B_{23}=e^{-m_1H}, B_{24}=e^{-m_2H}, B_{25}=B_{26}=0.$$

Eq. (26) is the dispersion relation for Love type wave propagation in a considered structure.

6 NUMERICAL EXAMPLE AND DISCUSSION

The dispersion relations (Eqs. (25) and (26)) have been used for numerical illustration and graphical representation. We have used the following data. The material constants of BaTiO3 ceramics are taken from Wang and Huang [8] and the material constants of elastic substrate are taken from Ristic [9].

Table 1
Material constants.

Materials	Elastic constant $C_{44}(10^{10}\ Nm^{-2})$	Mass density ρ (kgm^{-3})	Piezoelectric constant $e_{15}\ (Cm^{-2})$	Dielectric constant ε_{11} $(10^{-10}\ Fm^{-1})$
$BaTiO_3$	4.40	7.28	11.4	128.0
SiO_2	3.12	2.20	0.0	0.336

Figs. 2(a) and 2(b) are drawn to explore the influence of mechanical and electrical imperfect parameters on the profile of phase velocity of Love-type mechanical surface waves against dimensionless wave number for electrically open case. In the Figs. 2(a) and 2(b) dimensionless phase velocity is represented by vertical axis and dimensionless wave number is represented by horizontal axis. The variation of dimensionless phase velocity gets increases with the increment of imperfect parameters i.e. mechanical and electrical parameters in a fashion of arithmetic progression of common difference 4 in the Figs. 2(a) and 2(b) respectively. Also, this increment in the imperfect parameters increases the angular frequency. Imperfect parameter strongly increases the phase velocity in both cases which is a very significant result. In comparison, mechanically imperfect parameter influences more in increasing the phase velocity as compared to the electrical imperfect parameter. Both the Figs. 2(a) and 2(b) give valuable information for the choice of FGPM plate to be used in SAW devices.

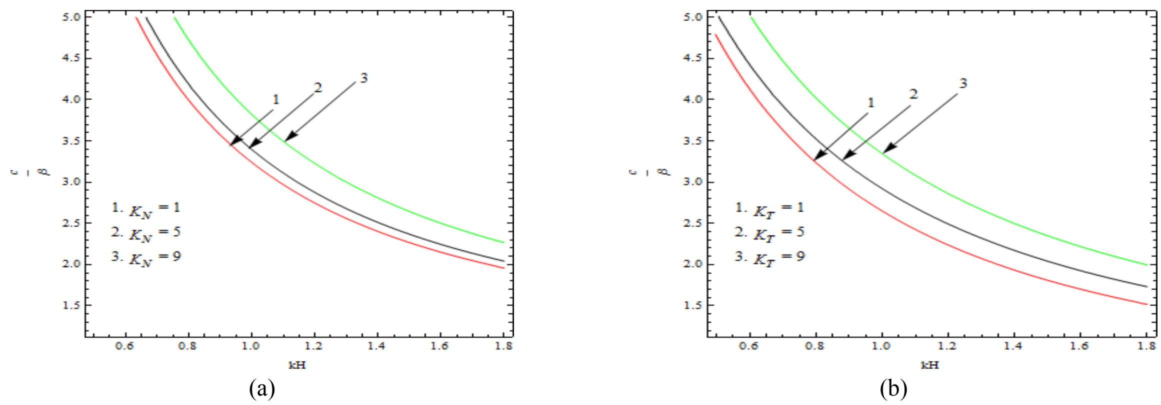


Fig.2
Variation of dimensionless phase velocity with respect to dimensionless wave number for different values of imperfectness parameters for electrically open case.

Figs. 3(a) and 3(b) are drawn to explore the influence of mechanical and electrical imperfect parameters on the profile of phase velocity of Love-type mechanical surface waves against dimensionless wave number for electrically short case. In the Figs. 3(a) and 3(b) dimensionless phase velocity is represented by vertical axis and dimensionless wave number is represented by horizontal axis. The variation of dimensionless phase velocity gets increases with the increment of imperfect parameters i.e. mechanical and electrical parameters in a fashion of arithmetic progression of common difference 4 in the Figs. 3(a) and 3(b) for electrically short case respectively. Also, this increment in the imperfect parameters increases the angular frequency. Imperfect parameter strongly increases the phase velocity in both cases which is a very significant result. Both the Figs. 3(a) and 3(b) give valuable information for the choice of FGPM plate to be used in SAW devices.

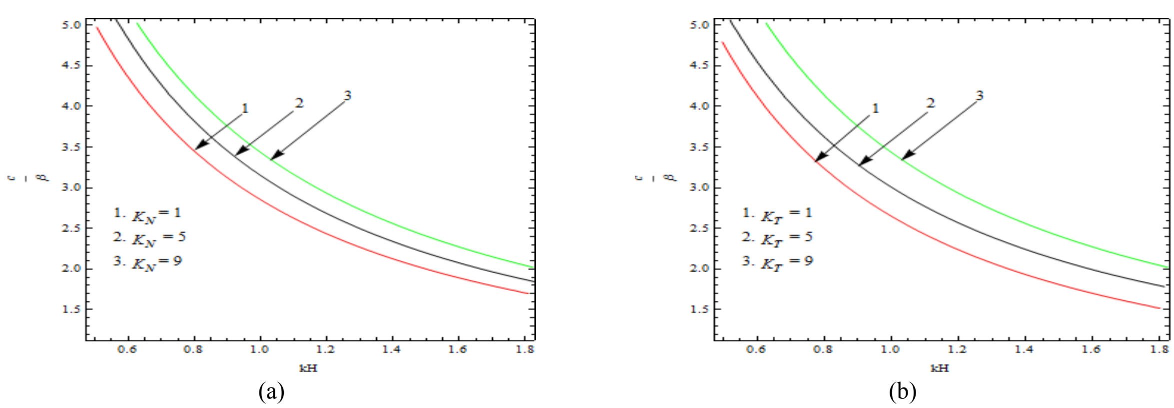
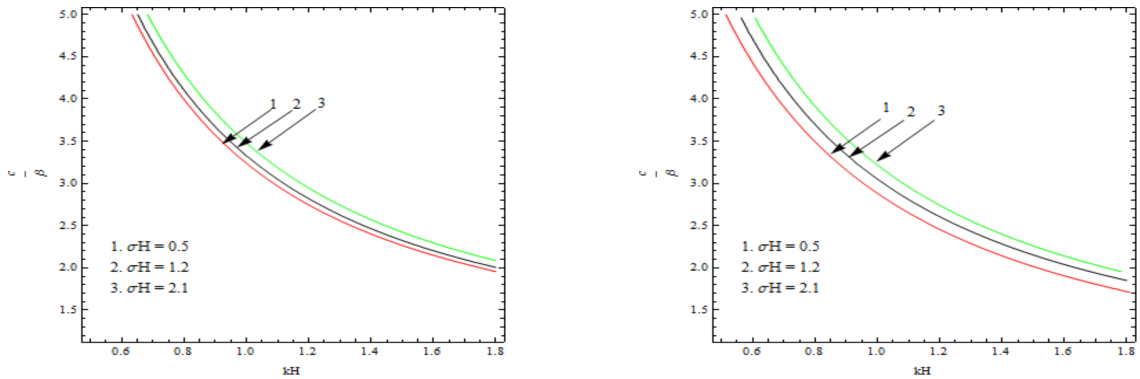


Fig.3
Variation of dimensionless phase velocity with respect to dimensionless wave number for different values of imperfectness parameters for electrically short case.

In consecutive Figs. 4(a) and 4(b), the variations in the dimensionless phase velocity of Love-type mechanical surface wave (effect of material gradient coefficients) are shown. It is significantly noticed that the dimensionless phase velocity curves are monotonically increasing with increasing value of material gradients with the interval value of 7 along the dimensionless wave number. Furthermore, the effect of material gradient coefficients is remarkably shown under electrically open case on dispersive curves of Love-type wave is more sensitive as compared to an electrically short case. All the self-explanatory figures suggest that the choice of FGPM plate to be used in seismic devices which are very significant result. Also, the outcomes of the problem suggest that FGPM with high dielectric gradient values should be preferred for designing SAW devices to reduce ultrasonic high-frequency wave's effect.



(a) (b)

Fig.4
Variation of dimensionless phase velocity with respect to dimensionless wave number for different values of material gradient for (a) electrically open case (b) electrically short case.

7 CONCLUSIONS

An analytical approach is used to to investigate the propagation of Love-type mechanical surface waves in FGPM layer deposited over an elastic substrate with imperect interface. The influence of imperfectness parameters has been shown significantly. Outcomes of the following study are as follows:

- Mechanical imperfect parameter influences more to increase the phase velocity in electrically open case comparison to electrically imperfect parameters in the same case respectively.
- Wave propagation analysis following some numerical examples suggests that the phase velocity increases with increasing value of imperfectness parameters and this result clearly indicate that imperfectness parameters significantly affect the dispersion relation.
- Moreover, interface with mechanical imperfect parameters affects the phase velocity curves more in comparison to electrically short case.
- The expressions for dispersion relations are obtained and matched with existing results.
- Obtained results of the present study may be useful in many application of piezoelectric/FGPM composites or structures and SAW devices.

The present research paper confirms that material gradient coefficients have same *x*-coordinate variation which comprehend the effect of material gradient coefficients on the Love-type mechanical surface wave properties and suggests knowledge for better and more enhancing designing of SAW devices.

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REFERENCES

[1] Love A.E.H., 1911, *Some Problems of Geodynamics*, Cambridge University Press.
[2] Du J., Jin X., Wang J., Xian K., 2007, Love wave propagation in functionally graded piezoelectric material layer, *Ultrasonics* **46**: 13-22.

[3] Qian Z., Jin F., Wang Z., Kishimoto K., 2007, Transverse surface waves on a piezoelectric material carrying a functionally graded layer of finite thickness, *International Journal of Engineering Science* **45**: 455-466.
[4] Eskandari M., Shodja H.M., 2008, Love waves propagation in functionally graded piezoelectric materials with quadratic variation, *Journal of Sound and Vibration* **313**: 195-204.
[5] Cao X.S., Jin F., Jeon I., Lu T.J., 2009, Propagation of love waves in a functionally graded piezoelectric material (FGPM) layered composite system, *International Journal of Solids and Structures* **46**: 4123-4132.
[6] Singhal A., Sahu S.A., Chaudhary S., 2018, Approximation of surface wave frequency in Piezo-composite structure, *Composite Part B: Engineering* **144**: 19-28.
[7] Singhal A., Sahu S.A., Chaudhary S., 2018, Liouville green approximation: An analytical approach to study the elastic wave vibrations in composite structure of piezo material, *Composite Structure* **184**: 714-727.
[8] Fan H., Yang J.S., Xu L.M., 2006, Antiplane piezoelectric surface wave over a ceramic half-space with an imperfectly bonded layer, *Ferroelectric and Frequency Control* **53**(9): 1695-1698.
[9] Li L., Wei P.J., Guo X., 2016, Rayleigh wave on the half-space with a gradient piezoelectric layer and imperfect interface, *Applied Mathematical Modelling* **40**(19): 8326-8337.
[10] Chaudhary S., Sahu S.A., Singhal A., 2017, Analytical model for Rayleigh wave propagation in piezoelectric layer overlaid orthotropic substratum, *Acta Mechanica* **228**: 529-547.
[11] Wang Z.K., Shang F.L., 1997, Cylindrical buckling of piezoelectric laminated plates, *Acta Mechanica Solida Sinica* **18**: 101-108.
[12] Du J., Jin X., Wang J., Xian K., 2007, Love wave propagation in functionally graded piezoelectric material layer, *Ultrasonics* **46**(1): 13-22.
[13] Saroj P.K., Sahu S.A., Chaudhary S., Chattopadhyay A., 2015, Love-type waves in functionally graded piezoelectric material (FGPM) sandwiched between initially stressed layer and elastic substrate, *Waves in Random and Complex Media* **25**(4): 608-627.
[14] Cao X., Jin F., Jeon I., Lu T.J., 2009, Propagation of Love waves in a functionally graded piezoelectric material (FGPM) layered composite system, *International Journal of Solids and Structures* **46**(22-23): 4123-4132.